Title: The Ins and Outs of Functions

Brief Overview:
   In these lessons, students will explore what a function is, how to determine if a relation is a function and different ways a function can be represented. Prior knowledge should include patterns, sequences and relations.

NCTM Content Standard/National Science Education Standard:

   Algebra -- understand patterns, relationships, and functions

Grade/Level:

   Grade 7/ 8; All learning levels

Duration/Length:

   Three 50-minute periods

Student Outcomes:

   Students will:
   • Identify the rule for a function
   • Define functions
   • Calculate a value for a designated function

Materials and Resources:

   • Student resource pages 1-9 (SR)
   • Teacher resource pages 1-11 (TR)
   • Internet (optional)

Development/Procedures:

Lesson 1 – Patterns and Rules

Pre-assessment – Have the students independently complete *What's Missing?* (SR1). This activity will activate their prior knowledge of sequences and finding the next term in a pattern.   Discuss with the students how they came up with the next term in each sequence. Also use the word "series" to describe each problem. Note that some students may come up with different patterns. Take the opportunity to discuss how each pattern may have more than one outcome.

Launch – Using the pre-assessment *What's Missing?* (SR1), discuss how each student came up with his or her *rule* for each sequence. Emphasize that each situation has a rule.

Teacher Facilitation – Give students two other examples of a sequence, and have them develop a rule for each situation. Present *Snack Time* (TR2) on overhead transparency and discuss what needs to go into the vending machine (coins only), what the "rule" is (the vending machine), and then what comes out (their snack). Make mention of the question, "*Is this a function?*" Do not answer the question until Lesson 2. Ask students what would come out for different coin amounts. Make connection that the vending machine is the function.

Student Application – Have the students complete *Machines at Work* (SR2). Students should begin to feel confident about identifying rules and finding the input and output for each function

Embedded Assessment – Have each student create his or her own machine. Instruct the students to model their machines after the completed *Machines at Work* (SR2) worksheet. When they have completed their machines, instruct the students to have a neighbor find the output or input using their machine. Circle around the classroom to make sure the students are not only creating their machines correctly, i.e. functions that have an individual output for each input, but are also finding the correct results for their neighbor's machine.

Re-teaching/Extension – Ask the students who understand the material to pair up with students who are still uncertain about the machines. Have the students who understand the material explain it to those who do not.

<u>**Lesson 2**</u> **– Function Definitions**

Pre-assessment – Have the students complete *What's the Rule?* (SR3). This activity will review and assess the information they learned in Lesson 1.

Launch – Discuss *What's the Rule?* (SR3) with the students. Have several students state the rule they chose for each table and compare it with the rules other students found. Then, discuss possibilities for future terms such as the 20th, 50th or 99th term.

Teacher Facilitation - Develop definitions for function, domain, range, input, output, and ordered pair from *Snack Time* (TR2) in lesson 1. Discuss input (domain) as the items going into the function and output (range) as the items coming out of the function. Discuss how a function may only have one output for each unique input. Further discuss the definitions using *Snack Time* (TR3) noting that this machine does not represent a function because if you put $0.75 into the machine, you could get two different items back.

Go back to *Machines at Work* (SR 2) from lesson 1 and discuss student examples. Have students identify and explain which student produced examples represent functions (#7). Discuss the properties that prohibit a machine from being a function.

Student Application – Have the students complete *Function Criss-Cross* (SR4). This will help students learn the definitions for the various terms. Math glossaries may be used at teachers' discretion.

Embedded Assessment – Ask students to complete *Ins and Outs of Functions* (SR5). This will help them to start seeing functions in a table format for Lesson 3.

Re-teaching/Extension – Ask students to complete a journal prompt, *The most important Thing about Functions* (SR6). Encourage students to list as many things that they can remember from the prior two lessons as possible. This prompt will help students sort what they know and don't know about functions from the lessons. Allow appropriate time to complete prompt. After the prompt has been completed, randomly have student share things they know by reading one important thing from their list. After the majority of ideas have been shared, have students volunteer to share their prompt with the class. This allows students to hear important ideas that were discussed that they may have missed.

Lesson 3 – Calculations with Functions

Pre-assessment – Have students complete *Number Detective* (SR7). This will allow a review and assessment of Lesson 2. Emphasize that the top row represents the entry number for each term in the bottom row. Also, the N value is the generalized function for the pattern in the bottom row.

Launch – Discuss *Number Detective* (SR7) with the students and introduce the algebraic form $f(x)$ to represent the rule for each table using the N input value.

Teacher Facilitation – Have students take notes on their own notebook paper by going over multiple examples of completing $f(x)$ tables, identifying range [$f(x)$ or (y)] and domain (x). Be sure to include how to take a y value and find the x value. Also, show them that they can pick their own "convenient" values (-1,0,1,2 . .) for x to complete a function table.

Student Application – The students will complete *Function Tables* (SR8). As an optional activity, students can explore on-line with "The Function Machine."
(http://score.kings.k12.ca.us/lessons/functions/machine.html)

Embedded Assessment – *Function Tables* (SR8) (same as above). This will help them practice function tables on their own. Students may complete entire worksheet or #1-5, then have #6-10 as optional challenge problems.

Re-teaching/Extension – To extend and re-teach this lesson, graph the ordered pairs from the lesson using the input as the *x*-value and the output as the *y*-value on overhead of graph paper (TR10). Seeing the relationship visually on a graph will help those students who are still uncertain about calculations with functions. Graphing the ordered pairs is also a great extension for those students who understand the material and are ready to proceed to the next level with functions.

Summative Assessment:

The students will complete *Functions Quiz* (SR9). They will apply their knowledge of sequences, patterns and functions by finding the missing elements in tables, identifying definitions and calculating algebraic functions.

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Key What's Missing?

Find the missing entry for each example.

