

# Deep Space Transportation System Using the Sun-Earth L2 Point

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## Abstract

Recently, various kinds of planetary explorations have become more feasible, taking the advantage of low thrust propulsion means such as ion engines that have come into practical use. The field of space activity has now been expanded even to the rim of the outer solar system. In this context, the Japan Aerospace Exploration Agency (JAXA) has started investigating a Deep Space Port built at the L2 Lagrange point in the Sun-Earth system. For the purpose of making the deep space port practically useful, there is a need to establish a method to making spaceship depart and return from/to the port. This paper first discusses the escape maneuvers originating from the L2 point under the restricted three-body problem. Impulsive maneuvers from the L2 point are extensively studied here, and using the results, optimal low-thrust escape strategies are synthesized. Furthermore, this paper proposes the optimal escape and acceleration maneuvers schemes using Electric Delta-V Earth Gravity Assist (EDVEGA) technique

## 1. Introduction

Since interplanetary voyage generally needs delta-V of over 10 km/s, interplanetary spacecraft uses high Isp propulsion, such as electric propulsion. On the contrary, to escape Earth gravity field, the spacecraft needs the propulsion of high thrust level, such as chemical propulsion. This means that deep space exploration needs a couple of propulsions which differ in kind. And so construction of bases of interplanetary flights and change of the spacecrafts at this port are effective. rt.

The L2(L1) point is considered as the leading candidate site for deep space port, for this point has some advantages that are applicable for the deep space port. L2(L1) point is located at the potential hill in the boundary of the gravity influence. And although L2 point is unstable point, this point is stable enough with keeping the position with respect to the Earth and the Sun with the small delta-V. In this context, the Japan Aerospace Exploration Agency (JAXA) has started investigating a Deep Space Port built at the L2 Lagrange point in the Sun-Earth system. For the purpose of making the deep space port practically useful, there is a need to establish a method to making spaceship depart and return from/to the port.

## 2. Dynamics

In this study, we consider the restricted three body system (see Fig. 1). The Sun and the Earth revolve around their center of mass in circular orbits under the influence of their mutual gravitational attraction and a spacecraft moves in the plane defined by them. The equation of motion of the spacecraft is written as follows,

$$\begin{aligned}\ddot{x} - 2\dot{y} &= \Omega_x \\ \ddot{y} + 2\dot{x} &= \Omega_y\end{aligned}\quad (1)$$

where

$$\Omega = \Omega(x, y) = \frac{x^2 + y^2}{2} + \frac{1 - \mu}{r_1} + \frac{\mu}{r_2} + \frac{\mu(1 - \mu)}{2}\quad (2)$$

and the subscripts of  $\Omega$  in Eq.1, denote the partial derivatives with respect to the coordinates of the spacecraft  $(x, y)$ . Also,

$$\begin{aligned}\mu &= \frac{m_E}{m_S + m_E} \\ r_1 &= \sqrt{(x + y)^2 + y^2}, \quad r_2 = \sqrt{(x - 1 + \mu)^2 + y^2}\end{aligned}\quad (3)$$

where  $m_E$  is the mass of the Earth and  $m_S$  is the mass of the Sun. Eq.1 are written in a rotating reference frame with the following conventions,

- The sum of the masses of the Sun and the Earth is  $m_S + m_E = 1$ .
- The distance between the Sun and the Earth is normalized to 1.
- The angular velocity of the Earth around the origin is normalized to 1.

The system has a first integral of motion, called Jacobi integral, which is given by

$$\frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - \Omega(x, y, z) = -C = const\quad (4)$$

