Statistical Evaluation and Improvement of Methods for Combining Random and Harmonic Loads

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TRADEMARKS

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CDF   cumulative distribution function

HCF   high-cycle fatigue

PDF   probability density function

PSD   power spectral density

rms   root mean square

SSME   Space Shuttle main engine

TP   Technical Publication

NOMENCLATURE

A	amplitude
$A_{\sin}$	amplitude of sine load
$F(z)$	cumulative distribution function
$k_{\frac{\alpha}{2}}$	tolerance band
N	normal distribution
n	number of samples
p_f	probability of failure
$p_{\sin}$	harmonic load
z	variable load
ε	very small value
σ	standard deviation
σ_{ran}	rms of random load
$\sigma_{\sin}$	rms of sine load
ϕ	phase relationship

STATISTICAL EVALUATION AND IMPROVEMENT OF METHODS FOR COMBINING RANDOM AND HARMONIC LOADS

1. INTRODUCTION

Many structural components in dynamic environments experience both sinusoidal and random loadings at the same time. One example of such a structure is a component of a rocket engine. There are several sources that produce the random component of the excitation, including the gas generator and main combustion chamber. These excitations are characterized by a power spectral density (PSD) spectrum of accelerations. These accelerations are used in a base excitation random response analysis, which results in a PSD response spectrum at any desired location, as shown by the example of the gas generator axial force response for NASA Marshall Space Flight Center's MC–1 engine in figure 1.[1] Although it can be in any desired quantity, the mean of this spectrum, defined simply as a load, is zero, and since the process can be assumed to be Gaussian,[2] the root mean square (rms) of the response is defined to be equal to one standard deviation, σ. Simultaneously, the turbomachinery in the engine generates large harmonic loads due to the unavoidable unbalance in their rotors. The frequency of the harmonic excitations are at the shaft speed of the turbines and their multiples. The excitation from this sinusoidal loading, which can also be determined by previous acceleration measurements, is applied independently in a frequency response analysis, which generates the amplitude of a sinusoidal response for any desired location for a specified range of harmonic excitation frequencies.
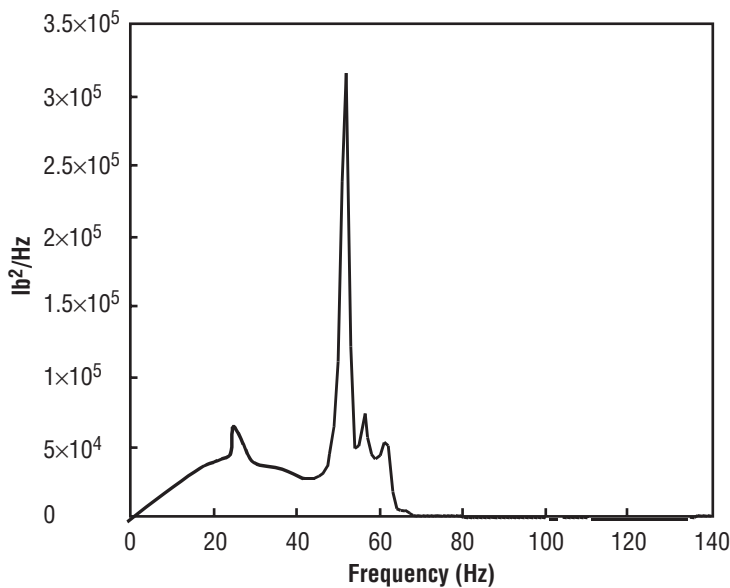

Figure 1. Typical results from random response analysis.

2. LOADS COMBINATION EQUATIONS

In general, the structural dynamic response analyses for the random and harmonic cases are performed separately, and the results are then combined to obtain a design load. These loads are used to determine a level of alternating stress in the component for design to ultimate or yield strength and for high-cycle fatigue (HCF). The calculation of this load has been a subject of some dispute in industry. Although the development of methods for combining different types of random loads has been a subject of extensive research,[3] detailed study of the use of similar equations for the combination of random and harmonically generated loads has not been performed. Perhaps because of this lack of examination, an industry standard loads combination equation has not been chosen. There are, therefore, several different methodologies that have been applied by various users. They are all based on the method used for obtaining design loads for purely random environments. Although there have been studies suggesting that the extreme-value distribution should be used for limit load design,[4] a widely accepted practice is to use 3 times the rms of the random load, σ_{ran}, for the design load where the distribution is assumed to be Gaussian. This value will exceed the response 99.865 percent of the time. It has also been suggested that a value less than 99.865 percent is acceptable for HCF design, but this Technical Publication (TP) will assume that both the Gaussian distribution and the 99.865-percent excedance level are valid.

The most frequently used method for combining the two dynamic load components, referred to here as the standard method, simply uses the sum of the amplitude of the sine load A_{sin} and $3 \times \sigma_{ran}$:[5]

$$\text{design load} = A_{sin} + 3\sigma_{ran} \ .$$

(1)

This technique assumes the sine peak value always occurs simultaneously with the peak random value. Another frequently applied method is to multiply 3 by the square root of the sums of mean square values of both the random and the sines, called the $3 \times$ ssMS method:[6]

$$\text{design load} = 3\sqrt{\left(\sigma_{sin}\right)^2 + \left(\sigma_{ran}\right)^2} \ ,$$

(2)

where σ_{sin}, the rms of the sine wave, is equal to the sine amplitude divided by $\sqrt{2}$. This method treats the sine as if it had a normal probability distribution, which has a much larger range than a sine function. The $3 \times$ ssMS approach is similar to equations written to combine loads on launch vehicles, which do not have a substantial harmonic component but do have random loads from several sources.[7] A third approach, proposed by Steinberg,[8] is to take the root sum square of the peak values (called the peak method):

$$\text{design load} = \sqrt{\left(A_{sin}\right)^2 + \left(3\sigma_{ran}\right)^2} \ .$$

(3)

Although it is stated in reference 8 that the standard method is too conservative and that it is convenient to use the suggested peak method, no mathematically rigorous justification is presented for using it for either ultimate/yield or HCF design. None of these closed-form methods produce a design load that can be consistently associated with a particular probability level.