1) 2   4   6   8   10   12   __14__

2) Alabama Alaska Arizona __Arkansas__

3) S   M   T   W   T   __F__

4) △   ▭   ⬠   ⬡___
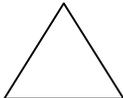
5) 1   4   9   16   25   36   __49__

6) a   b   c   d   e   f   g   h   __i__

7) $\dfrac{1}{2}$ $\dfrac{1}{4}$ $\dfrac{1}{8}$ $\dfrac{1}{16}$ $\dfrac{1}{32}$

8) Mercury Venus Earth Mars __Jupiter__

9) 1   5   1   5   1   5   __1__

10) 1   8   27   64   __125__

Snack Time

$0.55	$0.75	$0.45
$0.30	$0.25	$0.90
$0.70	$0.50	$0.85

S
N
A
C
K
S

Money
IN!!
☐ Change
Only

Get Your Snack OUT!

Is this a Function??

Snack Time

$0.75	$0.75	$0.45
$0.30	$0.25	$0.90
$0.50	$0.50	$0.85

S
N
A
C
K
S

Money
IN!!

☐ Change
Only

Get Your Snack OUT!

Is this a Function??

Name:_____KEY_____Date:_____

Machines at Work

Use the example below to complete the following machines.

EXAMPLE:

Input

Short

The Opposite

Tall

Output

1.

3 sides

The shape name

Triangle

6 sides

The shape name

hexagon

2.

Arizona

The first letter of the state

A

Florida

The first letter of the state

F

3.

Meow

Animal that makes the noise

Cat

Oink

Animal that makes the noise

Pig

4.

Texas
USA capital city
Austin

Maryland
USA capital city
Annapolis

5.

10
Add 4 to the number
14

5
Add 4 to the number
9

6.

