Charm elliptic flow from parton coalescence dynamics

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The plan

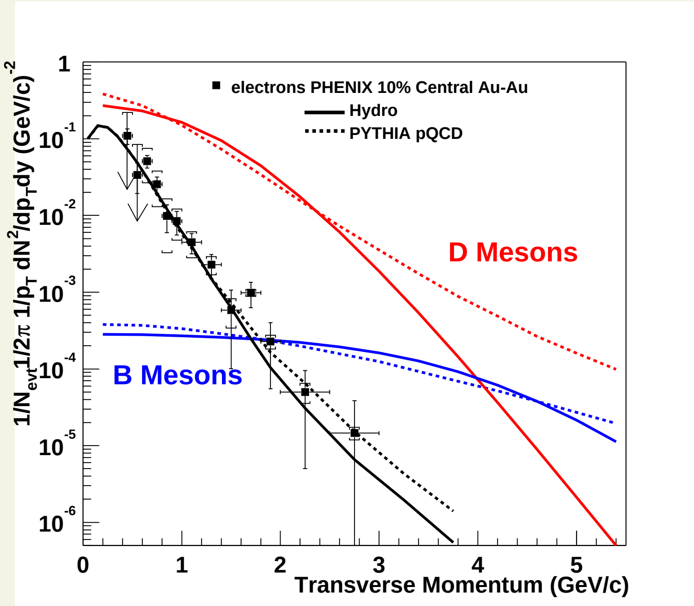
- Motivation
- (Rafting) theory
  - parton coalescence + transport theory
- Results (on/in the water)
  - parton chemistry & elliptic flow
  - charm meson flow
  - a few puzzles

Why charm?

- additional, heavy, probe of the same partonic medium
  - perturbative QCD: expect weaker jet-quenching for heavy quarks

    [Djordjevic, Gyulassy ('03), Dokshitzer, Kharzeev ('01)]

- charm thermalization or purely jet production?
  - both scenarios seem to be consistent with PHENIX electron spectra [Batsouli et al ('03)]



$\longrightarrow$ charm quark elliptic flow is the key: tells amount of rescattering

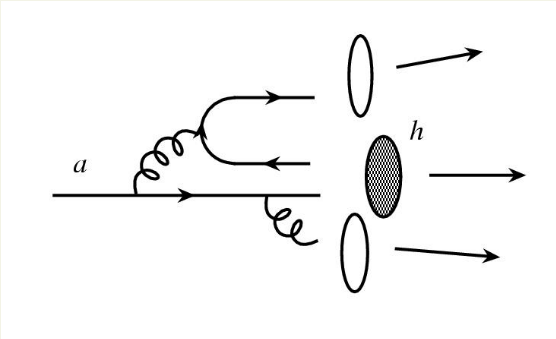
- Also: additional test for quark coalescence models (flow addition formulas)

First some theory:

- Parton coalescence
- Covariant parton transport

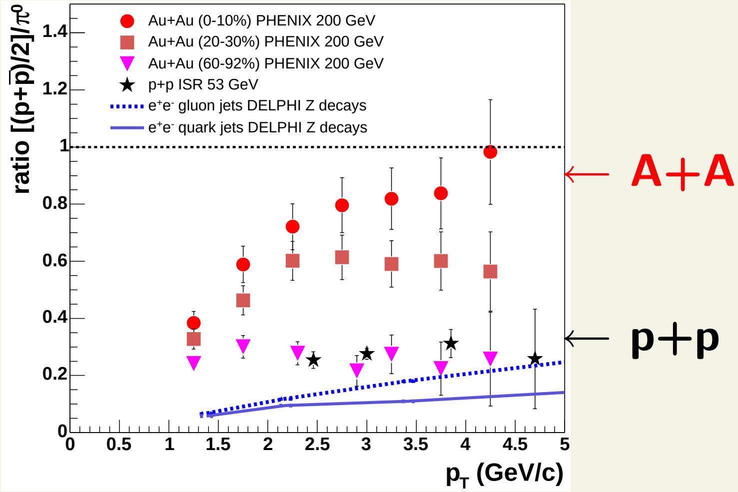
Parton coalescence

- **independent fragmentation:** $D_{a\to h}(z) \to$



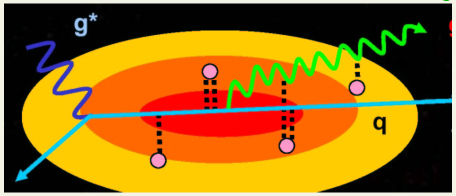
gives $p/\pi \approx 0.2 \Rightarrow$ **fails for baryons at RHIC**

Esumi [PHENIX]:



problem persists for jet quenching:

Wang, Gyulassy, Dokshitzer, Mueller, Levai, Vitev, Wiedemann et al, Guo, Djordevic, ...



- **parton coalescence**

Hwa, Yang, Biró, Zimányi, Lévai, Csizmadia, Ko, Lin, Voloshin, D.M., Greco, Fries, Müller, Nonaka, Bass, ...

heavy-ion collisions - large parton density $\Rightarrow$ multi-parton processes also relevant

lowest-order $q\bar{q} \to M$, $qqq \to B$ **(valence quarks only)**

Applications

hadron multiplicity: Das & Hwa, PLB68 ('77)
Biró et al, PLB347 ('95) - ALCOR
Csizmadia & Lévai, JPG28 ('02) - MICOR

baryon/meson ratio: Hwa & Yang, PRC65 ('02)
Greco, Ko, Levai, PRL90 ('03); PRC68 ('03)
Fries, Müller, Nonaka, Bass, PRL90 ('03); PRC68 ('03)
Hwa, Yang, PRC67 ('03)
Fries, Müller, Nonaka, Bass, JPG30 ('04)
Hwa & Yang, nucl-th/0401001

resonances: Nonaka, Müller, Asakawa, Bass, Fries, PRC69 ('04),
Zimányi & Lévai, nucl-th/0404060

elliptic flow: Ko & Lin, PRL89 ('02)
Voloshin, NPA715 ('02)
D.M. & Voloshin, PRL91 ('03)
Nonaka, Fries, Bass, PLB583 ('04)
D.M., JPG30 ('04)
Greco, Ko, nucl-th/0404020
D.M., nucl-th/0403035

charm hadron elliptic flow: Lin & D.M., PRC68 ('03)
Greco, Ko, Rapp, nucl-th/0312100

Simple coalescence formula

- **based on:** $n + p \to d$

  Butler & Pearson, PR129 ('63); Schwarzschild & Zupancic, PR129 ('63); Sato & Yazaki, PLB98 ('81);
  Gyulassy, Frankel & Remler, NPA402 ('86); Dover et al PRC44 ('91); Nagle et al PRC53 ('96)
  Kahana et al PRC54 ('96); Scheibl & Heinz, PRC59 ('99); ...

- **basic equations:** $q\bar{q} \to meson$, qqq (or $\bar{q}\bar{q}\bar{q}$) $\to baryon$

$$\frac{dN_M(\vec{p})}{d^3p} = g_M \int (\prod_{i=1,2} d^3x_i d^3p_i) W_M(x_1 - x_2, \vec{p}_1 - \vec{p}_2) f_\alpha(\vec{p}_1, x_1) f_\beta(\vec{p}_2, x_2) \delta^3(\vec{p} - \vec{p}_1 - \vec{p}_2)$$

$$\frac{dN_B(\vec{p})}{d^3p} = g_B \int (\prod_{i=1,2,3} d^3x_i d^3p_i) W_B(x_{12}, x_{13}, \vec{p}_{12}, \vec{p}_{13}) f_\alpha(\vec{p}_1, x_1) f_\beta(\vec{p}_2, x_2) f_\gamma(\vec{p}_3, x_3) \delta^3(\vec{p} - \sum \vec{p}_i)$$

hadron yield   space-time   hadron wave-fn.   quark distributions

$\approx$ **"partons close in phasespace can coalesce"**

assumes: **weak binding, no 2-body or 3-body correlations, rare process,**
sudden hadronization

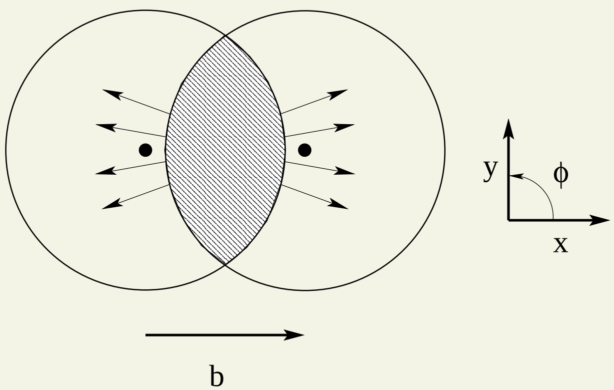
few→many mapping: 8 f_q's (u, d, s, c) give spectra for <u>all</u> hadron species

Two approaches

- **Parameterize f_q:** $\sim$ spirit of HBT source params, or Blastwave
  - this is what have been mostly done so far

successes:

- baryon/meson ratio enhancement at RHIC [Greco et al, Fries et al, Hwa et al, ...]
- elliptic flow (v_2) scaling with quark number [Voloshin, D.M., Lin,...]