Although the methodology discussed in this TP is relatively straightforward, it has not been generally applied in industry. In a paper by DebChaudhury et al., advanced probabilistic techniques were used to evaluate the stochastic nature of the parameters of the loading and the structure for the Space Shuttle main engine (SSME), but the sine and random loads were combined using the simple closed-form 3×ssMS method.[2] A similar study by Newlin et al., in which stochastic techniques were used to evaluate HCF problems in the SSME, generated a simulated random time history and added it to a harmonic time history.[9] The combined loading was then used for each time step. This methodology is similar to that presented in this TP but is too complex to use in a routine manner. Schock[10] was apparently able to analytically calculate the joint cumulative density function for $\sigma=1$ and was able to generate charts for different cummulative distribution function (CDF) values for this level of random σ. Ferebee[11] added to this work; however, the results do not appear to be consistent with the work presented below, and the methodology for performing the integration is not discussed.

3. LOADS COMBINATION USING PROBABILITY DENSITY FUNCTIONS

Since a sine wave considered by itself is not a random signal, it has always been unclear how to combine harmonic analysis results with output from a random excitation analysis to obtain a design load. The harmonic signal can, however, be defined as a stationary random process when combined with an independent Gaussian random signal since its phase relationship, ϕ, with the random signal is random.[12] If the harmonic signal is defined as

$$p_{\sin} = A\sin(\omega t + \phi) \ ,$$
(4)

then the argument $(\omega t + \phi)$ is therefore a random variable with a uniform distribution over the range $-\pi/2$ to $\pi/2$. The probability density function of the sine function of this distribution is

$$f(y) = \frac{1}{\pi A \sqrt{1 - \left(\frac{y}{A}\right)^2}} \ ,$$
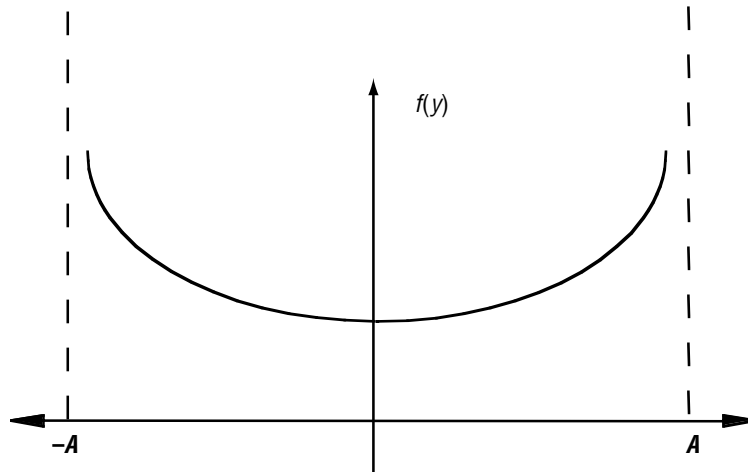(5)

shown graphically in figure 2.[13]

Figure 2. Probability density function $f(y)$=sin(x).

The equivalent event technique can now be used to calculate the CDF, $F(z)$, of functions of random variables.[14] For the function $z = x + y$, where x and y are random variables, the resultant $F(z)$ is defined in terms of the probability density functions (PDFs) f_x and f_y as

$$F(z) = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{z-y} f_x(x)dx \right) f_y(y)dy \ . \tag{6}$$

If x represents the random load, it can be characterized by a normal distribution with mean zero and standard deviation equal to the rms from the random analysis. Substituting the normal PDF and the PDF shown in equation (5) into equation (6) results in

$$F(z) = \int_{-A}^{A} \left(\frac{1}{\sigma_{ran}\sqrt{2\pi}} \int_{-\infty}^{z-y} \exp\left(\frac{-\left(\frac{x}{\sigma_{ran}}\right)^2}{2} \right) dx \right) \frac{1}{\pi A \sqrt{1 - \left(\frac{y}{A}\right)^2}} dy \ , \tag{7}$$

where the integral over y is only evaluated from $-A_{sin}$ to A_{sin} since the function is undefined outside of that range. A closed-form solution for this integral cannot be obtained; however, recent improvements in the software code Mathematica® 4.1 have enabled it to be evaluated numerically for any value of A, z, and σ_{ran}. The routine crashes if the integration is carried out over the range exactly from $-A$ to A, so the limits are instead set at $-A+\varepsilon$ to $A-\varepsilon$, where ε is a very small value. The value of ε was obtained by performing a series of runs with decreasing ε until the solution converged; the final value obtained was 1×10^{-11}.

It is also of interest to obtain the solution for the inverse problem from that described above; i.e., what would the design load be that would yield a specific CDF value. In particular, if the 99.865-percent level has traditionally been chosen as an acceptable level of probability of exceedance for designing to random loads, then it would be consistent to use that level to design for the combination of all dynamic loads. The Mathematica Runge-Kutta "FindRoot" command can be implemented to calculate this load (z) given the probability $F(z)$, A_{sin}, and σ_{ran}.