-2
Multiply the number by 4
-8

4
Multiply the number by 4
16
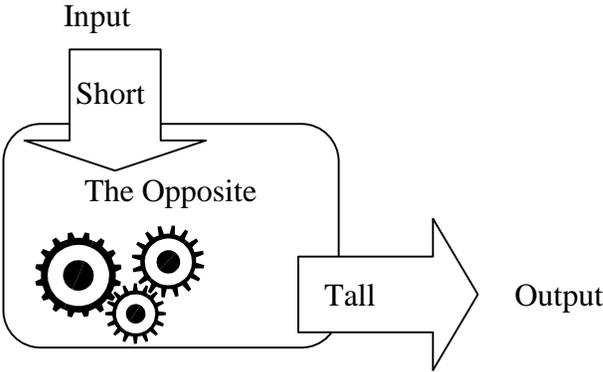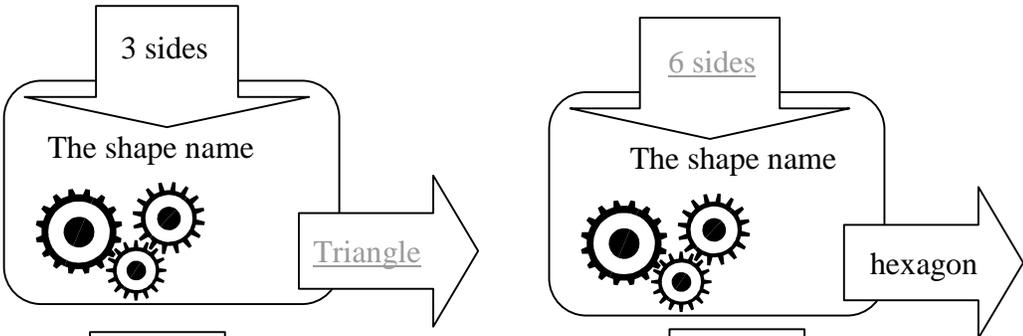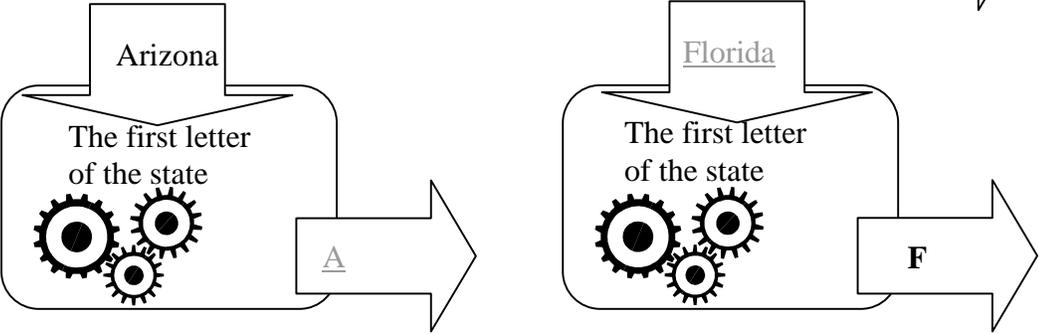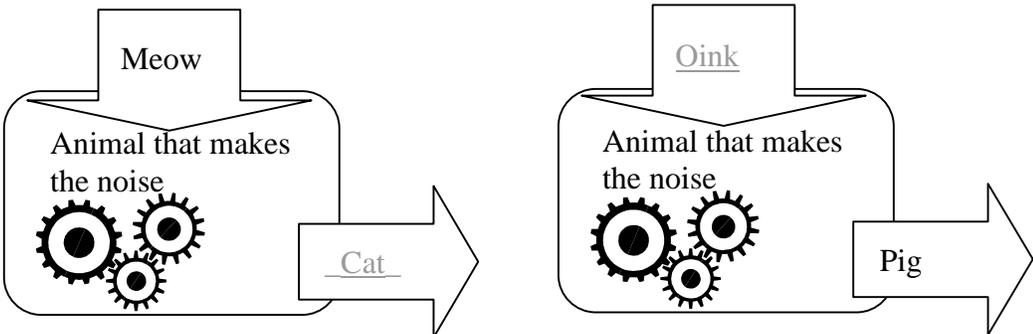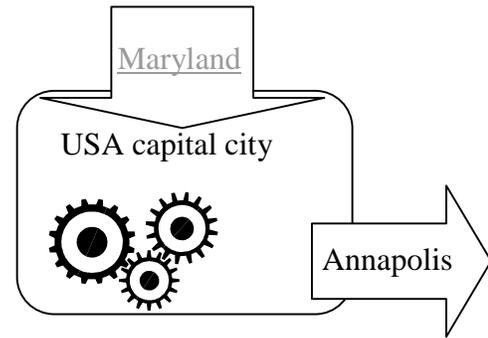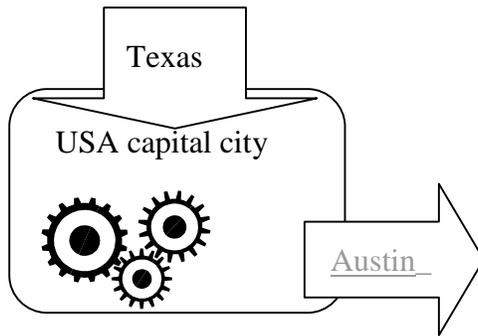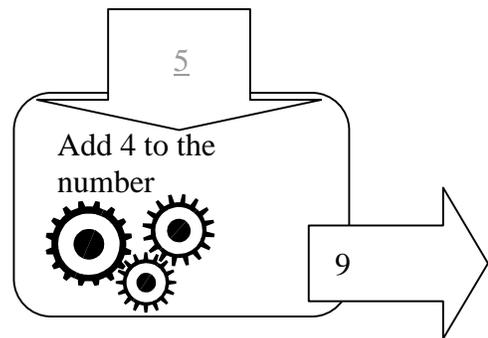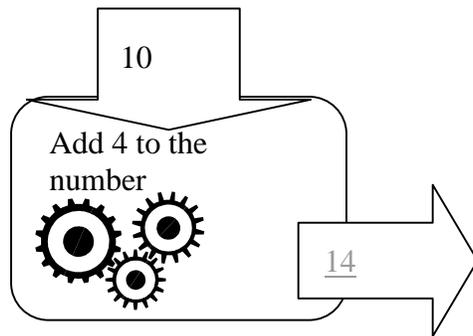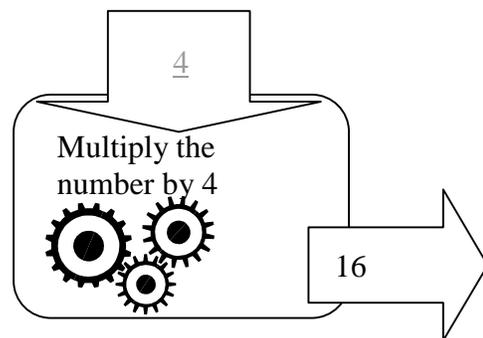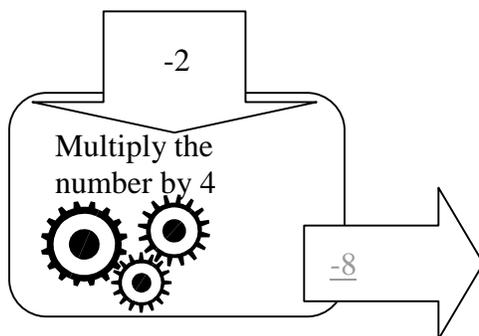
7. Make 2 <u>different</u> machines of your own below for a friend to complete. Answers will vary.
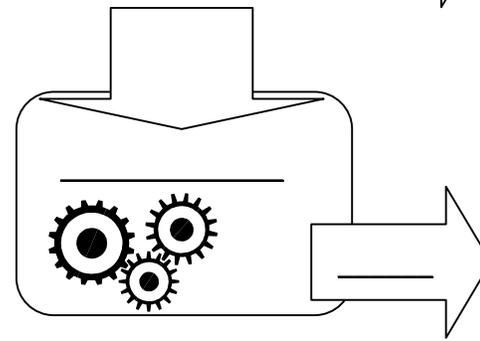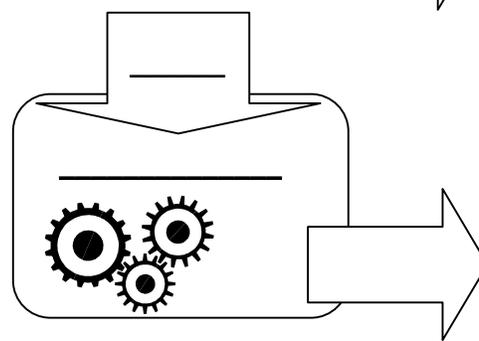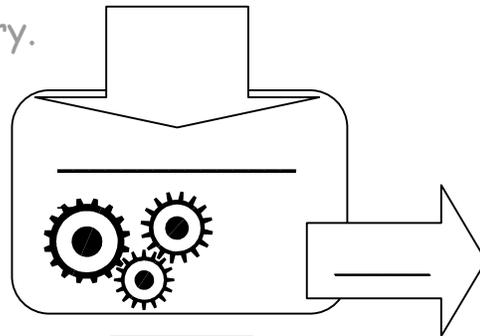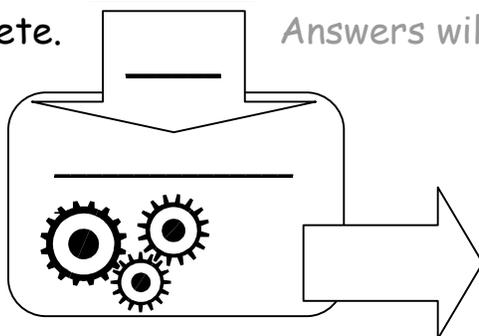
KEY What's the Rule?

Please fill in the missing values and state the rule that relates each number in the top row to the entry below it.

1.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20

Rule: Multiply the entry in the top row by 2.

2.

1	2	3	4	5	6	7	8	9	10
1	4	9	16	25	36	49	64	81	100

Rule: Square each entry in the top row.

3.

5	10	15	20	25	30	35	40	45	50
0	5	10	15	20	25	30	35	40	45

Rule: Subtract five from each entry in the top row.

4.

1	2	3	4	5	6	7	8	9	10
-1	-2	-3	-4	-5	-6	-7	-8	-9	-10

Rule: Take the opposite of each number.