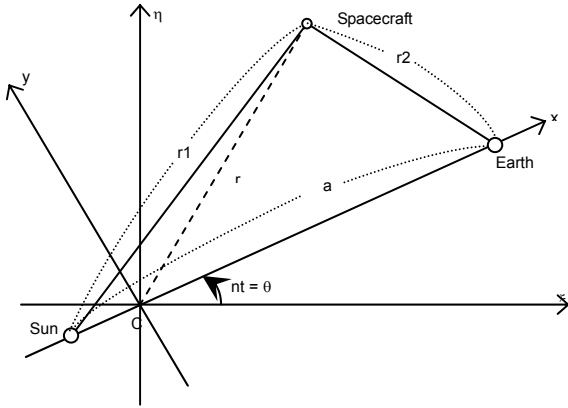


Fig. 1. The restricted three body system

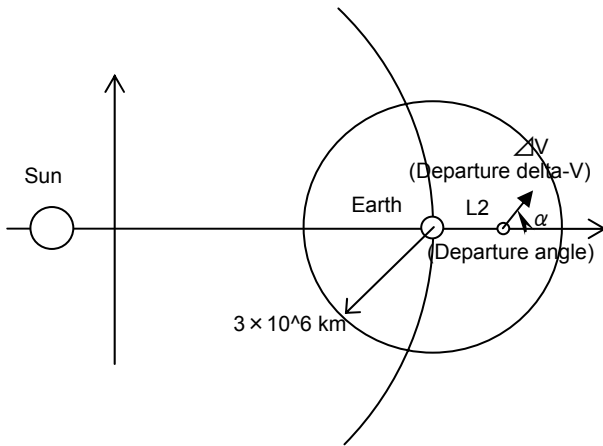


Fig. 2. Setting of the departure velocity and angle

### 3. Escape trajectory using impulsive transfer

First of all, impulsive escape paths from the equilibrium point where the quay anchored is are examined. A single impulse trajectory is not necessarily an optimal in terms of delta-V budget. This examination is aimed at examining the features of those trajectories.

#### 3.1 Single impulse trajectory

Single impulse trajectories from the L2 point are examined. The departure velocity and angle are taken for the parameters (see Fig. 2). This problem is calculated with the following assumptions,

- The Earth's sphere of influence is assumed 3 million km. (In this study, we define this distance as "infinity", and this boundary sphere is used evaluation point of potential energy.)
- The maximum flight time is one year.

#### 3.2 Results

Fig. 3 shows inertial velocity at infinity and Fig. 4 shows the direction of inertial velocity at infinity. Fig. 5 and Fig. 6 show examples of trajectory of region I and II

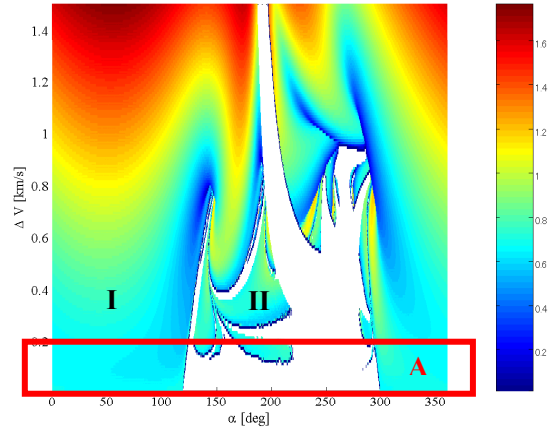


Fig. 3. Single impulse trajectory : Inertial velocity at infinity (Vinf) [km/s]

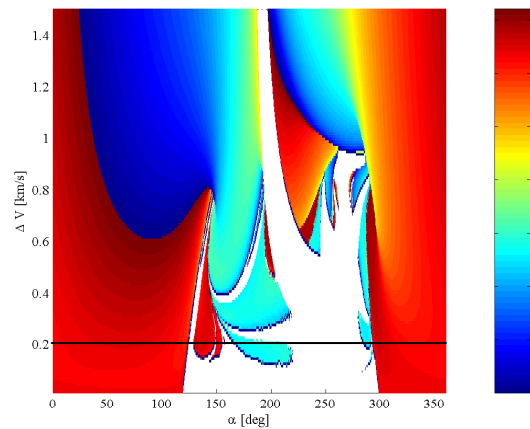


Fig. 4. Single impulse trajectory : Direction of inertial velocity at infinity [deg]

in Fig. 3. The region A in Fig. 3 is only one color. This means that, in the region of low departure delta-V, the escape direction is limited to one direction. The V-infinity of 0.6km/s can be obtained from departure delta-V of a few m/s. In other words, if the escape direction is limited to one direction, the spacecraft can escape from L2 point by infinitely small delta-V. And this direction is +x axis. On the contrary, the escape trajectory in the tangential direction of Earth's orbit needs high departure delta-v. Therefore, these trajectories are inefficient.