4. LOADS COMBINATION USING MONTE CARLO

Monte Carlo simulation can also be used to obtain the CDF value of interest. The random load is first simulated by generating a vector of points, $\{r\}$, that fall within a Gaussian (or normal) distribution of mean zero and standard deviation equal to the rms value resulting from the random analysis. This can be expressed as $\{r\} \sim N(0.0, \sigma_{ran})$. For illustrative purposes, a simulated history for such a loading is shown in figure 3. The vectors representing the results from the harmonic analysis are then generated. The method can account for analysis results from multiple independent harmonic excitations $\{y\}_i$, $i=1, ..., n$. Since the frequencies of each sine wave are different and the relative phasing is unknown, each sine wave should be considered independently.[15] Examples of sine loading time histories are shown in figure 4. This method is only applicable for independent harmonic sources, such as excitations from different turbopumps. The loads coming from multiples of the primary shaft rotational speed for a single turbopump would be correlated and, therefore, not independent.
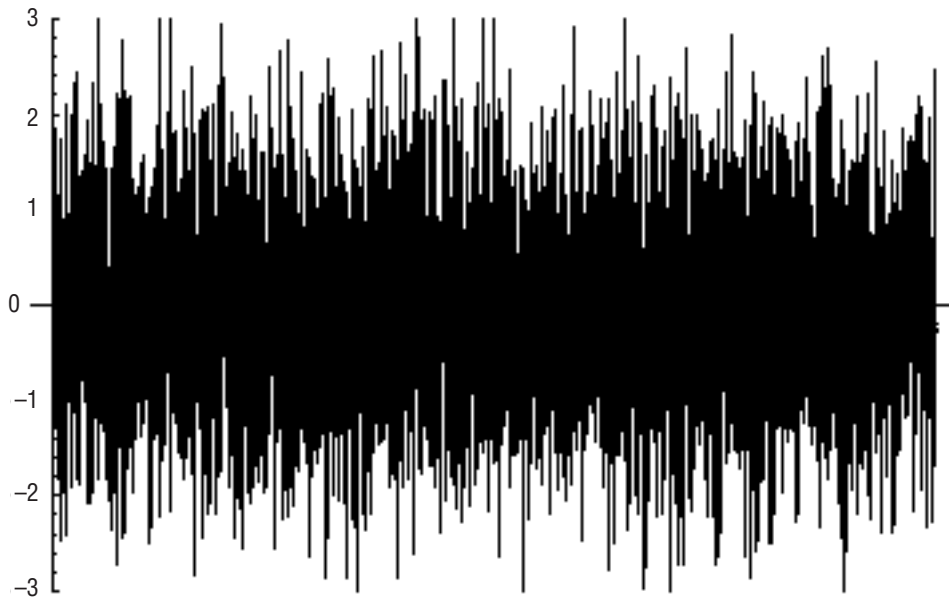
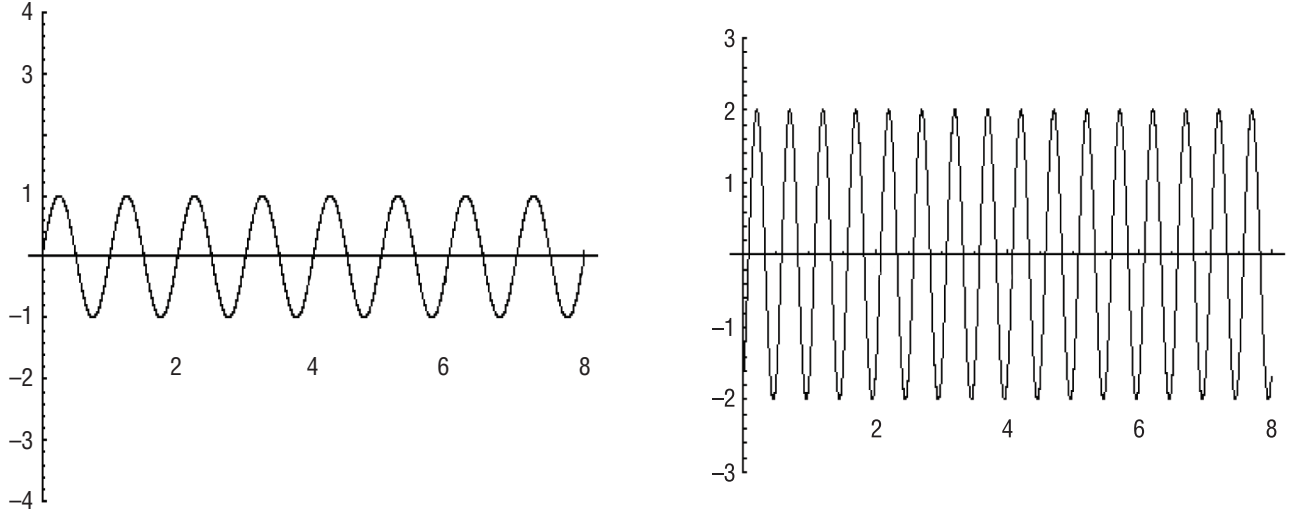Figure 3. Sample time history of random loading, rms =1.

Figure 4. Sample time history of sine loading.

For the independent case, a vector for each wave of the same length as $\{r\}$ is therefore created using the PDF defined previously, resulting in

$$\{y\}_i = A_i \sin\left(2\pi\{x\}_i\right) , \tag{8}$$

where $\{x\}$ is a random vector uniformly distributed between zero and 1. The resulting vector, $\{y\}$, will contain points that have the correct probability distribution of a sine wave, but are related randomly in phase (unlike the sequential time history shown in fig. 4). This procedure allows the frequencies of different harmonic signals to be rigorously removed from the calculation. The summation of all the harmonic signals and the random signal is performed for each sample to create a new vector, $\{z\}$, which is equal to a sample set of total responses of the structure to the multiple simultaneous excitations,

$$\{z\} = \{r\} + \{y\}_1 + \{y\}_2 + \ldots + \{y\}_n . \tag{9}$$

A sample of such a loading is shown in figure 5, which is formed by adding the curves in figures 3 and 4. The CDF of $\{z\}$ is then generated for the sample case, as shown in figure 6. A search procedure is performed to obtain the value from this CDF that is greater than 99.865 percent of the responses.