KEY FUNCTION CRISS-CROSS
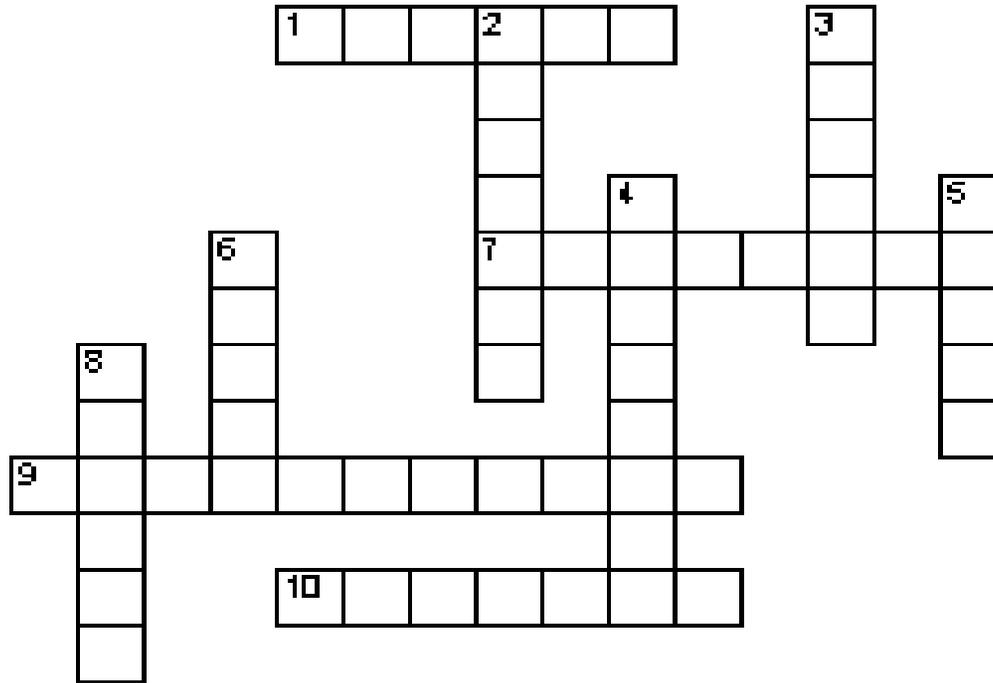
Across

1. The resulting value from the function (output)
7. A mathematical rule written using symbols (Equation)
9. A set of two numbers in which the order has an agreed upon meaning (Ordered Pair)
10. A device that performs a specific task (machine)

Down

2. A repeated design or recurring sequence (Pattern)
3. Another word for input (domain)
4. A Mathematical relationship between two values where the second value depends on the first. (Function)
5. The value going into the function (input)
6. Another word for output (range)
8. A list of numbers that follows a certain pattern (Series)

Word Bank

Function	Pattern	Ordered Pair	Equation	Output
Range	Domain	Series	Machine	Input

Name:_____KEY_____ Date:_____

Ins & Outs of Functions

Fill in each table, then complete the rule, and a description of the domain & range.

1.

In	Out
Apple	A
Banana	**B**
Kiwi	K
Orange	O
Grapes	G

Rule: Take the first letter of each fruit
Domain: Fruit names
Range: The first letter

2.

In	Out
Night	Day
Heavy	**Light**
Short	Tall
Big	Little
Skinny	**Fat**

Rule: Opposite
Domain: Any word that has an opposite
Range: The opposite of the word

3.

In	Out
Valentines	February
Christmas	Dec.
Independence Day	July
Labor Day	Sept.
Halloween	Oct.

Rule: Month of the holiday
Domain: A holiday that has a specified month
Range: The month of the holiday

4.

In	Out
Soccer	
Football	
Baseball	
Volleyball	
Basketball	

Rule: The image of the ball
Domain: A sport that uses a ball
Range: The image of the ball used

5.

In	Out
10	4
12	6
8	2
4	-2
-2	-8

Rule: Subtract 6 from IN
Domain: An integer
Range: An integer

6.

In	Out
11	46
8	26
-3	-10
2	10
0	2

Rule: **Multiply each number by 4, then add 2**
Domain: An integer
Range: An integer

KEY **Number Detective**

Please mentally identify the rule and then find the missing term.

1.

1	2	3	4	5	20	99	N
3	6	9	12	15	60	297	3N

2.

1	2	3	4	5	20	99	N
-4	-3	-2	-1	0	15	94	N-5

3.

1	2	3	4	5	20	99	N
1	4	9	16	25	400	9801	N^2

4.

1	2	3	4	5	20	99	N
16	17	18	19	20	35	114	N+15

5.

1	2	3	4	5	20	99	N
-2	-4	-6	-8	-10	-40	-198	-2N

Name:_____KEY_____ Date:_____

Function Tables

Directions: Complete the following tables. If the x value is given, evaluate the function to find y.   If y is given, work backwards to find x. If there is no value for x or y, use your own value for x to find y. SHOW ALL WORK!

1. $f(x)=x+1$

X	Work	Y
8	$f(8)=8+1$	9
-1		0
2		3
0		1

3. $f(x)=2x-3$

X	Work	Y
-2		-7
-1		-5
0		-3
1		-1

2. $f(x)=3x$

X	Work	Y
0		0
1		3
2		6
3		9

4. $f(x)=-2x+1$

X	Work	Y
4		-7
3		-5
0		1
	Answers will vary	

5. $f(x)=\dfrac{x}{2}+1$

X	Work	Y
0		1
2		2
4		3
	Answers will vary	

8. $f(x)=2x-2$

X	Work	Y
	Answers will vary	
	Answers will vary	
	Answers will vary	
	Answers will vary	

6. $f(x)=x+1$

X	Work	Y
4	$5=x+1$ $x=4$	5
-1		0
1		2
0		1

9. $f(x)=8x+1$

X	Work	Y
	Answers will vary	
	Answers will vary	
	Answers will vary	
	Answers will vary	

