Multigrid methods for primitive variable MHD (elliptic/hyperbolic) equations

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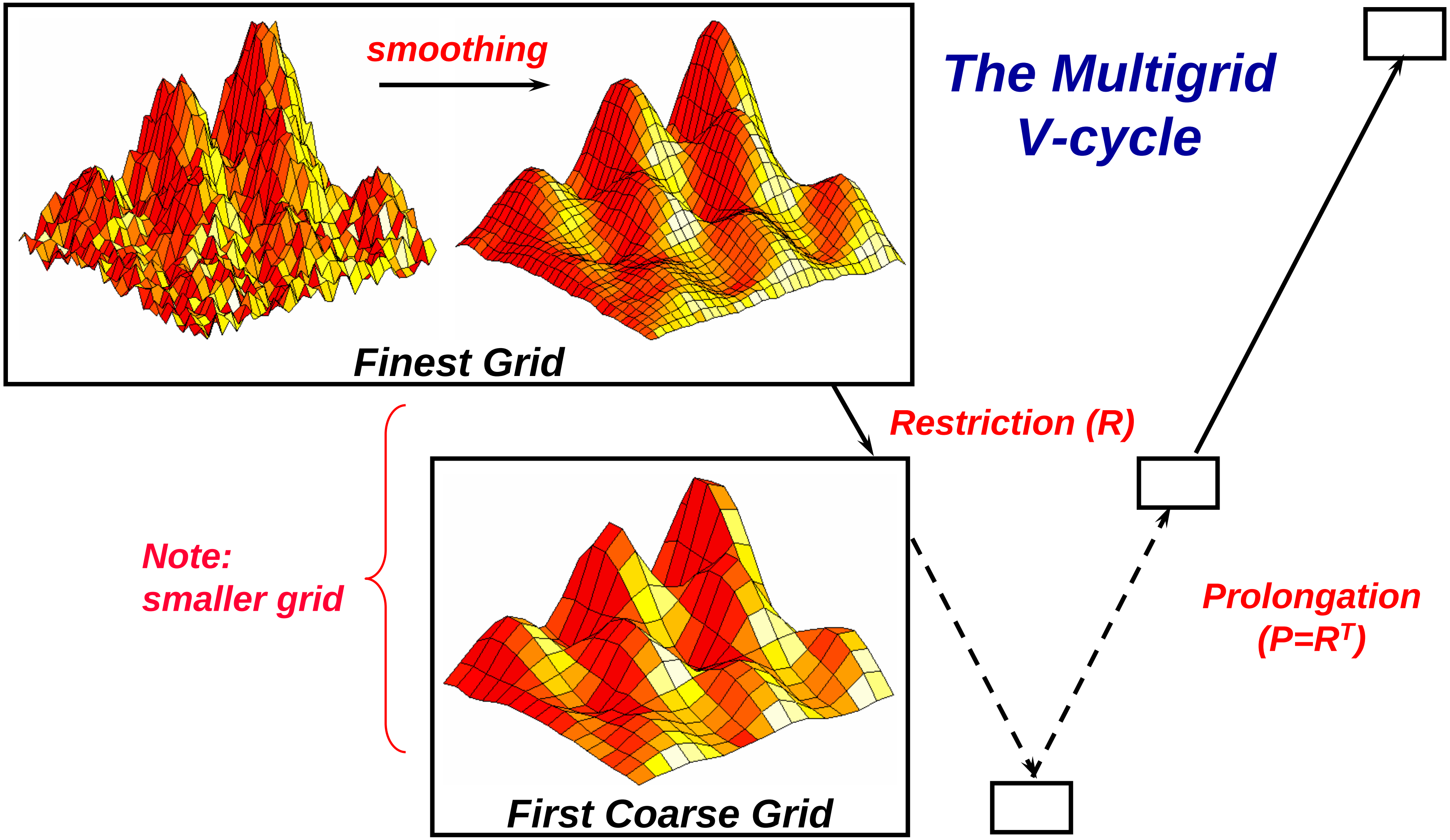
Multigrid for **elliptic** operators



Multigrid V - cycle

- Function u = **MG-V**(A,f)
  - if A is small
    - $u \leftarrow A^{-1}f$
  - else
    - $u \leftarrow S^{\nu 1}(f, u)$ -- $\nu 1$ steps of smoother (pre)
    - $r_H \leftarrow P^T(f - Au)$
    - $u_H \leftarrow$ **MG-V**(P^TAP, r_H) -- recursion (Galerkin)
    - $u \leftarrow u + Pu_H$
    - $u \leftarrow S^{\nu 2}(f, u)$ -- $\nu 2$ steps of smoother (post)
- Need only
  - 1) Coarse grid space (columns of P) for (Galerkin) correction
  - 2) (iterative) solver (aka, smoother) to actually solve the problem
    - Must complement the coarse grid correction
    - For isotropic Laplacian this “smoothes” the solution

Multigrid for MHD (hyperbolic/elliptic)

- Multigrid ideal elliptic op … but hyperbolic?
- 1) Coarse grid correction doesn't resolve shocks
  - Tokamak MHD doesn’t have shocks …
- 2) Solver (smoother) to relax coarse grid sol.
- MG used for elliptic/hyperbolic systems in fluid flow (Euler equations)
  - Jameson, van Leer, Brandt, Yavneh, Dendy, LeVeque,
- And to some extent for Navier Stokes
  - Mavriplis, Guillard ….

Multigrid for MHD (highly anisotropic)

- (highly) anisotropic equations are generically very hard to solve
- Two classic methods for MG:
  - 1) Course grid correction
    - Semi-coarsening
  - 2) Smoother
    - Line and plane smoothers
      - Use power of direct solvers judiciously

Non-linear multigrid

- The full non-linear problem can included in MG
- Incorporate full non-linear physical model on each (coarse) grid
- Potential of being fast
- Tricky to get to work because of close association to physics model
  - Eg, stability on the fine grid does not automatically imply stability on coarse grid