1 Brief Summary of Current Experimental Situation

At present there is increasingly strong evidence for neutrino masses and lepton mixing. All solar neutrino experiments (Homestake, Kamiokande, SuperKamiokande, GALLEX, and SAGE) show a significant deficit in the neutrino fluxes coming from the Sun [?]. This deficit can be explained by oscillations of the ν_e's into other weak eigenstates. Measurements now include not just the overall rates of ^{8}B neutrinos in the Homestake, Kamiokande, and SuperKamiokande experiments and rates of pp and ^{7}Be neutrinos from SAGE and GALLEX, but also, from SuperK, the recoil electron energy distribution (consistent with being flat), day-night asymmetry (a 1.3 σ effect), and seasonal variation (nominal). Fits to the SuperK and other experiments current favor the large-mixing angle (LMA) solution, with large $\sin^2(2\theta_{12})$ and Δm^2_{sol} between about 2×10^{-5} eV^2 and 10^{-4} eV^2 [?].

Especially strong evidence comes from the high-statistics data on atmospheric neutrinos from SuperKamiokande [?], strengthening earlier results from Kamiokande [?] and IMB [?]. The SuperK data includes the measurement of the ratio $R = N(\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$ as compared with a Monte Carlo calculation, and the zenith-angle distribution of events. The measurements by the Soudan-2 [?] and MACRO [?] experiments are consistent with SuperK. The SuperK experiment finds a good fit of their data to $\nu_\mu \to \nu_\tau$ oscillations with [?]

$$\Delta m^2_{atm} = |m(\nu_3)^2 - m(\nu_2)|^2 \simeq 3.5 \times 10^{-3} \ \ \mathrm{eV}^2 \tag{1}$$

and maximal mixing,

$$\sin^2(2\theta_{atm}) = \sin^2(2\theta_{23}) = 1 \tag{2}$$

These values are also consistent with the K2K long baseline neutrino experiment [?]. In their analysis of atmospheric neutrino data, the SK experiment excludes dominant contributions from $\nu_\mu \to \nu_e$ [?] (also excluded for the value of Δm^2 in eq. (??), by the CHOOZ reactor experiment [?]) or $\nu_\mu \to \nu_s$, where ν_s denotes an electroweak-singlet ("sterile") neutrino [?].

In addition, the LSND experiment has reported observing $\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ oscillations with $\Delta m^2_{LSND} \sim 0.1 - 1$ eV^2 and a range of possible mixing angles, depending on Δm^2_{LSND} [?]. This result is not confirmed, but also not completely ruled out, by a similar experiment, KARMEN [?].

There are currently intense efforts to confirm and extend the evidence for neutrino oscillations in all of the various sectors – solar, atmospheric, and accelerator. Some of these experiments are running; these include the Sudbury Neutrino Observatory, SNO, and the above-mentioned K2K long baseline experiment between KEK and Kamioka. Others are in development and testing phases, such as mini-BooNE, MINOS, the CERN - Gran Sasso program, KamLAND, and Borexino [?]. Among the long baseline neutrino oscillation experiments, the approximate distances are $L \simeq 250$ km for K2K, 730 km for both MINOS, from Fermilab to Soudan and the proposed CERN-Gran Sasso experiments. The sensitivity of these experiments is projected to reach down roughly to the level $\Delta m^2 \sim 10^{-3}\text{eV}^2$.