7. $f(x)=-3x-1$

X	Work	Y
1		-4
0		-1
	Answers will vary	
	Answers will vary	

10. $f(x)=\dfrac{x}{3}$

X	Work	Y
-3		-1
0		0
3		1
	Answers will vary	
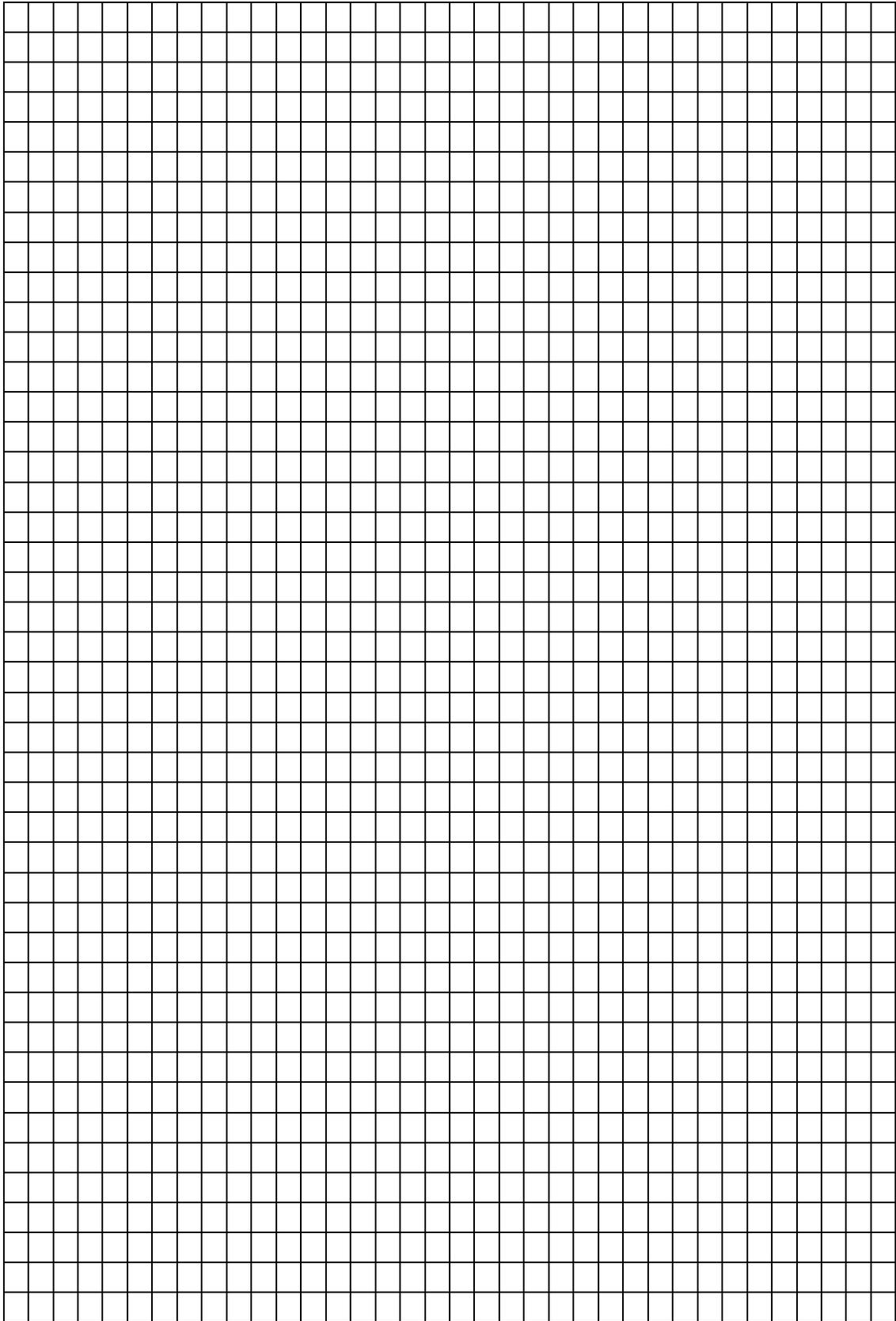
Name: _____

Date: _____

Class/ Teacher: _____

Period: _____

Functions Quiz

1.   Identify the rule for each function.

a.

X	1	2	3	4
F(x)	10	20	30	40

Rule: _____Multiply each number by 10._____

b.

X	1	2	3	4
F(x)	12	13	14	15

Rule:_____Add 11 to each number._____

2.   Matching: Match each term with the appropriate definition.

__c__	Function	a.	A set of two numbers in which the order has an agreed upon meaning
__d__	Series	b.	Another word for input
__b__	Domain	c.	A Mathematical relationship between two values where the second value depends on the first
__e__	Output	d.	A list of numbers that follows a certain pattern
__a__	Ordered Pair	e.	The resulting value from the function

3.   Complete the following tables

a.   *f*(*x*) = 3*x*-2   **b.   *f*(*x*) = 5 + 2x**

x	Work	*f*(*x*)
-2	f (-2) = 3(-2)-2	-8
0	f (0) = 3(0)-2	-2
5	f (5) = 3(5)-2	13
26	f (26) = 3(26)-2	76

x	Work	*f*(*x*)
-30	f (-30) = 5 + 2(-30)	-55
-2	f(-2) = 5 + 2(-2)	1
0	f(0) = 5 + 2(0)	5
5	f(5) = 5 + 2(5)	15

Bonus:

4.   List the ordered pairs created by the function in example 3(a).

(5 , 13)   (0 , -2)   (-2 , -8)   (26 , 76)

5.   List the ordered pairs created by the function in example 3(b).

(-2 , 1)   (0 , 5)   (5 , 15)   (-30 , -55)

What's Missing?

Find the missing entry for each example.