7
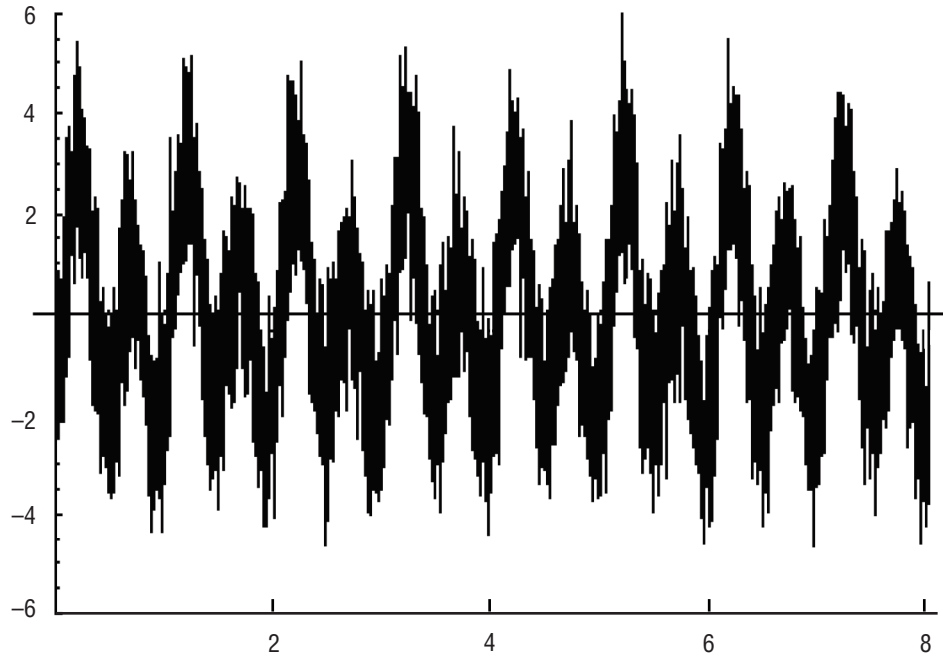
Figure 5. Sum of sine and random signals.
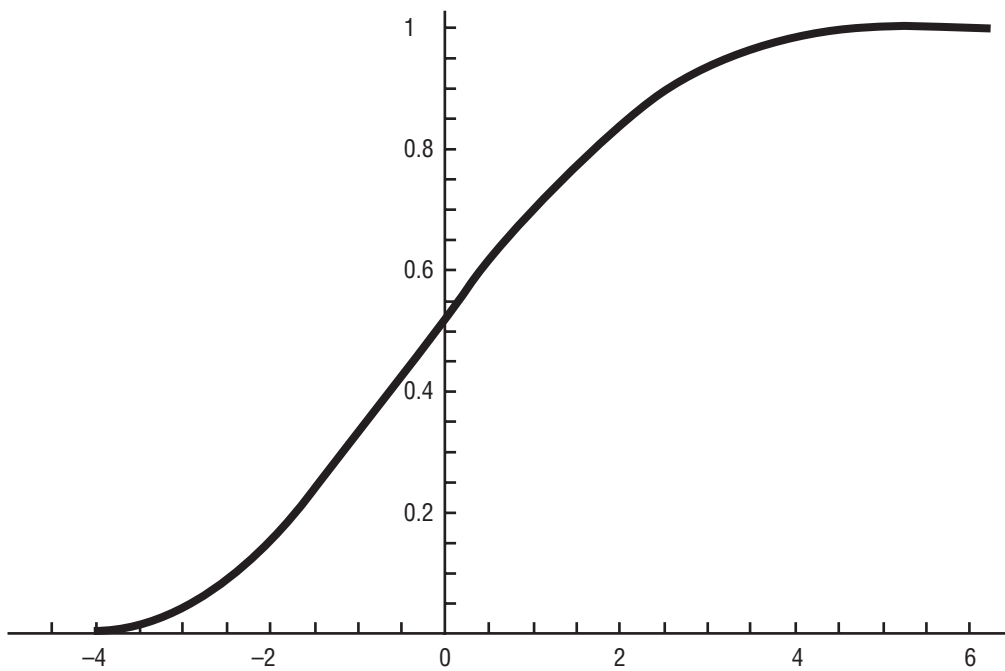
Figure 6. CDF of combined loadings.

This procedure was implemented in a FORTRAN program running on an SGI workstation.[16] It was applied successfully in the MC–1 engine program in 1998 to determine design loads used to evaluate stresses in interface brackets, fasteners, lugs, and other structural members. Since a large number of combinations were needed for each of a large number of load cycles, the speed of the program was increased by using efficient numerical routines found in standard FORTRAN libraries, such as IMSL, and by automating the process for multiple combinations. Using these methods, 100,000 samples for each of 792 separate sine-random load combinations were calculated in a total of 1,573 CPU s (26 min) on a Cray T90. For more general use, a generic, user-friendly computer program implementing this method was also written and incorporated into the NASA/Techtrac national software library.

5. ACCURACY AND SPEED OF MONTE CARLO

The accuracy of the Monte Carlo simulation is dependent on the number of samples selected. A convergence study performed here for a single harmonic wave with a random signal indicates that the values converge for $\approx$400,000 samples. There are several analytical methods for determining accuracy. Applying the method described by Ang and Tang,[17] the variation in the probability of failure estimate, p_f, for a given number of samples, n, with a specified confidence is

$$p_f \pm k_{\frac{\alpha}{2}} \sqrt{\frac{p_f(1-p_f)}{n}} \quad . \tag{10}$$

Charts for various confidence levels are used to determine $k_{\alpha/2}$. For a 95-percent confidence; i.e., 95/100 simulations would yield a probability estimate within the tolerance band, $k_{\alpha/2}$=1.96. The accuracy using 400,000 samples is therefore 0.99865 ± 0.000114 (99.854 – 99.876 percent). If 200,000 samples are used instead (doubling the speed), the error would be 0.99865 ± 0.000161, showing only a small loss of accuracy.