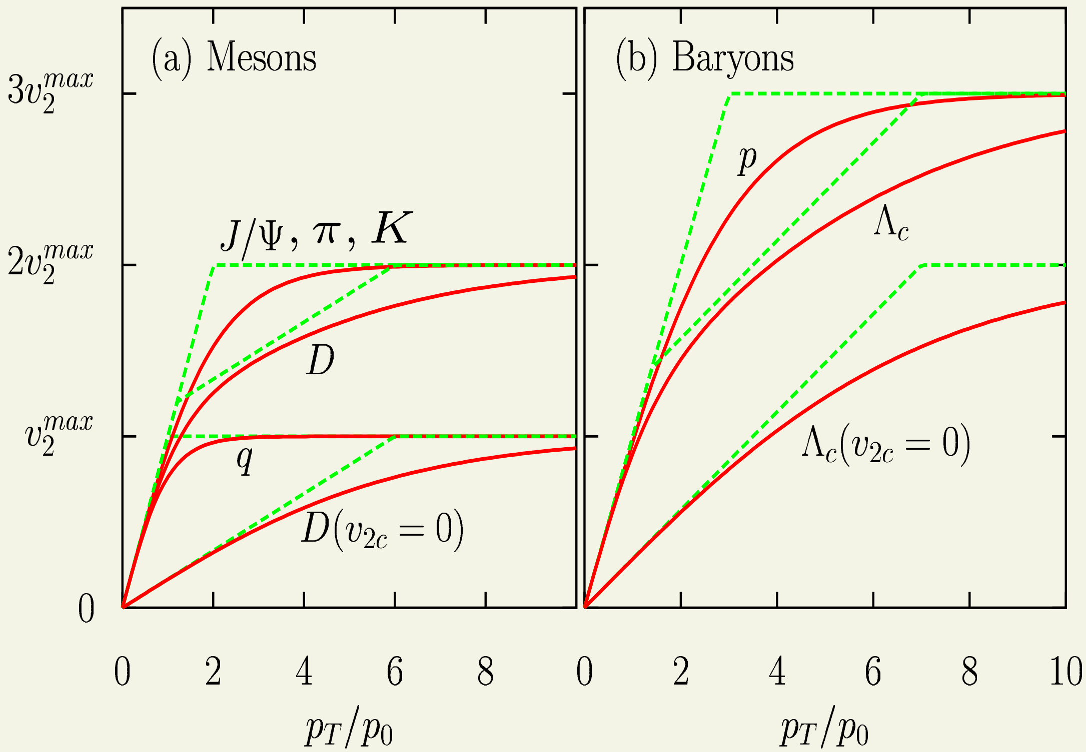
$$v_2^M(p_\perp) \approx v_2^a(z_a p_\perp) + v_2^{\bar{b}}(z_b p_\perp)$$
$$v_2^B(p_\perp) \approx v_2^a(z_a p_\perp) + v_2^b(z_b p_\perp) + v_2^c(z_c p_\perp)$$

$$z_a : z_b[: z_c] \approx m_a : m_b[: m_c]$$

- • **Compute f_q:** the only way to make predictions
  - requires a dynamical model
  - here choose: covariant transport theory

Coalescence expectations for charm

Lin & D.M. ('03)



$m_q : m_s : m_c = 3 : 5 : 15$

$$v_2^D(p_\perp) \approx v_2^c(\frac{5p_\perp}{6}) + v_2^q(\frac{p_\perp}{6})$$
$$v_2^{D_s}(p_\perp) \approx v_2^c(\frac{3p_\perp}{4}) + v_2^q(\frac{p_\perp}{4})$$
$$v_2^{\Lambda_c}(p_\perp) \approx v_2^c(\frac{5p_\perp}{7}) + 2v_2^q(\frac{p_\perp}{7})$$
$$v_2^{J/\Psi}(p_\perp) \approx 2v_2^c(\frac{p_\perp}{2})$$

simplistic (linear & flat); more realistic (based on parton cascade MPC)

- $v_2(p_\perp)$ **rises slower, saturates later for asymmetric systems** D, D_s, Λ_c
  - **heavy quark carries most of hadron momentum (momentum $\propto$ constituent mass)**
- **nonzero** v_2 **for** D, D_s, Λ_c, **even for zero charm** v_2 **(no thermalization)**

Covariant transport theory

[Pang, Zhang, Gyulassy, D.M., Vance, Csizmadia, Pratt, Cheng, ...]

- **Transport equation:** - kinetic theory of a parton gas $\sim$ Boltzmann

$$p^\mu \partial_\mu f_i(x, \vec{p}) = S_i + \sum_{jkl} C^{2\to 2}_{ij\to kl}[f_i, f_j, f_k, f_l] + \ldots C^{2\leftrightarrow 3}\ldots + \ldots$$

f_i : 1-particle phasespace distr., $C^{n\leftrightarrow m}$: contains the $n{\to}m$ matrix elements

S_i: initial conditions

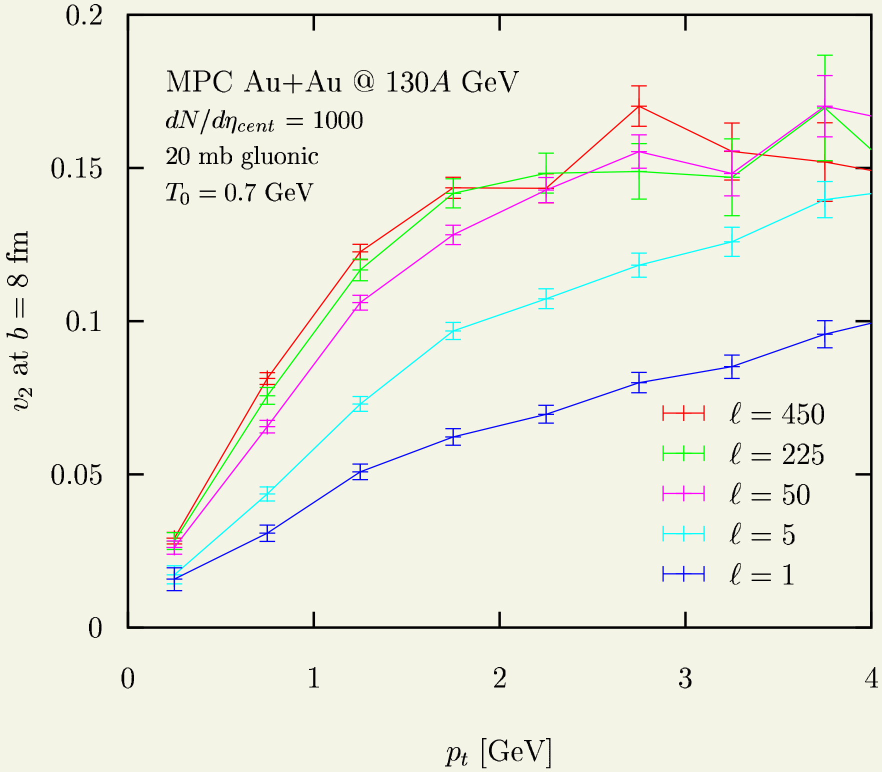
- relevant parameter is the mean free path $\lambda_{MFP} \sim 1/\rho\sigma$
- interpolates between ideal hydro $\lambda = 0$ and free streaming $\lambda = \infty$

- **solvable only numerically:** nonlinear integro-diff eq. in 6D phasespace
  - few Lorentz-covariant algorithms: ZPC, MPC, PSYCHE, GROMIT
  - common element: parton subdivision - essential to maintain covariance

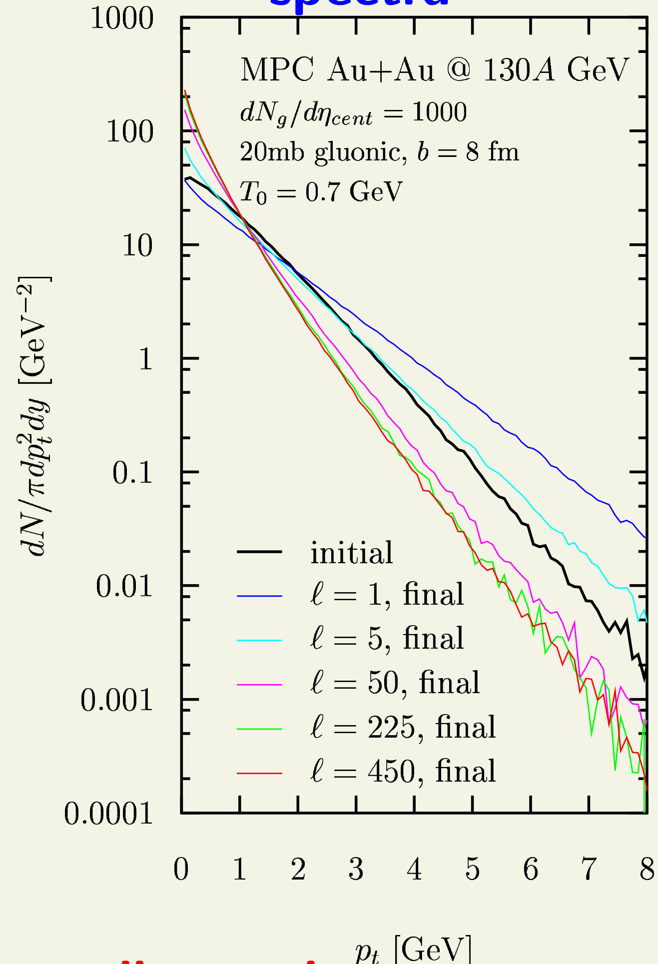
Subdivision matters

rescaling trick $f \to f \cdot \ell$, $\sigma \to \sigma/\ell$ - $\ell \to \infty$ **gives covariant answer**

D.M. & Gyulassy ('02): $v_2(p_T)$



spectra



- **naive cascade ($\ell = 1$): numerical artifacts - smaller v_2, hotter spectra**
- **need subdivision $\ell \sim 200$**

Transport + coalescence

Combine parton transport ($\rightarrow$ charm quark v_2) & coalescence ($\rightarrow$ charm hadron v_2)

- run parton cascade until freezeout (g, d, u, s, c) **(MPC)**
- then do parton coalescence **(Gyulassy-Frankel-Remler formula,** NPA 402 ('83)**)**
- hadronize partons w/o coalescence partner via indep. fragmentation **(JETSET)**

Processes: **leading-order perturbative QCD**

elastic: $gg \rightarrow gg$, $gq \rightarrow gq$, $qq \rightarrow qq$, $q\bar{q} \rightarrow q\bar{q}$, $\bar{q}\bar{q} \rightarrow \bar{q}\bar{q}$
inelastic: $gg \leftrightarrow q\bar{q}$, $q\bar{q} \rightarrow q'\bar{q}'$

Initial conditions: **Au+Au at $\sqrt{s} = 200A$ GeV, $b = 8$ fm:**

$p_T > 2$ **GeV: minijets from leading-order pQCD**
$p_T < 2$ **GeV: soft component via smoothly extrapolated spectra**
normalized to yield $dN_{parton}(b=0)/dy = 2000$

Processes

- **drive calculation with $\sigma_{gg\to gg} = 10$ mb**
- **relative "strength" of channels:**

elastic:

$gg \to gg \equiv 1$ **unit**

$gq \to gq = (4/9) \approx 0.5$, $\quad qq \to qq = (4/9)^2 \approx 0.2$

roughly same for $q = c$

inelastic:

$gg \to q\bar{q} \le 1/27 \approx 0.04$, $\quad q\bar{q} \to gg \le (16/6)^2 \times 1/27 \approx 0.3$

$q\bar{q} \to q'\bar{q}' \le 16/243 \approx 0.07$

$gg \to c\bar{c} \approx 0.07\mu_D^2/M_c^2 \approx 0.02$, $\quad c\bar{c} \to gg \approx 0.5\mu_D^2/M_c^2 \approx 0.2$