1) 2   4   6   8   10 12 _____

2) Alabama Alaska Arizona _____

3) S   M   T   W   T   _____

4) △   ▭   ⬠   _____
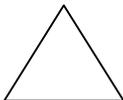
5) 1   4   9   16 25 36 _____

6) a   b   c   d   e   f   g   h   _____

7) $\dfrac{1}{2}$ $\dfrac{1}{4}$ $\dfrac{1}{8}$ $\dfrac{1}{16}$ _____

8) Mercury Venus Earth Mars _____

9) 1   5   1   5   1   5   _____

10) 1   8   27 64 _____

Name:_____Date:_____

Machines at Work

Use the example below to complete the following machines.

EXAMPLE:

Input

Short

The Opposite

Tall   Output

1.

3 sides

The shape name

_____

The shape name

hexagon

2.

Arizona

The first letter
of the state

_____

The first letter
of the state

F

3.

Meow

Animal that makes
the noise

_____

Animal that makes
the noise

Pig

4.

Texas

USA capital city

_____ →

_____

USA capital city

Annapolis →

5.

10

Add 4 to the number

_____ →

_____

Add 4 to the number

9 →

6.

-2

Multiply the number by 4

_____ →

_____

Multiply the number by 4

16 →
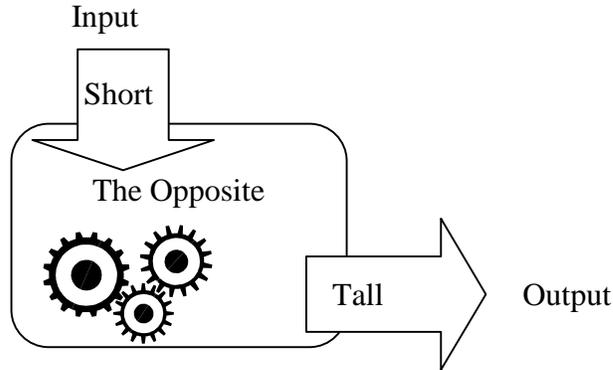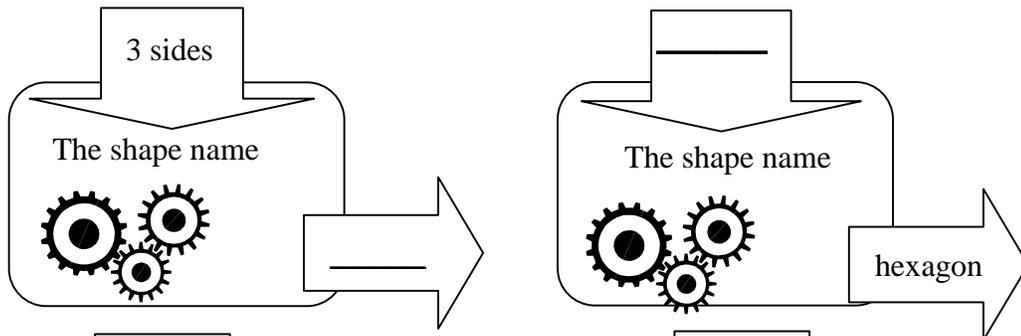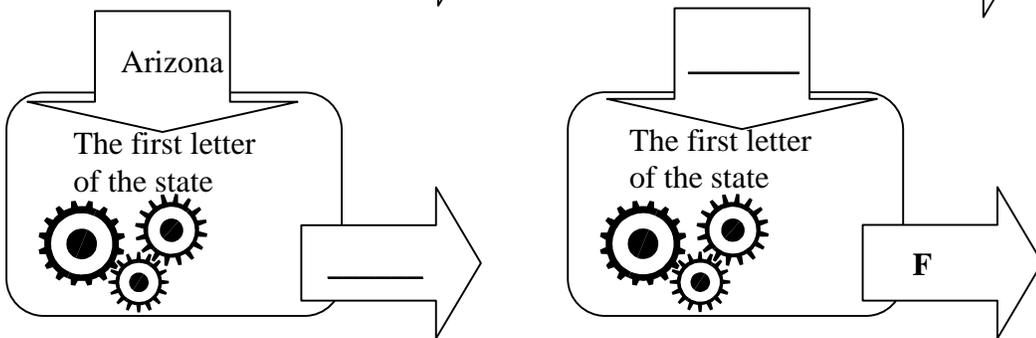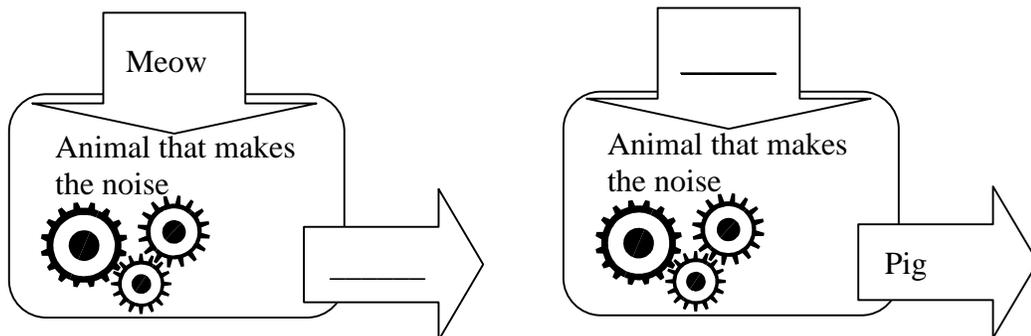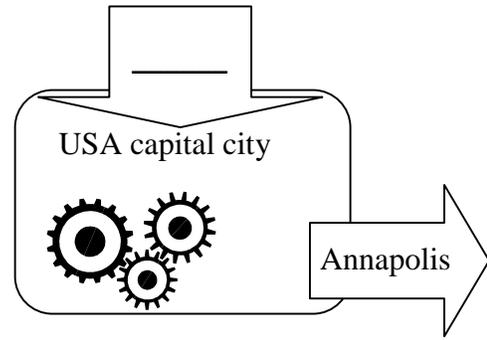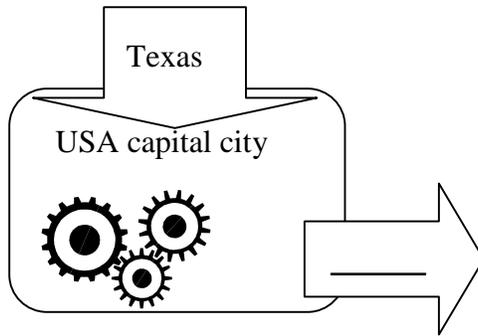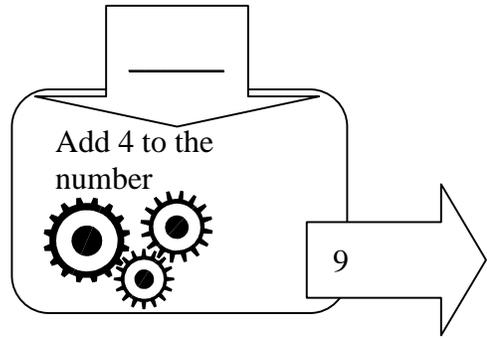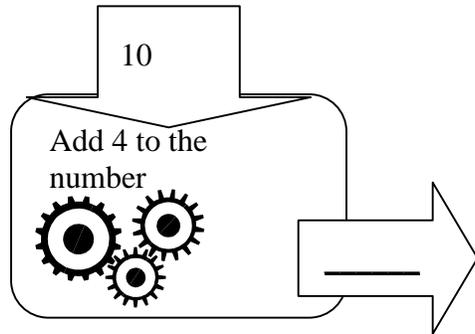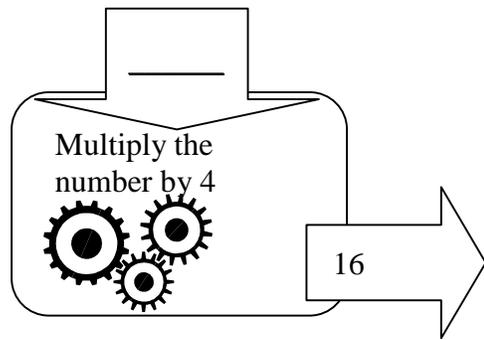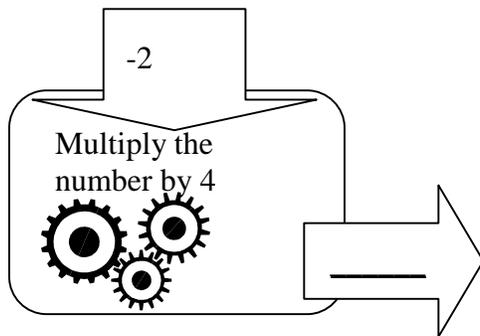
7. Make 2 __different__ machines of your own below for a friend to complete.