6. IMPLEMENTATION IN MICROSOFT EXCEL

To facilitate ease of use, both the PDF inverse-integration technique, discussed in Section 3, and the Monte Carlo technique were implemented as Visual Basic macros launched from Microsoft Excel. A complete listing of the macros and associated batch runstreams are presented in appendices A and B. The listings are also available in electronic format from the authors. In each case, any number of different combinations of random 1σ values and sine amplitudes are listed in columns, and these columns, along with a blank column adjacent, are chosen, the macro is selected, and then launched. The inverse-integration technique automatically writes the input values to a text file and launches the Mathematica kernel in the background, which reads the data, performs the root search for the chosen probability level (hard-coded as 99.865 percent), and writes the result into a text file that is then read into the result column in Excel. This process takes ≈4.5 s for each combination on an Intel Pentium 4 1.8 GHz workstation with 500 Mb RAM running Windows® 2000.

The Monte Carlo macro is based upon the FORTRAN code written previously, but can only combine a single sine load with the random load. It is run entirely within Visual Basic for Excel to enable maximum portability, and takes ≈51 s per combination.

7. RESULTS

The integration method was used to evaluate the actual CDF values using each of the three closed-form methods for a range of sine and random inputs chosen from actual MC–1 analysis results as well as some hypothetical cases. A representative subset of these CDF results is shown in table 1. The combination load values are also compared with loads obtained for a 99.865-percent probability level using the inverse-integration method and the Monte Carlo method, with the inverse-integration method being chosen as the baseline. The Monte Carlo method shows very close agreement resulting from the large number of samples arrived at from the convergence study. As expected, both the standard and the 3×ssMS methods always exceed the baseline value by varying amounts. The peak method always yields a value less than the baseline.

Table 1. Comparison of all methods.

Amp. Sine	1σ Random	Random/ Sine	Load Using Integration Method for 99.865% (baseline)	Load, Monte Carlo Method for 99.86% 400,000 Samples	Load, Standard Method A+3σ	Overshoot From Baseline (%)	CDF Value From Integration (%)
10	5	0.5	22.034	22.031	25	13.5	99.970
5	5	1	17.668	17.653	20	13.2	99.957
5	20	4	60.915	60.888	65	6.7	99.919
26	4	0.154	34.760	34.772	38	9.3	99.983
97	14.67	1.151	129.081	129.195	141.01	9.2	99.986
50	98.7	1.974	313.047	313.422	346.1	10.6	99.951
64	109.33	1.708	352.240	353.079	391.99	11.3	99.955

Amp. Sine	1σ Random	Load, 3×ssMs	Overshoot (%)	CDF Value From Integration (%)	Load, Peak Method ss(σ, A)	Overshoot (%)	CDF Value From Integration (%)
10	5	25.981	17.9	99.990	18.028	−18.2	93.930
5	5	18.371	4.0	99.912	15.811	−10.5	94.896
5	20	60.930	0.03	99.865	60.208	−1.2	95.758
26	4	56.445	62.4	100.000	28.636	−17.6	94.291
97	14.67	210.422	63.0	100.000	106.517	−17.5	94.316
50	98.7	314.524	0.5	99.871	300.292	−4.1	95.534
64	109.33	354.978	0.8	99.875	334.176	−5.1	95.443

8. EXAMINATION OF CLOSED-FORM METHODS

If a user chooses to apply one of the simple closed-form techniques, it would be desirable to use the technique that yields an answer with a CDF above 99.865 percent, but not excessively so. As seen in table 1, for some combinations, this rationale would point to the standard method, while for others, the 3×ssMS would apply. The exact ratio defining which is better to use can be calculated. First, equate equations (1) and (2):

$$A_{\sin} + 3\sigma_{\mathrm{ran}} = 3\sqrt{(\sigma_{\sin})^2 + (\sigma_{\mathrm{ran}})^2} \quad . \tag{11}$$

Using the relationship $A_{\sin} = \sqrt{2}\ \sigma_{\sin}$, equation (11) is satisfied when

$$\sigma_{\sin} = 1.21218\sigma_{\mathrm{ran}} , \quad \text{or} \quad A_{\sin} = (1.71429)\sigma_{\mathrm{ran}} \quad . \tag{12}$$

Therefore, if $A_{\sin} > 1.71429\ \sigma_{\mathrm{ran}}$, the standard method will yield a probability value greater than 99.965 percent but less than the value obtained using the 3×ssMS method, and vice-versa. This result is demonstrated in figure 7, which is a plot of the percentage overshoot versus the ratio, $x = \sigma_{\mathrm{ran}}/A_{\sin}$, for each of the above methods.
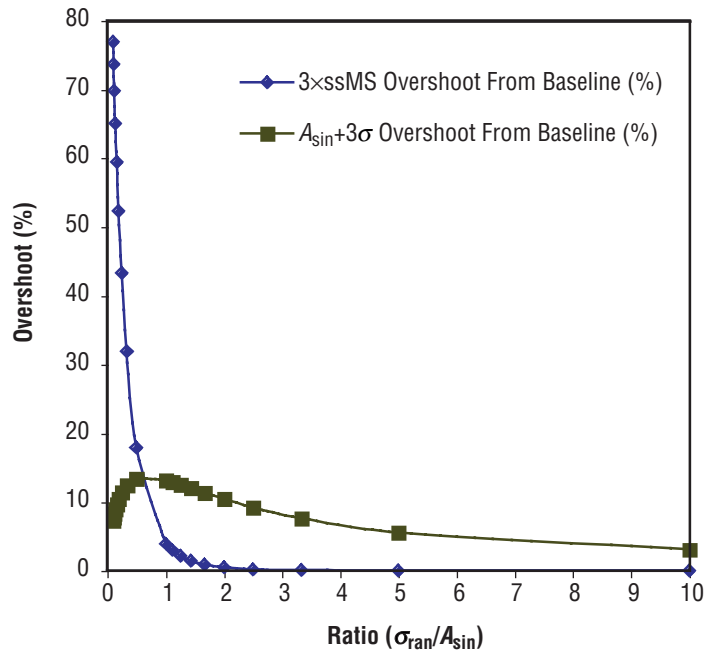

Figure 7. Percent overshoot from baseline using traditional methods.

