Viscosity and its effect on elliptic flow

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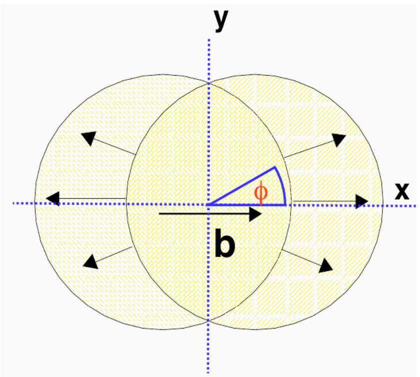
Success of Ideal Hydrodynamics

- Elliptic Flow:

$$v_2 \equiv \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle \approx 20\%$$



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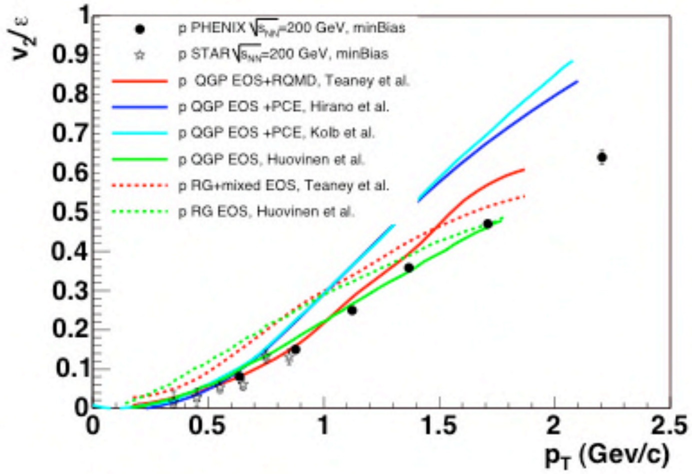
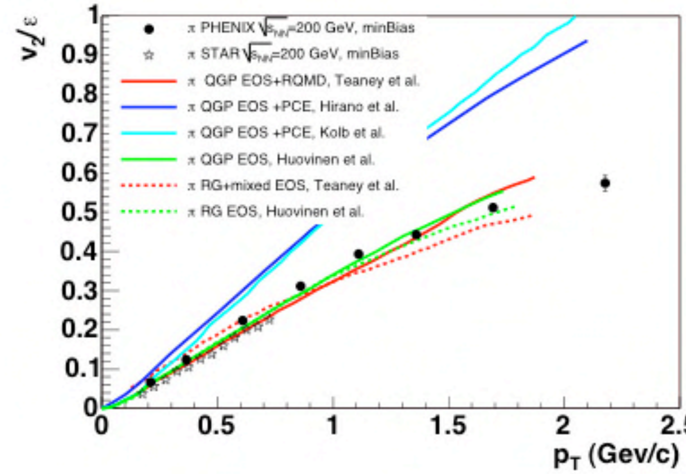
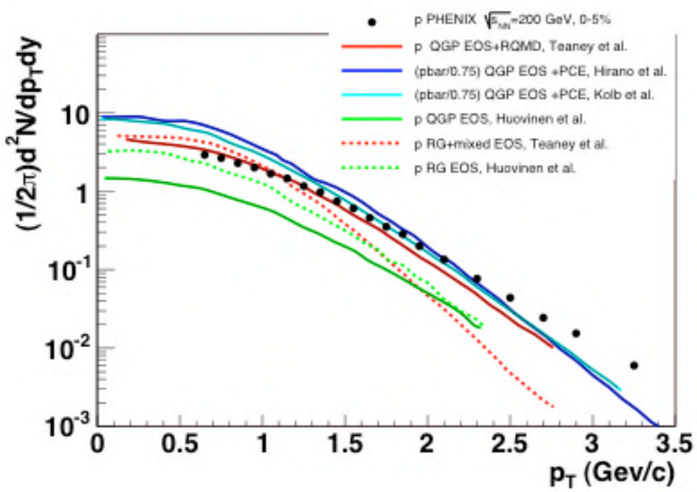
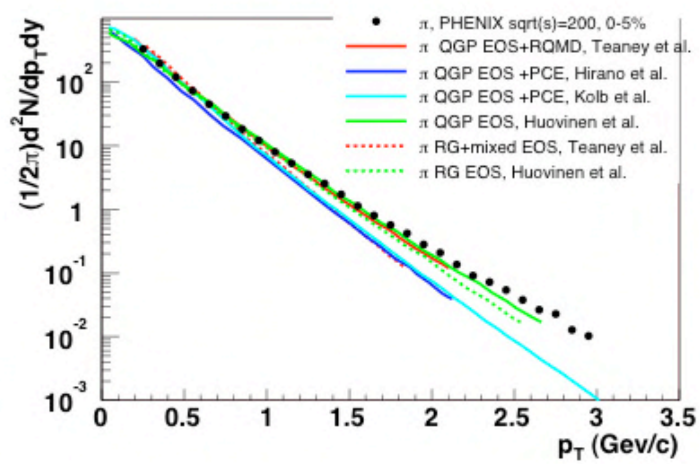




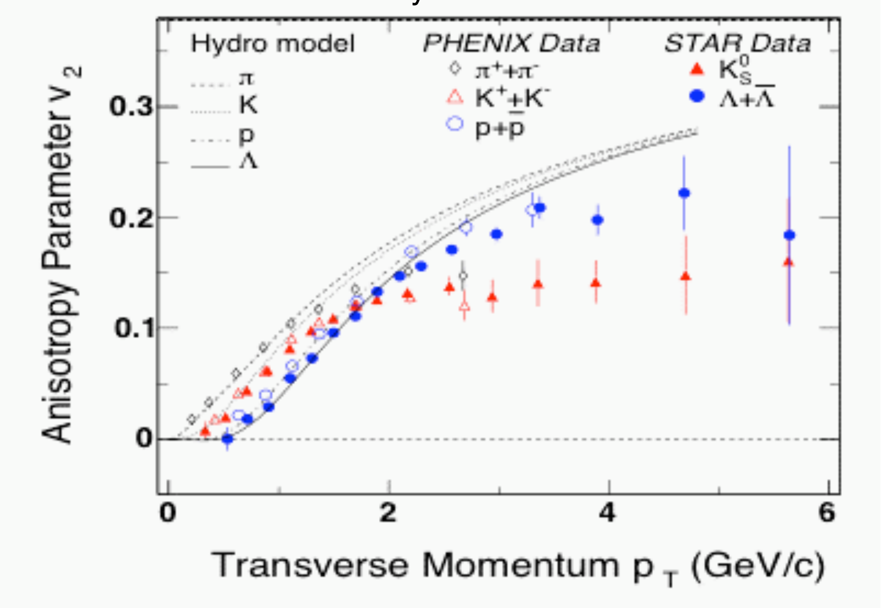




Limitations of Ideal Hydrodynamics

- Deviations from Ideal Results seen at:
  - Large p_T
  - Peripheral Collisions
  - Lower Energies
  - Smaller System Sizes
  - Forward Rapidites

Hydro Predictions: Huovinen P, Kolb P F, Heinz U, Ruuskanen P V and Voloshin S A 2001 Phys. Lett. B 503 58.



- Next natural step is to develop simulations incorporating viscosity to understand first correction.

When is hydrodynamics applicable?

- Viscosity sets a length scale for thermalization: $l_{mfp} \times \text{Expansion Rate} << 1$

$$\frac{\eta}{\varepsilon + p}\frac{1}{\tau} << 1 \quad \text{or} \quad \frac{\eta}{s}\frac{1}{T\tau} << 1$$

  - Condition on Medium: $\frac{\eta}{s}$
  - Condition of experiment: $\frac{1}{T\tau}$
  - Example: $T_0 \sim 300\, MeV \quad \text{and} \quad \tau_0 \sim 1\, fm/c \quad \rightarrow \quad \frac{1}{T_0\, \tau_0} \sim \frac{2}{3}$
  - Since experimental conditions are unfavorable to hydrodynamics, η/s must be small; $\eta/s<0.3$

When is hydrodynamics applicable?

- How does $\frac{\eta}{\varepsilon + p}\frac{1}{\tau}$ vary in time?
- Look at 1D Bjorken expansion: $T \sim \frac{1}{\tau^{1/3}}, \quad n \sim \frac{1}{\tau}$
- and consider two extreme models of the viscosity:
  - Weak coupling: $\eta \propto T^3$

$$\frac{\eta}{s\tau T} \propto \frac{1}{\tau T} \propto \left(\frac{1}{\tau}\right)^{2/3}$$

  - Hard sphere model: $\eta \propto \frac{T}{\sigma}$

$$\frac{\eta}{s\tau T} \propto \frac{1}{n_0 \sigma_0 \tau_0} \propto const$$

When is hydrodynamics applicable?

- Look at 3D expansion: $s = const, \; V \sim \frac{1}{\tau^3} \;\; \rightarrow \;\; T \sim \frac{1}{\tau}, \; n \sim \frac{1}{\tau^3}$
- Summary:

	1D Expansion	3D Expansion
$\eta \propto T^3$	$\left(\frac{\tau_0}{\tau}\right)^{2/3}$	$\left(\frac{\tau_0}{\tau}\right)^{0}$
$\eta \propto T/\sigma$	$\left(\frac{\tau_0}{\tau}\right)^{0}$	$\left(\frac{\tau_0}{\tau}\right)^{-2}$

- Introduction of mass scale hinders thermalization
- Problem is 3D after $\tau \sim R_A$ …… Rapid Breakup

Relativistic Navier-Stokes Equations (RNSE)

- RNSE difficult to solve
  - Unstable modes [Hiscock and Lindblom, PRD 31, 725 (1985).]
  - Violates Causality

- RNSE stress tensor changes instantly

$$T^{ij}_{vis}\Big|_{\text{instantly}} = \eta\left(\partial^i v^j + \partial^j v^i - \frac{2}{3}\delta^{ij}\partial_i v^i\right)$$

- There are a number of models which relax to RNSE

$$T^{ij}_{vis}\Big|_{\omega \to 0} \sim \eta\left(\partial^i v^j + \partial^j v^i - \frac{2}{3}\delta^{ij}\partial_i v^i\right)$$

- These models should agree with each other and with RNSE when hydrodynamics is applicable
  - Made Precise by Lindbolm

A Simple Relaxation Model

- Bjorken Expansion - Normal Viscous Hydro

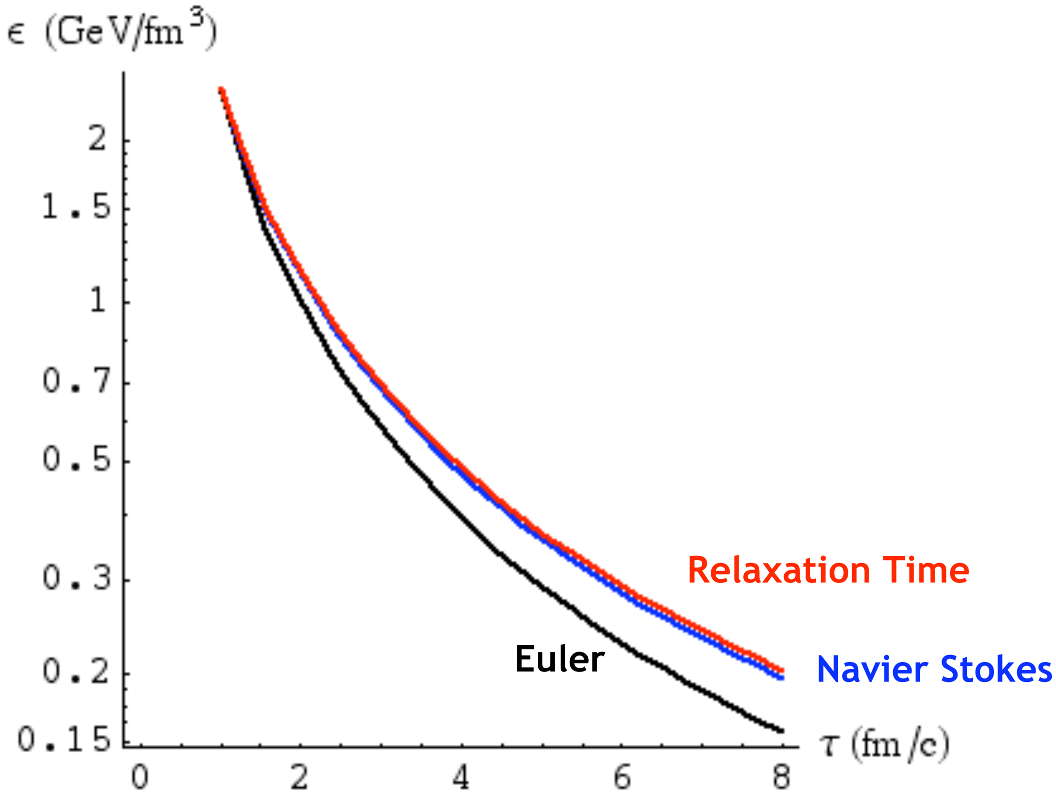
$$\frac{d\varepsilon}{d\tau} = -\frac{\varepsilon + T^{zz}_{eq}}{\tau} \qquad T^{zz}_{eq} = p - \frac{4}{3}\frac{\eta}{\tau}$$

- Bjorken Expansion - Relaxation Time Approximation

$$\frac{d\varepsilon}{d\tau} = -\frac{\varepsilon + T^{zz}}{\tau} \qquad \frac{dT^{zz}}{d\tau} = -\frac{T^{zz} - T^{zz}_{eq}}{\tau_R}$$

- What are the appropriate initial conditions for this second equation?
  - Answer: $T^{zz} = T^{zz}_{eq}$

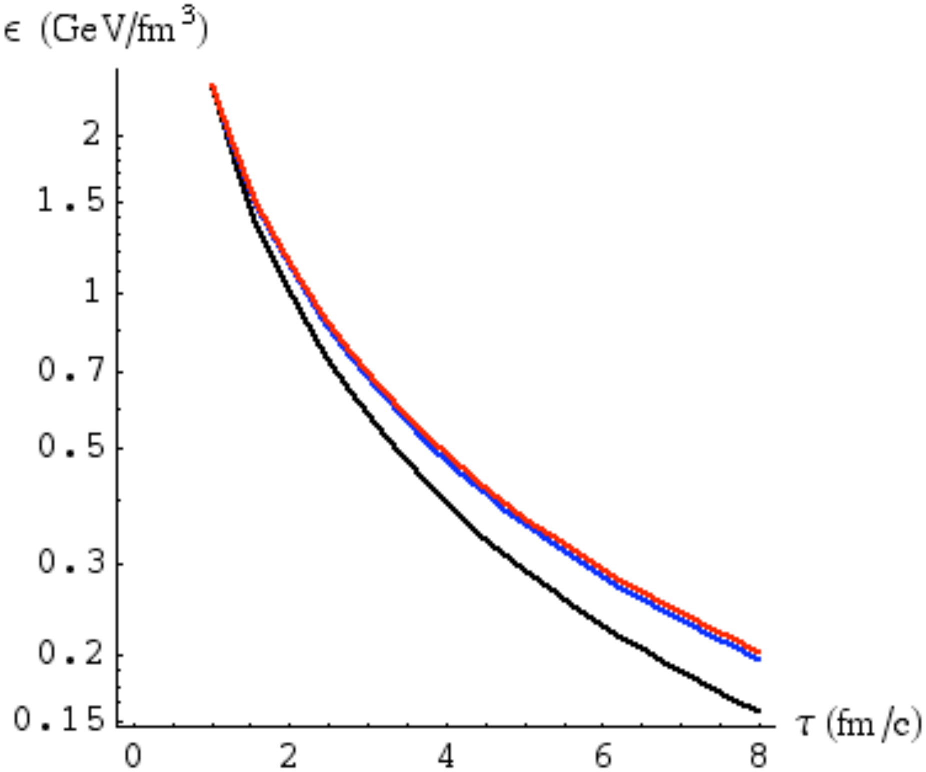
Solution of Relaxation Time Equations



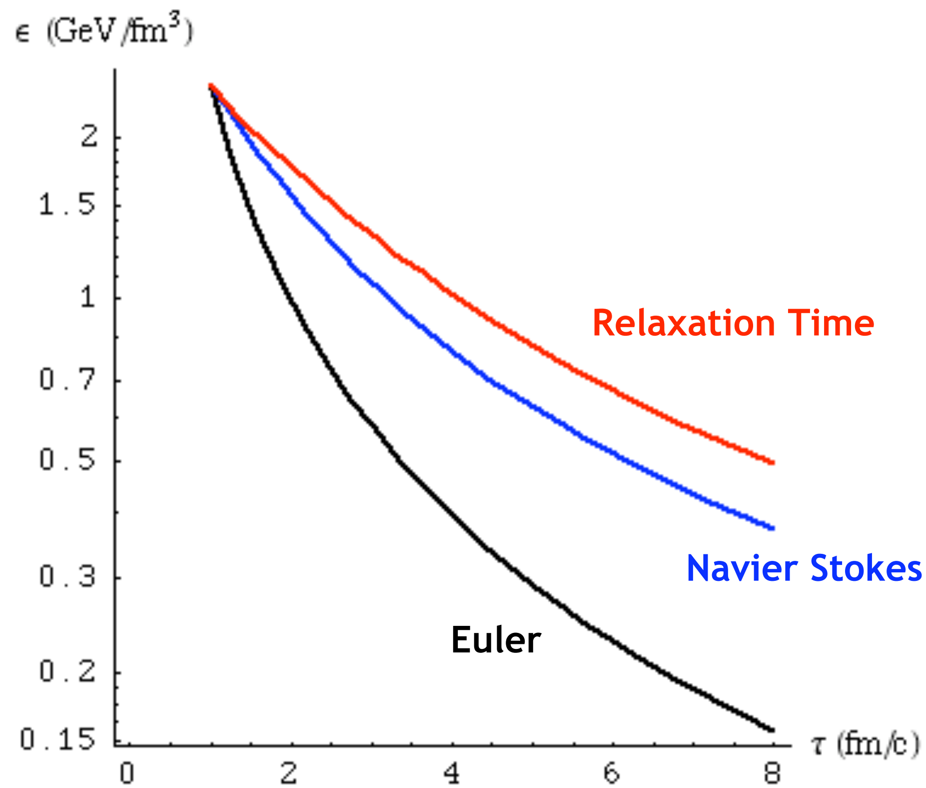
- Relaxation is practically the same as Navier Stokes

Made precise by L. Lindblom

Can’t believe hydro when viscosity is large!



$$\frac{\eta}{s} = 0.1$$



$$\frac{\eta}{s} = 0.4$$

Lessons

- Identify short and long time parameters in the relaxation schemes
  - The simulation should be sensitive to the long time parameters only

- Make sure the stress tensor is always close to the form expected from "first order" hydrodynamics
  - Otherwise the result is making more assumptions then kinetic theory

Model Equations

- Same principal as IS

- For Details see: H.C. Ottinger, Physica A 254 (1998) 433.