There is strong motivation for another generation of experiments with even higher sensitivity that can confirm the $\nu_\mu \to \nu_\tau$ transition with the values of Δm^2_{atm} and $\sin^2 2\theta_{atm}$ reported so far and carry out new measurements of various neutrino oscillation channels to gain further information, e.g. on θ_{13} and other properties of neutrino masses and mixing. Recently, there has been considerable interest in the idea of a muon storage ring that would serve as a "neutrino factory", i.e., a source of quite high intensity, flavor-pure neutrino and antineutrino beams: $\nu_\mu + \bar{\nu}_e$ ($\bar{\nu}_\mu + \nu_e$) from stored μ^-'s (μ^+'s) respectively. Given the very high intensities, one could envision neutrino oscillation experiments with quite long baselines of order several thousand km, with commensurate sensitivity to several neutrino oscillation channels . Some early papers on matter effects, three-flavor neutrino oscillations, the capabilities of long baseline neutrino oscillation experiments using atmospheric neutrinos and accelerators, include [?]-[?]. More recent discussions, including capabilities of a neutrino factory, are [?]-[?]. Fermilab commissioned working groups to report on the physics and accelerator issues for a neutrino factory [?]. Subsequently, Fermilab commissioned the present follow-up study to assess the capabilities of neutrino oscillation experiments using conventional neutrino beams with luminosity increased, perhaps by a factor of 4, relative to the design of the NUMI beam for MINOS. In particular, the present study addresses to what extent some of the physics program of an entry level ($E_\mu = 20$ GeV, $\mathcal{L} = 10^{19}$ μ decays/yr) can be achieved by experiments with higher-luminosity conventional neutrino beams (and upgraded detectors). Other papers attempting to address this question include [?, ?].

2 Theoretical Background on Neutrino Masses and Mixing

We briefly review here the theoretical background concerning neutrino masses and mixing. In the standard SU(3) $\times$ SU(2)$_L\times$ U(1)$_Y$ model (SM) neutrinos occur in SU(2)$_L$ doublets with $Y=-1$:

$$\mathcal{L}_{L\ell} = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix} \ , \quad \ell = e,\ \mu,\ \tau \tag{3}$$

There are no electroweak-singlet neutrinos (often called right-handed neutrinos) $\chi_{R,j}$, $j = 1, ..., n_s$. Equivalently, these could be written as $\overline{\chi^c}_{L,j}$. There are three types of possible Lorentz-invariant bilinear operator products that can be formed from two two Weyl fermions

ψ_L and χ_R:

- Dirac: $m_D \bar{\psi}_L \chi_R + h.c.$ This connects opposite-chirality fields and conserves fermion number.
- Left-handed Majorana: $m_L \psi_L^T C \psi_L + h.c.$ where $C = i\gamma_2\gamma_0$ is the charge conjugation matrix.
- Right-handed Majorana: $M_R \chi_R^T C \chi_R + h.c.$

The Majorana mass terms connect fermion fields of the same chirality and violate fermion number (by two units). Using the anticommutativity of fermion fields and the property $C^T = -C$, it follows that a Majorana mass matrix appearing as

$$\psi_i^T C (M_{maj})_{ij} \psi_j \tag{4}$$

is symmetric in flavor indices:

$$M_{maj}^T = M_{maj} \tag{5}$$

Thus, in the SM, there is no Dirac neutrino mass term because (i) it is forbidden as a bare mass term by the gauge invariance, (ii) it cannot occur, as do the quark and charged lepton mass terms, via spontaneous symmetry breaking (SSB) of the electroweak (EW) symmetry starting from a Yukawa term because there are no EW-singlet neutrinos $\chi_{R,j}$. There is also no left-handed Majorana mass term because (i) it is forbidden as a bare mass term and (ii) it would require a $I = 1$, $Y = 2$ Higgs field, but the SM has no such Higgs field. Finally, there is no right-handed Majorana mass term because there is no $\chi_{R,j}$. The same holds the minimal supersymmetric standard model (MSSM) and the minimal SU(5) grand unified theory (GUT), both for the original and supersymmetric versions.

However, it is easy to add electroweak-singlet neutrinos χ_R to the SM, MSSM, or SU(5) GUT; these are gauge-singlets under the SM gauge group and SU(5), respectively. Denote these theories as the extended SM, etc. This gives rise to both Dirac and Majorana mass terms, the former via Yukawa terms and the latter as bare mass terms. In the extended SM:

$$-\mathcal{L}_{Yuk} = \sum_{i=1}^{3} \sum_{j=1}^{n_s} h_{ij}^{(D)} \bar{\mathcal{L}}_{L,i} \chi_{R,j} \phi + h.c. \tag{6}$$