The functional relationships identified in figure 7 were examined for various values of the parameters and were seen to be only a function of the ratio. This independent functional relationship identified the possibility of creating a closed-form equation that would exactly match the values obtained using the inverse-integration method for this specific probability level. The Mathematica 4.1 curve fit routine was used to examine different categories of functions for fitting the overshoot, resulting in equation (13), with the comparison of the function with the data shown in figure 8:

$$\text{overshoot} = e^{-1.53546/x} \begin{pmatrix} 4.60691\times10^{-11}x^7 - 1.193679\times10^{-8}x^6 \\ +1.19197\times10^{-6}x^5 - 6.04117\times10^{-5}x^4 \\ +1.69965\times10^{-3}x^3 - .0271317x^2 + .241154x \end{pmatrix}. \tag{13}$$

The desired closed-form equation is therefore

$$\text{design load} = \frac{3\sqrt{\dfrac{A_{\sin}^2}{\sqrt{2}} + \sigma_{\text{ran}}^2}}{1+\text{overshoot}}. \tag{14}$$
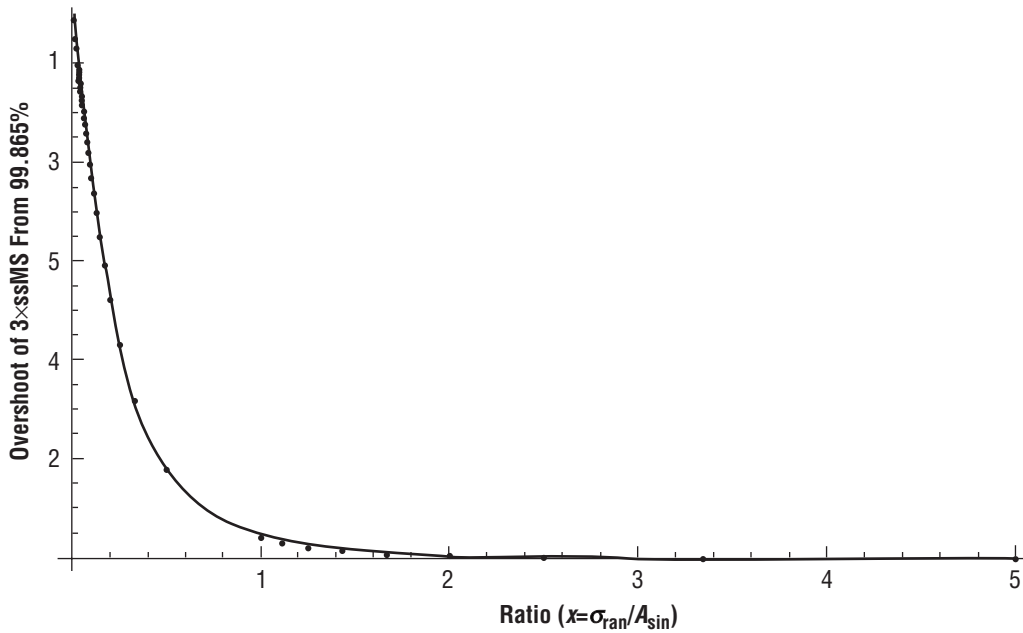

Figure 8. Curve fit of overshoot of 3×ssMS method over CDF of 99.865 percent.

A comparison of the baseline values with those obtained using this new closed-form solution is shown in table 2. The agreement is excellent, with only one of the points showing an overshoot of greater than 1 percent. This equation can be directly used in a spreadsheet. If the ratio σ_{ran} to A_{sin} is outside of the range 0.025 to 10, or if either component is zero, the curve fit is not valid; therefore, an Excel user-defined function has been written (see app. C), which will automatically apply the curve fit within the acceptable range or the appropriate traditional method outside of it (where they are accurate).

Table 2. Comparison of derived closed-form equation with 99.865-percent baseline.

Amp. Sine	1σ Random	Ratio σ_{ran}/A_{sin}	99.865% CDF Using Inverse Integration (Baseline)	Closed-form Curve Fit Based on $3\times ssMs$ Method	Overshoot From Baseline (%)
1	10	10.000	30.0811	30.0749	−0.02
2	10	5.000	30.2986	30.2979	0.00
3	10	3.333	30.6500	30.6547	0.02
4	10	2.500	31.1169	31.1152	−0.01
6	10	1.667	32.3175	32.2508	−0.21
8	10	1.250	33.7557	33.5979	−0.47
10	10	1.000	35.3340	35.1116	−0.63
20	10	0.500	44.0679	44.0599	−0.02
40	10	0.250	62.8036	63.1242	0.51
60	10	0.167	82.0509	82.0168	−0.04
80	10	0.125	101.5086	101.2031	−0.30
100	10	0.100	121.0833	120.8167	−0.22
200	10	0.050	219.7239	219.6545	−0.03
300	10	0.033	318.9018	319.8996	0.31
400	10	0.025	418.3045	417.9850	−0.08
50	1	0.020	51.7833	51.7960	0.02

9. CONCLUSION AND AREAS OF FUTURE WORK

The probability values associated with several widely-used methods for combining harmonic and random loads have been determined. In addition, two new Microsoft Excel macros have been written that calculate the combined load value for a specified probability level. Because of the speed of desktop computers, these macros can be executed easily and quickly. A closed-form solution approximating the value for a level of 99.865 percent has also been determined. The comparisons presented in this TP can assist the analyst in choosing a method for combining these loads; obtaining an accurate value can be critical for the design of weight-sensitive aerospace structures. A version of the macro using Monte Carlo simulation was successfully applied to the determination of loads for 44 separate structural components in the FASTRAC/MC–1 engine program for 10 or more load cycles, resulting in large weight savings. One avenue of future work in this area is incorporating the effect of the correlation of the sine loads for harmonics of a single rotating shaft. Another will be to implement the methodology discussed here to the extreme-value distribution.

