Theoretical framework for a dynamic cone-beam reconstruction algorithm based on a dynamic particle model

Pierre Grangeat, Anne Koenig, Thomas Rodet, Stéphane Bonnet

Abstract—**Dynamic cone-beam reconstruction algorithms are required to reconstruct 3D image sequences on dynamic 3D CT combining multi-row 2D detectors and ultra-fast rotating gantry. In order to compensate for time evolution and motion artifacts, we propose to use a dynamic particle model to describe the object evolution. One main interest is to process data acquisition on several half-turns in order to reduce the dose delivered per rotation with the same signal to noise ratio. We describe the dynamic particle model and its approximations, the dynamic cone-beam CT acquisition model and the dynamic cone-beam reconstruction algorithm based on a cone-beam to fan-parallel beam rebinning approach.**

Index Terms—**Dynamic tomographic imaging, fully four-dimensional image reconstruction, CT Fluoroscopy, cone-beam, particle model, motion compensation, time evolution.**

I. INTRODUCTION

THE purpose of dynamic computed tomography (CT) imaging is to reconstruct tomographic image sequences of dynamic organs in order to take into account the dynamic nature of a living human body. The description of dynamic organs includes both time evolution and motion. In this publication, we mainly focus on dynamic 3D Computed Tomography combining multi-row 2D detectors and ultra-fast rotating gantry. The main applications are 3D CT Fluoroscopy for interventionnal radiology, to help the radiologist to guide biopsy needles through soft tissues like the lung, radiotherapy planning to better delineate the tumour and healthy tissues during motion, heart diagnostic imaging to study heart kinetic or to reconstruct coronary arteries.

LETI is involved in the European project DynCT (IST - 1999 - 10515) dedicated to both real time and off-line motion compensated reconstruction and visualisation for dynamic computed tomography. Only the real time case is described here. In this presentation, we introduce the theoretical framework of a new dynamic cone-beam reconstruction algorithm based on a dynamic particle model. Standard approach tends to use a short scan acquisition over one half-turn without motion compensation [Taguchi, 2000]. But it implies to increase the dose delivered per rotation to preserve the signal to noise ratio. Using the dynamic particle model to compensate for dynamic evolution, it becomes possible to increase the acquisition time window over several half-turns, up to 4 in our case, in order to reduce the dose delivered per rotation to the patient with the same signal to noise ratio.

In [Hsieh, 1997] the impact of various reconstruction algorithms on 2D CT Fluoroscopy is investigated and inherent limitations of the CT Fluoroscopy in terms of time lag and delay is demonstrated. In [Taguchi, 1998], the authors propose to use a feathering technique to suppress an image artifact which rotates like a radar search line and sometimes hinders accurate observations. In [Ritchie, 1996], the reduction of in plane motion artifacts is achieved using a pixel-specific back projection technique in 2D. In [Schäffter, 1999] a motion compensated projection reconstruction algorithm is proposed for the reduction of blurring artifacts in MRI using motion estimation applied on a first set of low resolution reconstructed images. The new algorithm proposed here uses also this double reconstruction principle. It allows to compensate for both time evolution and motion.

II. THE DYNAMIC PARTICLE MODEL

We consider the image function f as the map of the physical property we want to study, in the present case the X-ray linear attenuation coefficient or the density. This function f is defined in a (O, x, y z) cartesian coordinate system. We represent by $f(M, t) = f(x, y, z, t)$ the value of the function f at the point M of coordinate (x, y, z), at the time instant t. We suppose here that the function f is sufficiently smooth and vanishes outside a ball Ω of radius R_0.

In order to define the particle model, we describe at the reference instant t_0 the object as a continuum of particles associated with each point M of the support Ω. We define the trajectory $\Gamma(M, t)$ associated with this point M as the set of positions this point M will take at each time value t within Ω. By definition, M corresponds to the initial position along the trajectory $\Gamma(M, t)$:

$$\Gamma(M, t_0) = M \tag{1}$$

In the following, the image sequence will be reconstructed at discrete time samples t_i.