The electroweak symmetry breaking (EWSB), with

$$\langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \tag{7}$$

where $v = 2^{-1/4} G_F^{-1/2} \simeq 250$ GeV, yields the Dirac mass term

$$\sum_{i=1}^{3} \sum_{j=1}^{n_s} \bar{\nu}_{L,i} (M_D)_{ij} \chi_{Rj} + h.c. \tag{8}$$

with

$$(M_D)_{ij} = h_{ij}^{(D)} \frac{v}{\sqrt{2}} \tag{9}$$

The Majorana bare mass terms are

$$\sum_{i,j=1}^{n_s} \chi_{Ri}^T C (M_R)_{ij} \chi_{Rj} + h.c. \tag{10}$$

For compact notation, define the flavor vectors $\nu = (\nu_e, \nu_\mu, \nu_\tau)$ and $\chi = (\chi_1, .., \chi_{n_s})$ and observe that one can equivalently write ν_L or ν_R^c and χ_R or χ_L^c, where $\psi^c = C\overline{\psi}^T$, $\overline{\psi} = \psi^\dagger \gamma^0$. One can write the full set of Dirac and Majorana mass terms in the compact matrix form

$$-\mathcal{L}_m = \frac{1}{2} (\bar{\nu}_L \ \overline{\chi^c}_L) \begin{pmatrix} M_L & M_D \\ (M_D)^T & M_R \end{pmatrix} \begin{pmatrix} \nu_R^c \\ \chi_R \end{pmatrix} + h.c. \tag{11}$$

where M_L is the 3×3 left-handed Majorana mass matrix, M_R is a $n_s \times n_s$ right-handed Majorana mass matrix, and M_D is the 3-row by n_s-column Dirac mass matrix. In general, all of these are complex, and $(M_L)^T = M_L$, $(M_R)^T = M_R$. Because the extension of the SM to include χ_R does not include a $I = 1$, $Y = 2$ Higgs field allowing a renormalizable, dimension-4 Yukawa term that would yield a left-handed Majorana mass, one may take $M_L = 0$ at this level (but see below for dimension-5 contributions). The diagonalization of this mass matrix yields the neutrino masses and corresponding transformation relating the neutrino weak eigenstates to the mass eigenstates.

The same comments apply to the extended MSSM and SU(5) GUT. In the extended SU(5) GUT, the Dirac neutrino mass term arises most simply from the Yukawa couplings of the 5_R with a 5-dimensional Higgs representation H^α (in terms of component fields):

$$\bar{\psi}_{R\alpha} M_D \chi_L^c H^\alpha + h.c. \tag{12}$$

and the bare Majorana mass term $\chi_R^T M_R \chi_R + h.c.$.

In the extended SM, MSSM, or SU(5) GUT, one could consider the addition of the χ_R fields as *ad hoc*. However, a more complete grand unification is achieved with the (SUSY) SO(10) GUT, since all of the fermions of a given generation fit into a single representation of SO(10), namely, the 16-dimensional spinor representation ψ_L. In this theory the states

χ_R are not *ad hoc* additions, but are guaranteed to exist. In terms of SU(5) representations (recall, SO(10) $\supset$ SU(5) $\times$ U(1))

$$16_L = 10_L + \bar{5}_L + 1_L \tag{13}$$

so for each generation, in addition to the usual 15 Weyl fermions comprising the 10_L and 5_R, (equivalently $\bar{5}_L$) of SU(5), there is also an SU(5)-singlet, χ^c_L (equivalently, χ_R). So in SO(10) GUT, electroweak-singlet neutrinos are guaranteed to occur, with number equal to the number of SM generations, inferred to be $n_s = 3$. Furthermore, the generic scale for the coefficients in M_R is expected to be the GUT scale, $M_{GUT} \sim 10^{16}$ GeV.

