23 Gauge Mediation: Part I

23.1 Modules

The basic idea of gauge mediation is that there are three sectors in the theory, a dynamical SUSY breaking sector, a messenger sector, and the MSSM. SUSY breaking is communicated to the messenger sector so that the messengers have a SUSY breaking spectrum. They also have SM gauge interactions, which then communicate SUSY breaking to the ordinary superpartners. We will take a model with N_f messengers ϕ_i, $\overline{\phi}_i$ and a Goldstino multiplet X

$$\langle X \rangle = M + \theta^2 F \tag{23.1}$$

with a superpotential

$$W = X\overline{\phi}_i\phi_i \ . \tag{23.2}$$

In order to preserve gauge unification, ϕ_i and $\overline{\phi}_i$ should form complete GUT multiplets. This shifts the coupling at the GUT scale by

$$\delta\alpha_{\mathrm{GUT}}^{-1} = -\frac{N}{2\pi}\ln\left(\frac{\mu_{\mathrm{GUT}}}{M}\right) \ , \tag{23.3}$$

where

$$N = \sum_{i=1}^{N_f} 2T(r_i) \ . \tag{23.4}$$

For the unification to remain perturbative we need

$$N < \frac{150}{\ln\left(\frac{\mu_{\mathrm{GUT}}}{M}\right)} \tag{23.5}$$

The VEV of X gives each messenger fermions a mass M, and the scalars squared masses $M^2 \pm F$. We will be interested in the case that $F \ll M^2$. We can construct an effective theory by integrating out the messengers.

23.2 RG Calculation of Masses

The pure gauge part of the Lagrangian is given by:

$$\mathcal{L}_G = -\frac{i}{16\pi}\int d^2\theta\,\tau(X,\mu)W^\alpha W_\alpha \tag{23.6}$$

Taylor expanding in the F component of X we find a gaugino mass

$$\begin{aligned} M_\lambda &= \frac{i}{2\tau}\frac{\partial\tau}{\partial X}|_{X=M}F \\ &= \frac{i}{2}\frac{\partial\ln\tau}{\partial\ln X}|_{X=M}\frac{F}{M} \end{aligned} \tag{23.7}$$

Since

$$\tau(X,\mu) = \tau(\mu_0) + i\frac{b'}{2\pi}\ln\left(\frac{X}{\mu_0}\right) + i\frac{b}{2\pi}\ln\left(\frac{\mu}{X}\right) \tag{23.8}$$

where

$$b' = b - N \tag{23.9}$$

So

$$M_\lambda = \frac{\alpha(\mu)}{4\pi}N\frac{F}{M} \tag{23.10}$$

Next consider the wavefunction renormalization for the matter fields of the MSSM

$$\mathcal{L} = \int d^4\theta Z(X, X^\dagger)Q^\dagger Q \tag{23.11}$$

where Z must be real. Taylor expanding we have

$$\begin{aligned} \mathcal{L} = \int d^4\theta \quad & \left(Z + \frac{\partial Z}{\partial X}F\theta^2 + \frac{\partial Z}{\partial X^\dagger}F^\dagger\theta^{\dagger 2} + \frac{\partial^2 Z}{\partial X\partial X^\dagger}F\theta^2F^\dagger\theta^{\dagger 2}\right)|_{X=M} \\ & Q^\dagger Q \end{aligned} \tag{23.12}$$

Canonically normalizing we have:

$$Q' = Z^{1/2}\left(1 + \frac{\partial Z}{\partial X}F\theta^2\right)|_{X=M}Q \tag{23.13}$$

$$\begin{aligned} \mathcal{L} = \int d^4\theta \quad & \left[1 - \left(\frac{\partial\ln Z}{\partial X}\frac{\partial\ln Z}{\partial X^\dagger} - \frac{1}{Z}\frac{\partial^2 Z}{\partial X\partial X^\dagger}\right)F\theta^2F^\dagger\theta^{\dagger 2}\right]|_{X=M} \\ & Q'^\dagger Q' \end{aligned} \tag{23.14}$$

So we have a sfermion mass term:

$$m_Q^2 = -\frac{\partial^2 \ln Z}{\partial \ln X \partial \ln X^\dagger}|_{X=M} \frac{FF^\dagger}{MM^\dagger} \tag{23.15}$$

Rescaling the matter fields also introduces an A term in the effective potential from Taylor expanding the superpotential:

$$Z^{-1/2} \frac{\partial \ln Z}{\partial X}|_{X=M} FQ' \frac{\partial W}{\partial Z^{-1/2} Q'} \; , \tag{23.16}$$

which is suppressed by a Yukawa coupling. To calculate Z, we do a SUSY calculation and replace M by $\sqrt{XX^\dagger}$. At l loops an RG analysis gives

$$\ln Z = \alpha(\mu_0)^{l-1} f(\alpha(\mu_0) L_0, \alpha(\mu_0) L_X) \tag{23.17}$$

where

$$L_0 = \ln\left(\frac{\mu^2}{\mu_0^2}\right) \tag{23.18}$$

$$L_X = \ln\left(\frac{\mu^2}{XX^\dagger}\right) \tag{23.19}$$

so

$$\frac{\partial^2 \ln Z}{\partial \ln X \partial \ln X^\dagger} = \alpha(\mu)^{l+1} h(\alpha(\mu) L_X) \tag{23.20}$$

So the two-loop scalar masses are determined by a one-loop RG eq.

At one-loop we have

$$\frac{d \ln Z}{d \ln \mu} = \frac{C_2(r)}{\pi} \alpha(\mu) \tag{23.21}$$

so

$$Z(\mu) = Z_0 \left(\frac{\alpha(\mu_0)}{\alpha(X)}\right)^{\frac{2C_2(r)}{b'}} \left(\frac{\alpha(X)}{\alpha(\mu)}\right)^{\frac{2C_2(r)}{b}} \; , \tag{23.22}$$

where

$$\alpha^{-1}(X) = \alpha^{-1}(\mu_0) + \frac{b'}{4\pi} \ln\left(\frac{XX^\dagger}{\mu_0^2}\right) \tag{23.23}$$

$$\alpha^{-1}(\mu) = \alpha^{-1}(X) + \frac{b}{4\pi} \ln\left(\frac{\mu^2}{XX^\dagger}\right) \tag{23.24}$$

So

$$m_Q^2 = 2C_2(r)\frac{\alpha(\mu)^2}{16\pi^2}N\left(\xi^2 + \frac{N}{b}(1-\xi^2)\right)\left(\frac{F}{M}\right)^2 \ , \tag{23.25}$$

where

$$\xi = \frac{1}{1 + \frac{b}{2\pi}\alpha(\mu)\ln\frac{M}{\mu}} \tag{23.26}$$

References

[1] G.F. Giudice and R. Rattazzi hep-ph/9706540, hep-ph/9801271.