APPENDIX A—MONTE CARLO TECHNIQUE

This appendix details the Microsoft Excel macro for performing combination of random and sine loads using Monte Carlo technique. Place A_{sin} in column 1, σ_{ran} in column 2, leave column 3 blank, select all three columns, and run macro "montecarlo". If a probability level other than 99.865 percent is chosen for use, this value can be inserted into line 24.

```
Option Base 1
Sub montecarlo ()

maxsam = 1000000
Dim combo(1000000) As Double
Dim pdf(10001) As Double
Dim div(10001) As Double
Dim cdf(10001) As Double
Dim cum(10001) As Double
Dim xlo As Double
Dim xhi As Double
Dim delta As Double
Dim rnorm As Double
numsam = 400000
Pi = 4.0*atn(1.0)
current_selection = Selection.Address
For I = 1 To Range(current_selection).Rows.Count
    a = Range(current_selection).Cells(I, 1)
    sigma = Range(current_selection).Cells(I, 2)
    For J = 1 To numsam
      rnorm = Application.WorksheetFunction.NormInv(Rnd, 0#, sigma)
      combo(J) = a * Sin(2 * Pi * Rnd) + rnorm
    Next J
    Call Sort(numsam, combo)
    ivalue = Int(0.99865 * numsam)
    Range(current_selection).Cells(I, 3) = combo(ivalue)
Next I
End Sub

Public Function Sort(N, RA) As Integer[18]
      Dim RRA As Double
      Dim L As Long
      Dim I As Long
```

```
      Dim IR As Long
      Dim J As Long
      Sort = 0
      L = N / 2 + 1
      IR = N
CONT10:
        If (L > 1) Then
          L = L - 1
          RRA = RA(L)
        Else
          RRA = RA(IR)
          RA(IR) = RA(1)
          IR = IR - 1
          If (IR = 1) Then
            RA(1) = RRA
            Sort = 1
            Exit Function
          End If
        End If
        I = L
        J = L + L
CONT20:
        If (J <= IR) Then
          If (J < IR) Then
            If (RA(J) < RA(J + 1)) Then J = J + 1
          End If
          If (RRA < RA(J)) Then
            RA(I) = RA(J)
            I = J
            J = J + J
          Else
            J = IR + 1
          End If
        GoTo CONT20:
        End If
        RA(I) = RRA
      GoTo CONT10:
End Function
```

APPENDIX B—INVERSE-INTEGRATION TECHNIQUE

This appendix details the Microsoft Excel macro, DOS batch runstream, and Mathematica Kernal text input file for performing combination of random and sine loads using inverse-integration technique. Place $A_{\sin}$ in column 1, σ_{ran} in column 2, leave column 3 blank, select all three columns, and run macro "eq3sig".

Microsoft Excel Macro "eq3sig"

```
Sub eq3sig()
current_selection = Selection.Address
For i = 1 To Range(current_selection).Rows.Count
    a = Range(current_selection).Cells(i, 1)
    sigma = Range(current_selection).Cells(i, 2)
    Open "C:\Program Files\Wolfram Research\Mathematica\4.1
     \excel_to_math.txt" For Output As #1
Open file for output.
    Print #1, a
    Print #1, sigma
    Close #1
    shell_sync ("C:\Program Files\Wolfram Research\Mathematica\4.1
    \mat_autorun.bat")
    Open "C:\Program Files\Wolfram Research\Mathematica\4.1
    \math_to_excel.txt" For Input As #3
    Input #3, y
    Range(current_selection).Cells(i, 3) = y
    Close #3
    Kill "C:\Program Files\Wolfram Research\Mathematica\4.1
    \math_to_excel.txt"
    Kill "C:\Program Files\Wolfram Research\Mathematica\4.1
    \excel_to_math.txt"
    Next i
End Sub

Public Function shell_sync(cmd As String) As Integer
'shell function runs application asynchronously
'for short duration apps where assurance of completion is desired this
'function enters a do loop activating the application until an error
'occurs signifying the app is no longer there and therefore complete
ireturn = Shell(cmd, vbMinimizedFocus)
If (ireturn <> 0) Then
```

```
'cmd successfully initiated
'set error trap and activate app in loop
  On Error Resume Next
  Do While Err.Number = 0
'idle time loop
    For i = 1 To 1000
    Next i
'check for system events that require action
    nfrm = DoEvents()
'attempt to activate app
    AppActivate ireturn
  Loop
'turn off error trap
  On Error GoTo 0
'shell-sync successful
  shell_sync = 0
Else
'cmd initiation unsuccessful
'shell-sync unsuccessful
  shell_sync = 1
End If
End Function
```

DOS batch runstream "mat_autorun.bat". Place file in directory with Mathematica Kernel executable.

```
C:
chdir C:\Program Files\Wolfram Research\Mathematica\4.1
Math <randsin_calc.txt >math_output.txt
```

Mathematica Kernel text input file "randsin_calc.txt". Place file in directory with Mathematica Kernel executable". If a probability level other than 99.865 percent is chosen for use, this value can be inserted into line 6.

```
AppendTo[$Echo, "stdout"]
Off[General::spell1]
v=Import["excel_to_math.txt","List"];
a=v[[1]];sigma=v[[2]];
\!\(max = 3*sigma\  + a; eps=1.0*10^-11; min =  .85*max;\n
  FindRoot[ .99865 == \(1\/\(a\ *
            sigma\ \@\(2  \[Pi]\^3\)\)\) \(\[Integral]\_\(-\((a -
\eps)\)\)\)\%\((a -  eps)\)\((\[Integral]\_\(\(-6\)*sigma\)\%\(z -
\y\)\(\[ExponentialE]\^\(\(-\((x\/sigma)\)\^2\)\/2\)\)
\[DifferentialD]x)\) \
```

```
\(1\/\@\(1 - \((y\/a)\)\^2\)\) \[DifferentialD]y\), {z, {min,max}}]\)
equiv3sigma=z/.%
Export["math_to_excel.txt",equiv3sigma,"List"];
Quit[ ];
```