There is an important mechanism, which originally arose in the context of GUT's, but is more general, that naturally predicts light neutrinos. This is the seesaw mechanism [**?**]. The basic point is that because the Majorana mass term $\chi^T_R C M_R \chi_R$ is an electroweak singlet, the associated Majorana mass matrix M_R should not be related to the electroweak mass scale v, and from a top-down point of view, it should be much larger than this scale. Denote this generically as m_R. This has the very important consequence that when we diagonalize the joint Dirac-Majorana mass matrix above, the eigenvalues (masses) will be comprised of two different sets: n_s heavy masses, of order m_R, and 3 light masses. We illustrate this in the simplest case of a single generation and $n_s = 1$. Then the mass matrix is simply

$$-\mathcal{L}_m = \frac{1}{2}(\bar{\nu}_L \ \bar{\chi^c}_L) \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu^c_R \\ \chi_R \end{pmatrix} + h.c. \tag{14}$$

The diagonalization yields the eigenvalues

$$\lambda = \frac{1}{2}\left[m_R \pm \sqrt{m_R^2 + 4m_D^2}\ \right] \tag{15}$$

Since $m_D \sim h^{(D)}v$ while m_R is naturally $>> v$ and hence $m_R >> m_D$, we can expand to get

$$\lambda_> \simeq m_R \tag{16}$$

and

$$\lambda_< \simeq -\frac{m_D^2}{m_R}\left[1 + O\Big(\frac{m_D^2}{m_R^2}\Big)\right] \tag{17}$$

(The $-$ sign is not physically important.) The largeness of m_R then naturally explains the smallness of the masses (or, most conservatively, upper bounds on masses) of the known neutrinos. This appealing mechanism also applies in the physical case of three generations and for $n_s \geq 2$.

At a phenomenological level, without further theoretical assumptions, there is a large range of values for the light m_ν, since (1) the actual scale of m_R is theory-dependent, and

(2) it is, *a priori*, not clear what to take for m_D since the known (Dirac) masses range over 5 orders of magnitude, from $m_e, m_u \sim$ MeV to $m_t = 174$ GeV, and this uncertainty gets squared. However, in the SO(10) GUT scheme, where one can plausibly use $m_D \sim m_t$ for the third-generation neutrino, and $m_R \sim M_{GUT} \sim 10^{16}$ GeV for the scale of masses in the right-handed Majorana mass matrix, one has

$$m(\nu_3) \sim \frac{m_t^2}{m_R} \sim 10^{-3} \text{ eV} \tag{18}$$

which is close to the value $m(\nu_3) = 0.06$ eV obtained from Δm^2_{atm} if one assumes a hierarchical neutrino mass spectrum with $m(\nu_3) >> m(\nu_2)$. Thus, the seesaw mechanism not only provides an appealing qualitative explanation of why neutrino masses are much smaller than the masses of the other known fermions, but also, with plausible assumptions, predicts a value for $m(\nu_3)$ comparable to suggestions from current atmospheric neutrino data.

For the full case with 3 generations and $n_s > 1$, and assuming, as is generic, that $det(M_R) \neq 0$ so that M_R^{-1} exists, the set of 3 light neutrino mass eigenstates is determined by the matrix analogue of eq. (**??**):

$$M_\nu = -M_D M_R^{-1} M_D^T \tag{19}$$

A different way to get neutrino masses is to interpret the SM as a low-energy effective field theory, as is common in modern quantum field theory. One thus allows (nonrenormalizable) operators $\mathcal{O}$ in the Lagrangian of mass dimension $d_{\mathcal{O}} > 4$, provided that their coefficients, of dimension $4 - d_{\mathcal{O}}$ in mass units, are sufficiently small. Then one observes that the dimension-5 operator

$$\mathcal{O} = \frac{1}{M_X} \sum_{a,b} h_{L,ab}(\epsilon_{ik}\epsilon_{jm} + \epsilon_{im}\epsilon_{jk}) \Big[\mathcal{L}^{Ti}_{aL} C \mathcal{L}^{j}_{bL}\Big] \phi^k \phi^m + h.c. \tag{20}$$

