K-correction Errors

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ABSTRACT

The K-correction errors as a function of supernova redshift and color-error are determined. The size of the errors depends on the wavelength spacing between observer filters. The errors can be constrained to less than 2% with $1.15 \times \lambda$ spacing between filters.

1. The K-correction

Note that K-corrections would be trivial if we had spectra for each supernova at each epoch of interest. The K-corrections from each specific spectrum could then be calculated. In practice (and for SNAP) we expect only one spectrum near maximum light. Supernovae are assumed to be homogeneous enough that K-corrections of individual events can be inferred from a spectral library. The discussion below assumes the latter scenario.

It has been found by Peter Nugent and confirmed by my studies that the observed color of a supernova is a very robust indicator of K-correction. In fact, the K-corrections at two very different epochs but with similar colors are closer to identical. This is not unexpected as the spectrum of an SN Ia at earlier epochs can be considered as continuum with relatively small and smooth spectral features. Color is a better indicator of K-correction than epoch, considering that color can be obtained in a single nights observing and due to inhomogeneities in the time-scale of light curves.

I have taken a large sample of nearby supernova spectra of Type Ia's over a wide span of epochs and stretches. Although SNAP plans on using K-corrections based on specific supernova observables, this large and varied sample can give an upper-limit on expected K-correction uncertainties. I calculate the K-correction for a fixed B filter shifting each spectrum by small amounts in redshift. For each redshift, I fit the K-correction as a 3nd order polynomial as a function of observed $B-V$ color and calculate the residual RMS from that model. The polynomial fits, and RMS residuals for varying z are shown in Table 1.

Although the fit coefficients are generally continuous, there is a jump at $z \sim 0.35$. Calculation of K-corrections at differing redshifts requires different spectral coverage. The spectra in my library have vastly different ranges. The observed discontinuities result from the dropping of spectra at higher redshifts.

SNAP demands a K-correction uncertainty < 0.02 mag. Given my model for K-corrections, there are two sources of uncertainty. The first comes from measurement uncertainties of the supernova color while the second is the RMS which reflects supernova inhomogeneity. The error model is then

$$dK^2 = [(a_1 + 2a_2(B-V) + 3a_3(B-V)^2)d(B-V)]^2 + RMS^2 \tag{1}$$

where a_1 and a_2 are the linear and quadratic terms respectively of the fit. SNAP already requires d$(B-V) \sim 0.02$ mag for the extinction correction. The resulting K-correction error as a function of redshift is given in Figure 1.

The RMS depends on supernova inhomogeneity scaled by the absolute size of the K-correction; its error monotonically increases as you move away from $z = 0$ as seen in Figure 1. At $z = 0.1$ the error starts to fall again into a secondary minimum at $z \sim 0.25$. This is because the K-correction is related to the ratio of B flux of the redshifted and unredshifted source, whereas the color is the ratio B and V flux of the redshifted supernova. For the K-correction

$$\frac{\int B(\lambda) f_i((1+z)\lambda) d\lambda}{\int B(\lambda) f_i(\lambda) d\lambda} \tag{2}$$