- Why use GENERIC Structure?
  - Numerically easier to implement
  - Not necessarily restricted to small deviations from equilibrium … but in our case it is

- Introduce a tensor that evolves in time as:

$$u^{\lambda}\left(\partial_{\lambda}c_{\mu\nu}-\partial_{\mu}c_{\lambda\nu}-\partial_{\nu}c_{\mu\lambda}\right)=-\frac{1}{c\tau_{0}}\bar{c}^{\mu\nu}-\frac{1}{c\tau_{2}}\left\langle c^{\mu\nu}\right\rangle$$

$$\bar{c}_{ij}=\left(tr\ c\right)\delta_{ij}\ \text{ and }\ \left\langle c_{ij}\right\rangle=c_{ij}-\frac{1}{3}\bar{c}_{ij}$$

Model Equations

- The energy mtm tensor in the LRF is:

$$T_{LRF}^{ij} = p_0 \delta^{ij} - \phi\left[c^{ij} + \cdots\right] \qquad \phi = 4\frac{\partial \varepsilon}{\partial tr\ \mathrm{c}^2} \geq 0$$

- Assuming a specific thermodynamic relation of the form:

$$\varepsilon = \varepsilon_0 + \frac{1}{2}\alpha\, \mathrm{tr}\, c^2$$

- For small deviations from equilibrium:

$$T_{LRF}^{ij} = p_0\left(\delta^{ij} - 2\alpha c^{ij}\right)$$

Model Equations

- This new tensor relaxes to the velocity gradients for small relaxation time τ_0, τ_2:

$$c^{ij} \approx \tau_0 \frac{2}{3}\delta^{ij}\partial_k v^k + \tau_2\left(\partial^i v^j + \partial^j v^i - \frac{2}{3}\delta^{ij}\partial_k v^k\right)$$

- The stress energy tensor in the LRF is then:

$$T_{ij} \approx p\left(\delta_{ij} - a_1 c_{ij}\right)$$

- Compare this to the canonical form:

$$T_{ij} \approx p\delta_{ij} - \zeta\partial_i v^j - \eta\left(\partial_i v^j + \partial_j v^i - \frac{2}{3}\delta_{ij}\partial_l v^l\right)$$

- Can Map:

$$\left(\tau_0, \tau_2, a_1\right) \longrightarrow \left(\zeta, \eta, v_{th}^2\right)$$

Mapping to IS

- Viscous tensor in IS is given by: $\pi^{\mu\nu} + \Pi\Delta^{\mu\nu} = -\frac{1}{\alpha p}\left(c^{\mu\nu} + u^{\mu}u^{\nu}\right)$
- Substitute into evolution equation:

$$u^{\lambda}\left(\partial_{\lambda}c_{\mu\nu} - \partial_{\mu}c_{\lambda\nu} - \partial_{\nu}c_{\mu\lambda}\right) = -\frac{1}{c\tau_0}\bar{c}^{\mu\nu} - \frac{1}{c\tau_2}\left\langle c^{\mu\nu}\right\rangle$$

- And we get: $\pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \tau\left[{}^{\langle}\ D\pi^{\mu\nu}\ {}^{\rangle} + \frac{4}{3}\pi^{\mu\nu}(\nabla u)\right]$

$$+\ \tau\left[\frac{1}{\eta}\pi^{\langle\ \mu}{}_{\lambda}\pi^{\nu\ \rangle\lambda} - 4\pi^{\langle\ \mu}{}_{\lambda}\Omega^{\nu\ \rangle\lambda} + \cdots\right]$$

- Which is IS when: $\lambda_1 = \eta\tau \qquad \lambda_2 = 4\tau\eta$

$$Ads: \quad \lambda_1 = \frac{\eta\tau}{2 - \ln 2} \qquad \lambda_2 = -2\tau\eta$$

Equations of Motion

- Ideal Hydrodynamic Equations:

$$\partial_\tau T^{00} + \partial_x T^{01} + \partial_y T^{02} = \frac{-1}{\tau}(T^{00} + \tau^2 P^{33})$$

$$\partial_\tau T^{10} + \partial_x T^{11} + \partial_y T^{12} = \frac{-1}{\tau} T^{10}$$

$$\partial_\tau T^{20} + \partial_x T^{21} + \partial_y T^{22} = \frac{-1}{\tau} T^{20}$$

- Relaxation Equations:

$$(\partial_\tau + v_x \partial_x + v_y \partial_y) c^{11} + 2[(c^{11} - 1)\partial_x v_x + c^{12} \partial_x v_y] = \frac{-1}{\gamma \tau_0} \bar{c}^{11} - \frac{1}{\gamma \tau_2} \mathring{c}^{11}$$

$$(\partial_\tau + v_x \partial_x + v_y \partial_y) c^{22} + 2[(c^{22} - 1)\partial_y v_y + c^{21} \partial_y v_x] = \frac{-1}{\gamma \tau_0} \bar{c}^{22} - \frac{1}{\gamma \tau_2} \mathring{c}^{22}$$

$$(\partial_\tau + v_x \partial_x + v_y \partial_y) \tilde{c}^{33} + \frac{2}{\tau}(\tilde{c}^{33} - 1) = \frac{-1}{\gamma \tau_0} \bar{\tilde{c}}^{33} - \frac{1}{\gamma \tau_2} \mathring{\tilde{c}}^{33}$$

$$(\partial_\tau + v_x \partial_x + v_y \partial_y) c^{12} + c^{12}(\partial_x v_x + \partial_y v_y) + (c^{22} - 1)\partial_x v_y + (c^{11} - 1)\partial_y v_x = \frac{-1}{\gamma \tau_0} \bar{c}^{12} - \frac{1}{\gamma \tau_2} \mathring{c}^{12}$$

Running Viscous Hydro in Three Steps

- Run the evolution and monitor the viscous terms
- When the viscous term is about half of the pressure:
  - The models start to disagree with each other
  - T^{ij} is not asymptotic with $\sim \eta\left(\partial^i v^j + \partial^j v^i - \frac{2}{3}\delta^{ij}\partial_l v^l\right)$

Freezeout is signaled by the equations

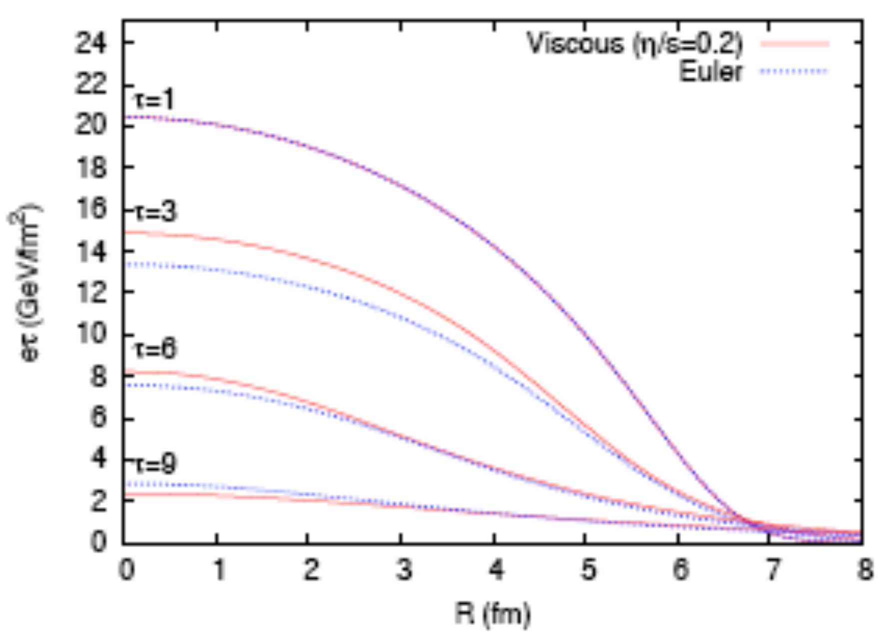
- Compute Spectra
  - Viscous correction to spectra grow with p_T

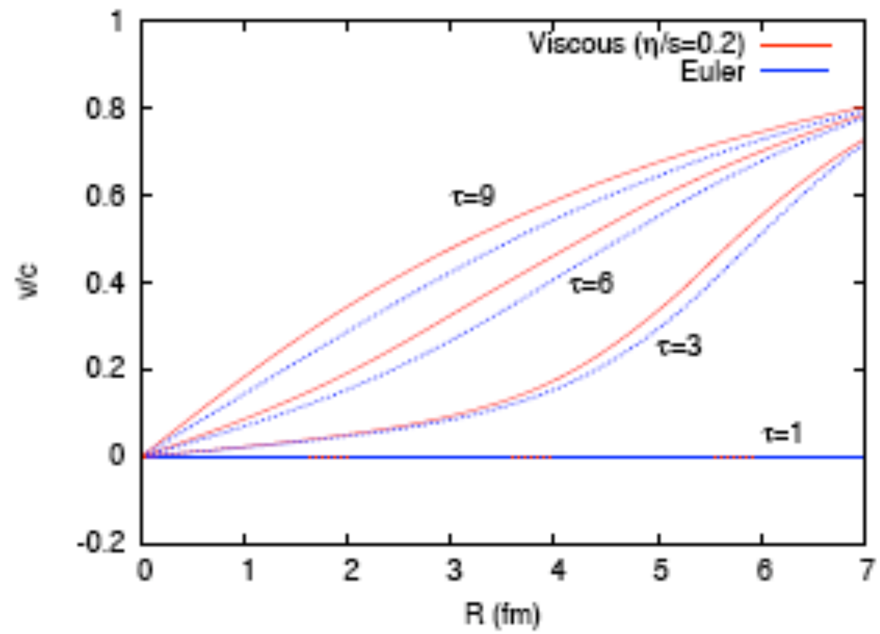
$$f_0 \rightarrow f_0 + \delta f$$

Maximum p_T also signaled by the equations

- Note: We don’t look at mixed / hadronic phase yet.

Hydrodynamic Results: 1D

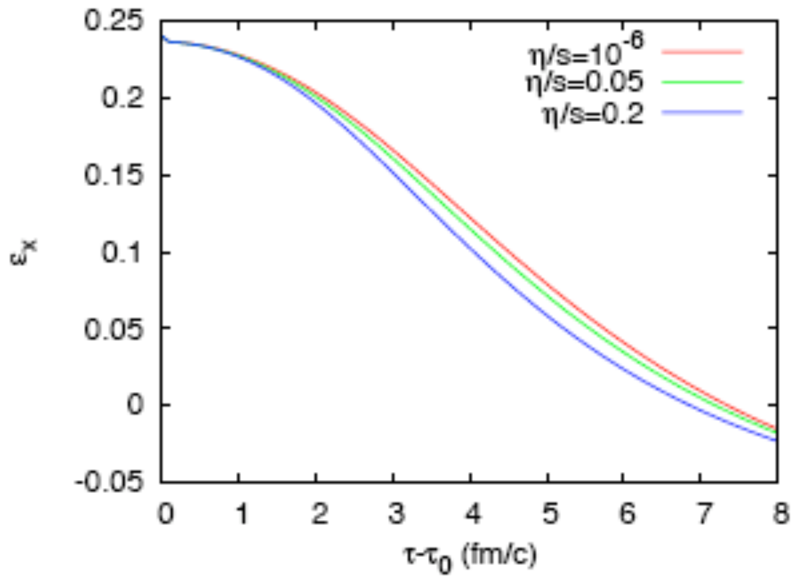




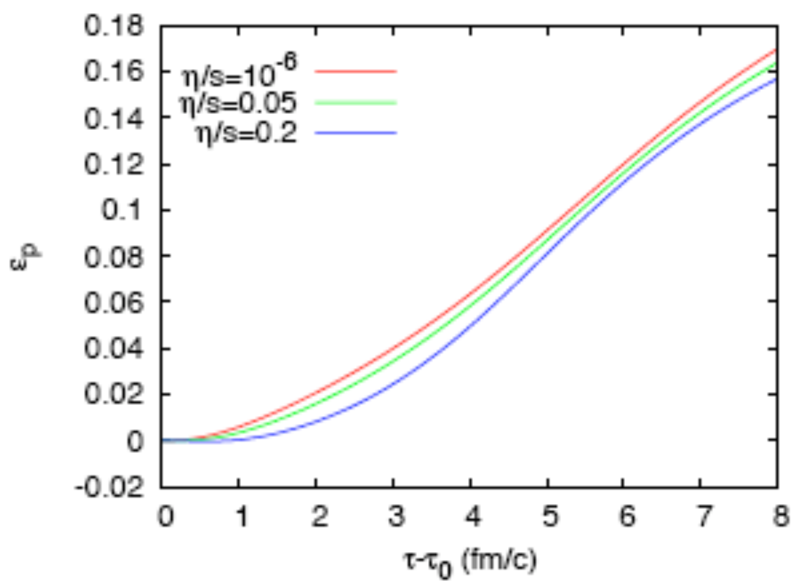
- Viscous solution does less longitudinal work
- The transverse pressure is larger leading to larger transverse velocities
- Larger velocities result in quicker reduction of energy density

Hydrodynamic Results: Anisotropy

$$\varepsilon_x = \frac{\langle\langle y^2 - x^2 \rangle\rangle}{\langle\langle y^2 + x^2 \rangle\rangle}$$



$$\varepsilon_p = \frac{\langle\langle T^{xx} - T^{yy} \rangle\rangle}{\langle\langle T^{xx} + T^{yy} \rangle\rangle}$$



Freezeout

- Viscous Hydrodynamics: $\tau_R \partial_\mu u^\mu << 1$

$$p \sim \varepsilon \langle v_{th}^2 \rangle$$

$$\eta \sim \varepsilon \langle v_{th}^2 \rangle \tau_R$$

- Freezeout signaled when: $\frac{\eta}{p} \partial_\mu u^\mu \sim \tau_R \partial_\mu u^\mu \sim \frac{1}{2}$

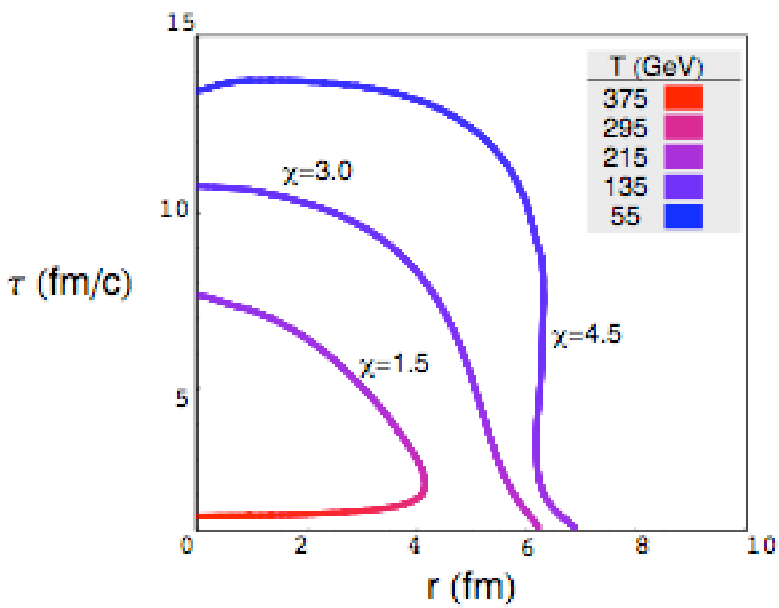
- Analogous surface for Ideal Case:
  - Keep surface fixed as $\eta/s \to 0$ and define: $\chi = \frac{4}{T} \partial_\mu u^\mu$