(where a, b are flavor indices, i, j, k, m are SU(2) indices) is an electroweak singlet. Upon electroweak symmetry breaking (EWSB), this operator yields a left-handed Majorana mass term

$$\sum_{a,b=1}^{3} \nu^T_{L,a} C (M_L)_{ab} \nu_{L,j} + h.c. \tag{21}$$

with

$$(M_L)_{ab} = \frac{(h_L)_{ab}(v/\sqrt{2})^2}{M_X} \tag{22}$$

Since the SM is phenomenologically very successful, one should have $M_X >> v$, so again these dimension-5 operators lead naturally to light neutrinos. The diagonalization of the above operator determines the unitary transformation relating the mass eigenstates to the weak eigenstates,

$$\nu_{\ell_a} = \sum_{i=1}^{3} U_{ai}\nu_i \ , \quad \ell_1 = e, \ \ell_2 = \mu, \ \ell_3 = \tau \tag{23}$$

i.e.,

$$\left(\begin{array}{c} \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \right) = \left(\begin{array}{ccc} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{array} \right) \left(\begin{array}{c} \nu_1 \\ \nu_2 \\ \nu_3 \end{array} \right) \tag{24}$$

For the case of electroweak-singlet neutrinos and resultant seesaw, because of the splitting of the masses into the light set and the heavy set, the observed weak eigenstates of neutrinos are again, to a very good approximation, linear combinations of the three light mass eigenstates, so that the full $(3+n_s)\times(3+n_s)$ mixing matrix breaks into block diagonal form involving the 3×3 U matrix and an analogous $n_s \times n_s$ matrix for the heavy sector. In terms of the flavor vectors, this is

$$\left(\begin{array}{c} \nu_\ell \\ \chi^c \end{array} \right) = \left(\begin{array}{cc} U & 0 \\ 0 & U_{heavy} \end{array} \right) \left(\begin{array}{c} \nu_i \\ \chi^c_{i,m} \end{array} \right) \tag{25}$$

If one accepts all of the data indicating neutrino masses, including the solar neutrino deficiency, atmospheric neutrinos, and LSND experiments, then one needs light sterile (electroweak-singlet) neutrinos with masses of $\sim$ eV or smaller. These are usually considered unnatural, because electroweak-singlet neutrinos naturally have masses $\sim m_R >> M_{ew} = v$. If such light χ_R's exist, this tends to undermine the reasoning on which the seesaw mechanism is based.

If the LSND claims are confirmed by mini-BOONE, there will be a strong motivation for experiments with high-intensity neutrino beams with rather low energy, $E_\nu \sim 1$ GeV and relatively short path lengths $L \sim 1$ km, in addition to long baseline neutrino oscillation experiments. Furthermore, in order to fit all of the neutrino data reasonly well, one will then need a light sterile neutrino, so that the lepton mixing will involve the full complexity of at least a 4×4 mixing matrix. However, since the LSND experiment has not so far been confirmed, we shall work within the framework of three active neutrinos. Although electroweak singlet neutrinos may be present in the theory, we will generally assume that they are much heavier than the mass scales of interest here, in accordance with the seesaw mechanism. With this assumption, the neutrino mixing can be described by the 3×3 unitary matrix

$$U = R_{23}KR_{13}K^*R_{12}K' \tag{26}$$

where R_{ij} is a rotation matrix in the ij subsector. Writing this out gives

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} K' \tag{27}$$

where R_{ij} is the rotation matrix in the ij subspace, $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, $K = diag(e^{-i\delta}, 1, 1)$ and K' involves further possible phases due to Majorana mass terms that will contribute here. Thus, in this framework, the neutrino mixing depends on the four angles θ_{12}, θ_{13}, θ_{23}, and δ, and on two independent differences of squared masses, $\Delta m^2_{atm.}$, which is $\Delta m^2_{32} = m(\nu_3)^2 - m(\nu_2)^2$ in the favored fit, and $\Delta m^2_{sol.}$, which may be taken to be $\Delta m^2_{21} = m(\nu_2)^2 - m(\nu_1)^2$. Note that these quantities involve both magnitude and sign; although in a two-species neutrino oscillation in vacuum the sign does not enter, in the three species oscillations relevant here, and including both matter effects and CP violation, the signs of the Δm^2 quantities do enter and can, in principle, be measured.