In the more general case where both time evolution and motion have to be taken into account, the function f may vary along the trajectory Γ. Thus, the general function expression associated with the dynamic particle model is given by the

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formula $f(\Gamma(M,t),t)$, where $\Gamma(M,t)\in\Omega$ and $t\in R$. This corresponds to a continuous expression of the discrete particle models used in computer graphics to describe deformable objects [Lombardo, 1996].

One example is a flying ball of constant radius which content might vary along time due to contrast product injection or to matter exchange with the neighbouring flying balls. In the special case of matter conservation and non-compressible material, it would be equivalent to assume the content remains constant along time.

The time evolution compensation will be based on the first order approximation of the expression $f(\Gamma(M,t),t)$, in the neighborhood of the discrete time sample t_i:

$$f(\Gamma(M,t),t)\approx f(\Gamma(M,t_i),t_i)+[<\nabla f, \frac{\partial\Gamma}{\partial t}(M,t_i)>+\frac{\partial f}{\partial t}(\Gamma(M,t_i),t_i](t-t_i) \quad (2)$$

where ∇f is the gradient of f and < , > the scalar product.

When the object fulfills the mass conservation principle and is irreducible, the term in bracket [] is null and we get:

$$f(\Gamma(M,t),t)=f(\Gamma(M,t_i),t_i) \quad (3)$$

However, in the reality, some important tissues like the lung are reducible and since the organs may move outside the field of view, the mass conservation principle is not always fulfilled. Thus we need to consider the general case.

In order to later simplify the computation, we introduce a cartoon like step-by-step motion law :

$$\Gamma(M,t) = \Gamma(M,t_i) \text{ for } t_i\leq t<t_{i+1} \quad (4)$$

Then, since $\frac{\partial\Gamma}{\partial t}$ is null, the first order approximation (2) of the dynamic model becomes:

$$f(\Gamma(M,t),t)=f(\Gamma(M,t_i),t_i)+\frac{\partial f}{\partial t}(\Gamma(M,t_i),t_i)(t-t_i) \quad (5)$$

In this case, this formula (6) can be approximated by a linear prediction law with respect to t between two time samples, for instance t_i and $t_i - T$, where T is the rotation period of the continuously rotating scanner:

$$f(\Gamma(M,t),t)\approx f(\Gamma(M,t_i),t_i)+\left[\frac{f(\Gamma(M,t_i),t_i)-f(\Gamma(M,t_i),t_i-T)}{T}\right]\cdot(t-t_i) \quad (6)$$

III. THE DYNAMIC CONE-BEAM CT ACQUISITION MODEL

1. *Dynamic cone-beam projection*

We consider the cone-beam geometry associated with a curved multi-row detector centred on the X-ray source F as described on the figure 1.

The dynamic cone-beam geometry is parametrized by the angle β between (F,O) and y axis, the angle γ between the detector column and the (F,O) axis, the row height q_d with respect to the trajectory plane, and the acquisition time t (see fig. 1). It is defined as follows :

$$X_c f(\beta,\gamma,q_d,t)=\int_{M\in D(\beta,\gamma,q_d)} f(M,t)dM \quad (7)$$

where $D(\beta,\gamma,q_d)$ is the straight line between the source point F and the detector cell A_d. In order to model a continuously rotating acquisition process, we state the following relation :

$$\beta(t) = \beta_0 + \omega.t \quad (8)$$

where ω defines the angular rotation speed. For a CT scanner with a 0.5 s rotation period, $\omega = 4\pi$ rad.s^{-1}.

In the following, we assume the angle γ belongs to $[-\gamma_m,+\gamma_m]$ where γ_m is the half fan angle. In our case we choose $\gamma_m=\frac{\pi}{6}$rad.