Freezeout Surfaces

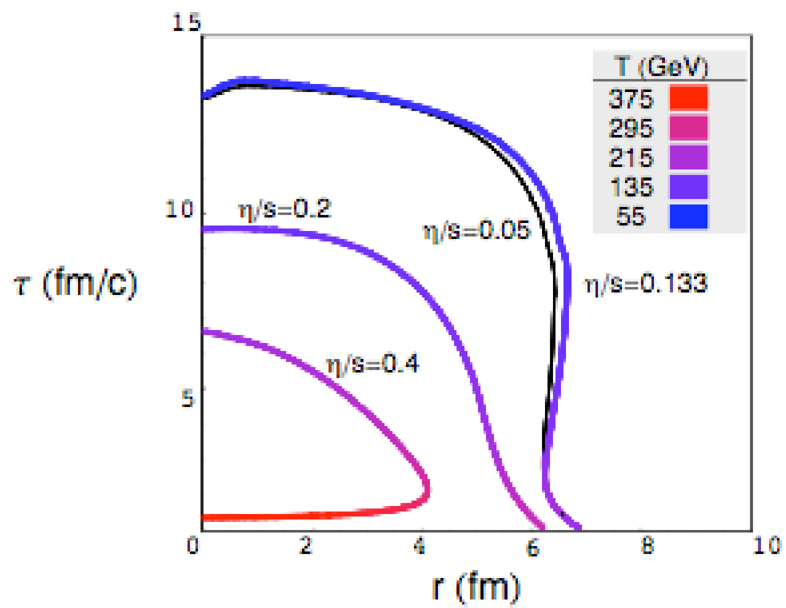
η/s	$\frac{\eta}{p}\partial_\mu u^\mu$	χ
0.05	0.6	12.0
0.05	**0.225**	**4.5**
0.05	0.15	3.0
0.2	0.9	4.5
0.2	**0.6**	**3.0**
0.133	**0.6**	**4.5**

- Surfaces the same:
  - Can easily compare viscous to Ideal hydro
  - From now on use: $\chi = \frac{4}{T}\partial_\mu u^\mu$

Ideal



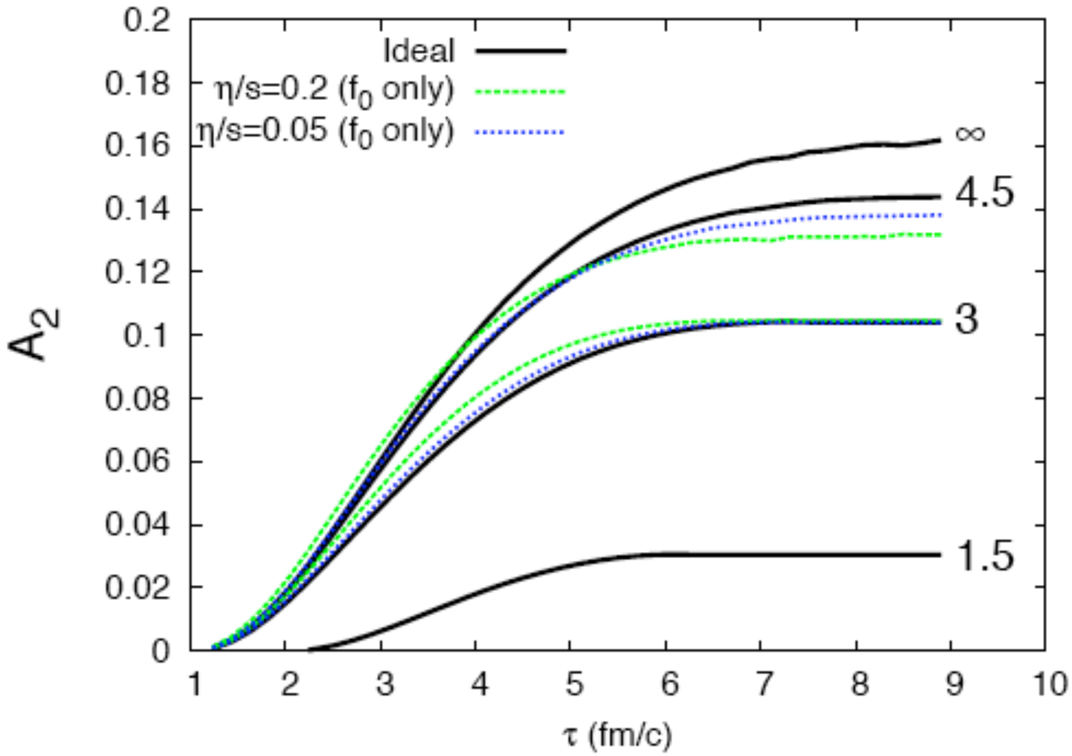
Viscous



Elliptic Flow versus Time - No δf

- First look at momentum anisotropy: Ollitrault
  - Independent of the particle content of theory
  - Depends on hydrodynamic fields (u^{μ}, $\pi^{\mu\nu}$) and moments of ideal distribution function

$$A_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \approx 2v_2$$



- Result without δf is insensitive to η/s

Viscous Correction to Distribution Function

- Corrections to thermal distribution function: $f_0 \rightarrow f_0 + \delta f$
  - Must be proportional to strains
  - Must be a scalar
  - General form in rest frame and ansatz

$$\delta f = F\left(|p|\right) p^i p^j \pi_{ij} \longrightarrow \delta f \propto f_0 p^i p^j \pi_{ij}$$

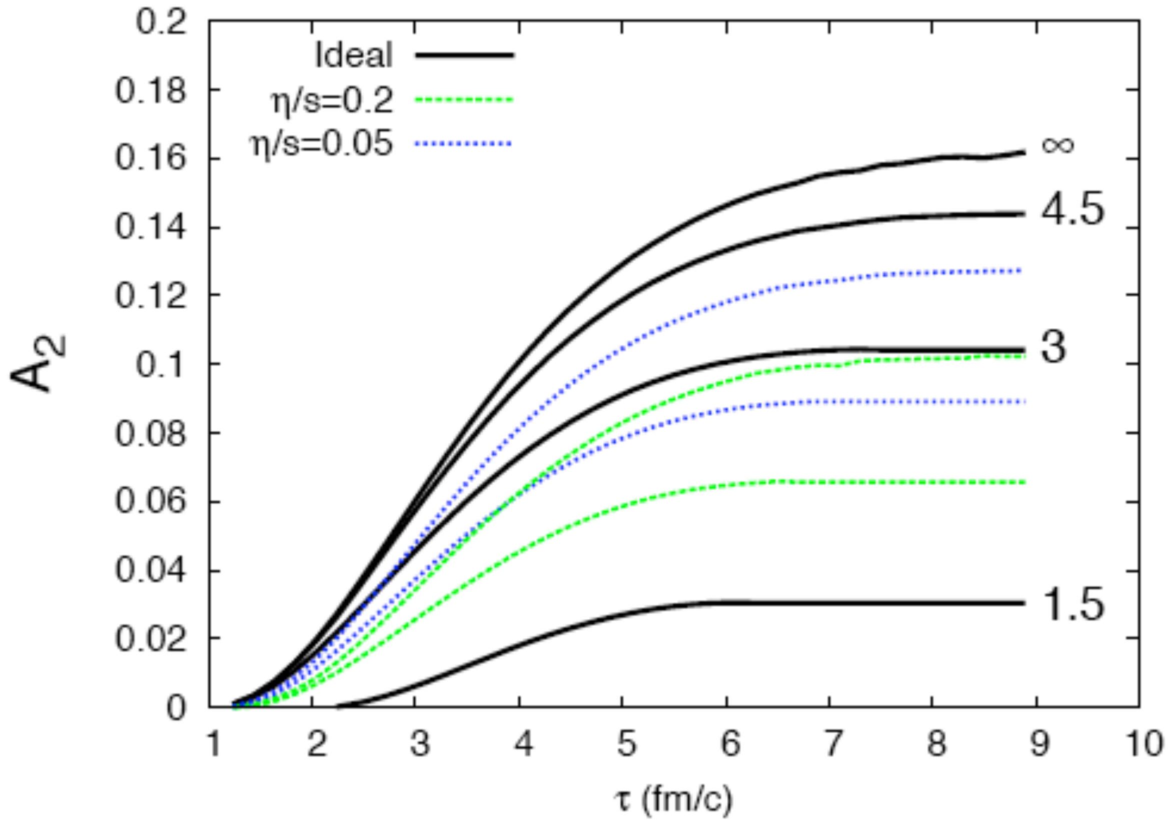
  - Constant can be fixed by:

$$T_0^{\mu\nu} + T_{vis}^{\mu\nu} = \int d^3 p \frac{p^\mu p^\nu}{E} (f_0 + \delta f)$$

  - Result:

$$\delta f = \frac{1}{2(e+p)T^2} f_0 p^i p^j \pi_{ij}$$

Elliptic Flow versus Time - with δf

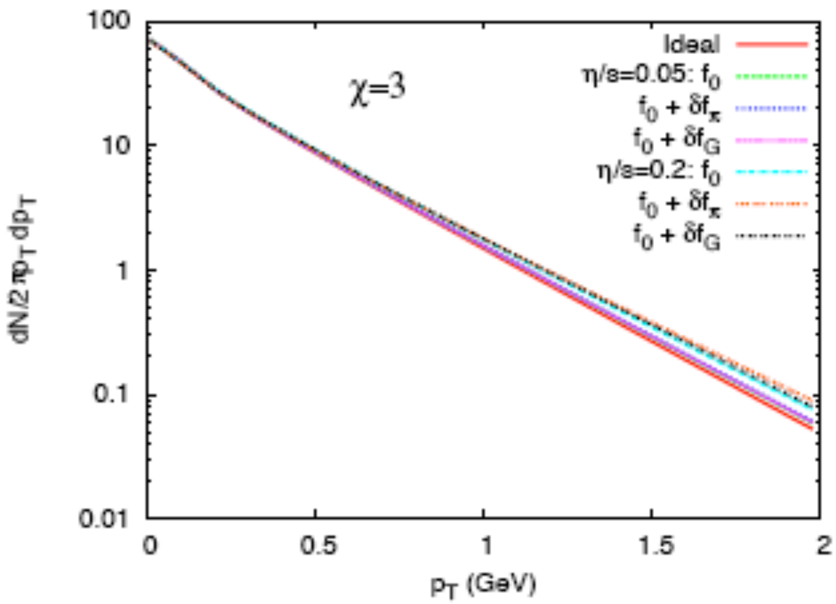


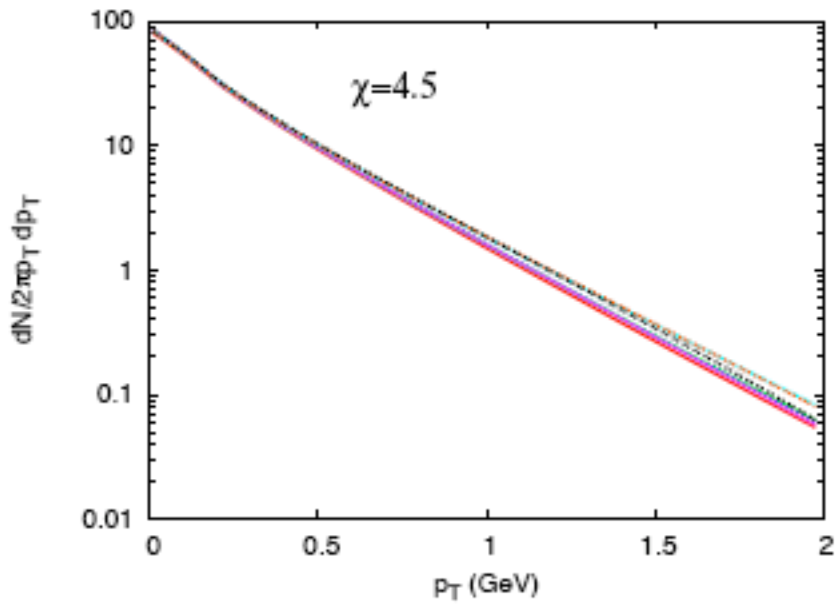
- δf strong modification to integrated elliptic flow

p_T Spectrum

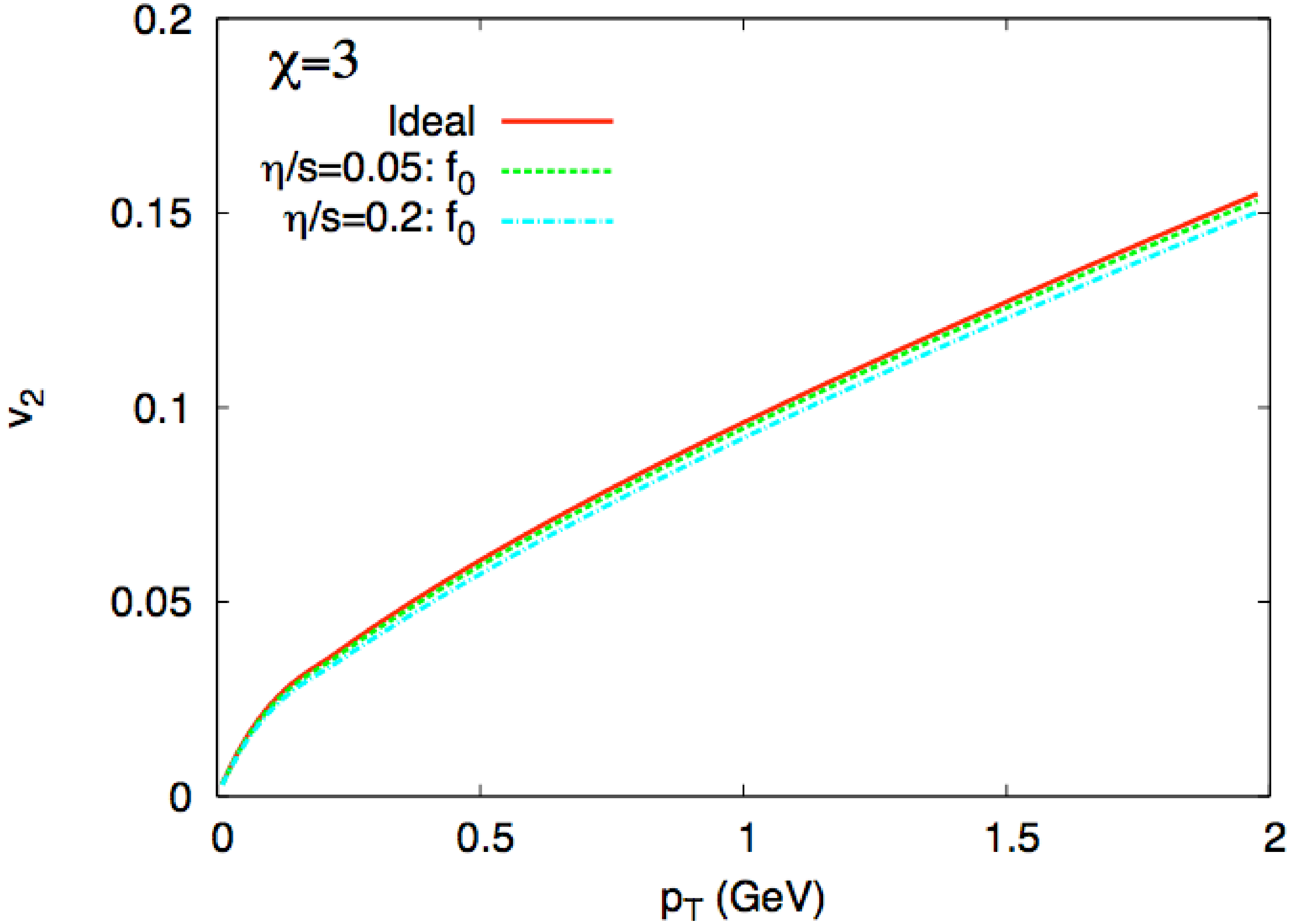
- Cooper Frye: $$E\frac{d^3N}{d^3p} = \frac{g}{2\pi^3}\int_\sigma f(p_\mu u^\mu, T)p^\mu d\sigma_\mu$$

$$\delta f = \frac{1}{2(e+p)T^2} f_o(1+f_o)\, p^\mu p^\nu \left[\pi_{\mu\nu} + \frac{2}{5}\Pi\Delta_{\mu\nu}\right]$$