#### 3.3 Double impulse trajectory

A single impulse trajectory is not necessarily an optimal in terms of delta-V budget. And so, here is considered the double impulsive maneuvers. The spacecraft is given the first delta-V (departure delta-V) at the L2 point and the spacecraft is given the second delta-V at the perigee where the spacecraft can obtain the energy efficiently. The departure velocity and angle are taken for the parameters (see Fig. 7). This problem is calculated with the following assumptions,

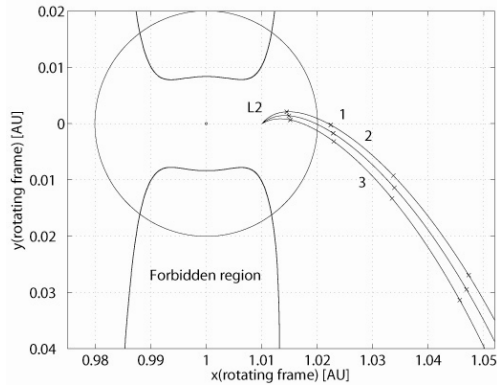


Fig. 5. Single impulse trajectory: region I (Fig. 3)

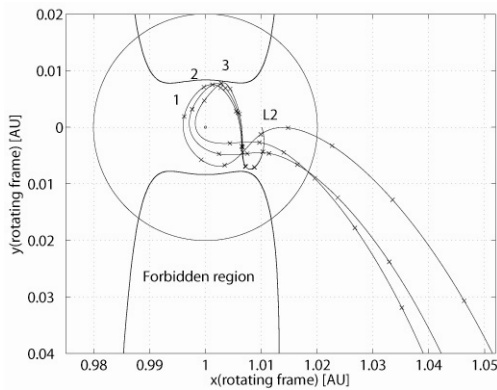


Fig. 6. Single impulse trajectory: region II (Fig. 3)

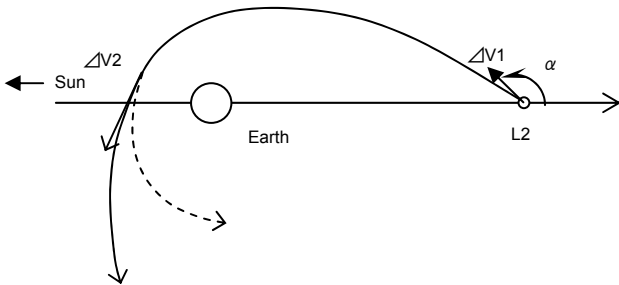


Fig. 7. Setting of the parameters

- The Earth's sphere of influence is assumed 3 million km. (In this study, we define this distance as "infinity", and this boundary sphere is used evaluation point of potential energy.)
- The maximum flight time is one year.
- Total  $\Delta V (= \Delta V1 + \Delta V2) = 50 \text{ m/s} \sim$  and  $200 \text{ m/s}$ .

### 3.2 Results

The simulation results of total  $\Delta V = 50 \text{ m/s}$  and  $200 \text{ m/s}$  are respectively shown in Fig. 8, 9. These figures show the relation between the escape velocity and the direction of escape velocity at the infinity. The conclusion is relatively simple and double impulse trajectories expand the direction of velocity at the infinity with less fuel.