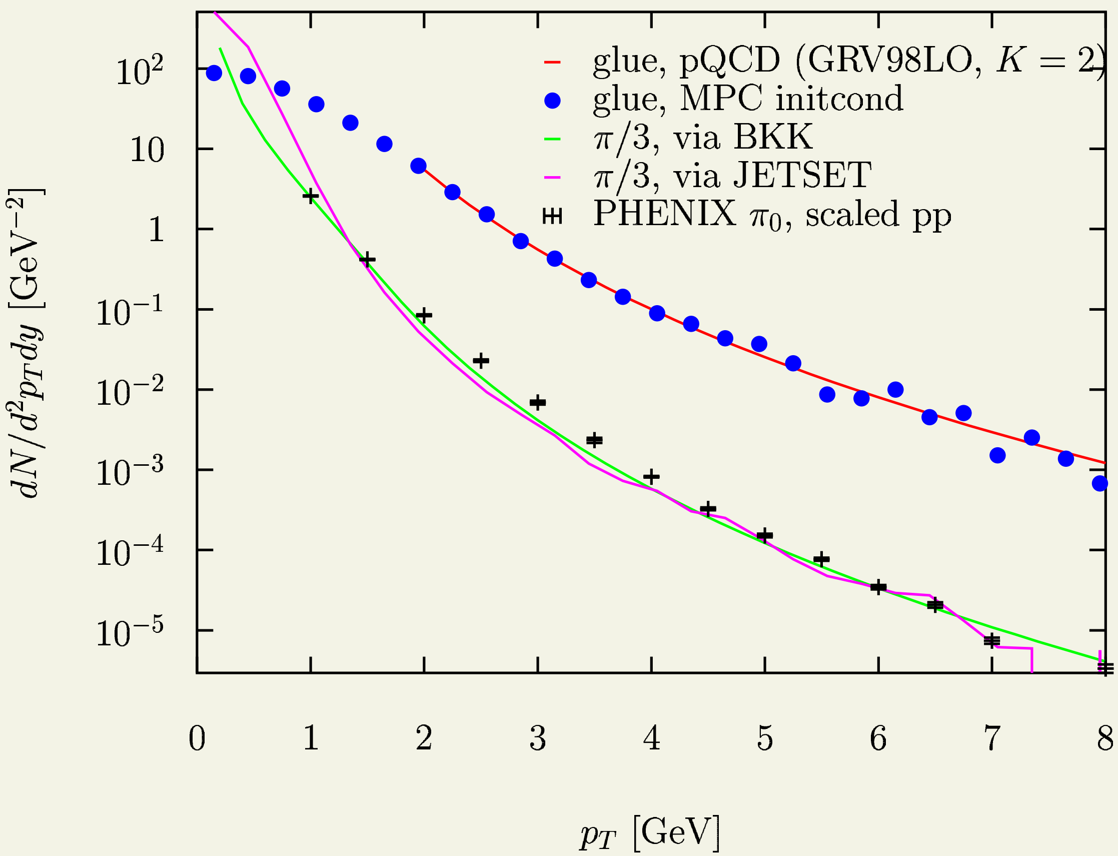
$q\bar{q} \to c\bar{c} \le 0.009\mu_D^2/M_c^2 \approx 0.003$, $\quad c\bar{c} \to q\bar{q} \le 0.009\mu_D^2/M_c^2 \approx 0.003$

Initial conditions

- **Au+Au at RHIC with $b = 8$ fm, i.e., 30% centrality**

- $p_\perp > 2$ **GeV: minijets(dijets) [GRV98LO, $K = 2$]**
  $p_\perp < 2$ **GeV: smoothly joined-on soft component, such that $dN^{parton}/dy(b = 0) = 2000$**

- **binary collision transverse profile, formation time $\tau_0 = 0.1$ fm$/c$**



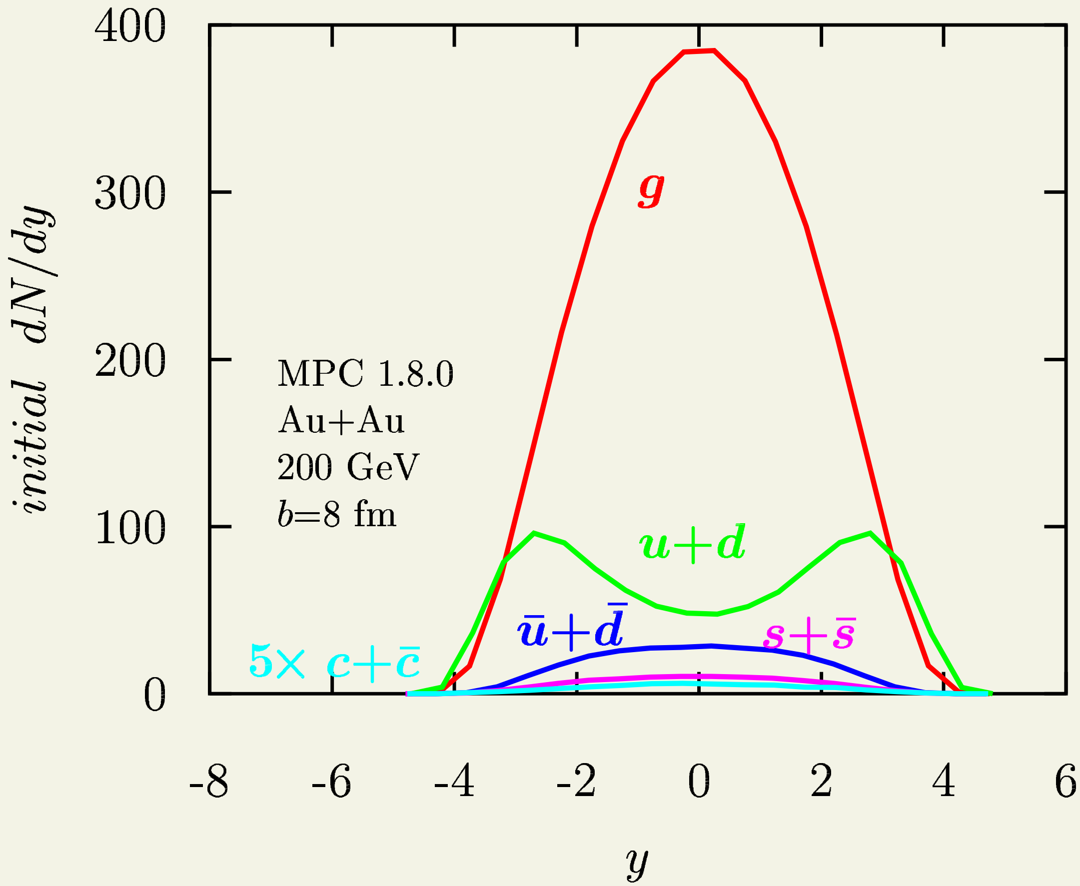
Ready to paddle! - results

- Chemistry - secondary charm production
- Charm quark elliptic flow
- Prediction for D mesons

Charm production

D.M. ('04):