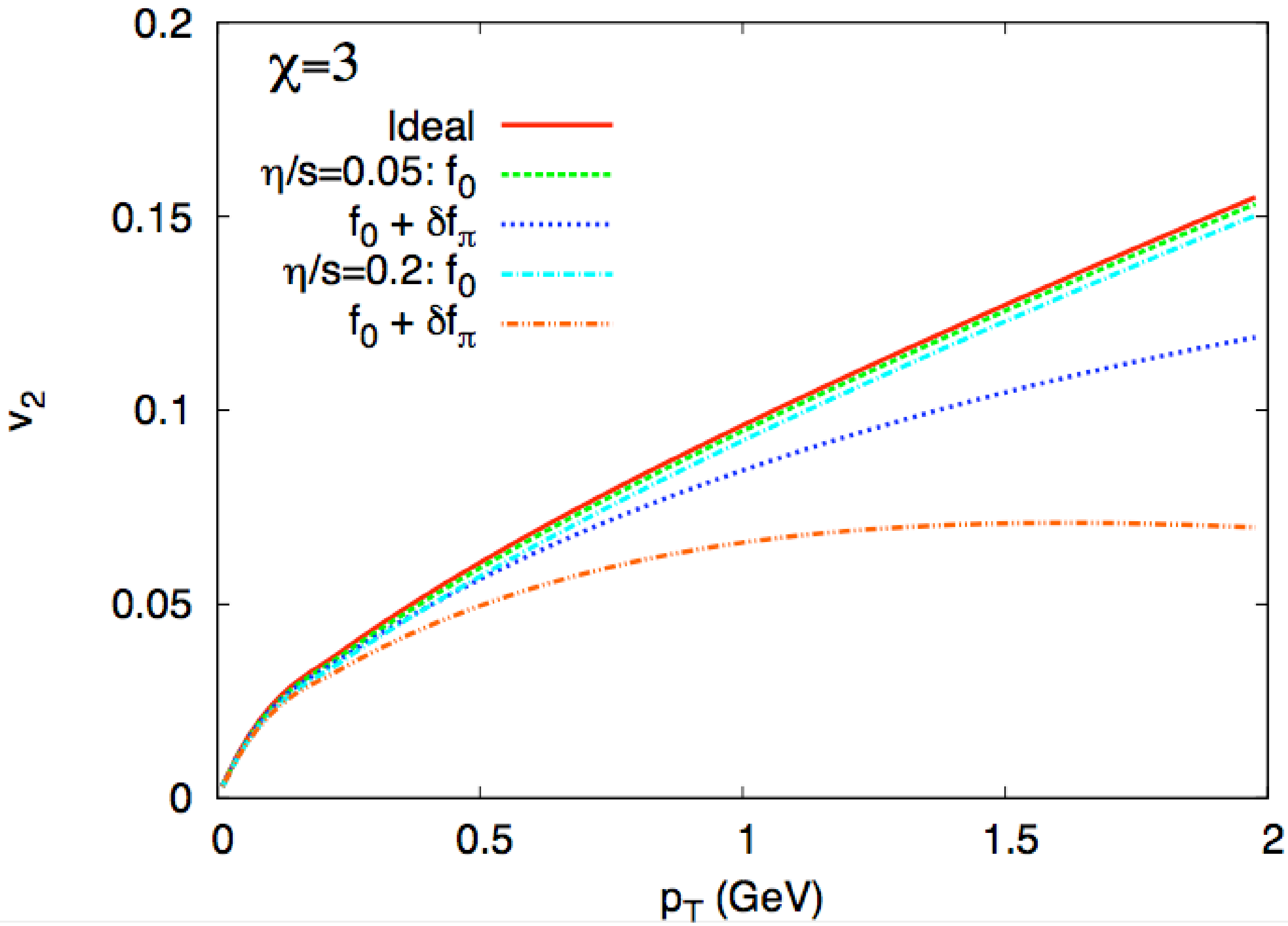


Elliptic Flow



- Without δf almost no change in elliptic flow

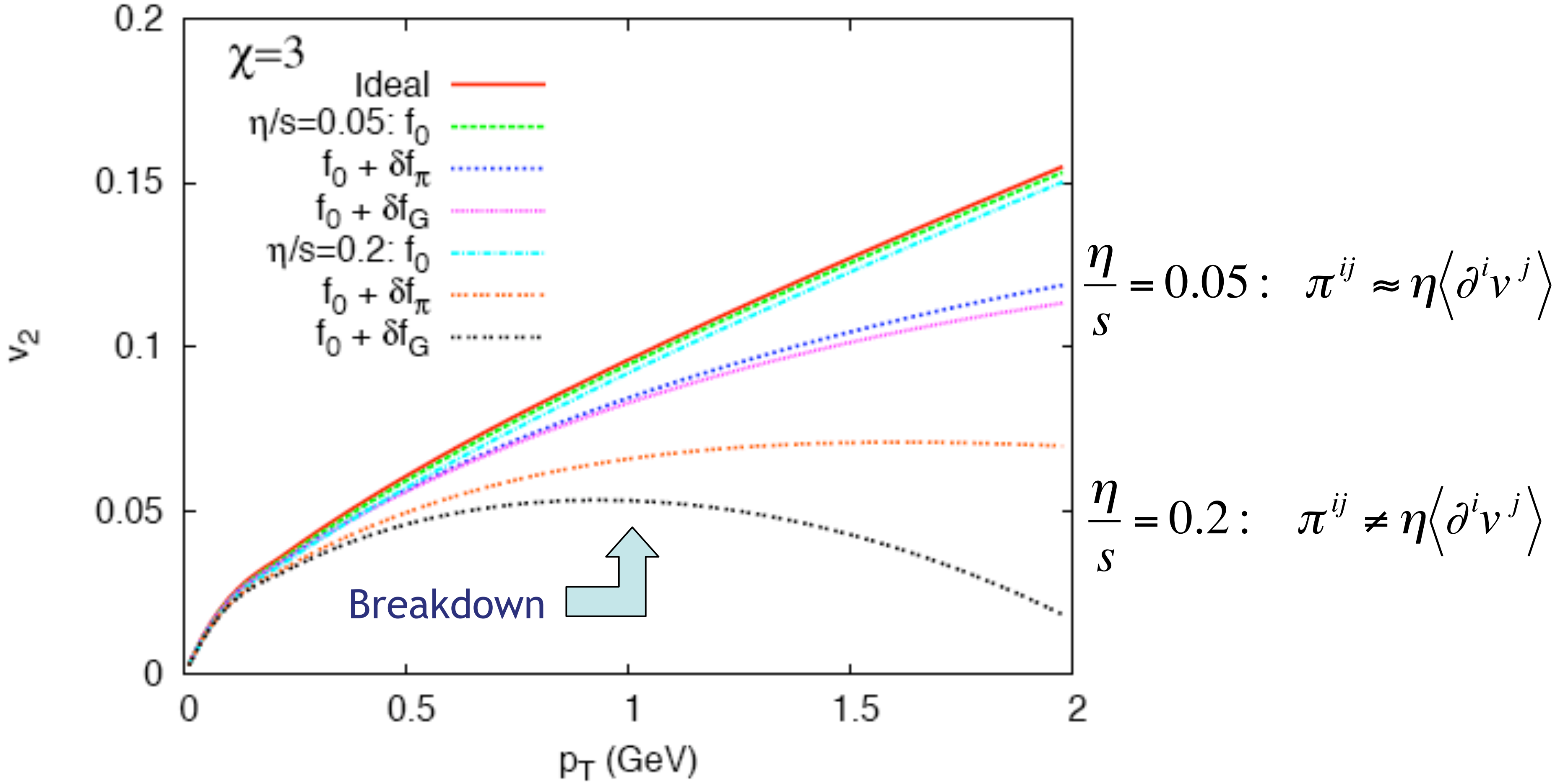
Elliptic Flow



$$\frac{\eta}{s} = 0.05$$

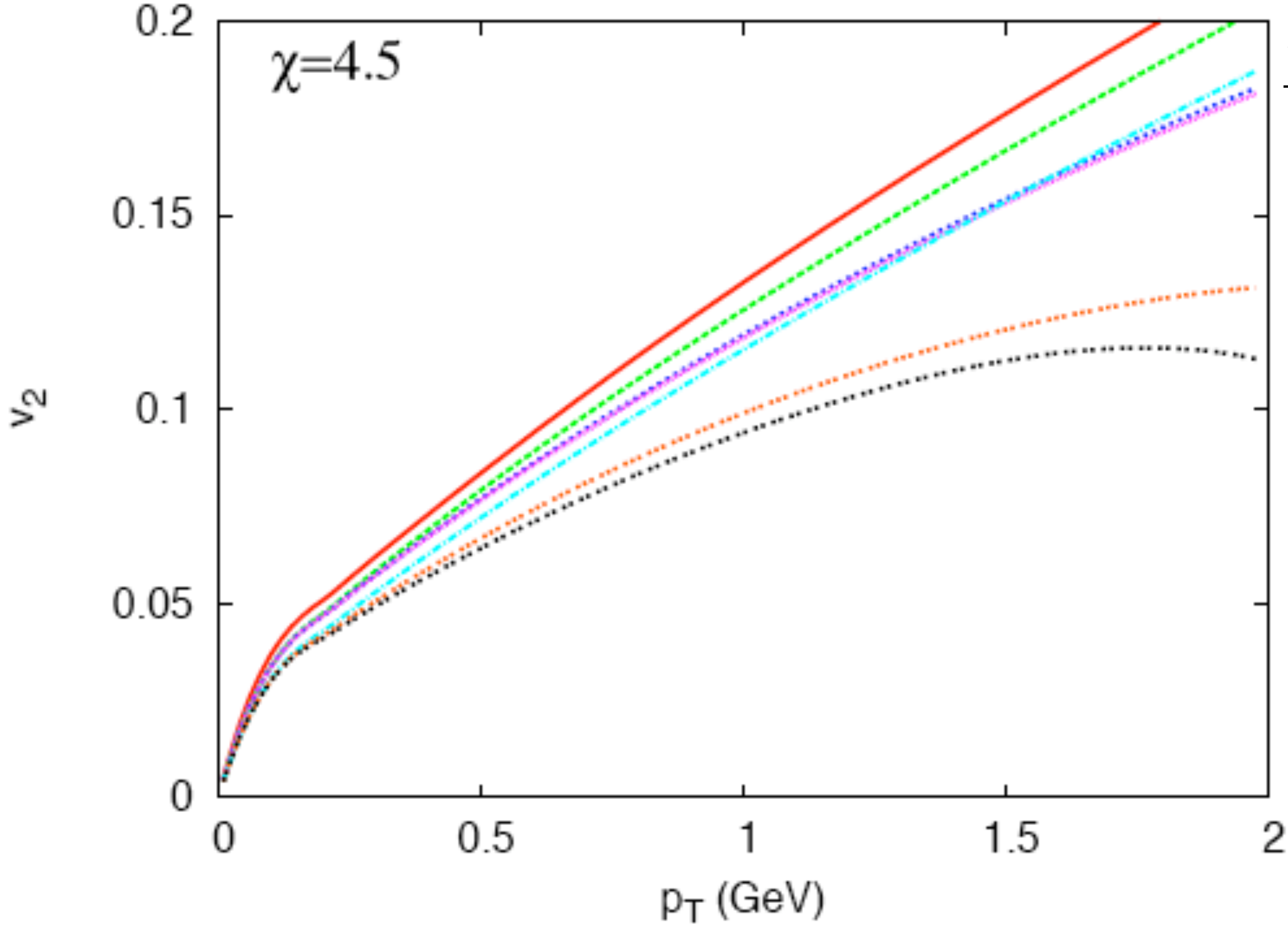
$$\frac{\eta}{s} = 0.2$$

Elliptic Flow



- For small viscosity: $\pi^{ij} \approx \eta \langle \partial^i v^j \rangle$
- Gradients signal breakdown of hydro at high p_T

Elliptic Flow

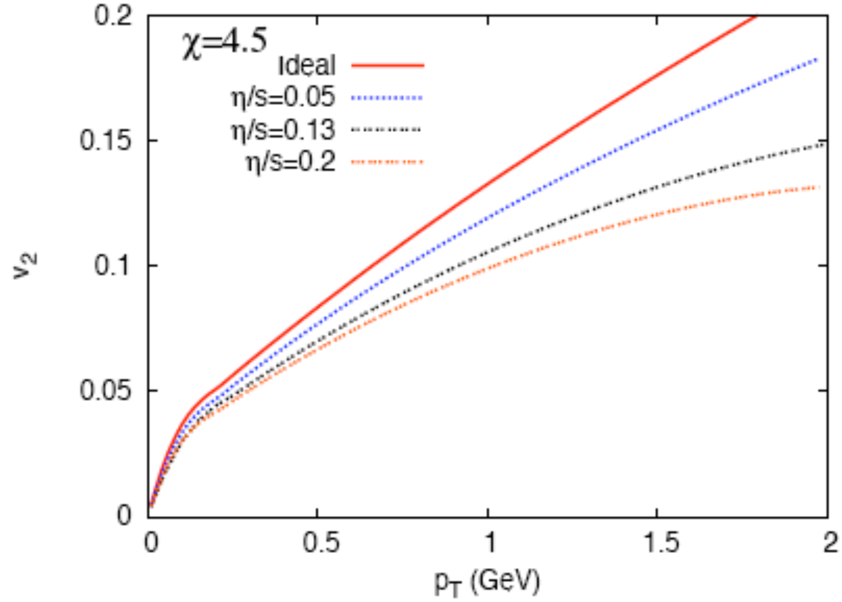


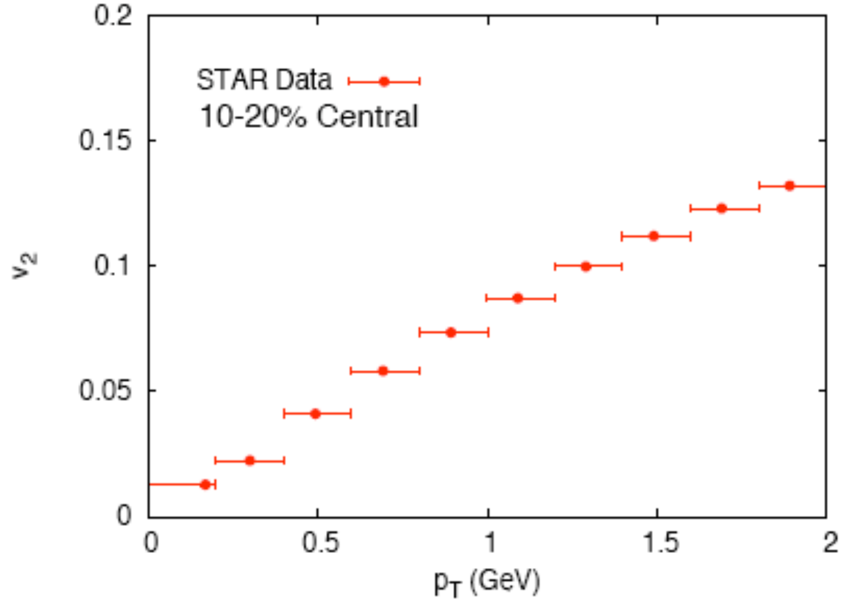
$$\frac{\eta}{s} = 0.05: \quad \pi^{ij} \approx \eta \langle \partial^i v^j \rangle$$

$$\frac{\eta}{s} = 0.2: \quad \pi^{ij} \neq \eta \langle \partial^i v^j \rangle$$

- Again, without δf almost no change in elliptic flow
- Viscous effects get steadily smaller with time

Summary of Elliptic Flow





Qualitative Summary of $v_2(p_T)$

- Two effects modify v2:
  - Flow Effects from changes to ideal EoM
  - Kinetic effects from δf
- Flow Effects:
  - Start small …. Build up gradually with time
  - Not sensitive to η/s close to decoupling time
- Kinetic Effects:
  - Start large …. Suppress with time (for η/s = const)
  - Very sensitive to η/s close to decoupling time
- What about bulk viscosity near T_c and $\eta/s \sim T^{-4}$ in HRG?
- May not change flow much but will greatly change δf !!!
- In my mind v2(pt) probes η/s on freezeout surface !!!

"Integrated" Elliptic Flow

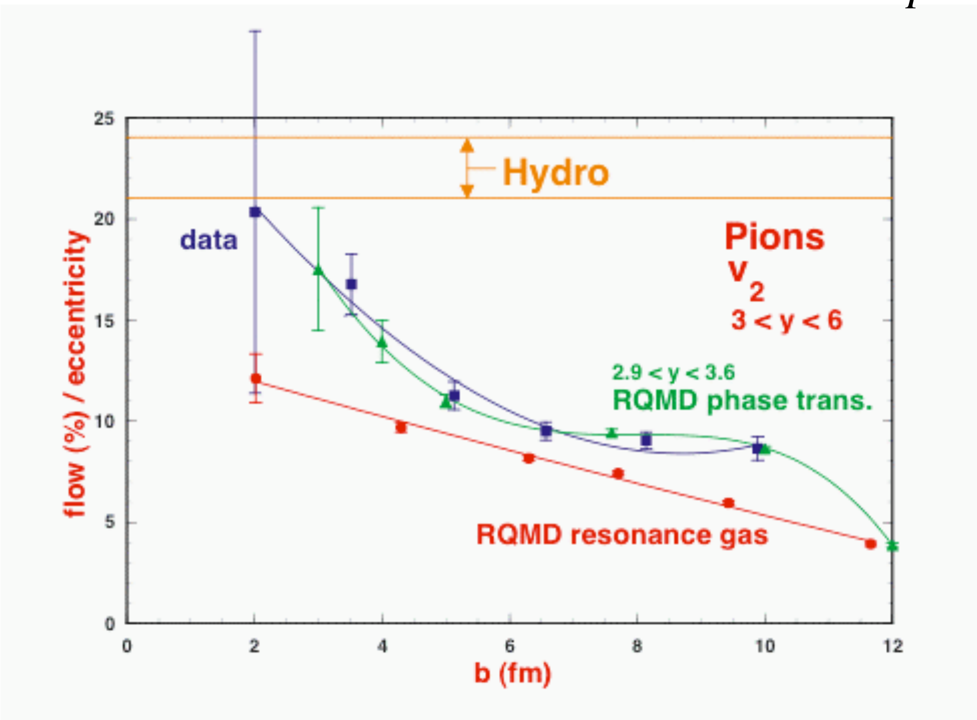
A.M. Poskanzer, et al., Nucl. Phys. A661, 341c (1999).