_____

_____ →

_____

_____

_____ →

_____

_____ →

_____

_____

_____ →
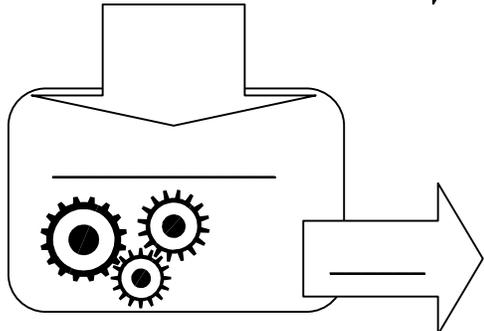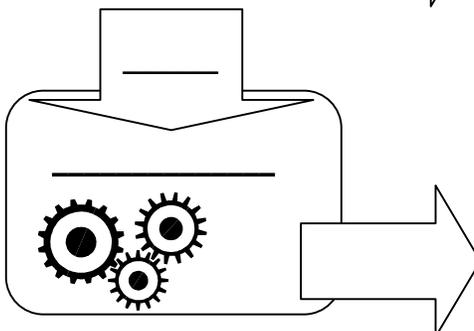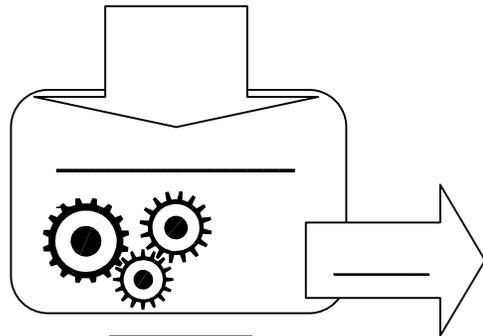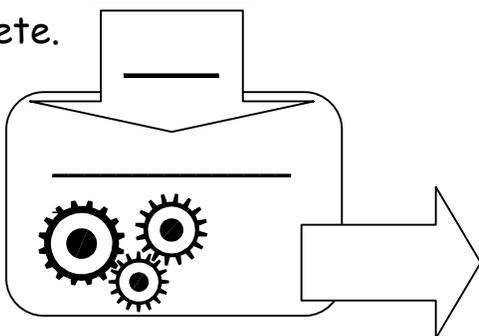
What's the Rule?

Please fill in the missing values and state the rule that relates each number in the top row to the entry below it.

1.

1	2	3	4	5	6		8	9	10
2	4	6	8			14		18	

Rule: _____

2.

1	2	3	4	5	6	7		9	10
1	4	9	16	25			64		

Rule: _____

3.

5	10	15	20	25	30	35		45	50
0	5	10	15	20			35		

Rule: _____

4.