TABLE 1

K-CORRECTION MODEL AND RESIDUALS FROM THE B-BAND

z	a_0	a_1	a_2	a_3	RMS
-0.20	-0.551	5.252	16.606	10.882	0.167
-0.18	-0.888	2.025	10.634	7.870	0.175
-0.16	-0.952	0.198	8.273	7.831	0.165
-0.14	-0.705	-0.575	3.229	3.238	0.160
-0.12	-0.481	-0.558	1.189	0.970	0.108
-0.10	-0.335	-0.471	0.638	0.449	0.077
-0.08	-0.221	-0.413	0.394	0.369	0.057
-0.06	-0.136	-0.286	0.220	0.195	0.028
-0.04	-0.082	-0.172	0.123	0.102	0.014
-0.02	-0.039	-0.076	0.048	0.035	0.007
-0.00	0.000	0.000	0.000	0.000	0.000
0.02	0.045	0.061	-0.024	-0.003	0.009
0.04	0.092	0.076	-0.004	0.139	0.019
0.06	0.142	0.102	0.002	0.256	0.028
0.08	0.180	0.104	-0.009	0.442	0.034
0.10	0.232	0.159	-0.082	0.527	0.035
0.12	0.279	0.255	-0.160	0.541	0.032
0.14	0.329	0.367	-0.248	0.536	0.030
0.16	0.370	0.502	-0.294	0.463	0.024
0.18	0.401	0.653	-0.319	0.360	0.019
0.20	0.400	0.817	-0.238	0.139	0.016
0.22	0.374	0.926	-0.094	0.004	0.013
0.24	0.333	0.965	0.075	-0.071	0.010
0.26	0.299	0.938	0.105	0.086	0.009
0.28	0.273	0.903	0.075	0.204	0.018
0.30	0.266	0.853	0.139	0.180	0.027
0.32	0.271	0.883	0.019	0.248	0.025
0.34	0.337	0.525	0.689	-0.119	0.031
0.36	0.451	-0.019	1.582	-0.545	0.036
0.38	0.548	-0.323	1.982	-0.707	0.024
0.40	0.661	-0.664	2.387	-0.844	0.057
0.42	0.787	-0.988	2.690	-0.917	0.081
0.44	0.938	-1.324	2.937	-0.956	0.109
0.46	1.220	-1.996	3.442	-1.055	0.139
0.48	2.517	-5.628	6.686	-1.959	0.167

and color:

$$\frac{\int B(\lambda) f_i((1+z)\lambda) d\lambda}{\int V(\lambda) f_i((1+z)\lambda) d\lambda} \tag{3}$$

Because $B(\lambda) \sim V(\lambda/(1+0.25)$ the unredshifted B flux is identical to the redshifted V flux, except for a fixed constant independent of input flux f_i.

As you move towards higher redshift, the K-correction can be better obtained from the next filter at higher wavelength. This is shown as the dotted-line in Figure 1. Given a series of filters, the K-correction errors will have a saw-tooth shape as a function of redshift as the spectrum moves in and out of matching filters. Note that the V-band K-correction does not go to zero error since it's shape does not exactly match B.

If the spacing between consecutive filters is smaller, you can reduce the peaks of the saw tooth. We consider the case where instead of the standard V band, we use a B filter redshifted to $z = 0.15$ to determine color and the next K-corrections.. Not only do we benefit with more frequent sweet-spots in redshift, but the smaller gap in color pushes the secondary minimum in as shown in Figure 2 where the secondary minimum is pushed back to $z \sim 0.1$. With this closer spacing, the K-correction errors are kept under 2%, the goal of the SNAP experiment. The $B(1.15\lambda)$ K-correction does not go to zero at $z = 0.15$ because I didn't do the calculation at high-enough resolution.

A choice of filters then for SNAP can thus be a set of redshifted-B filters at z =0, 0.15, 0.32, 0.52, 0.75, 1.01, 1.31, 1.65, 2.05, 2.5. The K-correction and residuals for this system is given in Table 2 and can be found on the SNAP BCSW website.

In this note, I have been discussing the errors involved in single filter K-corrections. When cross-filter K-corrections are performed, such as in the dashed V-to-B K-corrections in the Figures, a zeropoint error must also be included. This is discussed in a separate note.

2. Redshift errors

Uncertainty in the supernova redshift will also propagate into K-correction errors. Shown in Figure 3 are the derivatives of the K-corrections as a function of redshift for two different colors. Suppose that the limiting resolution of the spectrograph is 10Å. Depending on the spectral feature used to make the redshift determination, the redshift error will be $dz \sim 0.002$. As long as dK/dz is less than 5, which it is comfortably,

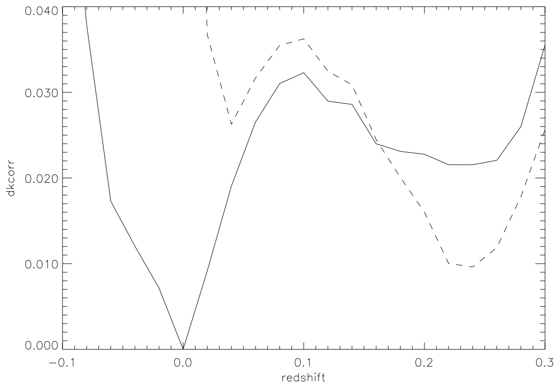

Fig. 1.— The K-correction error for an observed $(B-V) = 0$ with uncertainty of 0.02 magnitudes. The dashed line corresponds to the K_{BV} K-correction errors.