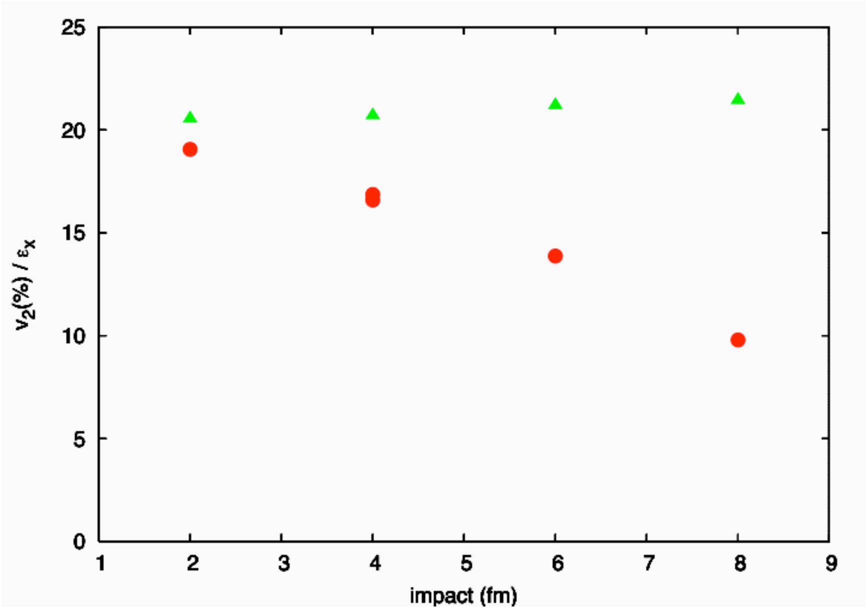
P.F. Kolb, J. Sollfrank, P.V. Ruuskanen and U. Heinz, Nucl. Phys. A661, 349c (1999).

$$v_2 \approx \frac{1}{2}\varepsilon_p(\tau_f)$$

$$\text{Ideal}: \quad \tau_f = \tau_f(T = 155\ \text{MeV})$$

$$\text{Vis}: \quad \tau_f = \tau_f(\frac{\eta}{p}\langle\nabla^\mu u^\nu\rangle \approx 0.5)$$





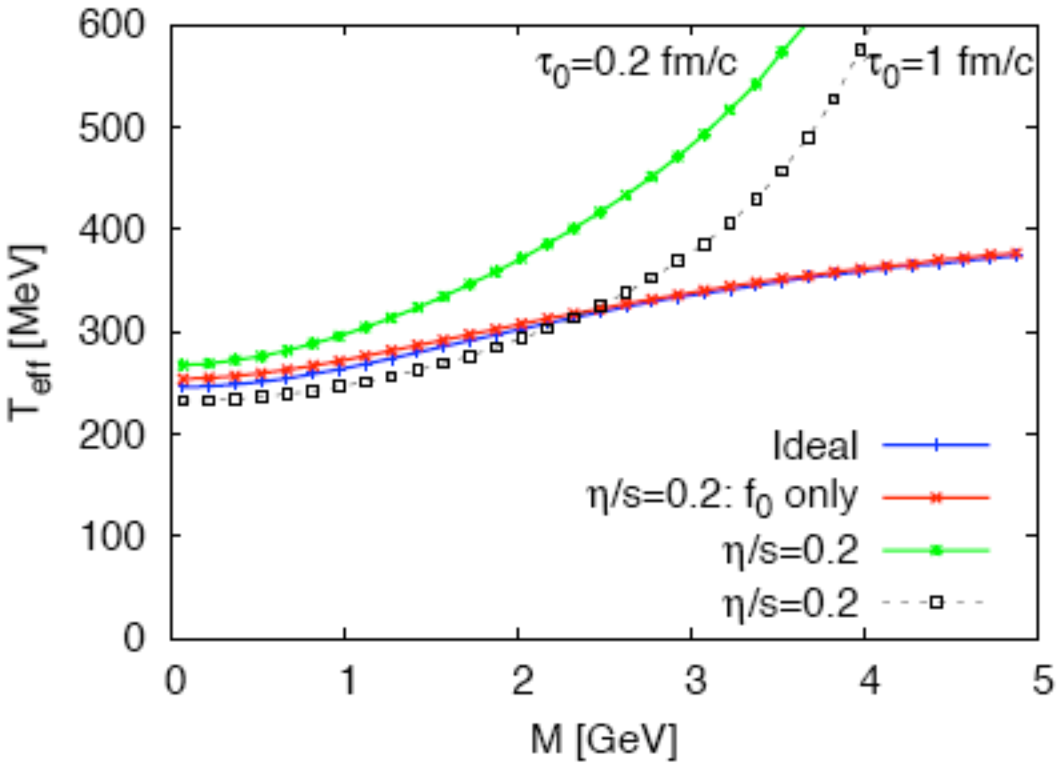
Dilepton Production

- Look at qq annihilation

$$\frac{dN}{d^4q} = \int \frac{d^3\mathbf{k}_1}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3} f(E_1,T)f(E_2,T)v_{12}\sigma(M^2)\delta^4(q-k_1-k_2)$$

- Replace quark distribution with viscosity modified:

$$f(p) \to f(p) + \frac{C_1}{2(\epsilon+p)T^2} f(p)[1-f(p)]p^\alpha p^\beta \pi_{\alpha\beta}$$



Huge Signal!
Not sensitive to Cooper Frye!

Conclusions

- Viscous Hydrodynamics (KD and Derek Teaney, arXiv:0710.5932)
  - Viscosity does not significantly change ideal hydrodynamic solution
  - Viscosity signals the boundary of applicability of hydrodynamics
  - v_2 very sensitive to viscous corrections
  - Use physical gradients to signal breakdown of hydrodynamics

- Thermal Dilepton Production (KD and Shu Lin, arXiv:0803.1262)
  - High mass T_{eff} sensitive to thermalization time and η/s
  - p_T spectrum key to understanding thermalization mechanism, thermalization time and viscosity

Dilepton Production (cont.)

- Result (for Boltzmann Statistics):

$$\frac{dN}{d^4q} = \underbrace{\frac{N_c \alpha^2 e_q^2}{12\pi^4} e^{-q^0/T} \left[1 + \right.}_{\text{Ideal Contribution}} \underbrace{\left. \frac{C_1}{3(\epsilon + p)T^2} q^\alpha q^\beta \pi_{\alpha\beta} \right]}_{\text{Viscous Correction}}$$

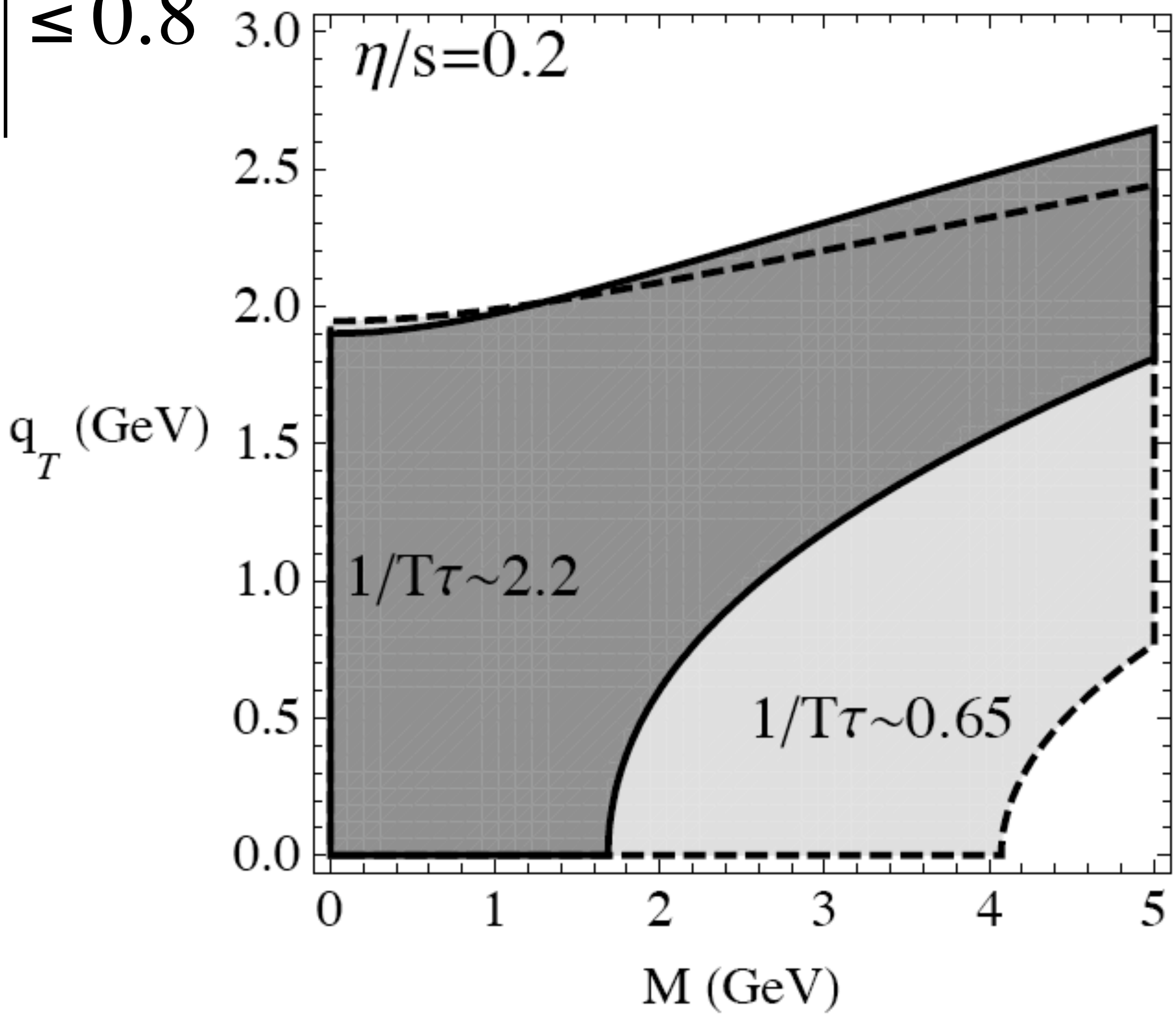
- Consider a simple model:
  - ➢ Bjorken expansion without transverse flow:

$$q^\alpha q^\beta \pi_{\alpha\beta} = \frac{2}{3\tau} q_\perp^2 - \frac{4}{3\tau} m_\perp^2 \sinh^2(y - \eta_s)$$

- Result: $\frac{dN}{dM^2 dq_\perp^2 dy} = \frac{N_c \alpha^2 e_q^2}{12\pi^3} K_0(x) \left(1 + \frac{2C_1}{9\tau T} \left(\frac{\eta}{s} \right) \left[\left(\frac{q_\perp}{T} \right)^2 - 2 \left(\frac{m_\perp}{T} \right) \frac{K_1(x)}{K_0(x)} \right] \right)$

When/Where is thermal production reasonable?

$$\left|\frac{\delta f}{f_0}\right| \leq 0.8$$

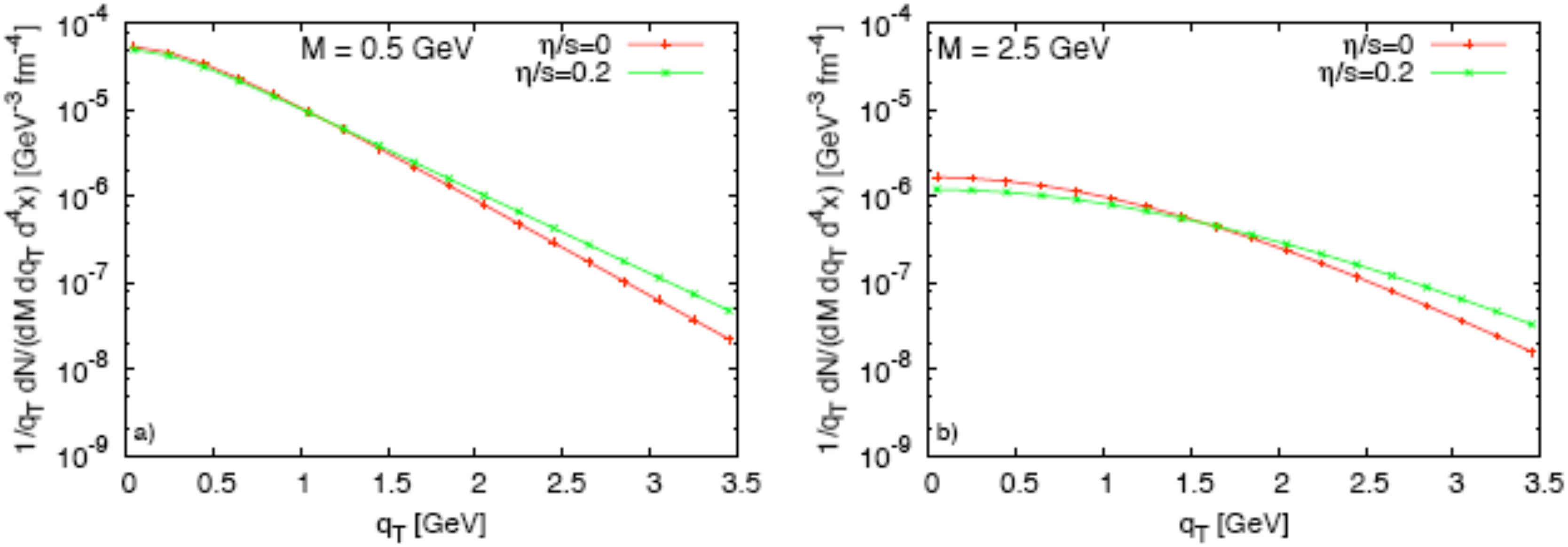


$$M_{Max} \approx \frac{2\tau_0 T_0^2}{\left(\eta/s\right)}$$

Dilepton Production (Results)

- Consider a simple model:
  - ➢ Bjorken expansion without transverse flow:

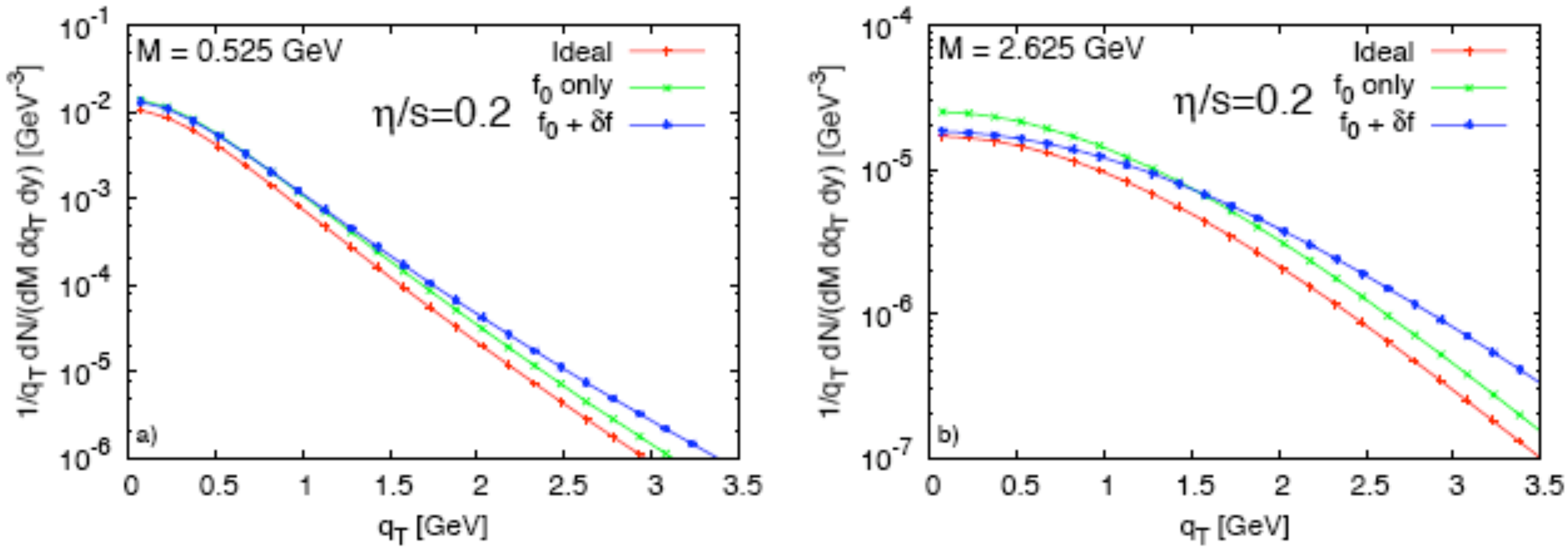
$$q^\alpha q^\beta \pi_{\alpha\beta} = \frac{2}{3\tau} q_\perp^2 - \frac{4}{3\tau} m_\perp^2 \sinh^2(y - \eta_s)$$



Dilepton Production

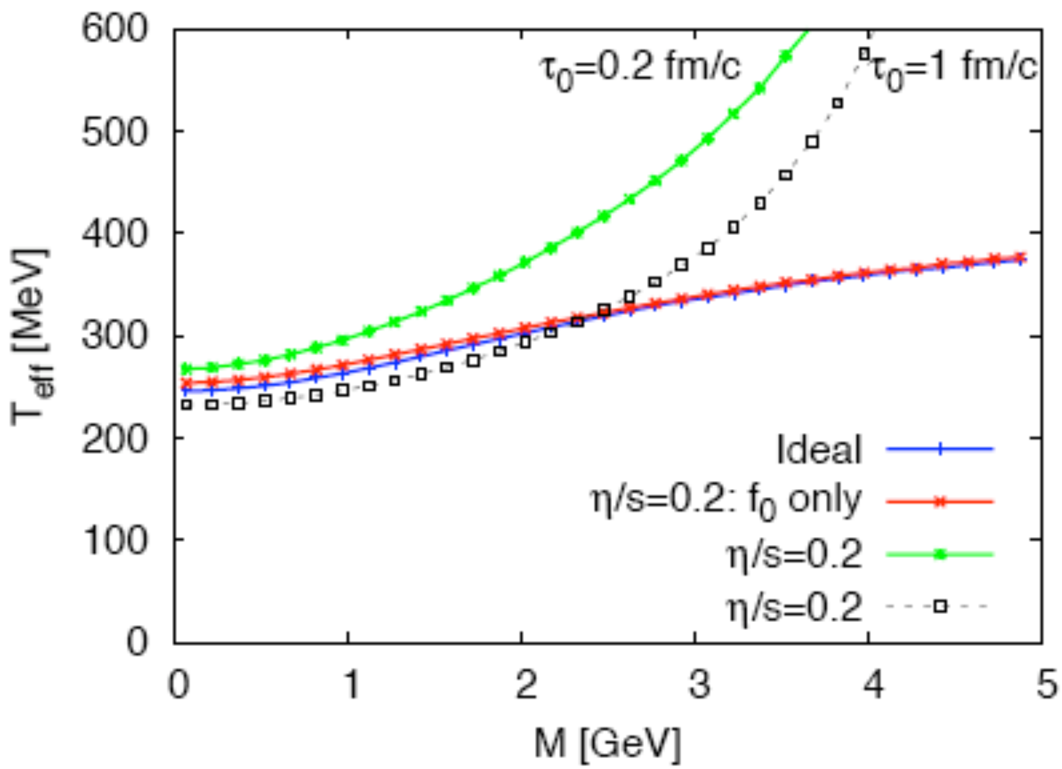
- Full space-time integration (now including arbitrary transverse flow):

$$q^\alpha q^\beta \langle \nabla_\alpha u_\beta \rangle = q_\perp^2 \cos^2(\theta)\pi^{rr} + q_\perp^2 \sin^2(\theta) r^2 \pi^{\phi\phi} + m_\perp^2 \sinh^2(\eta_s)\tau^2\pi^{\eta\eta} + m_\perp^2 \cosh^2(\eta_s) v^2 \pi^{rr} - 2 m_\perp \cosh(\eta_s) q_\perp \cos(\theta) v \pi^{rr}$$