In the 1980's, most theorists thought that lepton mixing would be hierarchical, i.e. the lepton mixing matrix U would differ from the identity by small entries, and these would be smaller as one moved further from the diagonal, as is established to be the case with quark mixing. This was, indeed, a large part of the appeal of the MSW mechanism: it could produce large mixing with small vacuum mixing angles. However, the results from the SuperK measurements of atmospheric neutrinos have forced a revision in this conventional picture, providing strong evidence for essentially maximal mixing, $\sin^2(2\theta_{23}) = 1$. A challenge to model-builders has thus been to get maximal $\sin^2(2\theta_{23})$. More recently, the SuperK solar neutrino data favors large $\sin^2(2\theta_{12})$. Bimaximal mixing schemes take $\theta_{23} = \theta_{12} = \pi/4$ and $\theta_{13} << 1$ [**?**]. There are no compelling theoretical suggestions concerning the magnitude of θ_{13}, and one of the important physics goals for neutrino oscillation experiments with conventional beams is to try to measure this angle.

For our later discussion it will be useful to record the formulas for the various relevant neutrino oscillation transitions. In the absence of any matter effect, the probability that a (relativistic) weak neutrino eigenstate ν_a becomes ν_b after propagating a distance L is

$$\begin{aligned} P(\nu_a \to \nu_b) &= \delta_{ab} - 4\sum_{i>j=1}^{3} Re(K_{ab,ij}) \sin^2\Big(\frac{\Delta m^2_{ij} L}{4E}\Big) + \\ &+ 4\sum_{i>j=1}^{3} Im(K_{ab,ij}) \sin\Big(\frac{\Delta m^2_{ij} L}{4E}\Big) \cos\Big(\frac{\Delta m^2_{ij} L}{4E}\Big) \end{aligned} \tag{28}$$

where

$$K_{ab,ij} = U_{ai}U^*_{bi}U^*_{aj}U_{bj} \tag{29}$$

and

$$\Delta m^2_{ij} = m(\nu_i)^2 - m(\nu_j)^2 \tag{30}$$

Recall that in vacuum, CPT invariance implies $P(\bar{\nu}_b \to \bar{\nu}_a) = P(\nu_a \to \nu_b)$ and hence, for $b = a$, $P(\bar{\nu}_a \to \bar{\nu}_a) = P(\nu_a \to \nu_a)$. For the CP-transformed reaction $\bar{\nu}_a \to \bar{\nu}_b$ and the T-reversed reaction $\nu_b \to \nu_a$, the transition probabilities are given by the right-hand side of (**??**) with the sign of the imaginary term reversed. (Below we shall assume CPT invariance, so that CP violation is equivalent to T violation.)

The solar and atmospheric neutrino data indicate that

$$\Delta m^2_{21} = \Delta m^2_{sol} \ll \Delta m^2_{31} \approx \Delta m^2_{32} = \Delta m^2_{atm} \tag{31}$$

In this case, CP (T) violation effects are very small, so that in vacuum

$$P(\bar{\nu}_a \to \bar{\nu}_b) \simeq P(\nu_a \to \nu_b) \tag{32}$$

$$P(\nu_b \to \nu_a) \simeq P(\nu_a \to \nu_b) \tag{33}$$

In the absence of T violation, the second equality (**??**) would still hold in matter, but even in the absence of CP violation, the first equality (**??**) would not hold. With the hierarchy (**??**), the expressions for the specific oscillation transitions are

$$\begin{aligned} P(\nu_\mu \to \nu_\tau) &= 4|U_{33}|^2|U_{23}|^2 \sin^2\Big(\frac{\Delta m^2_{atm} L}{4E}\Big) \\ &= \sin^2(2\theta_{23}) \cos^4(\theta_{13}) \sin^2\Big(\frac{\Delta m^2_{atm} L}{4E}\Big) \end{aligned} \tag{34}$$

$$\begin{aligned} P(\nu_\mu \to \nu_e) &= 4|U_{13}|^2|U_{23}|^2 \sin^2\Big(\frac{\Delta m^2_{atm} L}{4E}\Big) \\ &= \sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin^2\Big(\frac{\Delta m^2_{atm} L}{4E}\Big) \end{aligned} \tag{35}$$

$$\begin{aligned} P(\nu_e \to \nu_\tau) &= 4|U_{33}|^2|U_{13}|^2 \sin^2\Big(\frac{\Delta m^2_{atm} L}{4E}\Big) \\ &= \sin^2(2\theta_{13}) \cos^2(\theta_{23}) \sin^2\Big(\frac{\Delta m^2_{atm} L}{4E}\Big) \end{aligned} \tag{36}$$