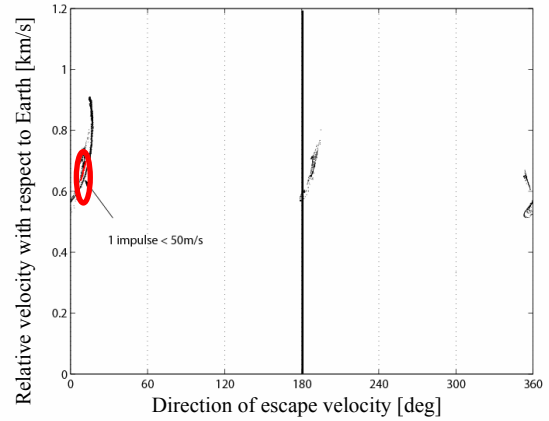


Fig. 8. Result of total  $\Delta V = 50 \text{ m/s}$

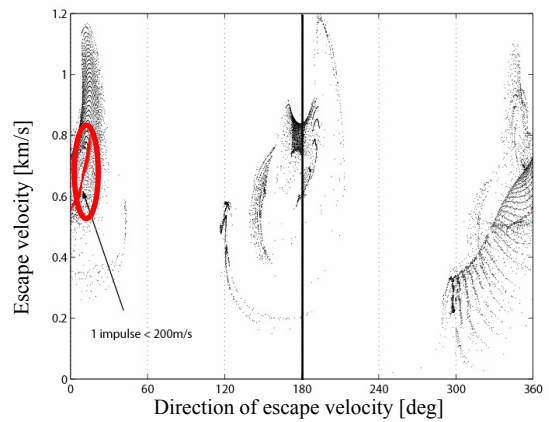


Fig. 9. Result of total  $\Delta V = 200 \text{ m/s}$

## 4. Escape trajectory using low thrust Transfer

In this section, based on the result of single impulse trajectory, escape trajectory using low-thrust transfer is optimized. In this study, the steering law is optimized by using DCNLP (Direct Collocation with Nonlinear Programming) method.

### 4.1 Escape trajectory optimization 1

First, escape trajectory optimization from Sun-Earth L2 point to the infinity is considered.

#### The assumptions for this problem

- Initial velocity :  $0 \text{ km/s}$  (The relative velocity of the spacecraft with respect to L2 point is  $0 \text{ km/s}$ .)
- Terminal position : infinity (Radius of 3 million km from the center of the Earth)
- Flight time : Free
- Terminal velocity at infinity : given

#### Formulation

- **Equation of motion**

The equation of motion of the spacecraft is given by Eqn. (1).

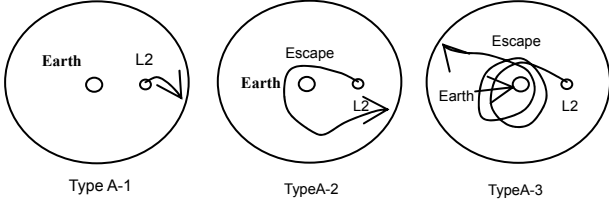


Fig. 10. Initial estimate for this problem

• **Criterion function**

The minimum fuel consumption problem is considered, so the criterion is written as follow,

$$J = \int_{t_0}^{t_f} (a_x^2 + a_y^2) dt \rightarrow \min \quad (5)$$

where  $a_x, a_y$  are the control input for x and y direction respectively.

• **Thrust constraint**

Suppose the spacecraft is equipped with electric propulsion, so that the thrust is very small. This thrust is assumed by

$$a_{\max} = (\sqrt{a_x^2 + a_y^2})_{\max} = 1.5 \times 10^{-4} [m/s] \quad (6)$$

• **Boundary conditions**

According to the assumptions for this problem, the spacecraft remain stationary with respect to L2 point at initial time. And terminal position of the spacecraft is infinity. Therefore the boundary conditions can be written as follows,

$$\begin{pmatrix} x_0 \\ y_0 \\ \dot{x}_0 \\ \dot{y}_0 \end{pmatrix} = \begin{pmatrix} 1 - \mu + \gamma_{L2} \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} x_f - (1 - \mu) \\ y_f \\ \dot{x}_f \\ \dot{y}_f \end{pmatrix} = \begin{pmatrix} x_e \\ y_e \\ V_{xe} \\ V_{ye} \end{pmatrix} \quad (7)$$

where  $\gamma_{L2}$  is the distance between the Sun and the Earth. The subscripts “0, f” mean initial and final condition respectively. And the subscript “ $\infty$ ” means condition at infinity.

Initial estimate

Optimization problem is heavily dependent on initial estimate, so an applicable initial estimate is needed. From the results of impulsive escape trajectories, we showed that high delta-V efficiency is obtained by the escape to  $\pm x$  axes. Therefore, in this study, the following trajectories are provided as the initial estimate.