initial **rapidity distributions**

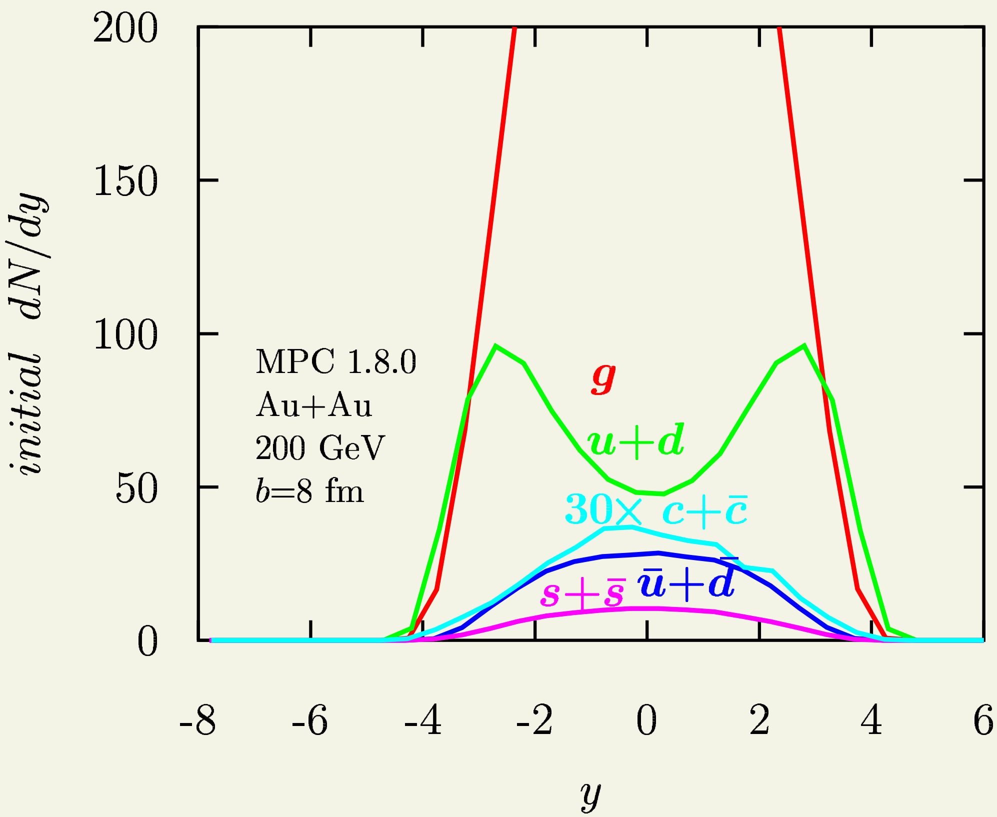


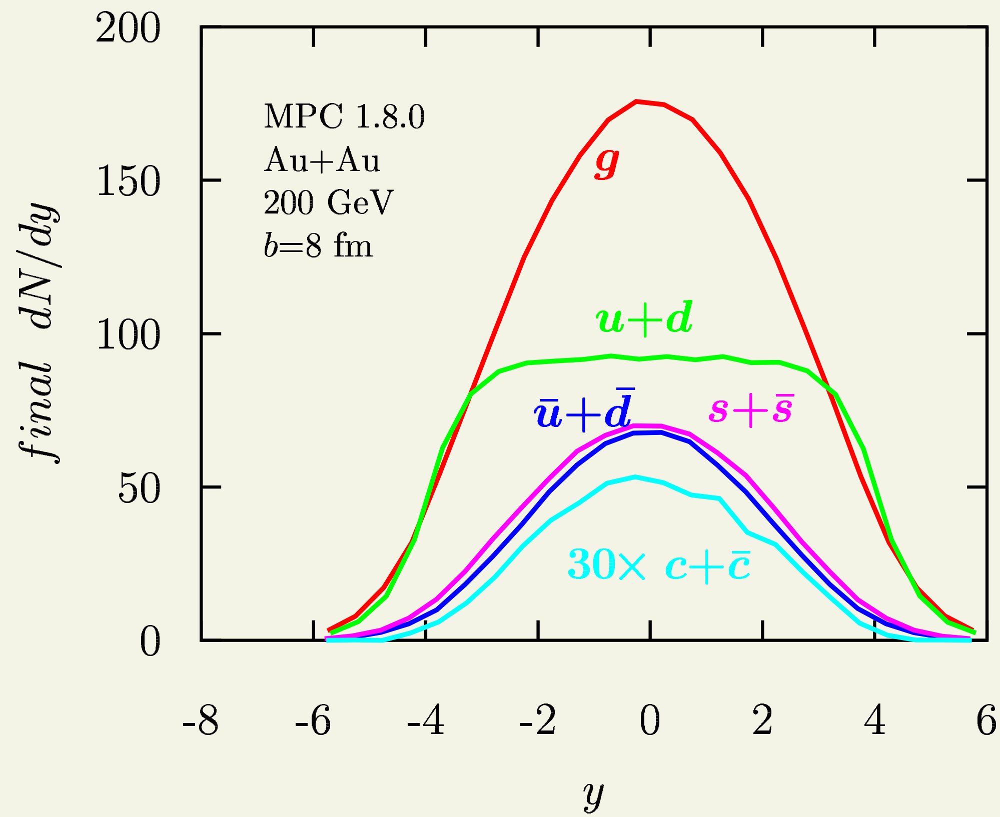
- **gluons dominate at RHIC**

Charm production

D.M. ('04):

initial vs. final rapidity distribution

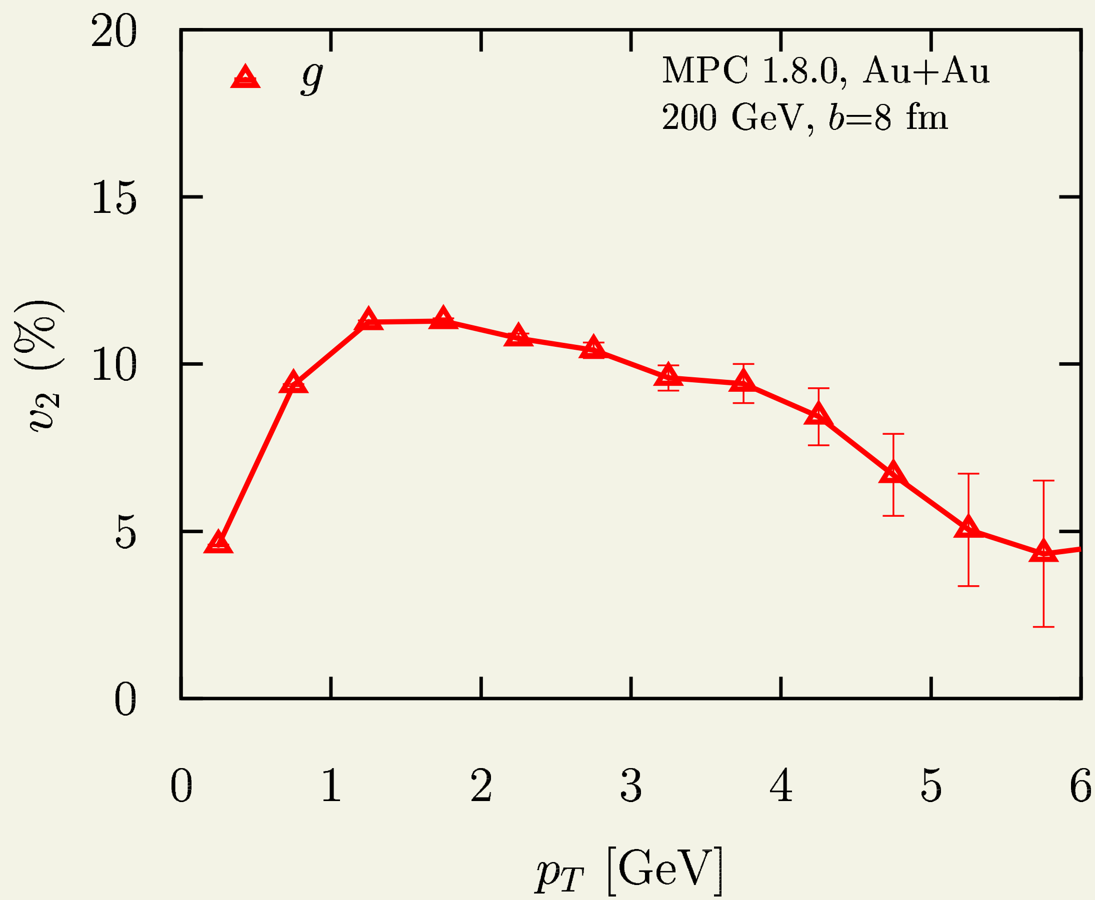




- **roughly half the glue fuse to** $q\bar{q}$
- **extra** $40-50$**% charm yield due to secondary production**
- **strangeness is up by much more, factor 5 or so**

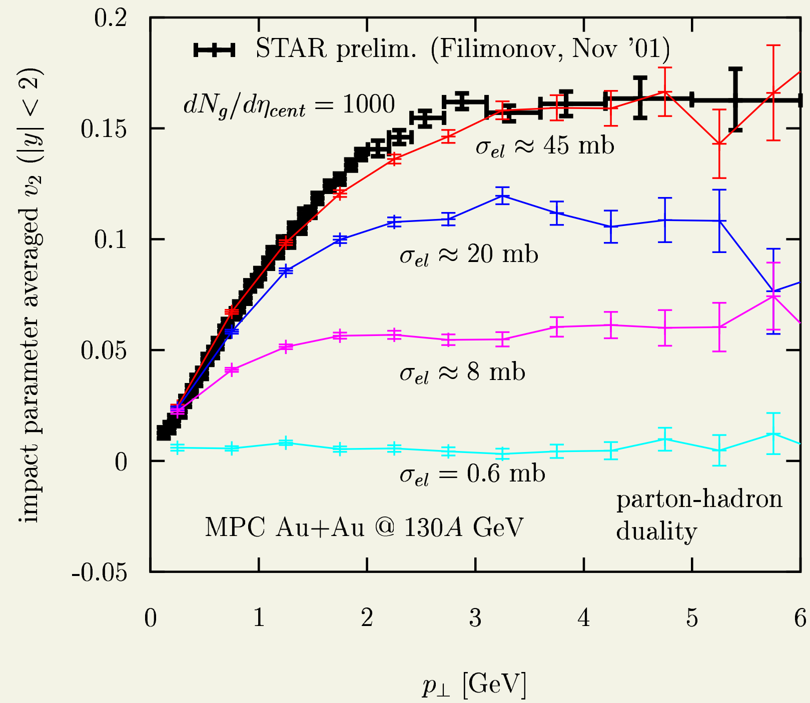
Charm quark elliptic flow

D.M. ('04):



D.M. & Gyulassy ('02):

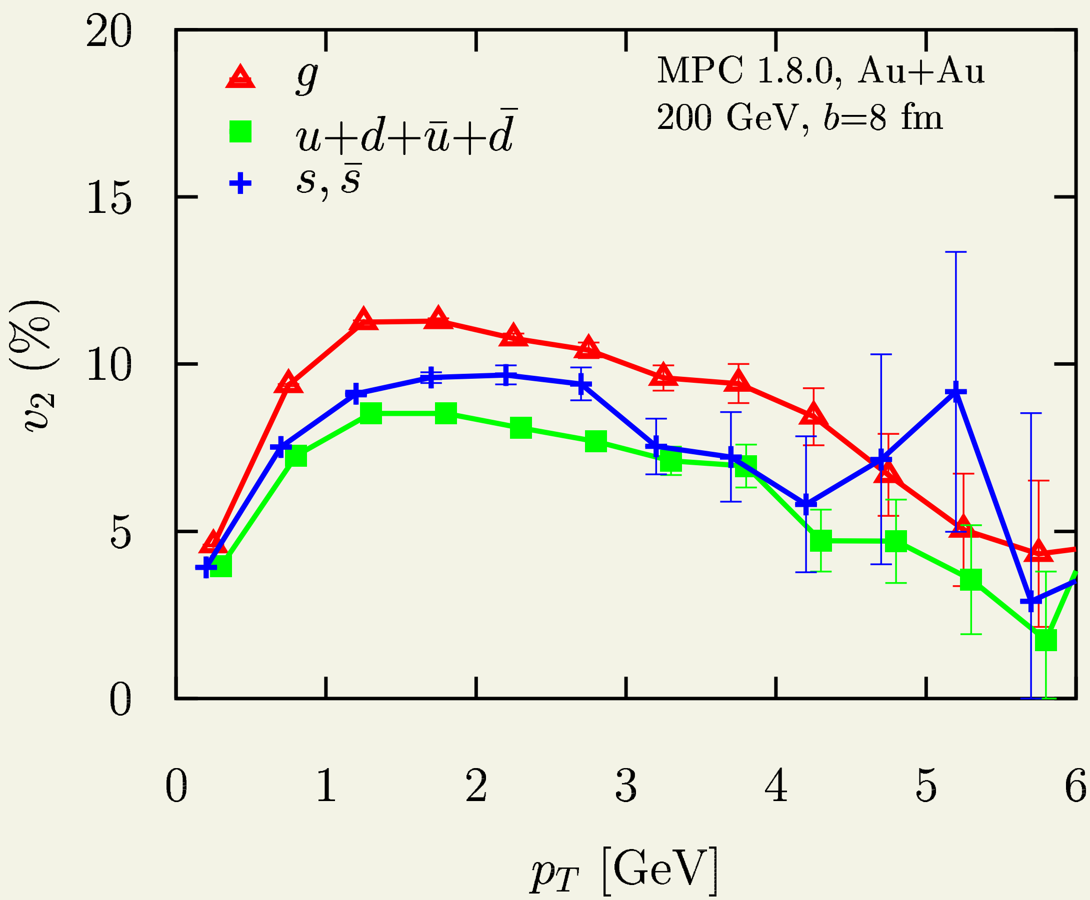
generic earlier results



- **turnover and decrease at high p_T seen for the first time**
  - **difference in initial conditions?**