Dilepton Effective Temperature

- Fit transverse mass spectrum to: $\frac{dN}{m_T dm_T} \propto \exp(-\frac{m_T}{T_{eff}})$



- Without δf, viscous corrections are negligible
- Viscosity and thermalization time set mass limit on thermal dilepton production

$$M_{Max} \approx \frac{2\tau_0 T_0^2}{\left(\eta/s\right)}$$

Compare to Free Streaming Quarks

- Collisionless Boltzmann Eqn: $p^\mu \partial_\mu f(p,x) = 0$

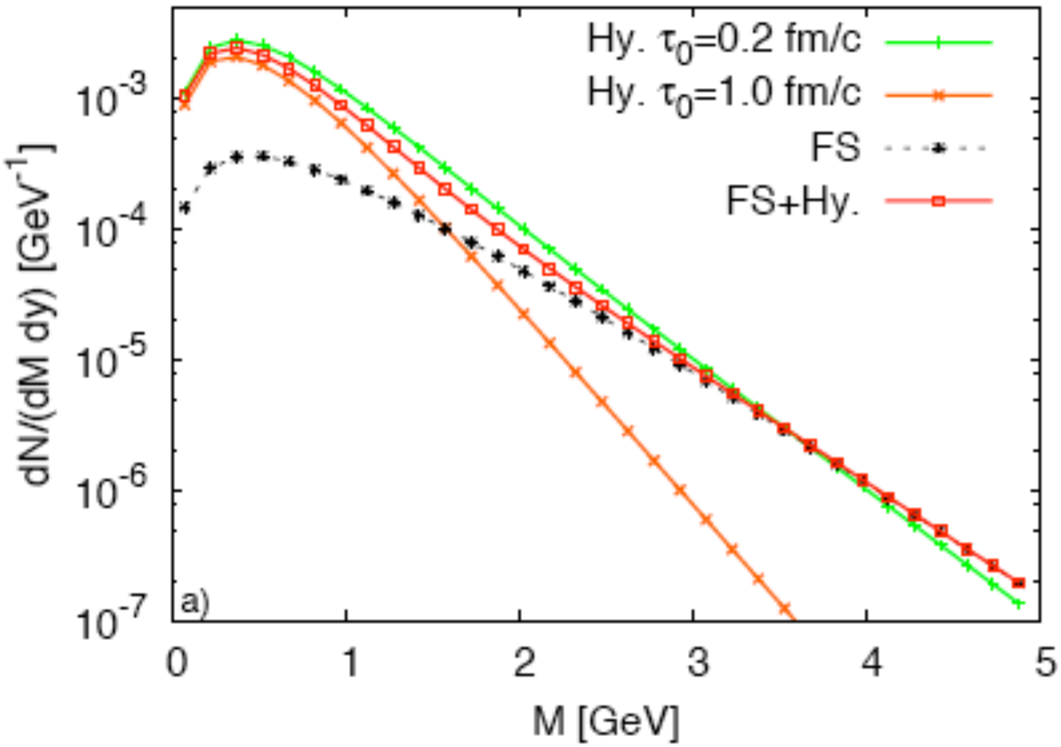
- Bjorken Geometry w/o transverse flow: $\partial_\tau f - \frac{\tanh\chi}{\tau}\partial_\chi f = 0$

- Solution: $f(p,x) = \frac{1}{e^{\frac{p}{T}\sqrt{1+\sinh^2(\chi)\left(\frac{\tau}{\tau_0}\right)^2}} + 1}$

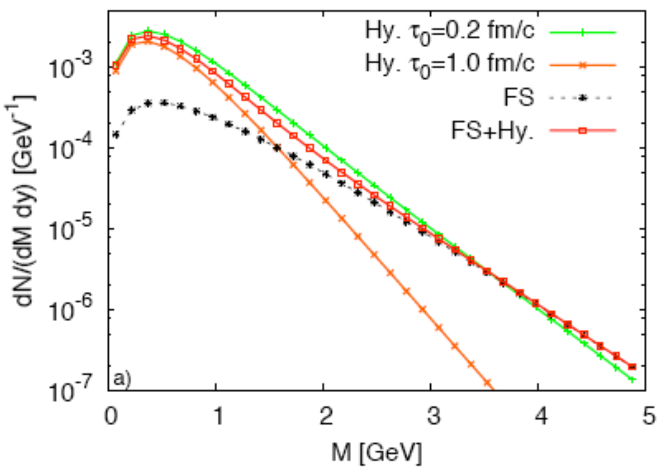
- Calculate Dilepton Yields:

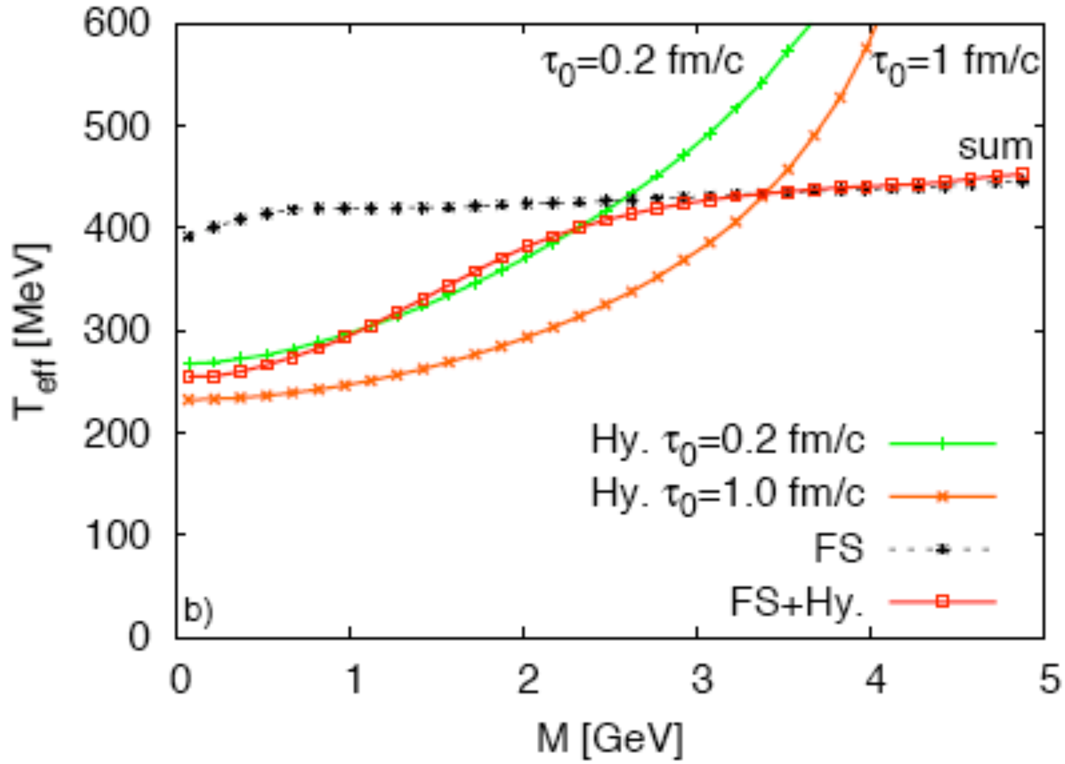
$$\frac{dN}{d^4q} = \int d^4x \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} f(p_1,x) f(p_2,x) v_{12}\sigma\delta^{(4)}(p_1 + p_2 - q)$$

Compare to Free Streaming Quarks



Teff: Free Streaming vs. Early Viscosity





$$M_{Max} \approx \frac{2\tau_0 T_0^2}{\left(\eta/s\right)} \approx \begin{cases} 4.5 \text{ GeV} & \text{for } \tau_0 = 1.0\, fm/c \\ 2.0 \text{ GeV} & \text{for } \tau_0 = 0.2\, fm/c \end{cases}$$

Backup

Viscous Correction to Distribution Function

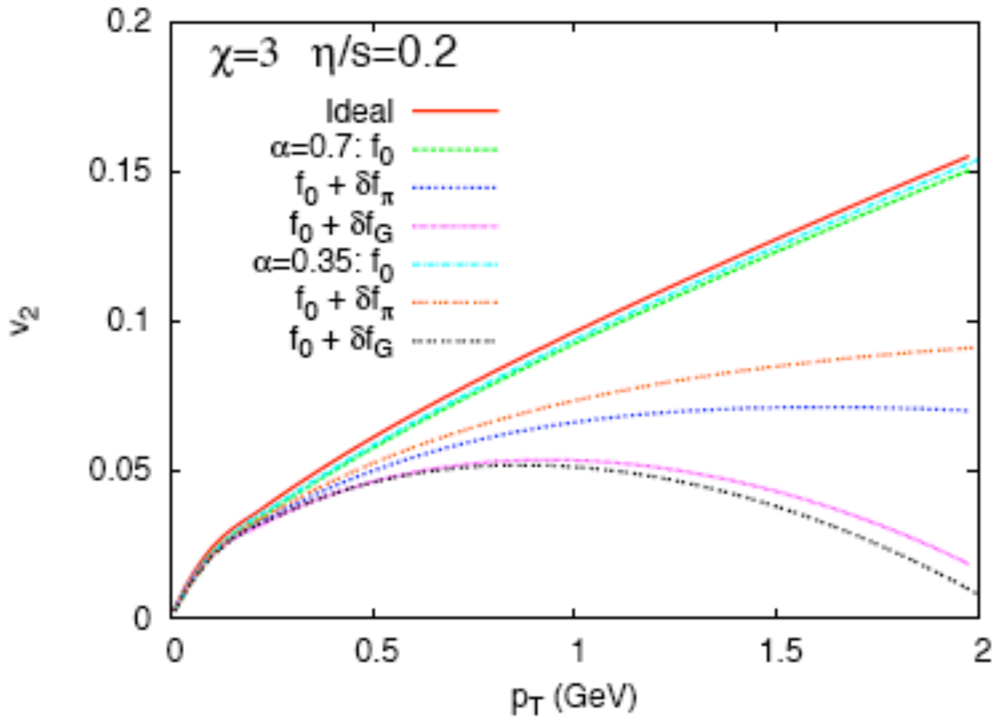
- Use 2nd moment ansatz for correction to thermal distribution function
  - P. Arnold, G. D. Moore, and L. G. Yaffe, J. High Energy Phys. 11, 001 (2000).
  - D. Teaney, Phys. Rev. C 68, 034913 (2003) [arXiv:nucl-th/0301099]
- Use linearized Boltzmann equation: $\frac{p^\mu}{E}\partial_\mu f_p = \int_{1,2,3} d\Gamma_{12\to 3p}(f_1 f_2 - f_3 f_p)$
  - Substitute: $f \to f_0(1+\delta f)$
  - Keep first order in gradients
  - Collision term with f_0 vanishes

$$\frac{p^\mu}{E}\partial_\mu f_p = \int_{1,2,3} d\Gamma_{12\to 3p}(\delta f_1 \delta f_2 - \delta f_3 \delta f_p)$$

  - Restrict δf to polynomial of degree less than 3: $f = f_0\left(1 + \frac{C}{2T^3} p^\mu p^\nu \left\langle \partial_\mu u_\nu \right\rangle\right)$
  - Get constant C from: $T_0^{\mu\nu} + T_{vis}^{\mu\nu} = \int d^3p \frac{p^\mu p^\nu}{E} f_0(1+\delta f)$
- Boost Invariant expansion w/o transverse flow:

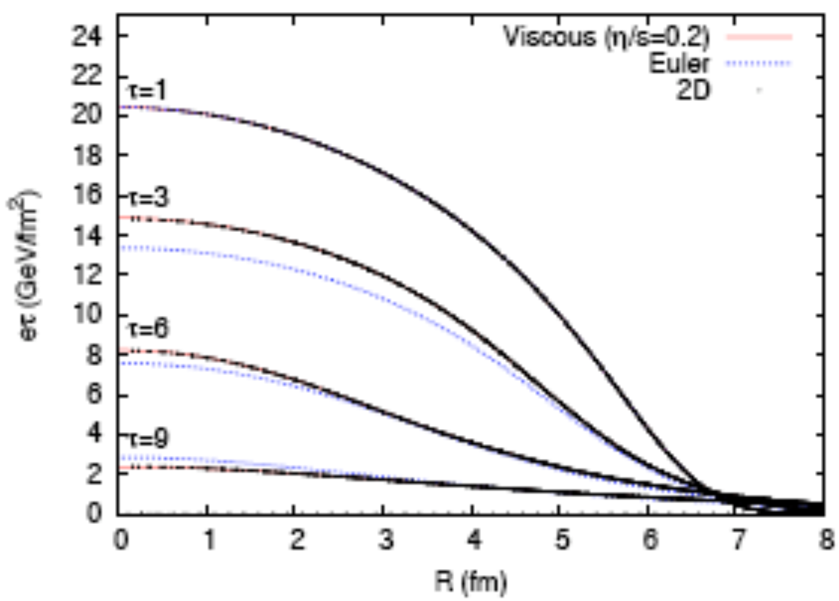
Dependence on small time parameter

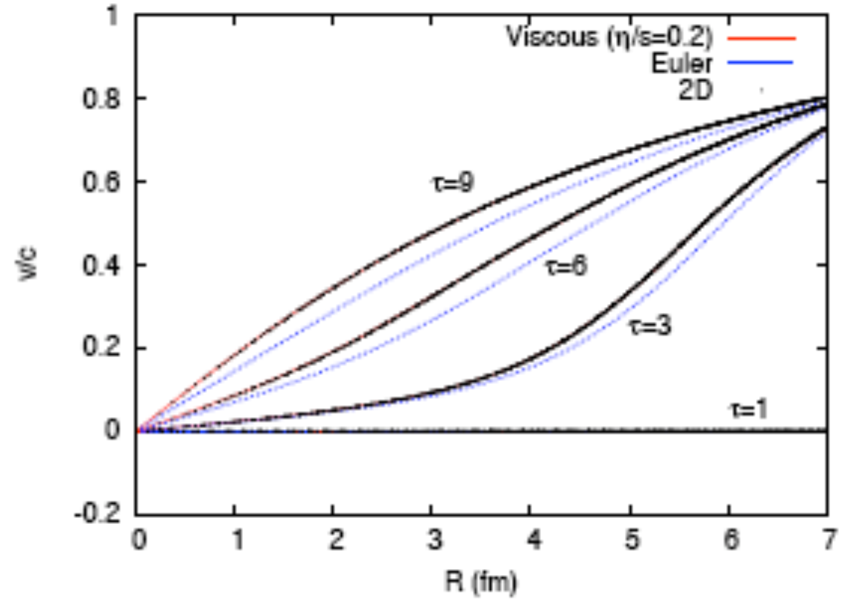
- Dependence on small time parameter



Scatter Plot

- 1D versus 2D





Dilepton Production (Fermi Statistics)

- Result:

$$\frac{dN}{d^4q} = -\frac{N_c \alpha^2 e_q^2}{12\pi^4}\Big[\underbrace{f_b(q_0,T)[1+\frac{2T}{|\mathbf{q}|}\ln(\frac{n_+}{n_-})]}_{\text{Ideal Contribution}} - \underbrace{\frac{C_1}{2(\epsilon+p)T^2} b_2(q_0,|\mathbf{q}|)q^\alpha q^\beta \pi_{\alpha\beta}}_{\text{Viscous Correction}}\Big]$$

$$b_2(q_0,|\mathbf{q}|) = \frac{1}{|\mathbf{q}|^5}\int_{E_-}^{E_+} f(E_1,T)f(q_0-E_1)(1-f(E_1))\Big[(3q_0^2-|\mathbf{q}|^2)E_1^2 - 3q_0E_1M^2 + \frac{3}{4}M^4\Big]$$

where $E_\pm = \frac{1}{2}(q_0 \pm |\mathbf{q}|)$.