- Type A-1 is the straightforward escape trajectory to the right. (see Fig. 10)
- Type A-2 is the escape trajectory to the right via the earth. (see Fig. 10)
- Type A-3 is the escape trajectory to the left via the earth. (see Fig. 10)

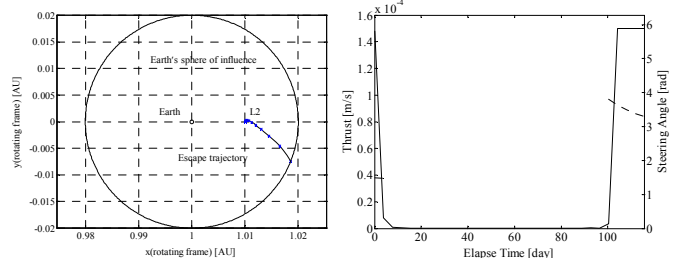


Fig. 11. Escape trajectory and Thrust history (Type A-1 :  $V_{\infty} = 0.307$  km/s)

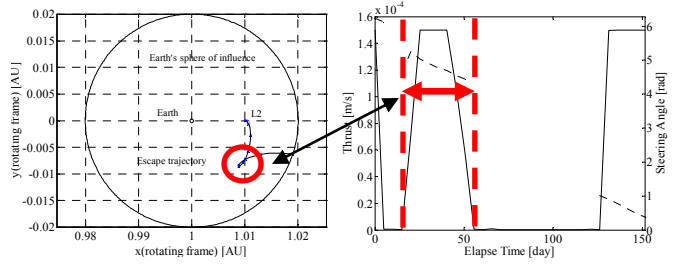


Fig. 12. Escape trajectory and Thrust history (Type A-1 :  $V_{\infty} = 0.857$  km/s)

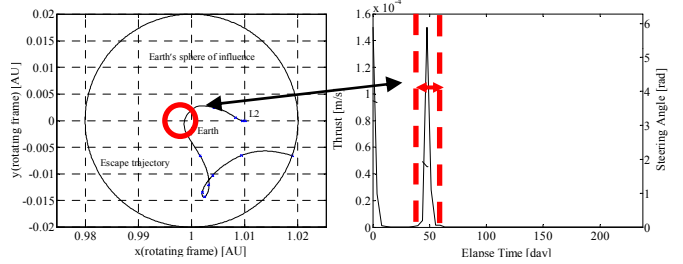


Fig. 13. Escape trajectory and Thrust history (Type A-2 :  $V_{\infty} = 0.972$  km/s)

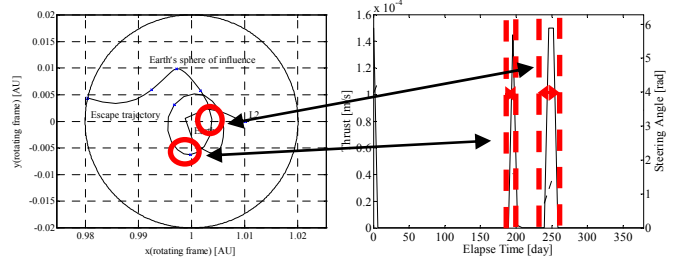


Fig. 14. Escape trajectory and Thrust history (Type A-3 :  $V_{\infty} = 0.857$  km/s)

**4.2 Results**

Fig. 11, 12 is trajectory and thrust history of type A-1 before the peak of efficiency is reached. Type A-1 has two thrust terms and one coasting term. Backward thrust term decreases gradually with the increasing of the V-infinity. And, when the efficiency is maximum, forward thrust term only remains. In the region where v-infinity is required over 0.6km/s, this type has three thrust terms. Second thrust term corresponds to the position in which the spacecraft is approaching zero velocity curve. This means that the spacecraft accelerates at the sensitive region of direction of relative velocity. Fig. 13 is result of type A-2 at the maximum efficiency. In this type, the spacecraft accelerates near the Earth.

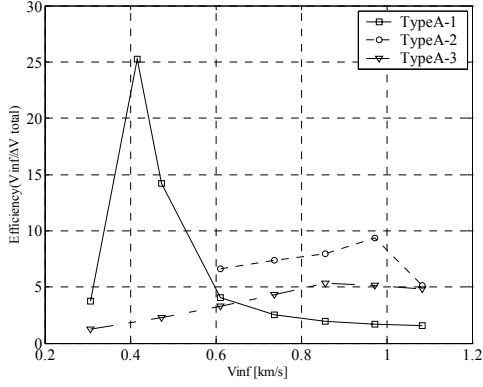


Fig. 15. Efficiency (velocity at infinity/ $\Delta V$ )