Charm quark elliptic flow

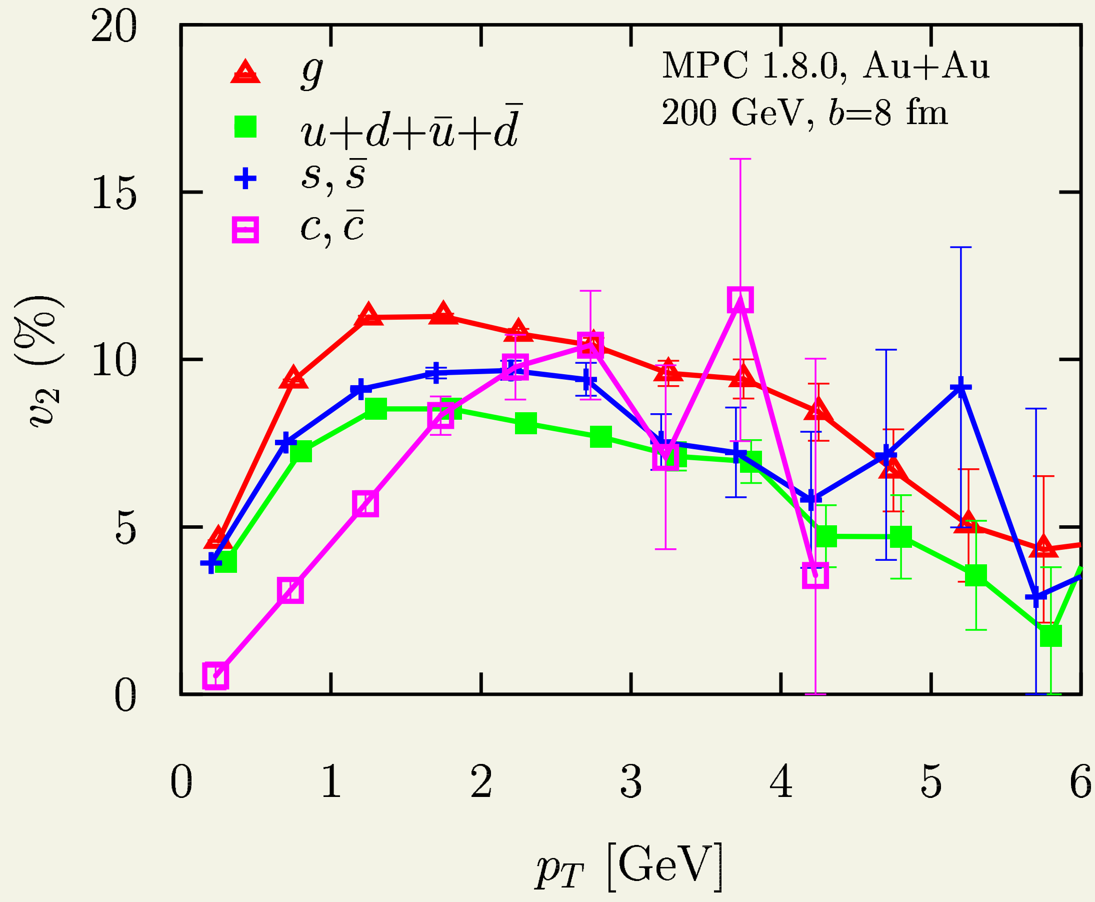
D.M. ('04):



- **light quark v_2's are smaller because mainly driven by $gq \to gq$**
  - **4/9 times weaker than $gg \to gg$**

Charm quark elliptic flow

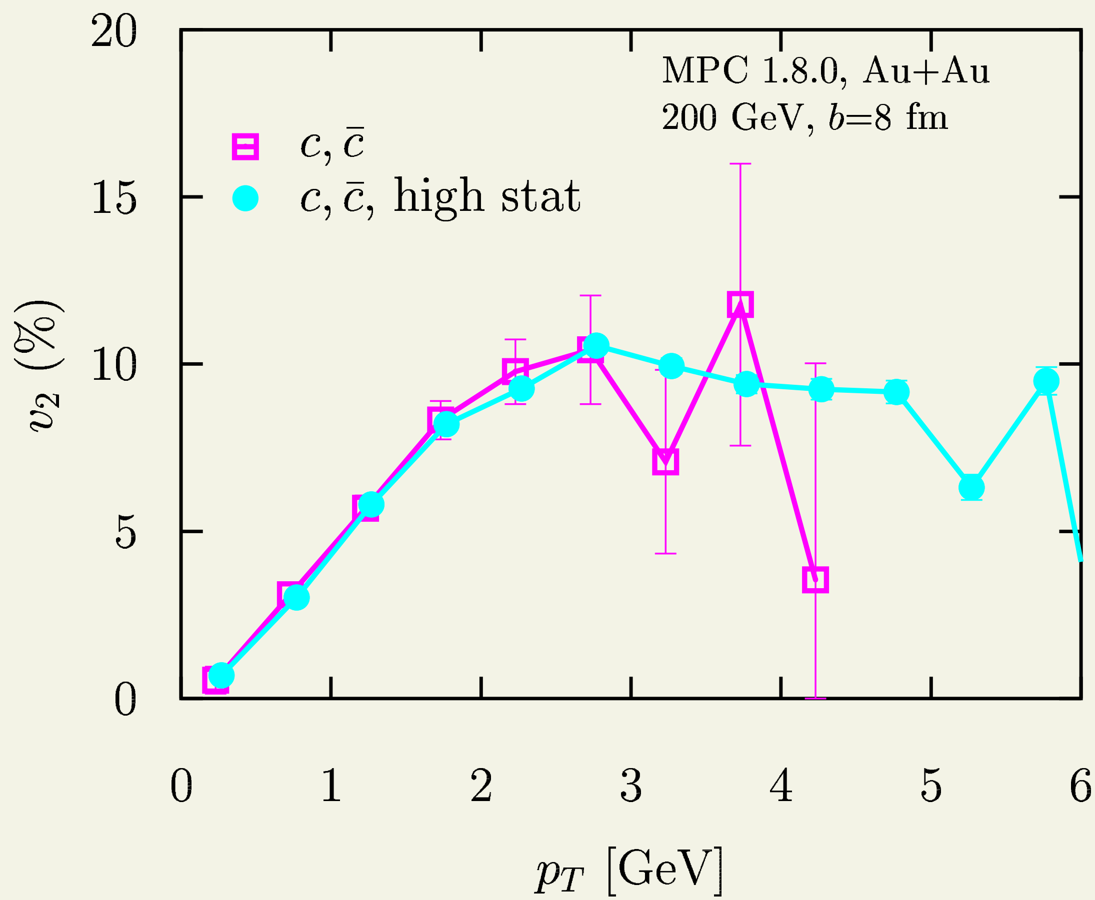
D.M. ('04):



- **charm v_2 rises slower at low pT $\sim$ hydro mass effect**
- **(!) but becomes comparable to light quark v_2 above $p_T \approx 2.5$ GeV (!)**

Charm quark elliptic flow

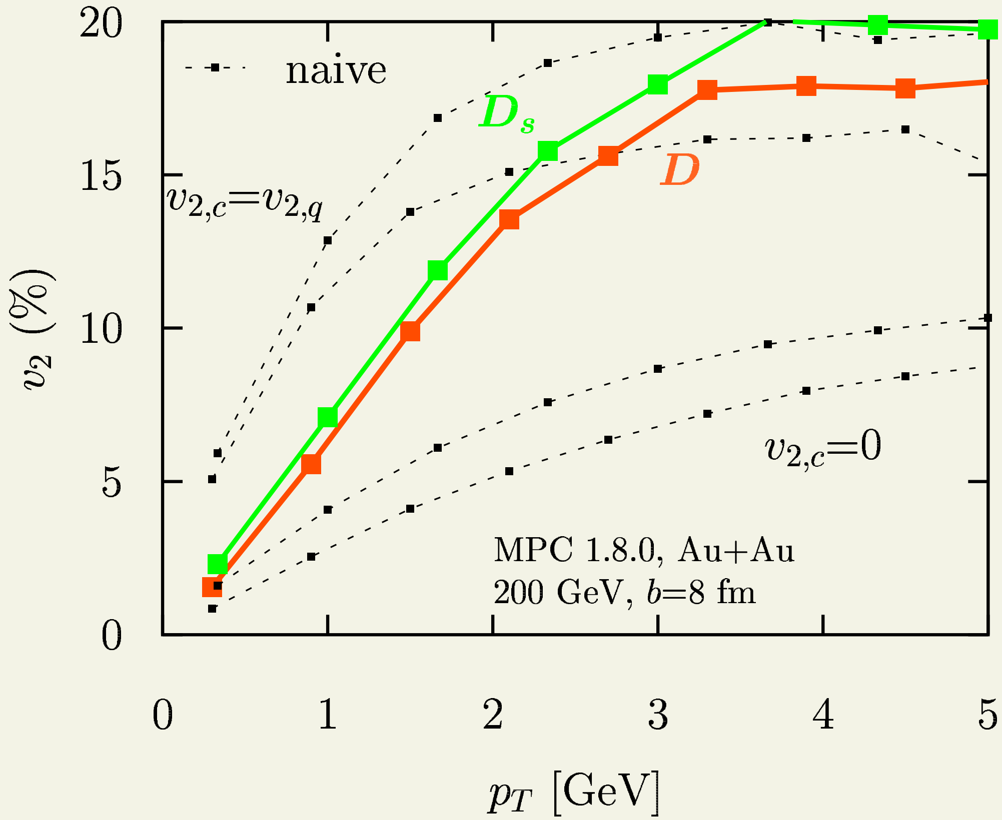
D.M. ('04):