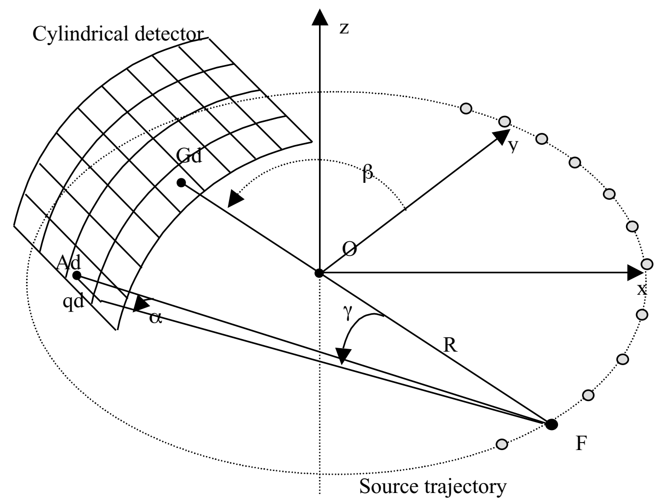

Figure 1 : cone-beam geometry.

2. *Dynamic fan-parallel projection*

The reconstruction will be done via the line rebinning approach in fan-parallel projection as suggested in [Grass, 2000].

The fan-parallel geometry is defined by a virtual planar detector placed on the rotation axis with coordinates p parallel to the trajectory and q parallel to the rotation axis. We define φ as the angle between the virtual detector and the (0,x) axis.

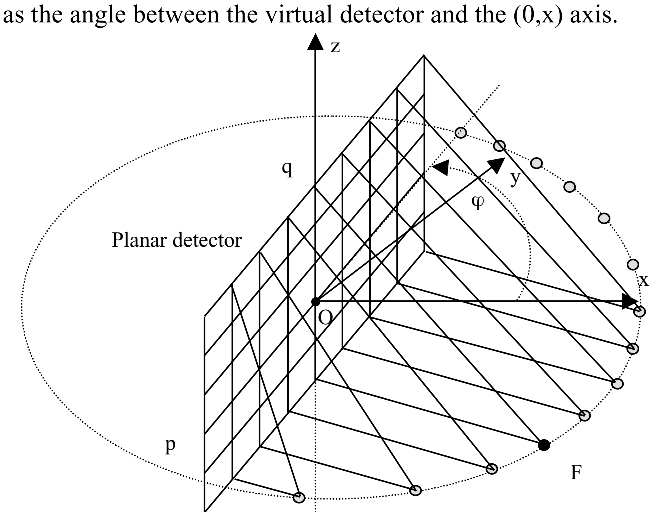

Figure 2 : fan-parallel-beam geometry.

The dynamic fan-parallel projection is defined as follows :

$$X_{fp}f(\varphi,p,q,t)=\int_{M\in D(\varphi,p,q)} f(M,t)dM \quad (9)$$

In the ideal case, the full fan-parallel projection data set is available for each t. Then, the short scan dynamic fan-parallel reconstruction formula is:

$$f(\Gamma(M,t),t) = \int_0^{\pi} HDYf\,(\varphi, A\,[\Gamma\,(M, t)], t)\, d\varphi \tag{10}$$

where Yf is the weighted fan-parallel projection:

$$Yf(\varphi,A,t) = X_{fp},f(\varphi,A,t).w(A) \tag{11}$$

and $w(A) = \dfrac{\sqrt{R^2 - p^2}}{\sqrt{R^2 - p^2 + q^2}}$

A [Γ (M, t)] is the fan-parallel projection of the point Γ(M,t) onto the virtual detector, with detector coordinates p,q.

HDYf is the weighted projection Yf convoluted along the transverse row by the ramp filter HD(p).

IV. The dynamic cone-beam reconstruction algorithm

The reconstruction algorithm described here is dedicated to 3D CT fluoroscopy, assuming a real-time reconstruction processing at a frame rate of 12 frames per second for a gantry rotation period T of 0.5 s.

1. The sliding window principle

Let us consider the discrete fan-parallel angles :

$$\varphi_i = i\,\Delta\varphi \tag{12}$$

where Δφ is the angular step between two reconstructed frames. In the following, we assume $\Delta\varphi = 2.\,\gamma_m$. It means we divide the full rotation in constant angular positions separated from the full fan angle $2.\,\gamma_m$ equal to $\frac{\pi}{3}$. For a 0.5 s gantry rotation period, this corresponds to a frame rate of 12 images per second.