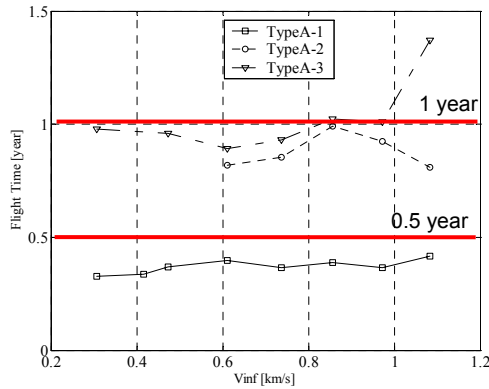


Fig. 16. Flight time

This means that the spacecraft accelerates at the sensitive region of magnitude of the velocity. Fig. 14 shows result of type A-3. This type has features which are combinations of the features of type A-1 and A-2.

Fig. 15 shows efficiency and Fig. 16 shows Flight time. From Fig. 15 efficiency of type A-1 is highest at the region where required v-infinity is less than 0.6 km/s. And at the region where v-infinity is required over 0.6 km/s, efficiency of type A-2 is highest. On the contrary, the escape trajectory from L2 point to the left is inefficient. Also, Fig. 16, flight times of type A-2 and A-3 are required twice as long as type A-1.

### 4.3 Escape trajectory optimization 2

In this section, trajectory which goes through the Earth synchronous orbit is optimized. And this study applies the EDVEGA technique to provide the high delta-V to the spacecraft.

#### The assumptions for this problem

- Initial velocity : 0km/s
- Terminal position : Earth's sphere of influence (Radius of 930,000km from the center of the Earth)
- Flight time : Free
- Terminal velocity : given

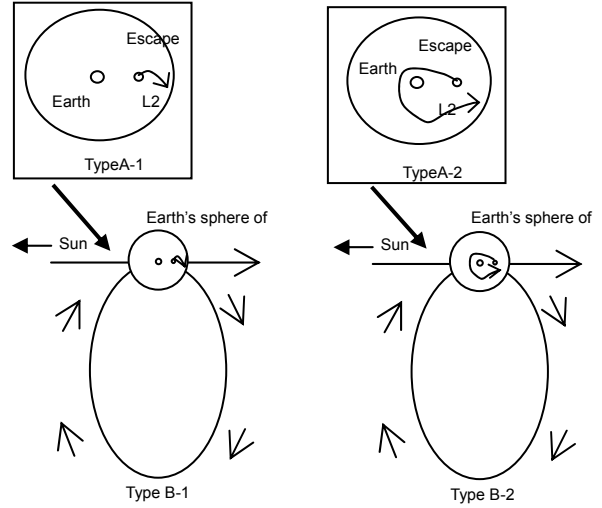


Fig. 17. Initial estimate for this problem

#### Formulation

As in the case of section 4.1, this problem is formulated.

- **Equation of motion**

The equation of motion of the spacecraft is given by Eqn. (1).

- **Criterion function**

The criterion is written as follow,

$$J = \int_{t_0}^{t_f} (a_x^2 + a_y^2) dt \rightarrow \min \quad (8)$$

- **Thrust constraint**

This thrust is assumed by

$$a_{\max} = (\sqrt{a_x^2 + a_y^2})_{\max} = 1.5 \times 10^{-4} [m/s] \quad (9)$$

- **Boundary conditions**

Terminal position of the spacecraft is Earth's sphere of influence. Therefore the boundary conditions can be written as follows,

$$\begin{pmatrix} x_0 \\ y_0 \\ \dot{x}_0 \\ \dot{y}_0 \end{pmatrix} = \begin{pmatrix} 1 - \mu + \gamma_{L2} \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} x_f - (1 - \mu) \\ y_f \\ \dot{x}_f \\ \dot{y}_f \end{pmatrix} = \begin{pmatrix} x_{\text{inf}} \\ y_{\text{inf}} \\ V_{\text{inf}} \\ V_{\text{inf}} \end{pmatrix} \quad (10)$$

where the subscript "inf" means the condition at Earth's sphere of influence.

#### Initial estimate

From the results of section 4.2, we showed that high delta-V efficiency is obtained by the escape to +x axes. Therefore, the escape trajectory connects to EDVEGA trajectory via the x direction. In this study, the following trajectories are provided as the initial estimate.