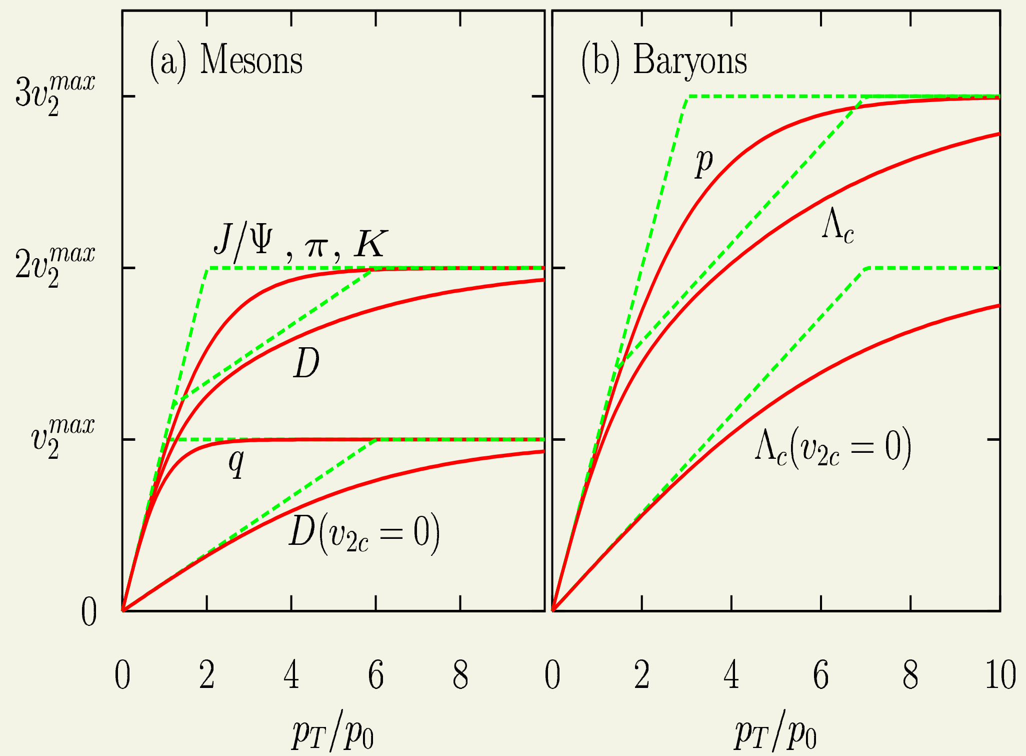
- **enhanced high-pT stat. using "perturbative" charm (no feedback on medium)**
- **consistent with equal-weight sampling**

D meson v_2 from coalescence

D.M. ('04):



[naive: Lin & D.M. ('03)]



- for D, later saturation ($p_T \sim 3$ GeV) than for light mesons ($p_T \sim 2$ GeV)
  - very different from scenario w/o charm flow
- however, eventually similar v_2 magnitude - twice that of quarks

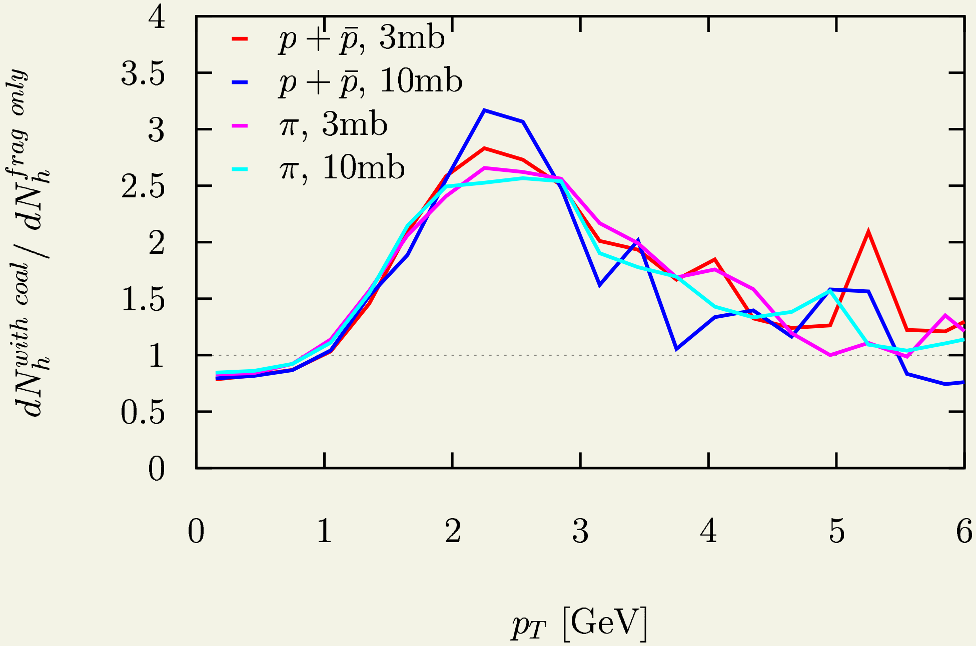
Not all stunts work - open issues

- Fragmentation contribution

Puzzles for light quarks

B/M **unchanged, same enhancement for** π **&** p

D.M., nucl-th/0403035 ('04):

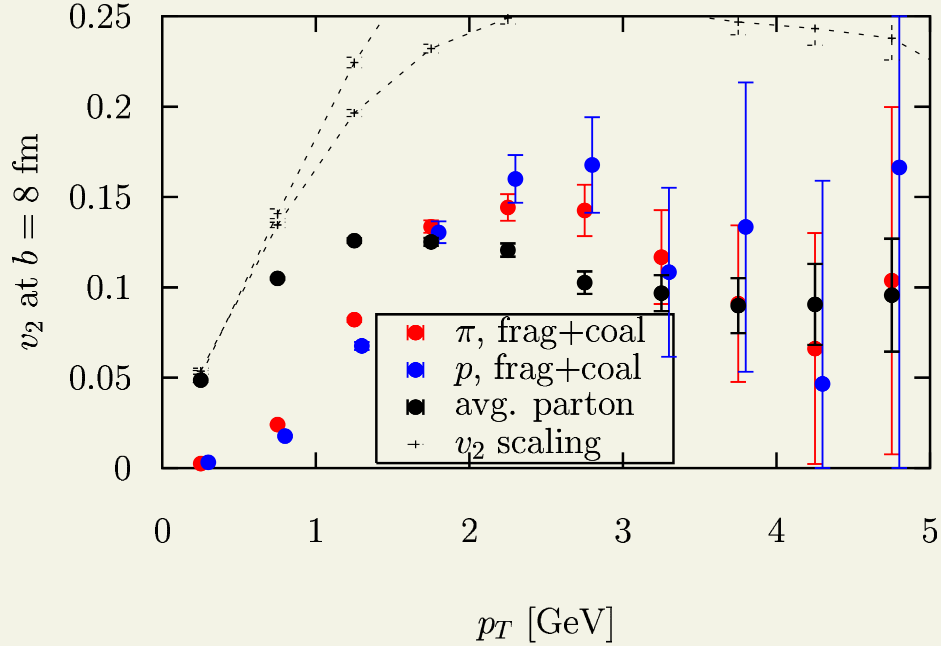


- $2-3\times$ **enhancement over fragmentation for** $1.5 < p_T < 4$ **GeV**

  but same for protons as pions $\Rightarrow p/\pi$ **stays low**

  $\rightarrow$ **baryons have more constituents** $\Rightarrow$ **are more "fragile"**

frag. contribution suppresses v_2



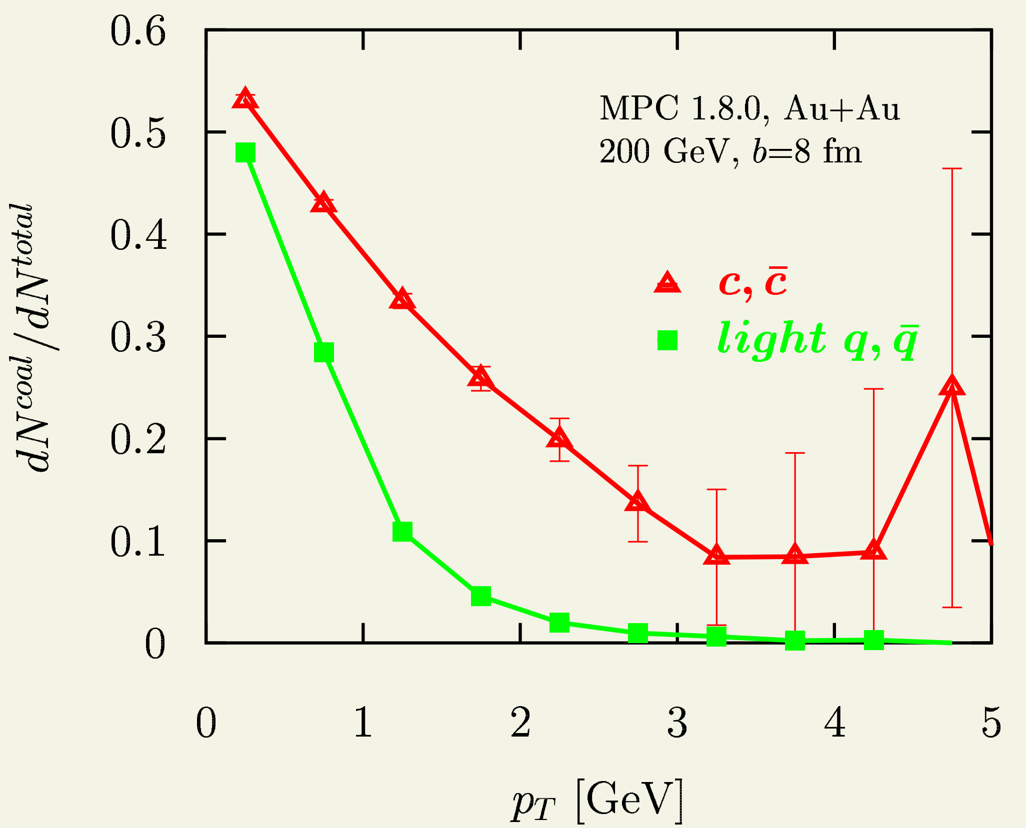
- **still 25-30% of partons fragment**

  $\rightarrow$ **reduced** v_2 **relative to pure coalescence**
- **baryon-meson splitting almost gone**

Need higher freezeout density (smaller σ**) - but how to keep then** v_2 **large??**

Same puzzle for charm

D.M. ('04): **key issue - fraction of partons that coalesce**



- **80-90% of charm quarks above $p_T > 3$ GeV fragment**
  - **fragment roughly preserving the quark momentum ($z \approx 1$)**
  - **for these hadrons elliptic flow is about the same as for charm quarks - no doubling**

$\Rightarrow$ coalescence v_2 contribution will be reduced above $p_T \sim 3 - 4$ GeV

Trip is over - remember the best parts

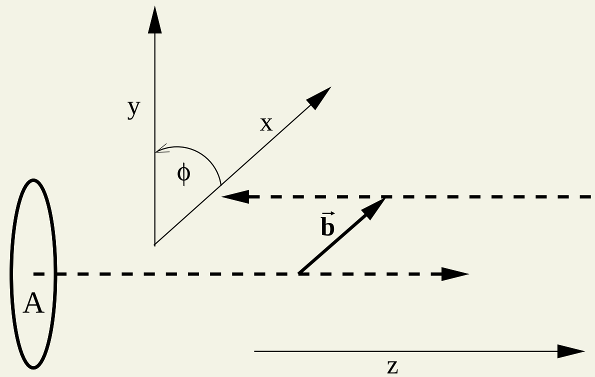
- 40-50% extra secondary charm from transport
- significant charm v_2 at high p_T
- D meson $v_2(p_T)$ may saturate at similar values as that of of light mesons, but at 50% higher p_T

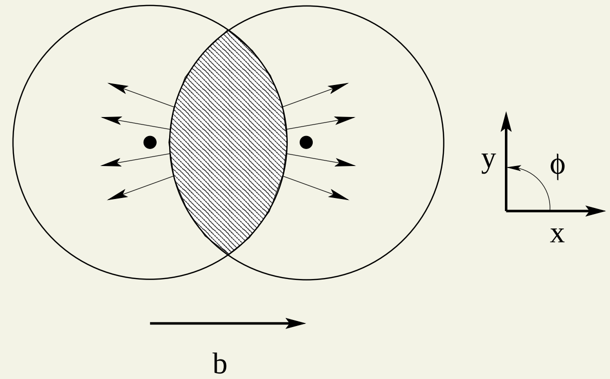
- already planning next one:
  - "best chance" scenario for coalescence
  - various other improvements

Additional ammunition

Azimuthal anisotropy (v_2, ...)