APPENDIX C—APPLYING CURVE FIT OF THE PROBABILITY DENSITY FUNCTION

This appendix details the Microsoft Excel user-defined function applying curve fit of PDF Integration. Create macro as shown below, then in spreadsheet, type function "curvefit(a,s)" with A_{sin} as the first argument and σ_{ran} as the second argument.

```
Function curvefit(a, s)
If a = 0 Then
    combo = 3 * s
ElseIf s = 0 Then
    combo = a
ElseIf (s >= 0.02 * a) Then
    x = s / a
    combo = (3 * Sqr(a ^ 2 / 2 + s ^ 2)) / _
(1 + Exp(-1.53546*x) * ((4.60691E-11/x^7)-(1.19367935364834E-08/x^6)+ _
(0.00000119197 /x^5) - (0.0000604117 /x^4) + (0.00169965 /x^3) - _
(0.0271317 /x^2) + (0.241154/x)))
Else
    combo = a + (3 * s)
End If
If (s > (10 * a)) Then
    combo = 3 * Sqr(a ^ 2 / 2 + s ^ 2)
End If
curvefit = combo
End Function
```

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REFERENCES

1. Christensen, E.; Frady, G.; Mims, K.; and Brown, A.: "Structural Dynamic Analysis of the X–34 Rocket Engine," AIAA Paper No. 98–2012, 39th AIAA Structures, Structural Dynamics, and Materials Conference, Long Beach, CA, April 20–23, 1998.

2. DebChaudhury, A.; Rajagopal, K.; Ho, H.; and Newell, J.: "A Probabilistic Approach to the Dynamic Analysis of Ducts Subjected to Multibase Harmonic and Random Excitation," AIAA Paper No. 90–1100–CP, Proceedings of the 31st Structures, Structural Dynamics, and Materials Conference, Long Beach, CA, pp. 104–1061, April 2–4. 1990.

3. Gaver, D.P.: "On Combinations of Random Loads," U.S. Navy Report No. AD–A085489, 1980.

4. Merchant, D.H.: "Methods for Combining Payload Parameter Variations with Input Environment," *NASA/CR–149939*, Marshall Space Flight Center, AL, June 30, 1976.

5. *Rocketdyne Structural Dynamics Manual*, Rocketdyne Corporation, Vol. 2, Sec. 2.2–6, 1989.

6. Wen, Y.K.: *Structural Load Modeling and Combination for Performance and Safety Evaluation*, Elsevier, Amsterdam, p. 211, 1990.

7. Macheske, V.; Womack J.; and Binkley, J.: "A Statistical Technique for Combining Launch Vehicle Atmospheric Flight Loads," AIAA Paper No. 93–0755, 31st Aerospace Sciences Meeting, Reno, NV, January 11–14, 1993.

8. Steinberg, D.S.: *Vibration Analysis for Electronic Equipment*, John Wiley & Sons, New York, p. 249, 1988.

9. Newlin, L.; Sutharshana, S.; et al.: "Probabilistic High Cycle Fatigue Failure Analysis with Application to Liquid Propellant Rocket Engines," AIAA Paper No. 90–1141–CP, Proceedings of the 31st AIAA Structures, Structural Dynamics, and Materials Conference, Long Beach, CA, pp. 1105–1114, April 2-4, 1990.

10. Schock, R.; and Tuell, L.: "An Investigation into the Probabilistic Combination of Quasi-Static and Random Accelerations," AIAA Paper No. 84-0908, 25th Structures, Structural Dynamics, and Materials Conference, Palm Springs, CA, May 14–16, 1984.

11. Ferebee, R.: "Loads Combination Research at Marshall Space Flight Center," *NASA/TM—2000–210331*, Marshall Space Flight Center, AL, 2000.

12. Wirshing, P.; Paez, T.; and Ortiz, K.: *Random Vibrations: Theory and Practice*, John Wiley & Sons, New York, p. 81, 1995.

13. Rheinfurth, M.; and Howell, L.W.: "Probability and Statistics in Aerospace Engineering," *NASA/TP—1998–207194*, Marshall Space Flight Center, AL, p. 53, March, 1998.

14. Rheinfurth, M.; and Howell, L.W.: "Probability and Statistics in Aerospace Engineering," *NASA/TP—1998–207194*, Marshall Space Flight Center, AL, p. 51, March, 1998.

15. Johnson, P.; and Ayyub, B.: *Probabilistic Structural Mechanics Handbook*, Sudararajan, C. (ed.), Chapman & Hall, New York, p. 652, 1995.

16. Brown, A.M.: "Combining Loads From Random and Harmonic Excitation Using the Monte Carlo Technique," *Journal of Spacecraft & Rockets*, Vol. 37, No. 4, pp. 541–543, July–August 200l.

17. Ang, A.; and Tang, W.: *Probability Concepts in Engineering Planning and Design*, Vol. 1, John Wiley & Sons, New York, p. 254, 1975.

18. Press, W.H.; et al.: *Numerical Recipes, The Art of Scientific Computing (FORTRAN)*, Cambridge University Press, Cambridge, England, pp. 229–232, 1989.

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13. ABSTRACT *(Maximum 200 words)*

Structures in many environments experience both random and harmonic excitation. A variety of closed-form techniques has been used in the aerospace industry to combine the loads resulting from the two sources. The resulting combined loads are then used to design for both yield/ultimate strength and high-cycle fatigue capability. This Technical Publication examines the cumulative distribution percentiles obtained using each method by integrating the joint probability density function of the sine and random components. A new Microsoft Excel spreadsheet macro that links with the software program Mathematica to calculate the combined value corresponding to any desired percentile is then presented along with a curve fit to this value. Another Excel macro that calculates the combination using Monte Carlo simulation is shown. Unlike the traditional techniques, these methods quantify the calculated load value with a consistent percentile. Using either of the presented methods can be extremely valuable in probabilistic design, which requires a statistical characterization of the loading. Additionally, since the CDF at high probability levels is very flat, the design value is extremely sensitive to the predetermined percentile; therefore, applying the new techniques can substantially lower the design loading without losing any of the identified structural reliability.

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