Initial Conditions

- N_f=3 ideal QGP equation of state: p=1/3ε
- Entropy distributed according to number of participants
  - ➢ C_s fixes initial temperature and total particle yield

$$s(x,y,\tau_0) = \frac{C_s}{\tau_0}\frac{dN_p}{dxdy}$$

- Also must specify viscous fields in 2nd order setup
  - ➢ Auxiliary fields should be in their relaxed form

$$\pi_{xx} = \pi_{yy} = -\frac{1}{2}\pi_{zz} = \frac{2}{3}\eta\,\partial_z u^z \qquad \Pi = 0$$

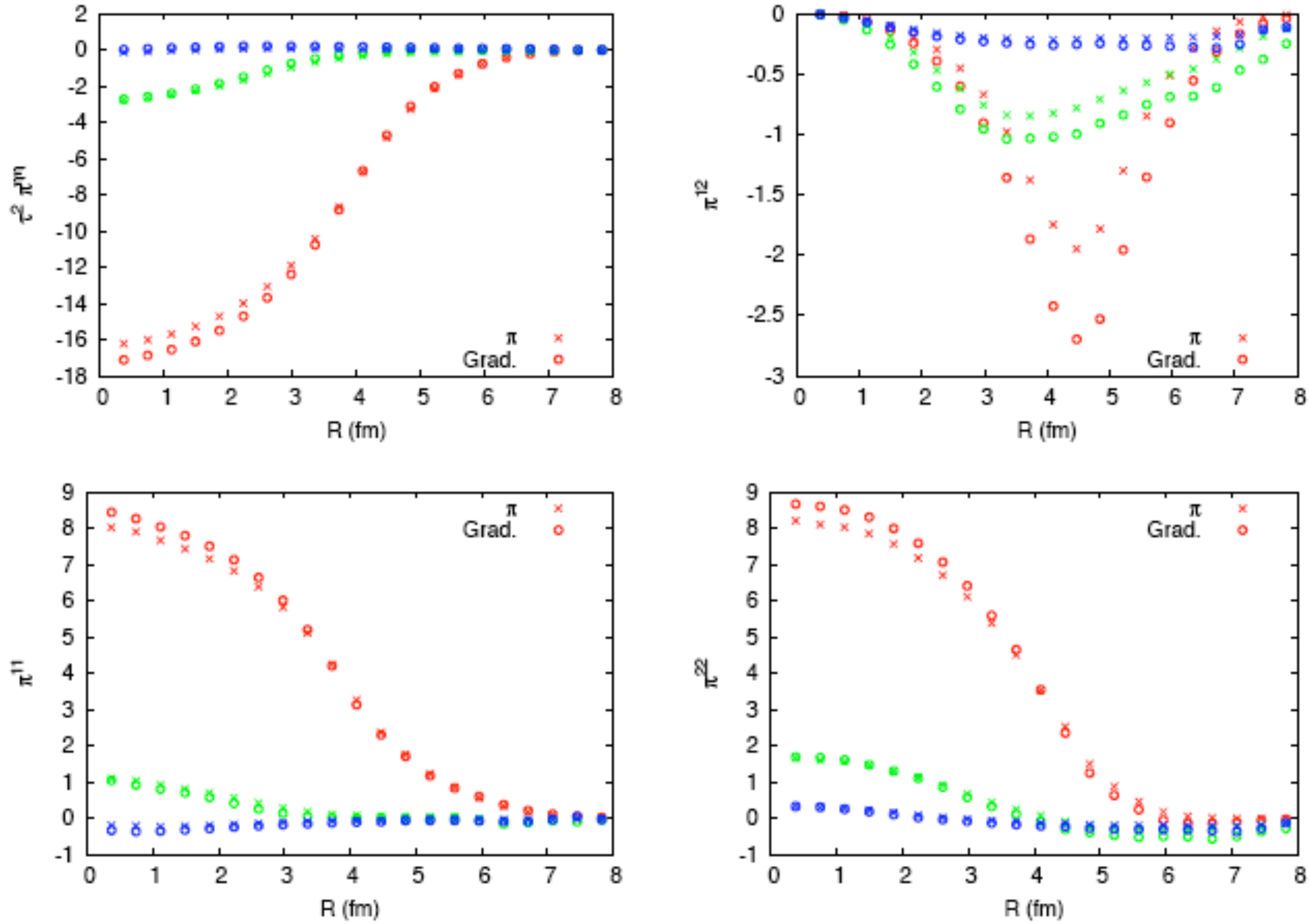
$$c^{11} = \frac{2}{3}\frac{\tau_0}{\tau} - \frac{2}{3}\frac{\tau_2}{\tau}$$

$$c^{22} = \frac{2}{3}\frac{\tau_0}{\tau} - \frac{2}{3}\frac{\tau_2}{\tau}$$

$$c^{33} = \frac{2}{3}\frac{\tau_0}{\tau} + \frac{4}{3}\frac{\tau_2}{\tau}$$

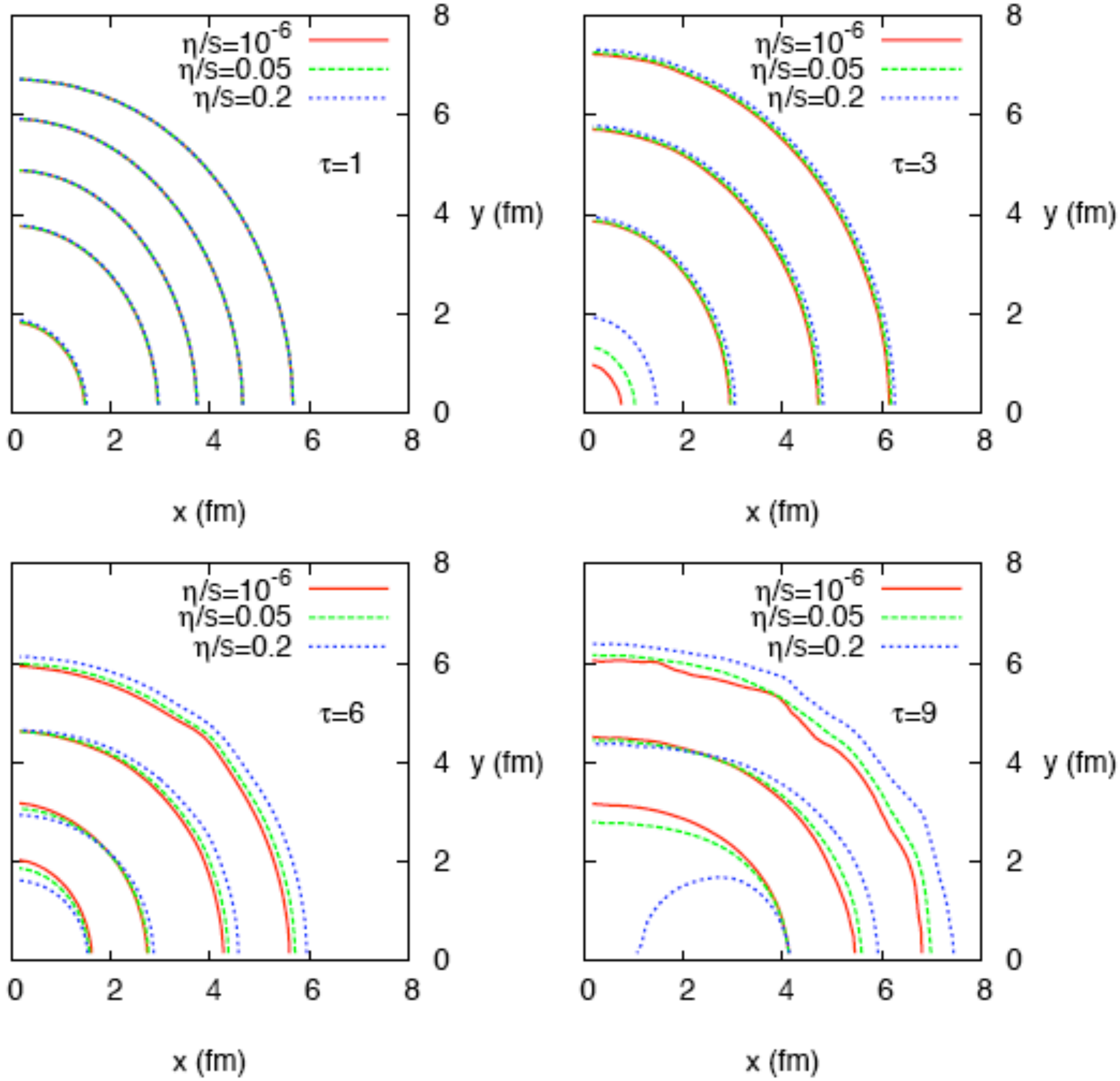
Hydrodynamic Result: Gradients

- Fluid Gradients:



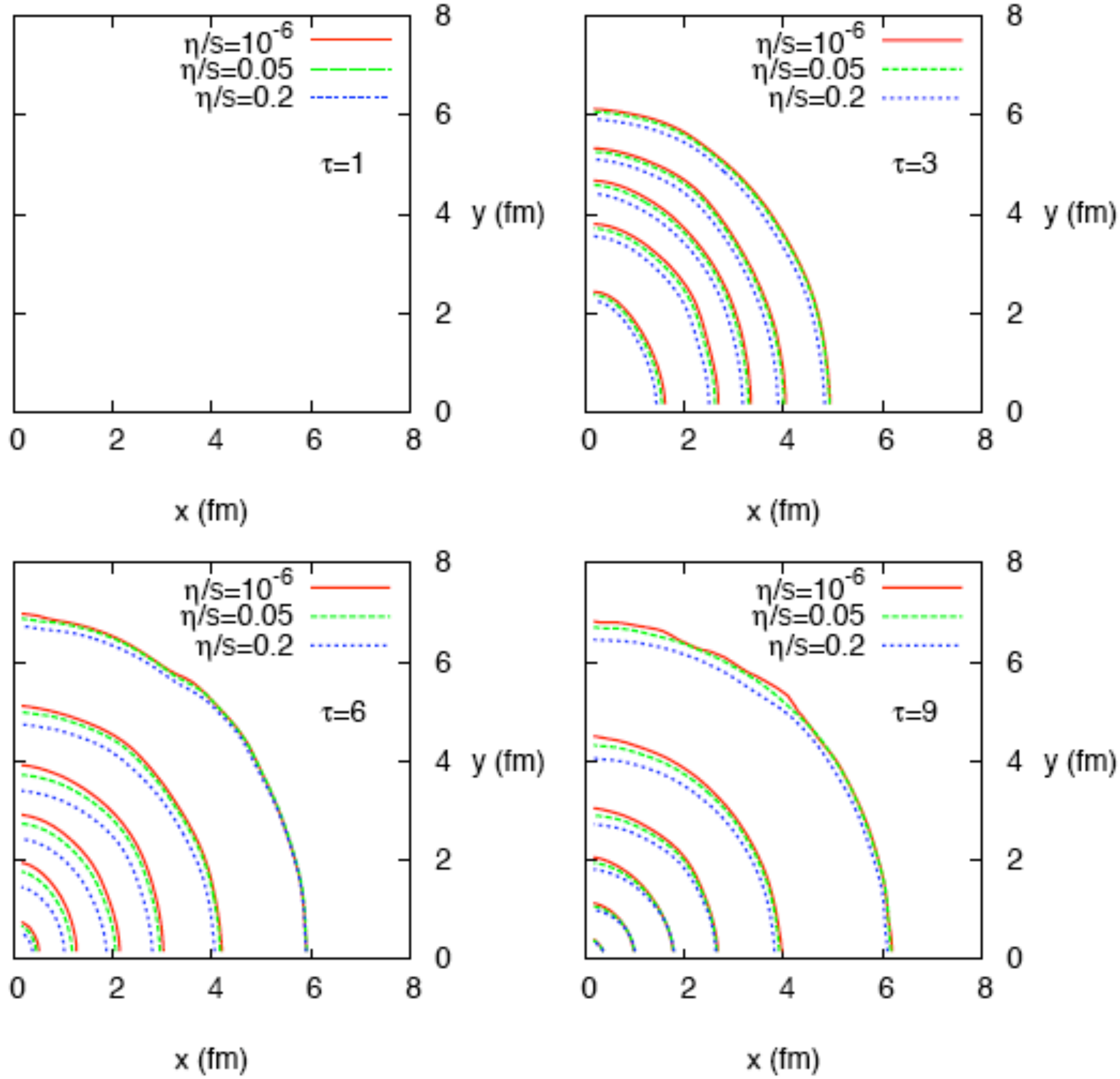
Hydrodynamic Results: 2D

- Energy Density per unit rapidity



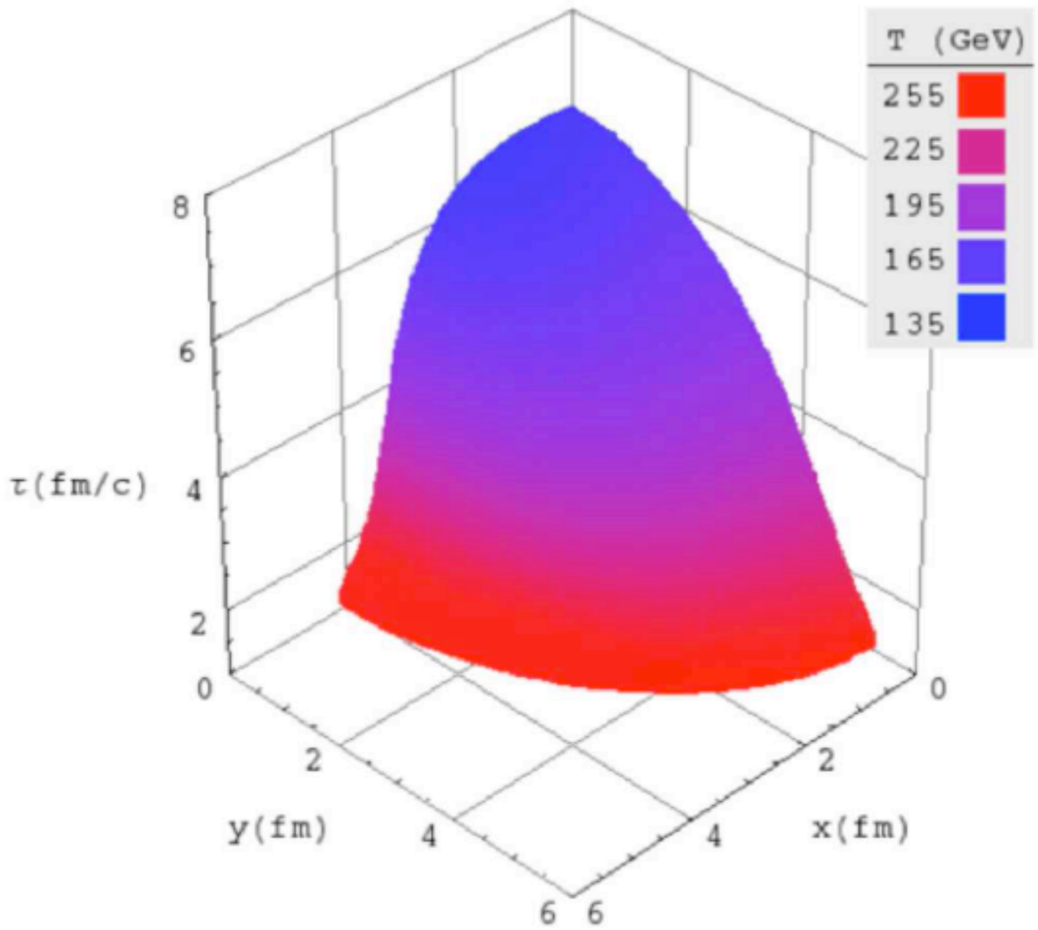
Hydrodynamic Results: 2D

- Transverse velocity contours



Freezeout Surfaces

- Sample 2D freezeout surface:

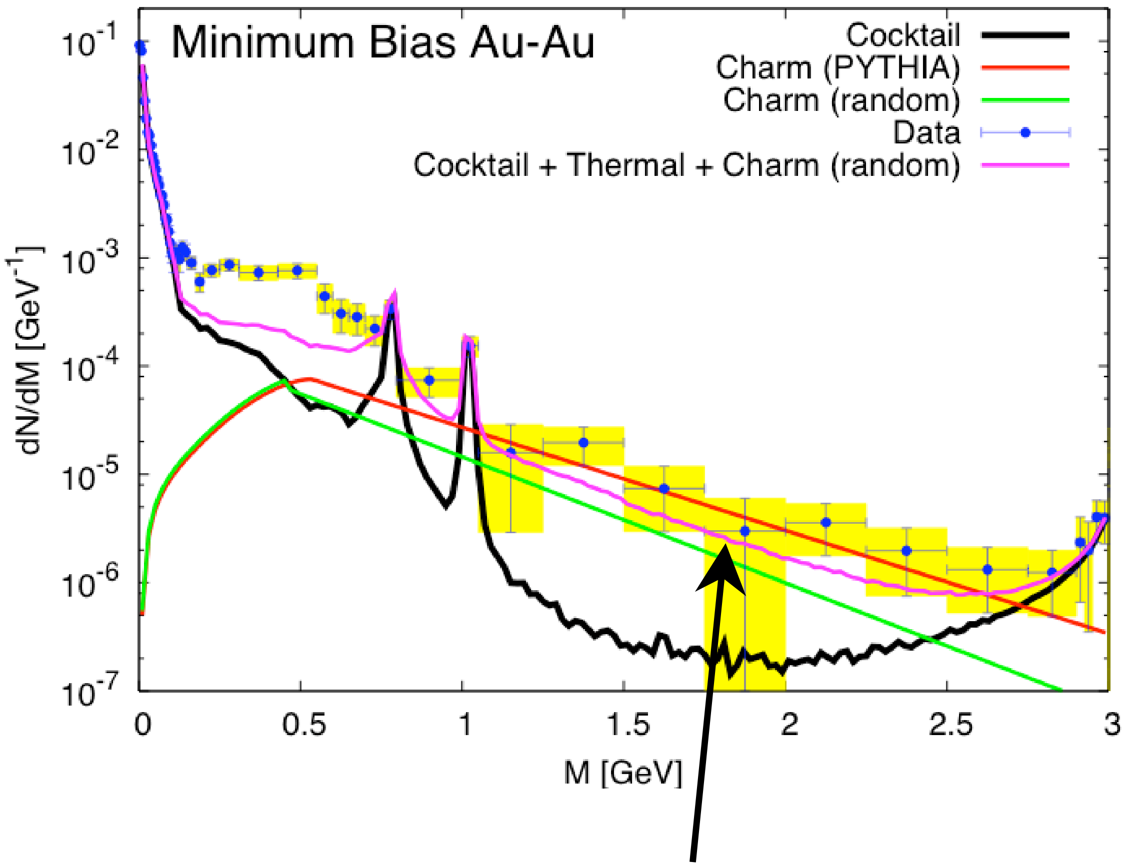


$$\chi = \frac{4}{T}\partial_\mu u^\mu = 3.0$$

$$\frac{\eta}{s} = 0.2$$

RHIC Di-electrons

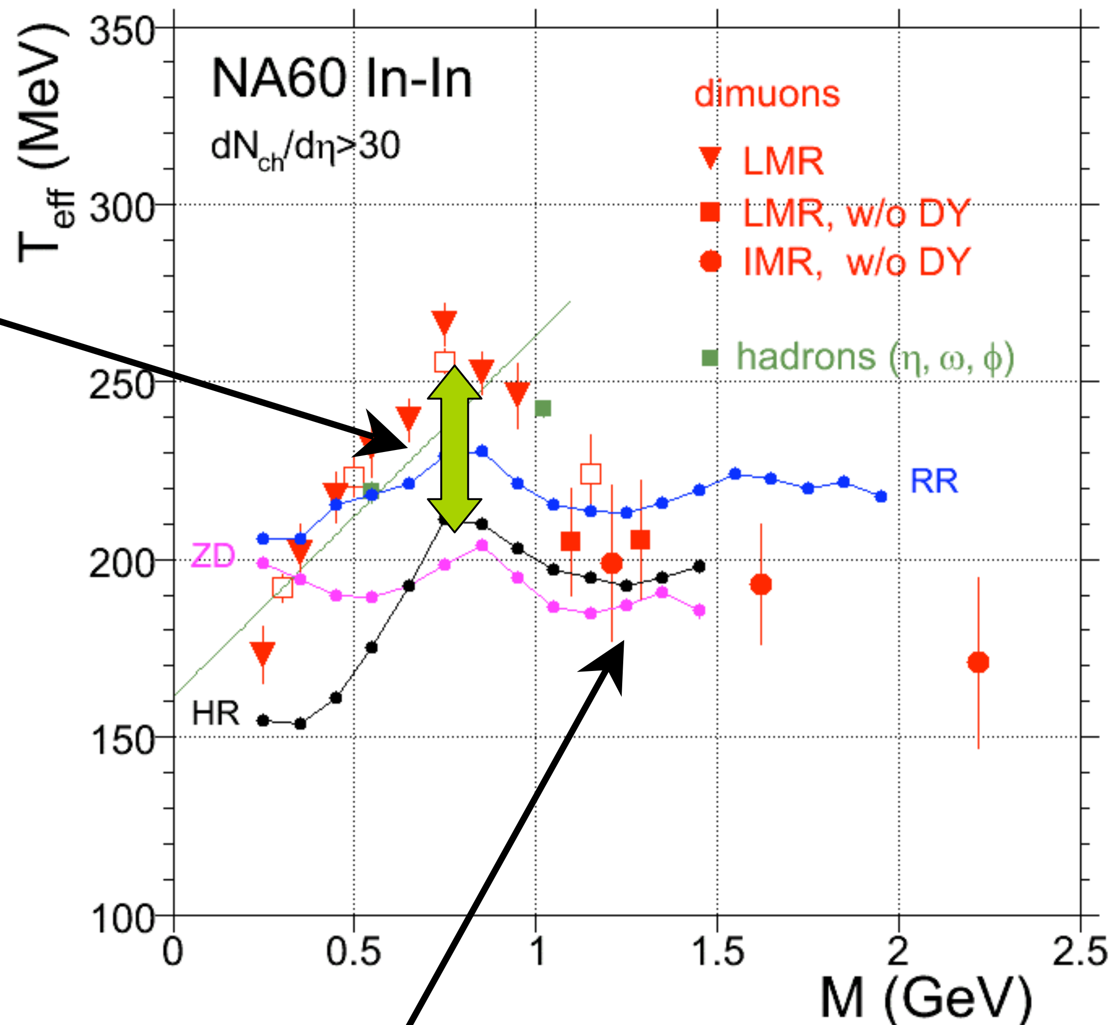
- Both RHIC and NA60 suggest that thermal emission can be separated from background sources in IMR



Hard to believe thermal emission in this region but need p_T spectrum to decipher!

NA60 Di-Muons

Viscous correction to Hadronic emission ???



Possibilities to probe thermalization time and η/s ???

Typical Relaxation Process: Diffusion

- Has similar problems to RNSE
  - Violates Causality
  - Breaks sum rules

- Continuity: $\partial_t n + \partial_x j = 0$
- Fick's Law: $j = -D\nabla n$

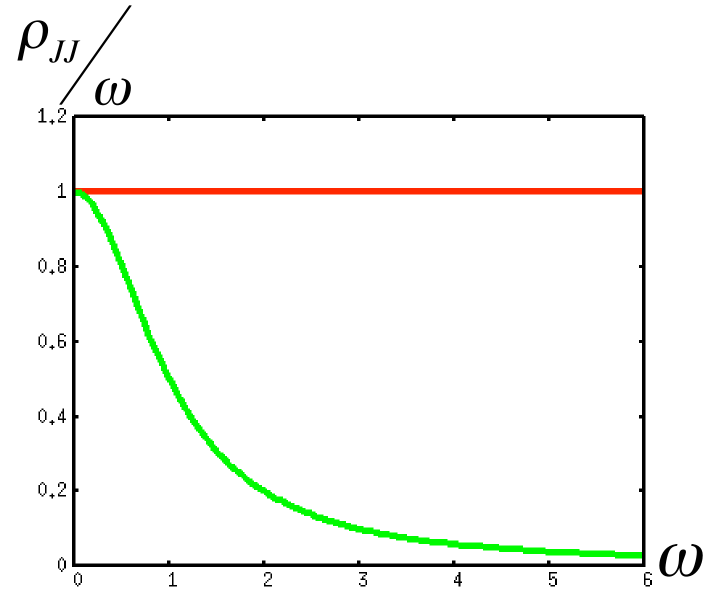
$$\partial_t n - D\nabla^2 n = 0$$

- Relaxation Time Approximation: $\partial_t j = -\dfrac{(j + D\nabla n)}{\tau_R}$

$$\tau_R \partial_t^2 n + \partial_t n - D\nabla^2 n = 0 \qquad \lambda = \pm\sqrt{\frac{D}{\tau_R}}$$

Diffusion (con't)

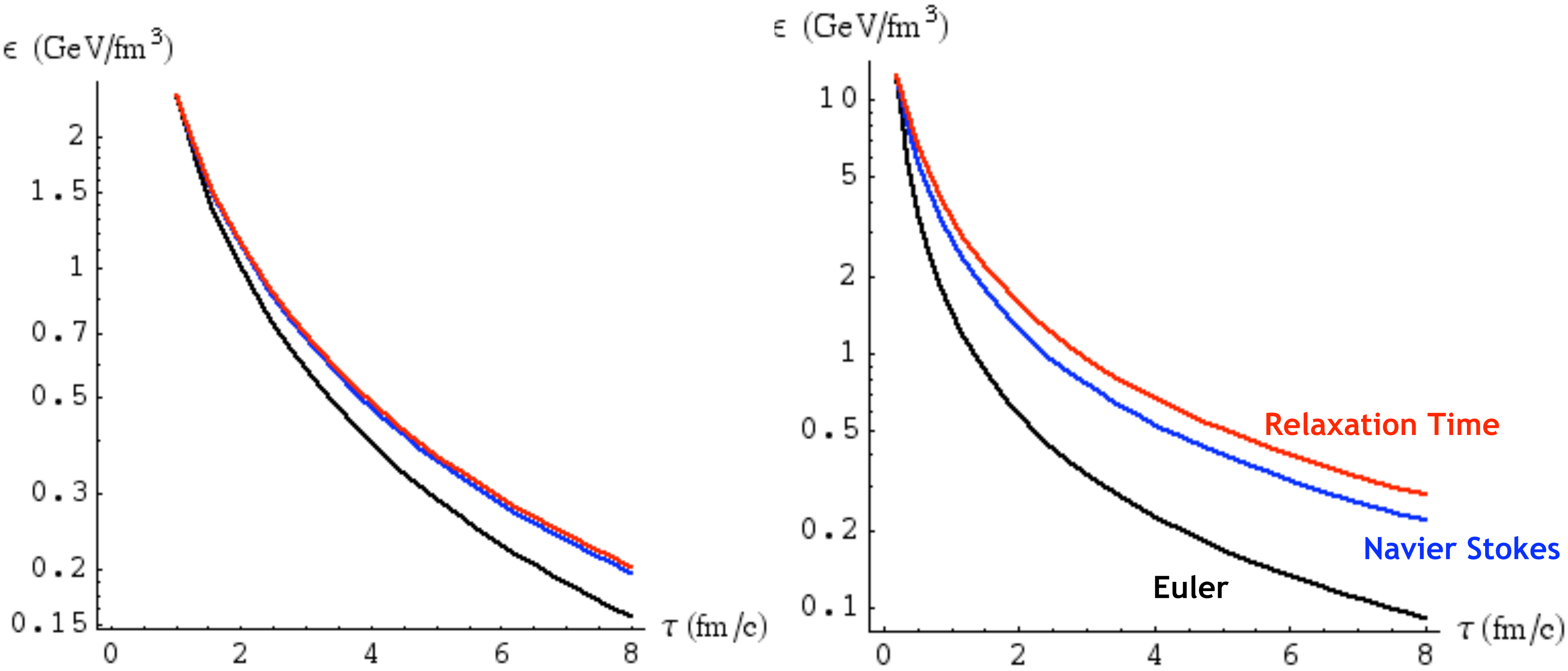
- Spectral Densities:
  - Ordinary Diffusion: $\dfrac{\pi\rho_{JJ}(\vec{k}=0,\omega)}{\omega} = \chi_S D$
  - Relaxation: $\dfrac{\pi\rho_{JJ}(\vec{k}=0,\omega)}{\omega} = \dfrac{\chi_S D}{1+(\omega\tau_R)^2}$



- Sum Rule: $\displaystyle\int \frac{\rho_{JJ}(\vec{k}=0,\omega)}{\omega}d\omega = \chi_S \langle v_{th}^2 \rangle$ (Weak Coupling)
  - Can separate short and long time parameters:
  - Long Time: D
  - Short Time: $\dfrac{D}{\tau_R} = \langle v_{th}^2 \rangle$

Results should (and will!) be insensitive to short time response.

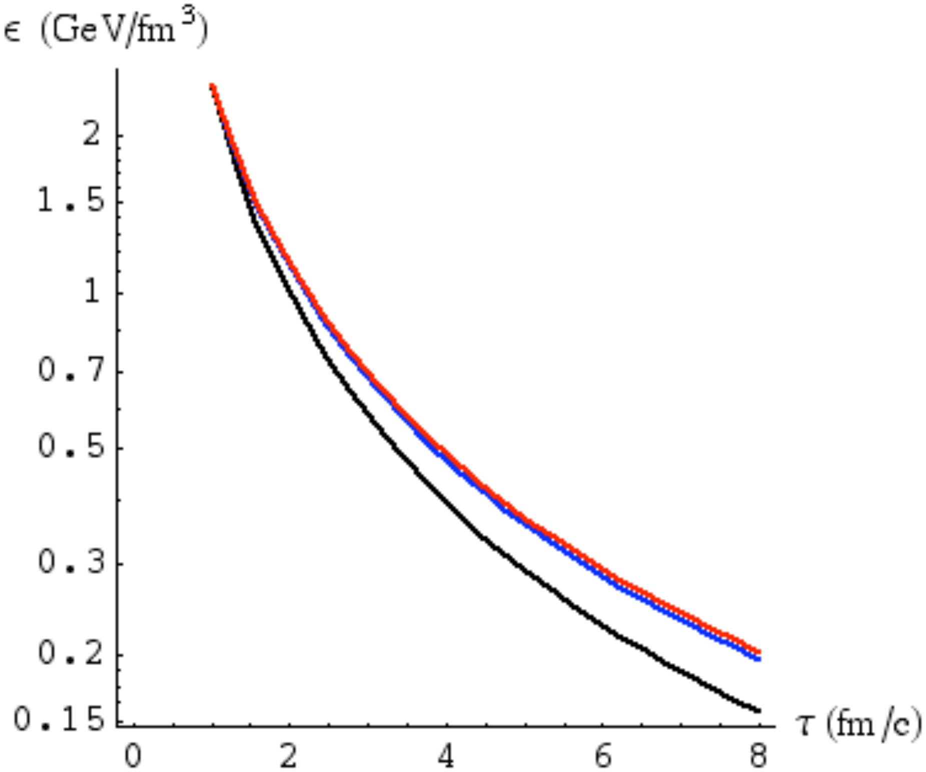
Can't believe Hydro too early!



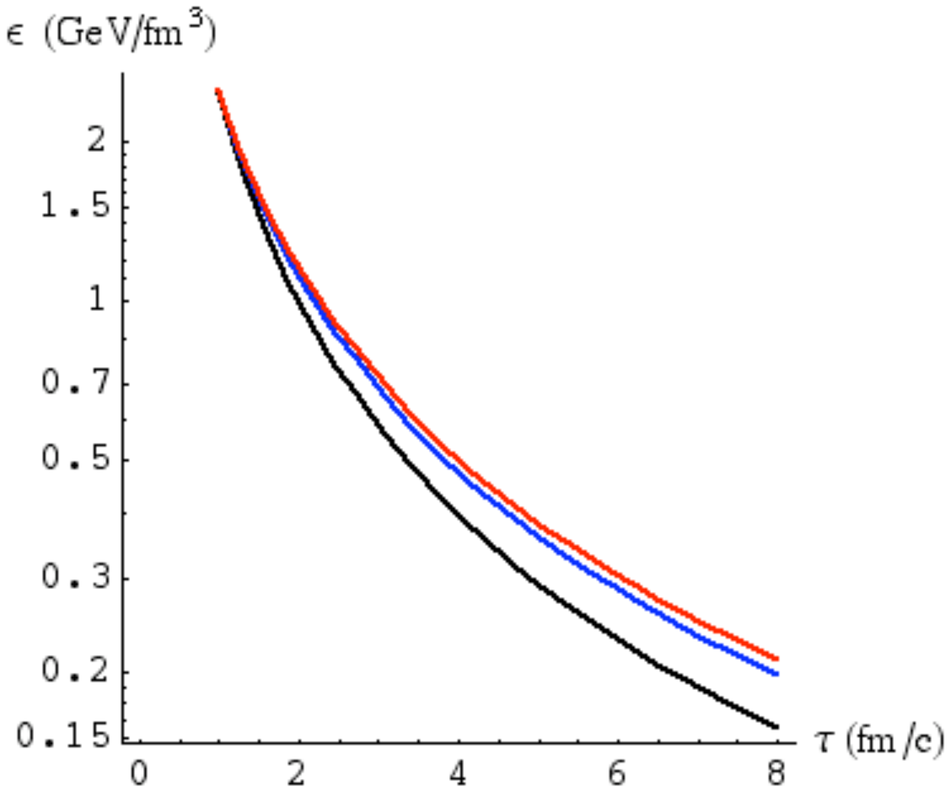
$\tau_0 = 1.0$

$\tau_0 = 0.2$

No dependence on small time parameter!



$$\tau_R = \frac{5\eta}{4p}$$



$$\tau_R = 2\tau_R$$

Viscosity and its effect on elliptic flow and thermal dileptons

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I present on the recent simulations of a viscous hydrodynamical model of non-central Au-Au collisions in 2+1 dimensions, assuming longitudinal boost invariance. The model fluid equations were proposed by Ottinger and Grmela. Freezeout is signaled when the viscous corrections become large relative to the ideal terms. Then viscous corrections to the transverse momentum and differential elliptic flow spectra are calculated. When viscous corrections to the thermal distribution function are not included, the effects of viscosity on elliptic flow are modest. However, when these corrections are included, the elliptic flow is strongly modified at large p_T. We also investigate the stability of the viscous results by comparing the non-ideal components of the stress tensor (π^{ij}) and their influence on the v_2 spectrum to the expectation of the Navier-Stokes equations ($\pi^{ij} = -\eta \llangle \partial_i u_j \rrangle$). We argue that when the stress tensor deviates from the Navier-Stokes form the dissipative corrections to spectra are too large for a hydrodynamic description to be reliable. For typical RHIC initial conditions this happens for $\eta/s \gsim 0.3$.

In the second part of this presentation I discuss the first correction to the leading order q\={q} dilepton production rates due to shear viscosity in an expanding gas. The modified rates are integrated over the space-time history of a viscous hydrodynamic simulation of RHIC collisions. The net result is a {\em hardening} of $q_\perp$ spectrum with the magnitude of the correction increasing with invariant mass. We argue that a thermal description is reliable for invariant masses less than $M_{max}\approx(2\tau_0 T_0^2)/(\eta/s)$. For reasonable values of the shear viscosity and thermalization time $M_{max}\approx 4.5$ GeV. Finally, the early emission from a viscous medium is compared to emission from a longitudinally free streaming plasma. Qualitative differences in $q_\perp$ spectrum are seen which could be used to extract information on the thermalization time, viscosity to entropy ratio and possibly the thermalization mechanism in heavy-ion collisions.

K.~Dusling and D.~Teaney, “Simulating elliptic flow with viscous hydrodynamics,”
Phys. Rev. C 77, 034905 (2008)
[arXiv:0710.5932 [nucl-th]].

K. Dusling and S. Lin, “Dilepton production from a viscous QGP,”
arXiv:0803.1262 [nucl-th].

May 2, 2008