- **momentum-space anisotropy of particle production in A+A collisions**





$$\frac{dN}{d\phi dX} \equiv \frac{1}{2\pi}\frac{dN}{dX}[1 + 2\sum_{n=1} v_n(X)\cos(n\phi)] \quad \rightarrow$$

directed flow: $v_1(X) \equiv \langle \cos\phi \rangle_X$

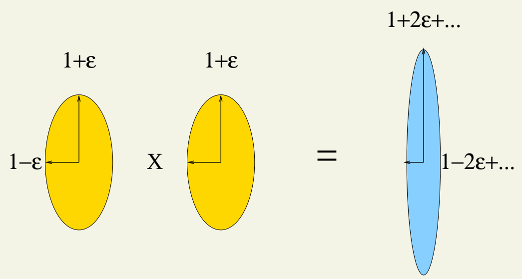
elliptic flow: $v_2(X) \equiv \langle \cos 2\phi \rangle_X$

X : **event and particle selection, e.g., centrality, transverse momentum**

- v_2 **is a measure of the opacity, or number of scatterings**
  - **w/o scatterings,** $v_2 = 0$

Anisotropy amplification [D.M. & Voloshin, PRL91 ('03)]

narrow wave fn. (in p): $q\bar{q} \Rightarrow\!\!\longrightarrow M$ $(2 \times \frac{\vec{p}}{2} \to \vec{p})$, $3q \Rrightarrow\!\!\longrightarrow B$ $(3 \times \frac{\vec{p}}{3} \to \vec{p})$



$$\frac{dN_M}{d\phi} \propto \left(\frac{dN_q}{d\phi}\right)^2, \quad \frac{dN_B}{d\phi} \propto \left(\frac{dN_q}{d\phi}\right)^3$$

$$\begin{aligned} v_2^M(p_\perp) &\approx v_2^a\left(\frac{p_\perp}{2}\right) + v_2^{\bar{b}}\left(\frac{p_\perp}{2}\right) \\ v_2^B(p_\perp) &\approx v_2^a\left(\frac{p_\perp}{3}\right) + v_2^b\left(\frac{p_\perp}{3}\right) + v_2^c\left(\frac{p_\perp}{3}\right) \end{aligned}$$

$\Rightarrow$ **hadron v_2 amplified at high $p_\perp$**

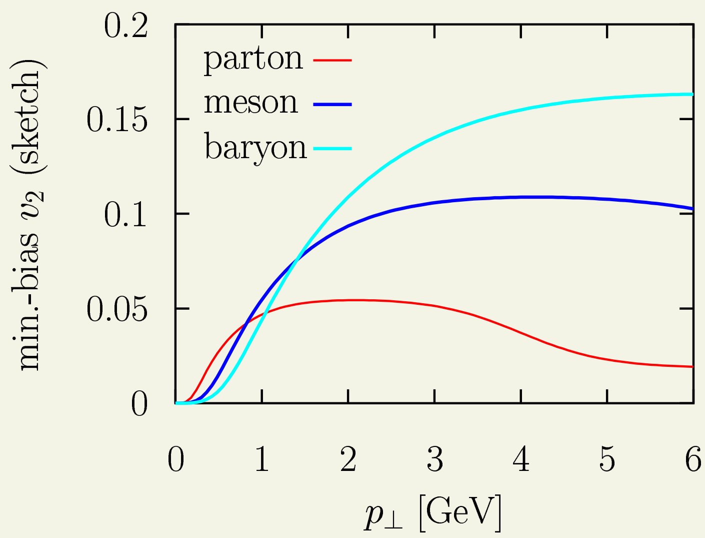
if all quarks have same v_2:

$3\times$ **for baryons** $\quad\Big\}$ **50% larger v_2 for baryons**

$2\times$ **for mesons**

" $v_2^h(p_\perp) \approx n \times v_2^q(p_\perp/n)$ "

$\rightarrow 5\times$ **for pentaquark,** $6\times$ **for deuteron**



- **if v_2 depends on quark flavor, further differentiation by flavor content**

Determination of parton v_2

Via measuring $v_2(p_\perp)$ for various hadrons, one can extract quark flows AND test consistency of coalescence model

Consider, e.g., $v_2^q \neq v_2^s$ (light vs. strange)

$$\begin{aligned} v_2^\pi(p_\perp) &\approx 2v_2^q(\frac{p_\perp}{2}) \\ v_2^K(p_\perp) &\approx v_2^q(\frac{p_\perp}{2}) + v_2^s(\frac{p_\perp}{2}) \\ v_2^\phi(p_\perp) &\approx 2v_2^s(\frac{p_\perp}{2}) \end{aligned}$$

$$\begin{aligned} v_2^p(p_\perp) &\approx 3v_2^q(\frac{p_\perp}{3}) \\ v_2^{\Lambda,\Sigma}(p_\perp) &\approx 2v_2^q(\frac{p_\perp}{3}) + v_2^s(\frac{p_\perp}{3}) \\ v_2^\Xi(p_\perp) &\approx v_2^q(\frac{p_\perp}{3}) + 2v_2^s(\frac{p_\perp}{3}) \\ v_2^\Omega(p_\perp) &\approx 3v_2^s(\frac{p_\perp}{3}) \end{aligned}$$

2 unknowns, 7 equations $\Rightarrow$ e.g.,

$$\begin{aligned} v_2^q(p_\perp) &= v_2^\pi(2p_\perp)/2 = v_2^p(3p_\perp)/3 = v_2^\Lambda(3p_\perp) - 2v_2^p(3p_\perp)/3 \\ v_2^s(p_\perp) &= v_2^\phi(2p_\perp)/2 = v_2^\Omega(3p_\perp)/3 = v_2^K(2p_\perp) - v_2^\pi(2p_\perp)/2 \end{aligned}$$

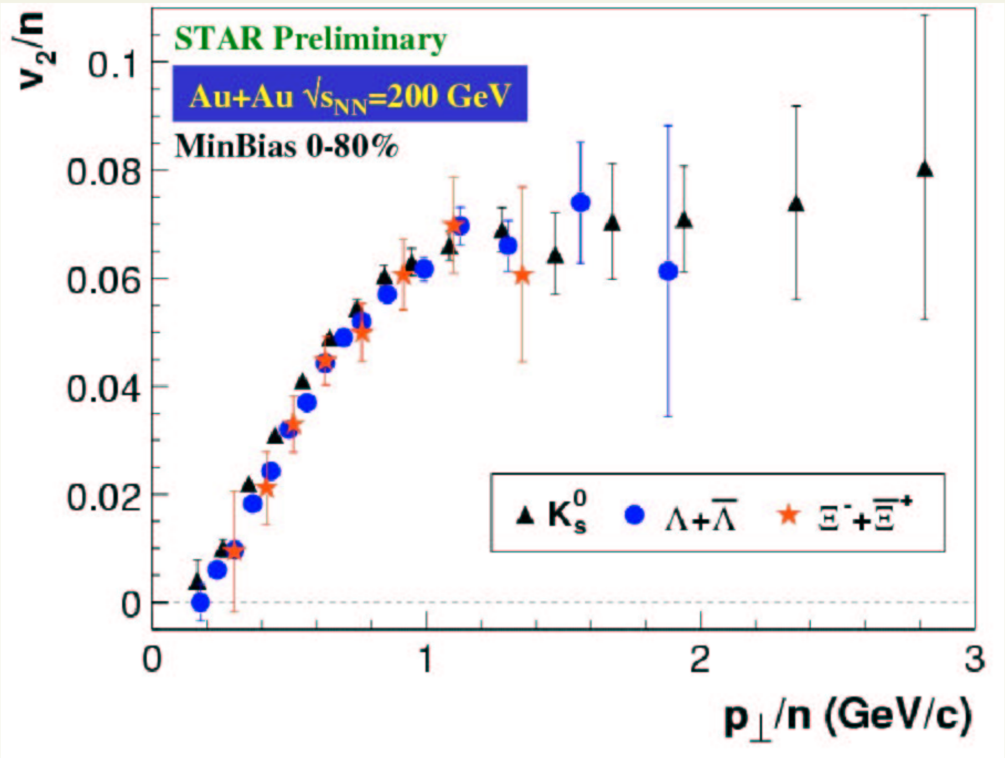
Simplest $v_2^q = v_2^s$ case: $v_2^{hadron}(np_\perp)/n$ universal fn. ($n = 2, 3$ for meson,baryon)

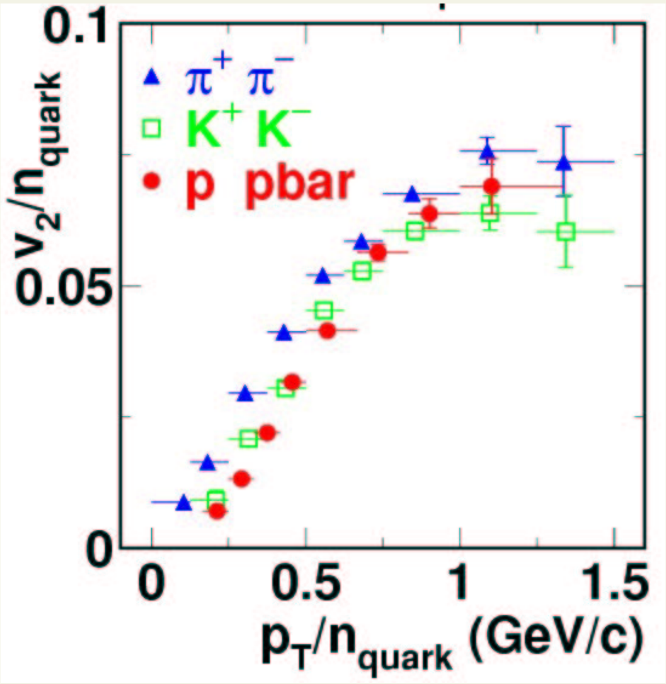
Experimental test of v_2 scaling

Simplest $v_2^q = v_2^s$ case: $v_2^{hadron}(np_\perp)/n$ universal fn. ($n = 2, 3$ for meson,baryon)