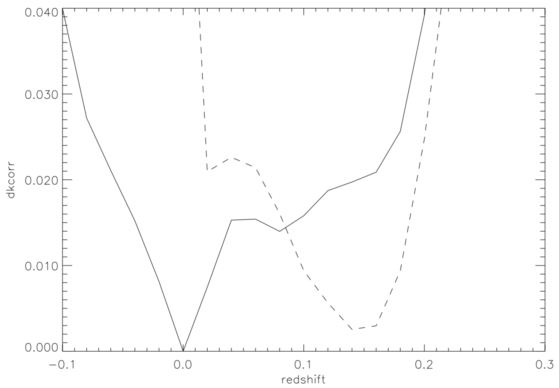

Fig. 2.— The K-correction error for an observed $(B - B(1.15\lambda)) = 0$ with uncertainty of 0.02 magnitudes. The dashed line corresponds to the $K_{BB(1.15\lambda)}$ K-correction errors.

TABLE 2
K-CORRECTION MODEL AND RESIDUALS FROM THE B-BAND WITH REDUCED FILTER SET

z	a_0	a_1	a_2	a_3	RMS
-0.20	-0.684	-5.236	5.636	42.599	0.145
-0.18	-0.263	-3.885	-1.568	22.303	0.135
-0.16	-0.067	-2.691	-2.670	12.241	0.080
-0.14	-0.146	-1.802	0.108	3.592	0.120
-0.12	-0.095	-1.406	0.025	2.395	0.079
-0.10	-0.062	-1.052	0.156	1.242	0.055
-0.08	-0.029	-0.792	-0.028	1.112	0.042
-0.06	-0.020	-0.531	0.044	0.584	0.028
-0.04	-0.015	-0.307	0.054	0.232	0.014
-0.02	-0.009	-0.130	0.037	0.027	0.008
-0.00	0.000	0.000	0.000	0.000	0.000
0.02	0.021	0.106	-0.090	0.143	0.007
0.04	0.053	0.206	-0.329	0.553	0.014
0.06	0.082	0.342	-0.664	1.038	0.019
0.08	0.089	0.518	-0.854	1.183	0.015
0.10	0.075	0.726	-0.718	0.816	0.008
0.12	0.046	0.886	-0.441	0.404	0.007
0.14	0.014	0.974	-0.112	0.075	0.003
0.16	-0.011	1.025	0.070	-0.032	0.003
0.18	-0.029	1.111	0.023	0.044	0.011
0.20	-0.051	1.420	-0.857	0.844	0.025
0.22	-0.061	1.380	-0.481	0.467	0.039
0.24	-0.016	0.760	1.081	-0.556	0.053
0.26	0.345	-1.583	5.099	-2.547	0.052
0.28	0.459	-1.917	5.228	-2.313	0.042
0.30	0.702	-3.134	7.202	-3.286	0.048
0.32	0.812	-3.514	7.750	-3.526	0.055
0.34	0.822	-3.468	7.780	-3.540	0.075
0.36	0.477	-1.790	5.503	-2.529	0.103
0.38	-0.109	0.907	1.845	-0.945	0.140
0.40	1.046	-3.286	6.744	-2.743	0.179

the K-correction errors due to redshift error will be less than 1%.

3. Conclusion

From this method of calculating K-corrections $K(z,(B-V),(B-V)\mathrm{zeropoint},\mathrm{filter}-\mathrm{spacing})$, assuming that fixed shapes for the filters. Thus for the purposes of the SNAP error budget, K-correction errors can be considered as a function of these variables. A tool now exists to generate the numbers for an error matrix.

Things to worry about for later are the effect of reddening on K-corrections and the use of redshifted B filters for V measurements. We need to study optimal filter shapes although the primary driver will be minimizing intrinsic supernova magnitude dispersion rather than K-correction errors.

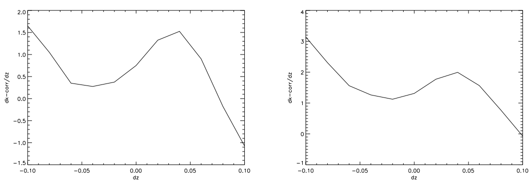

Fig. 3.— dK/dz for the redshifted B filters. The first plot is for $B-V=0$ and the second for $B-V=0.1$.