With units inserted, one has the general relation

$$\sin^2\Big(\frac{\Delta m^2 L}{4E}\Big) = \sin^2\Big(\frac{1.27(\Delta m^2/\mathrm{eV}^2)(L/\mathrm{km})}{(E/\mathrm{GeV})}\Big) \tag{37}$$

This makes it clear what the approximate sensitivity of an experiment with a given path-length is to a neutrino oscillation channel involving a given Δm^2, for a beam with an energy E.

In neutrino oscillation searches using reactor antineutrinos, i.e. tests of $\bar{\nu}_e \to \bar{\nu}_e$, the two-species mixing hypothesis used to fit the data is

$$P(\nu_e \to \nu_e) = 1 - \sin^2(2\theta_{reactor}) \sin^2\Big(\frac{\Delta m^2_{reactor} L}{4E}\Big) \tag{38}$$

where $\Delta m^2_{reactor}$ is the squared mass difference relevant for $\bar{\nu}_e \to \bar{\nu}_x$. In particular, in the upper range of values of Δm^2_{atm}, since the transitions $\bar{\nu}_e \to \bar{\nu}_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\tau$ contribute to $\bar{\nu}_e$ disappearance, one has

$$P(\nu_e \to \nu_e) = 1 - \sin^2(2\theta_{13}) \sin^2\Big(\frac{\Delta m^2_{atm} L}{4E}\Big) \tag{39}$$

i.e., $\theta_{reactor} = \theta_{13}$, and the Chooz reactor experiment yields the bound [**?**]

$$\sin^2(2\theta_{13}) < 0.10 \tag{40}$$

which is also consistent with conclusions from the SuperK data analysis [**?**].

Further, the quantity "$\sin^2(2\theta_{atm})$" often used to fit the data on atmospheric neutrinos with a simplified two-species mixing hypothesis, is, in the three-generation case,

$$\sin^2(2\theta_{atm}) \equiv \sin^2(2\theta_{23}) \cos^4(\theta_{13}) \tag{41}$$

The SuperK data implies that (up to redefinitions of quadrants, etc.)

$$\theta_{23} \simeq \frac{\pi}{4} \tag{42}$$

and $\sin^2(2\theta_{13}) << 1$. Thus, to good accuracy, $\theta_{atm} = \theta_{23}$.

The types of neutrino oscillations that can be searched for with a conventional neutrino beam include

- $\nu_\mu \to \nu_\mu$ (disappearance)
- $\nu_\mu \to \nu_e$, $\nu_e \to e^-$ (appearance)
- $\nu_\mu \to \nu_\tau$, $\nu_\tau \to \tau^-$; $\tau^- \to (e^-, \mu^-)$... (appearance)

and the conjugate oscillation channels using $\bar{\nu}_\mu$ beams. Since these have lower fluxes than ν_μ beams (and this difference can be made large with sign-selected π beams that are decaying), one can concentrate on oscillation channels with ν_μ beams.

For neutrino oscillation experiments with pathlengths of order 10^3 km, matter effects are significant. These have been studied in a number of papers, e.g., [?, ?, ?, ?, ?]. The constant density assumption provides a first approximation; realistic density profiles were included in the calculations of [?]. In the constant density approximation, for a simple two-species mixing, one has

$$P(\nu_a \to \nu_b) = \sin^2(2\theta_m)\sin^2(\omega L) \tag{43}$$

where

$$\sin^2(2\theta_m) = \frac{\sin^2(2\theta)}{\sin^2(2\theta) + \left[\cos(2\theta) - \frac{2\sqrt{2}G_F N_e E}{\Delta m^2}\right]^2} \tag{44}$$

$$\omega^2 = \left[\frac{\Delta m^2}{4E}\cos(2\theta) - \frac{G_f}{\sqrt{2}}N_e\right]^2 + \left[\frac{\Delta m^2}{4E}\sin(2\theta)\right]^2 \tag{45}$$

where N_e is the electron number density of the matter.

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