Castillo [STAR], HIC03: K_S^0, Λ, Ξ flow

PHENIX, PRL91 ('03): π, K, p





- **coalescence predictions confirmed for $\pi, K, K_0, p, \Lambda, \Xi$ - also for Ω, ϕ (poor statistics)**

  - pions are little off the curve, likely due to resonance decays [Greco & Ko]

- **surprisingly, RHIC data indicate $v_2^q \approx v_2^s$ $\longrightarrow$ next open question: charm v_2?**

Wave function

Hadron rest frame: - **consider** $|\vec{p}_1, \vec{p}_2\rangle = |\vec{q}/2, -\vec{q}/2\rangle$ (meson)

- **with** $\langle|\vec{q}|\rangle \sim \Lambda_{QCD}$

Fast, weakly bound hadron w/ momentum $\vec{n}p$: - **a kinematic estimate**

$$p'_i \equiv \vec{p}_i{}'\vec{n} = \frac{E'_i}{m_H}p + \vec{q}_i \cdot \vec{n}\frac{\sqrt{p^2 + m_H^2}}{m_H} \approx \frac{m_i}{m_H}p + \frac{\vec{q}_i\vec{n}p}{m_H}$$

$\rightarrow$ **with momentum fractions** $z_i \equiv p'_i/p$

$$z_i = \bar{z}_i + \delta z_i \approx \frac{m_i}{m_H} + \frac{\vec{q}_i\vec{n}}{m_H}$$

average spread

Two effects: • **lighter quark carries smaller fraction of momentum**

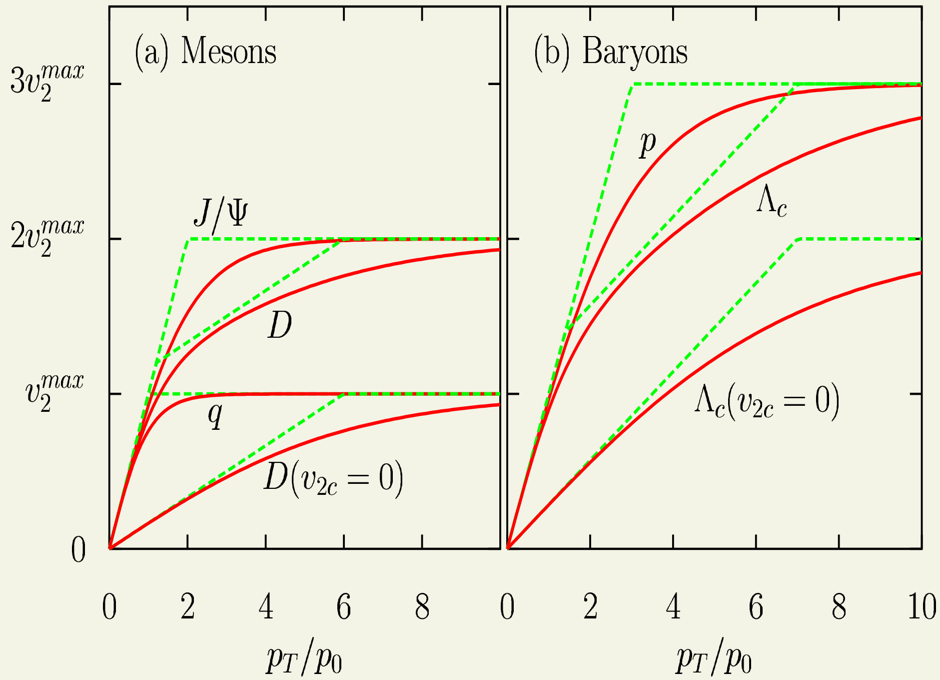
- **because similar velocities (not momenta) in moving frame**

• **wave function is narrower in δz for heavy quarks**

Elliptic flow for charm

generic expectations:

Lin & D.M. ('03)

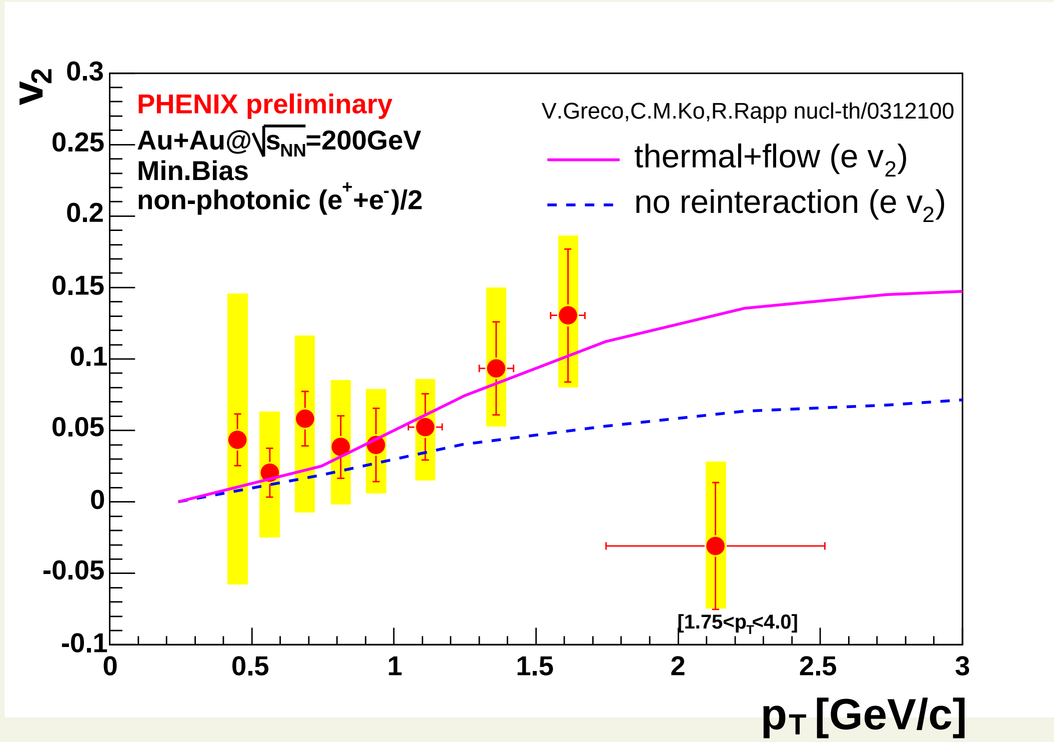


simplistic (linear & flat); more realistic (based on parton cascade MPC)

- $v_2(p_\perp)$ **rises slower, saturates later for asymmetric systems (D, D_s, Λ_c)**
  - **heavy quark carries most of the hadron momentum**
    (momentum proportional to constituent mass)
- **nonzero v_2 for D, D_s, Λ_c**
  even for zero charm v_2 (no thermalization)

decay electron v_2: $D \to K\,\nu\,e$, $D \to K^*\,\nu\,e$

Kaneta, nucl-ex/0404014; Greco & Ko, nucl-th/0312100:



e's from hadron decays and γ-conversion subtracted
$\equiv$ "non-photonic"

- **need better data to discriminate between scenarios with or without charm flow**

Coal. formalism for diffuse freezeout

Gyulassy, Frankel & Remler (GFR): [NPA 402, 596 ('83)]

- for each constituent pair/triplet, propagate particles to the latest of freezeout times and evaluate weight $W(\Delta x, \Delta p)$ there
- reason: any interaction would break up a weak bound state
- note, relative distance changes(!), e.g., if $t_2 > t_1$:

$$weight = W_M\left(\vec{x}_1(t_1) + (t_2 - t_1)\vec{v}_1 - \vec{x}_2(t_2), \vec{p}_1 - \vec{p}_2\right)$$

Naturally incorporates:

- "diffuse" 4D freezeout
- space-time and space-momentum correlations

Model ingredients

Processes: **ideally:** f - $2 \longrightarrow 2$ parton scatterings, showers ($1 \longrightarrow 2$, $1 \longrightarrow 3$), parton fusion ($2 \longrightarrow 1$, $3 \longrightarrow 1$), inelastic $n \longrightarrow m$, parton recombination t o hadrons, hadron breakup, ... etc.

here:
- **only $2 \longrightarrow 2$ (with $g, u, d, s, c, \bar{u}, \bar{d}, \bar{s}, \bar{c}$)**
- **no parton showers until freezeout**
- **coalescence rate computed over freezeout 4D volume via GFR**
- **partons with no coalescence partner fragment as in vacuum**

Coalescence part:
- **assume easy color neutralization - no color penalty factors**
- **but consider spin & flavor**
- **channels:** π, K, η, η'; ρ, K^*, ω, Φ, D, D^*, J/ψ, η_c
  p, n, Σ, Λ, Ξ; Δ, Ω
- **"spherical box" Wigner functions:** $W_M = \Theta(p_M - |\Delta p|)\Theta(x_M - |\Delta x|)$
  $$W_B = \prod_{k \neq i,j} \Theta(p_B - |\Delta p_{ij}|)\Theta(x_B - |\Delta x_{ij}|)$$
  $x_M = x_B = 1$ fm
- **convert g to a random q (extreme case of $q - \bar{q}$ splitting)**
- **when several coalescence final states, unbiased random choice of one**

Codes:
- **MPC 1.8.0 for parton transport**
- **JETSET 7.4.10 for fragmentation & decays ("out of box")**

Various issues regarding coalescence

- coalescence dynamics

- extend to low-p_T regime: simple formula not applicable (unitarity)
  - does coal w/ unitarity still reduce to statistical models/hydrodynamics?

- hadron correlations: coal dilutes jetlike correlations??
  - not necessarily: constituent correlations possible $\Rightarrow$ map uniquely to hadron correlations
  - a correlation source: parton showers [Hwa & Yang, nucl-th/0401001]

- binding energy: only relevant for Goldstones π & K
  - likely a small correction because resonance channels $K^* \to K$; $\rho, \omega, \Delta, \ldots \to \pi$ dominant

- constituents vs partons: so far - two, mostly independent components
  - Hwa & Yang: hadrons made of constituents, which in turn consist of partons limited to momentum space $\longrightarrow$ need extension to 6D phasespace

- entropy: $2 \to 1$, $3 \to 1$??
  - globally, problem is cured by resonance decays and larger s/n for massive particles