The associated discrete reconstruction time t_i associated with the last projection of the angular range $[\varphi_i, \varphi_i+\Delta\varphi]$ is :

$$t_i = \frac{\left(\varphi_i + \Delta\varphi\right) + \gamma_m - \beta_0}{\omega} \tag{13}$$

where $\frac{\gamma_m}{\omega}$ corresponds to the rebinning latency delay to begin the fan-parallel reconstruction.

We define the overscan φ angular range to compute $f(\Gamma(M,t_i),t_i)$ as the φ sliding window $[\varphi_i - n.\pi, \varphi_i+\Delta\varphi]$ where n represents the number of half-turns on which we want to smooth the data to improve the signal to noise ratio or to reduce the dose.

The associated β angular range defined by the following rebinning equation (14) is the β sliding window $[\varphi_i-(n.\pi+\frac{\Delta\beta}{2}),\varphi_i+\frac{3}{2}\Delta\beta]$, where $\Delta\beta=\Delta\varphi=2.\gamma_m$.

The sliding window principle is that a new frame is computed for each new t_i value. This new frame corresponds to a shift of the β sliding window from Δβ, and the associated shift of the φ sliding window from Δφ.

2. The cone-beam to fan-parall-beam rebinning

We get the following rebinning equation :

$$X_{fp}f(\varphi, p, q, t) = X_c f(\beta, \gamma, q_d, t) \tag{14}$$

with :

$$\begin{cases} \beta = \varphi - \gamma \\ \gamma = \arcsin\left(-\dfrac{p}{R}\right) \\ q_d = q\left(\dfrac{FGd}{\sqrt{R^2 - p^2}}\right) \end{cases} \tag{15}$$

However, we need to compute $X_{fp}f(\varphi, p, q\ t_i)$ at t_i, the reconstruction time associated with the time range $[t_{i-1}, t_i]$.

When the time difference $(t_i$-t) is small, we can use a nearest neighbour interpolation:

$$X_{fp}f(\varphi,p,q,t_i) = X_{fp}f(\varphi,p,q,t) \tag{16}$$

Otherwise, if the sliding window is larger than 2π, using the linear interpolation model associated with equation (6) we get the following extrapolation formula :

$$X_{fp}f(\varphi,p,q,t_i) = \frac{t_i - \left(t - T\right)}{T} X_c f(\beta,\gamma,q_d,t) + \frac{t_i - t}{T} X_c f(\beta,\gamma,q_d,t-T) \tag{17}$$

where T is the gantry rotation period.

3. Block reconstruction

We split the φ angular range into elementary projection blocks of size $\Delta\varphi = \frac{\pi}{3}$. We denote $BHDYf(\Gamma(M,t),t,\varphi_i)$ the partial block backprojection over the projection angular $[\varphi_i, \varphi_i + \Delta\varphi]$:

$$BHDYf(\Gamma(M, t),t,\varphi_i) = \int_{\varphi = \varphi_i}^{\varphi_i + \Delta\varphi} HDYf\left(\varphi, A\left[\Gamma\left(M, t\right)\right] t\right) d\,\varphi \tag{18}$$

For seak of simplicity, we assume here we want to reconstruct the function at the instant $t = t_0$. Thus $\Gamma(M,t) = M$. The following result can then be generalized for each t_i time by shifting the sliding window.

From the reconstruction formula (10), we get:

$$f(M,t_0) = \int_0^{\pi} HDY\,f\left(\varphi, A\,[M], t_0\right) d\varphi \tag{19}$$

Since $\Delta\varphi = \frac{\pi}{3}$, we can decompose this integral into 3 terms associated with 3 partial block backprojections sectors $[0, \frac{\pi}{3}]$, $[\frac{\pi}{3}, \frac{2\pi}{3}]$, $[\frac{2\pi}{3}, \pi]$:

$$f(M,t_0) = \sum_{i=0}^{2} BHDY\,f\,(M,t_0, \varphi_i) \tag{20}$$

The dynamic evolution compensation will take place in the estimation of each partial block backprojection as described in the next section.