- Type B-1 is the EDVEGA trajectory connected to type A-1. (see Fig. 17)
- Type B-2 is the EDVEGA trajectory connected to type A-2. (see Fig. 17)

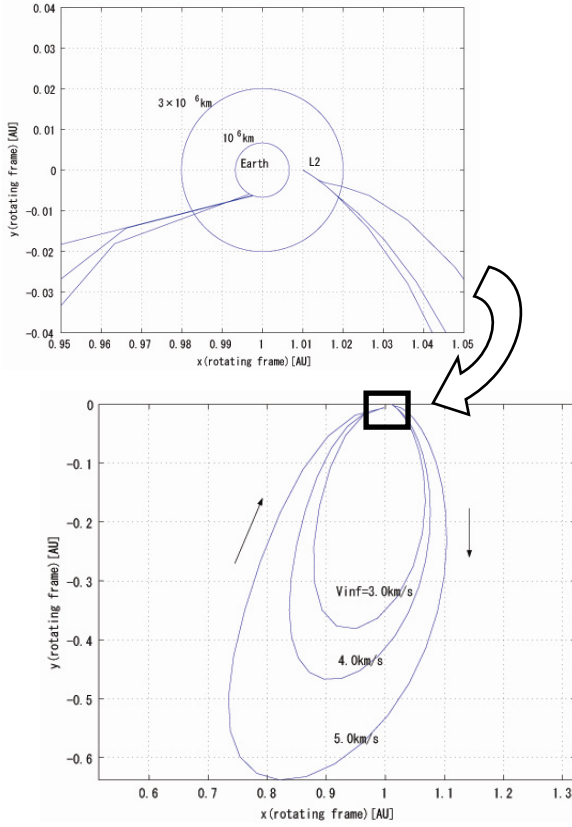


Fig. 18. EDVEGA trajectory (Type B-1)

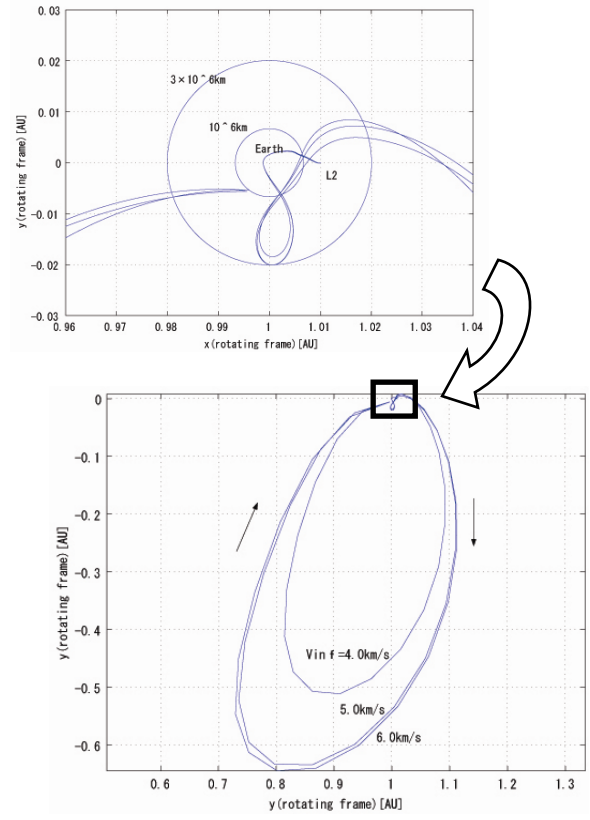


Fig. 19. EDVEGA trajectory (Type B-2)

#### 4.4 Results

Fig.18, 19 show the EDVEGA trajectories. Fig. 20 shows efficiency and Fig. 21 shows Flight time and approach angle. From Fig. 20, maximum efficiency of type B-1 is about 2.7 and maximum efficiency of type B-2 is about 3.0. And from Fig. 21, flight time of type B-1 is about 1.6 years and type B-2 is about 2 years.