1	2	3	4	5	6		8	9	10
-1	-2	-3	-4	-5		-7			

Rule: _____

FUNCTION CRISS-CROSS
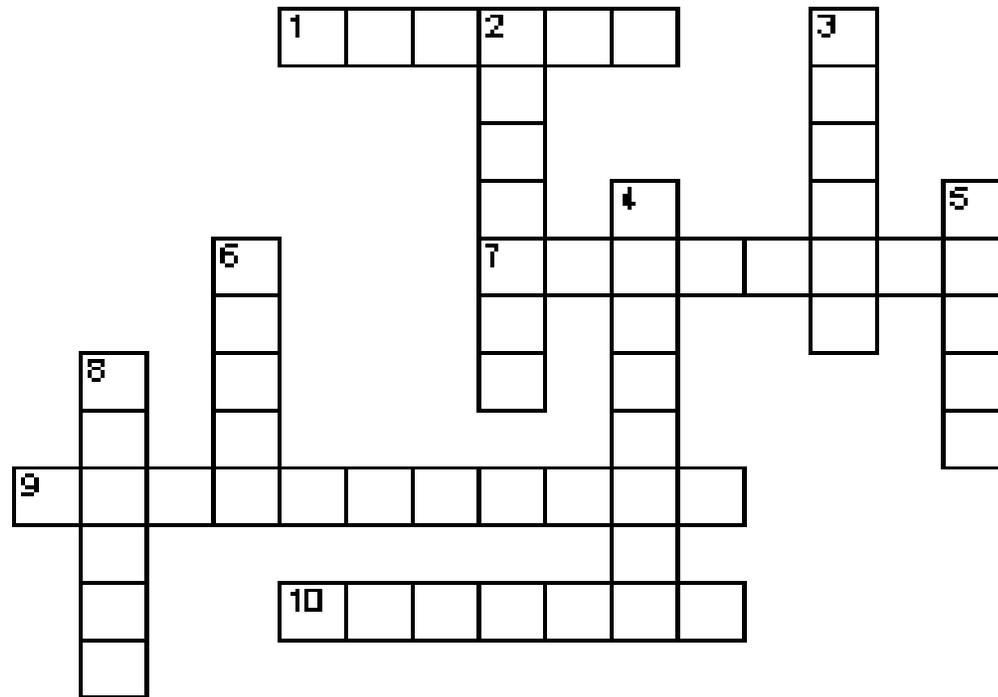
Across

1. The resulting value from the function
7. A mathematical rule written using symbols
9. A set of two numbers in which the order has an agreed upon meaning
10. A device that performs a specific task

Down

2. A repeated design or recurring sequence
3. Another word for input
4. A Mathematical relationship between two values where the second value depends on the first.
5. The value going into the function
6. Another word for output
8. A list of numbers that follows a certain pattern

Word Bank

Function	Pattern	Ordered Pair	Equation	Output
Range	Domain	Series	Machine	Input

Name:_____ Date:_____

Ins & Outs of Functions

Fill in each table, then complete the rule, and a description of the domain & range.

1.

In	Out
Apple	A
Banana	
Kiwi	
Orange	
Grapes	

Rule:

Domain:

Range:

4.

In	Out
Soccer	⚽
	🏈
Baseball	
	🏐
Basketball	

Rule:

Domain:

Range:

2.

In	Out
Night	Day
Heavy	
Short	
	Little
Skinny	

Rule:

Domain:

Range:

5.

In	Out
10	4
12	
8	
4	
-2	

Rule:

Domain:

Range:

3.

In	Out
Valentines	February
Christmas	
Independence Day	
Labor Day	
Halloween	

Rule:

Domain:

Range:

6.

In	Out
11	
8	
-3	
2	
0	

Rule: **Multiply each number by 4, then add 2**

Domain:

Range:

Name:_____**Date:**_____

The Most Important Thing about

FUNCTIONS!

<u>List</u> below the important things that we have studied about functions.

❖ _____
❖ _____
❖ _____
❖ _____
❖ _____
❖ _____
❖ _____
❖ _____

Now, write the MOST important thing you think that we have studied and why you picked it as the most important.

_____
_____
_____
_____
_____
_____
_____

Number Detective

Please mentally identify the rule and then find the missing term.

1.

1	2	3	4	5	20	99	N
3	6	9	12				

2.

1	2	3	4	5	20	99	N
-4	-3	-2	-1				

3.

1	2	3	4	5	20	99	N
1	4	9	16				

4.

1	2	3	4	5	20	99	N
16	17	18	19				

5.

1	2	3	4	5	20	99	N
-2	-4	-6	-8				

Name:_____ Date:_____

Function Tables

Directions: Complete the following tables. If the x value is given, evaluate the function to find y. If y is given, work backwards to find x. If there is no value for x or y, use your own value for x to find y. SHOW ALL WORK!

1. $f(x)=x+1$

X	Work	Y
8		
-1		
2		
0		

3. $f(x)=2x-3$

X	Work	Y
-2		
-1		
0		
1		

2. $f(x)=3x$

X	Work	Y
0		
1		
2		
3		

4. $f(x)=-2x+1$

X	Work	Y
4		
3		
0		

5. $f(x) = \dfrac{x}{2} + 1$

X	Work	Y
0		
2		
4		

8. $f(x) = 2x - 2$

X	Work	Y

6. $f(x) = x + 1$

X	Work	Y
		5
-1		
1		
0		

9. $f(x) = 8x + 1$

X	Work	Y

7. $f(x) = -3x - 1$

X	Work	Y
1		
0		

10. $f(x) = \dfrac{x}{3}$

X	Work	Y
-3		
0		
3		

Name: _____

Date: _____

Class/ Teacher: _____

Period: _____

Functions Quiz

1.   Identify the rule for each function.

a.

X	1	2	3	4
F(x)	10	20	30	40

Rule: _____

b.

X	1	2	3	4
F(x)	12	13	14	15

Rule: _____

2.   Matching: Match each term with the appropriate definition.

_____   Function

_____   Series

_____   Domain

_____   Output

_____   Ordered Pair

a.   A set of two numbers in which the order has an agreed upon meaning

b.   Another word for input

c.   A Mathematical relationship between two values where the second value depends on the first

d.   A list of numbers that follows a certain pattern

e.   The resulting value from the function

3. **Complete the following tables**

a. *f(x) = 3x-2*   **b.** *f(x) = 5 + 2x*

x	Work	*f(x)*
-2		
0		
5		
26		

x	Work	*f(x)*
-30		
-2		
0		
5		

Bonus:

4. List the ordered pairs created by the function in example 3(a).

(,   )   (,   )   (,   )   (,   )

5. List the ordered pairs created by the function in example 3(b).

(,   )   (,   )   (,   )   (,   )