4. Dynamic evolution compensation

Given an angular sector i, let us take a set of N_b partial block projection acquired in the past for the same angular range modulo π :

$\varphi_{ij} = \varphi_i - j.\pi \qquad j = 0,...., N_b - 1$

We get for each M point a set of values along the $\Gamma(M,t)$ trajectory for each associated block instant $BHDYf(\Gamma(M,t_{ij}),t_{ij}\ \varphi_{ij})$, where t_{ij} is the time associated with φ_{ij} according to equation (13). Using the first order approximation (2) of the dynamic particle model, we get :

$$f(\Gamma(M,t_{ij}),t_{ij}) \approx f(M,t_0)+[<\nabla f, \frac{\partial \Gamma}{\partial t}(M,t_0)>+\frac{\partial f}{\partial t}(M,t_0)](t_{ij}-t_0) \qquad (21)$$

Under the piecewise constant motion hypothesis (4), as no motion occurs during the block angular range, the same relation holds for the partial backprojection:

$$BHDYf(\Gamma(M,t_{ij}),t_{ij},\varphi_{ij}) \approx BHDYf(M,t_0,\varphi_{i0}) + a(M,\varphi_{i0}).(t_{ij}-t_0) \qquad (22)$$

Thus, the terms $BHDYf(M,t_0,\varphi_{i0})$ and $a(M,\varphi_{i0})$ can be computed by linear regression on the discrete sample set:

$$\left\{BHDYf\left(\Gamma\left(M, t_{ij}\right), t_{ij}, \varphi_{ij}\right)\right\} \ j\in\left\{0,\ldots, Nb\text{-}1\right\}.$$

In order to apply such a linear regression, we need to have at least two samples belonging to the sliding window. Otherwise using the same relation, we can only compensate for motion, and not for time evolution. The motion compensation equation is given by:

$$BHDYf(\Gamma(M,t_{ij}),t_{ij},\varphi_{ij}) = BHDYf(M,t_0,\varphi_{i0}) \qquad (23)$$

This is equivalent to choose a zero order regression model. In every case, it is important to note that using an overscan range within the sliding window, the dynamic evolution compensation by regression introduces a principle equivalent to the feathering technique used by different authors in CT fluoroscopy [Taguchi, 1998].

5. *Motion estimation*

In the previous sections, we have assumed we can measure the motion field between M and Γ (M, t_{ij}). However, as this motion field is unknown, we need to estimate it. We present here only the basic idea. Detailed explanation will be given in the final version.

The first order approximation (2) can be interpreted as the sum of a spatial shift term and a time evolution term. In first approximation, we will assume here that the shift can be approximated by a translation motion at constant speed:

$$\Gamma(M,t) = M + D(M).\frac{2\left(t-t_0\right)}{T} \qquad (24)$$

where D(M) is the displacement of the point M after a π rotation of the gantry and T is the gantry rotation period.

The reconstruction process without motion compensation will produce a blurred image. However, the reconstruction after a π rotation will produce the same blurred image, shifted from the displacement vector D(M). Thus, using a correlation principle, it will be possible to estimate this displacement vector. Such a correlation approach should not be too much disturbed by time evolution.

In order to reduce the blurring spread, the image sequence needed to evaluate the motion should be reconstructed with the smallest temporal resolution corresponding to a half-turn rotation.

In fact, we need to get the full motion field over the region to reconstruct. The actual technique we use is a block matching approach as for motion estimation in MPEG coding [Sikora, 1997].

One other important issue is the ability to detect when the particle trajectory goes outside the region of interest. In the approach described here, we can detect it since the correlation fails. The prediction should be applied only to those points which are inside the region of interest. To manage this issue, we estimate for each motion vector a confidence factor associated with the correlation factor, and we take it into account within the estimation of the prediction rule coefficients given by the equation (22).

V. CONCLUSION

In this paper, we present the outline of the theoretical framework for time evolution and motion compensation in dynamic cone-beam reconstruction algorithm using a dynamic particle model. The principle described here for a line rebinning algorithm can also be applied to direct cone-beam reconstruction algorithm such as Feldkamp algorithm.

However, according to [Grass, 2000], this line rebinning approach can be generalized to reconstruct a larger region of interest by correctly handling the line shadow zone region. The approach described here can also be extended to such a case. Further investigation are needed to extend this dynamic approach to indirect plane rebinning algorithm via the first derivative of the Radon transform using Grangeat formula.

Preliminary results will be presented at the conference.

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