#### 5. Evaluation of efficiency

In this section, the efficiency of trajectory using Low thrust transfer is estimated. The efficiency of trajectory which goes through the Earth synchronous orbit is designated  $\eta$ .  $\eta$  is written as follows,

$$\eta = \frac{V_{\infty 0} + 2\Delta V_1}{\Delta V_0 + \Delta V_1}, \quad V_{\infty 1} = V_{\infty 0} + 2\Delta V_1 \quad (11)$$

where  $\Delta V_0$  is total delta-V from L2 point to the infinity (3 million km) and  $V_{\infty 0}$  is the velocity which is obtained by  $\Delta V_0$ . Also,  $\Delta V_1$  is total delta-V for EDVEGA and  $V_{\infty 1}$  is the velocity which is obtained by  $\Delta V_1$ . The efficiency of the escape trajectory from Sun-Earth L2 point to the infinity is designated  $\kappa$ .  $\kappa$  is written as follows,

$$\kappa = V_{\infty 0} / \Delta V_0 \quad (12)$$

Eqn.12 is transformed into the following equation using kappa.

$$\eta = \frac{2V_{\infty 1}}{V_{\infty 1} + (2 - \kappa)\Delta V_0} \quad (13)$$

The second term of the denominator of Eqn.13 is dependent on the result of trajectory optimization of escape from Sun-Earth L2 point. Also, this equation shows that the trend of efficiencies is changed like Fig. 22 by the value of kappa.

And the solution of this equation corresponds to Fig. 20. This means that evaluation of delta-V can be solved algebraically.

#### 6. conclusions

First of all, impulsive escape trajectories from the equilibrium point where the quay anchored is were sought, and the features of those trajectories were examined. From these results, we showed that, when the departure velocity is small, the escape direction is limited to one direction (+x axis), and at this time efficiency is nearly infinity.

Based on the results of impulsive escape trajectories, escape trajectory using low-thrust transfer was optimized. First, the escape trajectory optimization from Sun-Earth L2 point to the infinity was considered. And the trajectory

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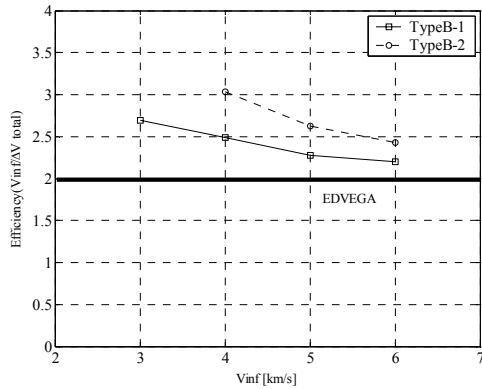


Fig. 20. Efficiency (velocity at infinity/ $\Delta V$ )

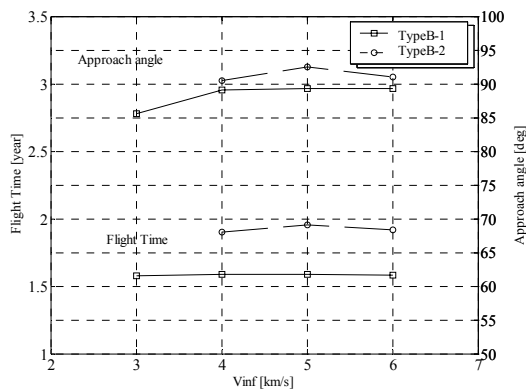


Fig. 21. Flight time and approach angle

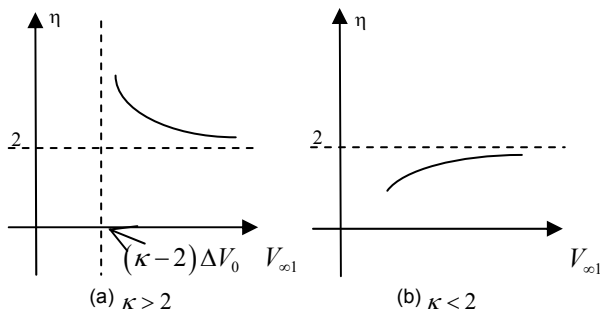


Fig. 22. trend of efficiency

which goes through the Earth synchronous orbit (using EDVEGA technique) is optimized. From these results, efficiency of each trajectory was estimated, and efficiency of trajectory to use EDVEGA technique could be solved algebraically. And we can estimate for required  $\Delta V$  of interplanetary voyage without optimization calculation.