Tree Unitarity Breakdown in Spontaneously Broken N=1 Supergravity Theories and Phenomenology of a Superlight Gravitino

by

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1. Statutory Declarations

Title of the thesis	:	Tree Unitarity Breakdown in Spontaneously Broken N=1 Supergravity Theories and Phenomenology of a Superlight Gravitino
Name of canditate	:	Tanmoy Bhattacharya
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Place of Research	:	Tata Institute of Fundamental Research, Bombay.
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Statements required under ordinanaces O.770 and O.771 of the University of Bombay:

1. The work presented in this thesis describes lower limit on the ratio of the mass of the gravitino to other masses in an $N = 1$ broken Supergravity theory. Experimental tests for detecting a superlight gravitino are also proposed.

2. This work is original. The work of other persons, whenever used, is duly acknowledged in the text through appropriate references.

3. This work has been done under the supervision of and in collaboration with Prof. Probir Roy. Part of the work described in chapter 6 was done in collaboration with Dr. Sourendu Gupta.

4. This work has not been submitted to any university or body in quest of a degree, diploma or any other academic award.

Signature of the supervisor

Signature of the candidate

March 24, 1996

2. Synopsis

Supersymmetry, relating bosons and fermions, is a beautiful idea. It also pertains to one of the most promising extensions[1] of the standard model that have been suggested so far. The motivation behind this extension is to solve some of the theoretical problems present (in particular the naturalness problem, more of which later) in the standard model, within the context of point particle field theory itself. Since supersymmetry is a spacetime symmetry, a consistent *local* formulation of it needs to include gravity. Hence, local supersymmetry is called supergravity. In a sense,this is an advantage since it provides a natural setting for gravity as an extension of ordinary gauge theories and is a reasonably unique theory. At the same time, the problems that besiege any attempts towards a quantum theory of gravity plague these supergravity models as well, though they may be somewhat softened on account of supersymmetry. In particular, one finds that within the perturbative domain, this theory is definable only with a cutoff which may reasonably be expected to be of the order of the Planck scale $\kappa^{-1} = M_{Pl}$, the natural scale of the theory.

Experimentally, of course, there is no evidence for supersymmetry so far. This may simply mean that supersymmetry breaking mass differences between the known particles and their superpartners are beyond the reach of the present accelerators. There are, nevertheless, theoretical limitations on how big such mass differences can be. On inclusion of gravity, the standard model becomes unnatural, since the scale of the electroweak symmetry breaking is much smaller ($\sim 10^{-17}$) than the natural Planck scale mentioned above. As a result, the coupling constants in the theory have to be readjusted drastically at every level of the perturbative calculation. The weak scale survives the onslaught of huge radiative corrections only on fine tuning. Supersymmetry alleviates the latter problem by arranging cancellations in the perturbative corrections to the weak

scale between the bosons and their superpartner fermions. In order to retain this nice feature of supersymmetry, we have to demand that the magnitudes of supersymmetry breaking mass differences cannot be too different (*i.e.* not more than a factor of ten) from the weak scale of ~ 100 GeV. While the experimental lower limits on these are in the tens of GeV range, the theoretical expectation, therefore, is that they should not exceed 1 TeV.

However, such a value of the supersymmetry breaking expectation values will naturally give rise to a large ($\sim 10^{120}$ times the observed value) cosmological constant. A class of supergravity models called no-scale models,[2] characterized by noncompact symmetries in the Kähler manifold and a flat potential in the scalar sector, were used to solve this problem. In this thesis, we shall study some features of supergravity theories with special relevance to no-scale models.

In any gauge-symmetric theory, we can arrange for symmetry breaking mass terms by realizing the theory in the Higgs phase. In such a case, the gauge particle also acquires a mass. In similar fashion, when supergravity, which can be looked upon as the gauge theory of supersymmetry, is 'spontaneously broken' through the super-Higgs mechanism, the corresponding gauge particle *i.e.* the gravitino $\tilde{G}$ becomes massive. However, in no-scale models, the mass of the gravitino is essentially an independent parameter,[2] almost uncorrelated with the laboratory supersymmetry breaking mass-differences. This leads to the interesting possibility that this gravitino can be superlight. In fact, this has been claimed to be a virtue because of the expectation by some that quantum corrections to the cosmological constant are predominantly controlled by the gravitino mass. In addition there are cosmological advantages, vis-a-vis the inflationary universe, of a superlight gravitino.

It is, however, known that theories involving a particle of spin more than half coupled

to a nonconserved current corresponding to a broken symmetry have singular massless limits. In other words, these theories yield cross-sections for processes (involving the longitudinal components of the particle's polarization) which increase and tend to infinity as the mass of the particle is decreased. This can lead to a large cross-section for small values of the particle mass. Fayet[3] had initially used the goldstino approximation on a superlight gravitino field enabling him to give a lower bound on the gravitino mass from available experimental data on $e^+e^- \to \gamma\tilde{\gamma}\tilde{G}$, $\tilde{\gamma}$ being the photino. We have studied[4] a different reaction, namely the two-photon process $\gamma\gamma \to \tilde{G}\tilde{G}$ and have seen that this process becomes measurably large when the superlight gravitinos are longitudinally polarized. This is because the polarization tensor of a longitudinal gravitino brings a factor of the gravitino mass $m_{3/2}$ in the denominator whereas the numerator contains a four vector which agrees with the energy-momentum four-vector of the gravitino except for additive terms of order $m_{3/2}^2$. Neglecting the latter when $m_{3/2}$ is small, an application of the reduction formula yields the matrix element of the divergence of the source of the gravitinos, *i.e.* the divergence of the spinorial supercurrent. The latter picks up the predominant supersymmetry-breaking mass difference in the 'matter or gauge' sector. In this particular problem, that turns out to be the mass of the photino. Hence, this amplitude will be proportional to $\kappa^2\omega^2 M_{\tilde{\gamma}}^2 m_{3/2}^{-2}$ where κ is Planck's constant, 2ω is the center-of-mass energy. $M_{\tilde{\gamma}}$ is the photino mass and $M_{\tilde{\gamma}} m_{3/2}^{-1}$ is squared owing to the appearance of two longitudinally polarized gravitinos.

We have calculated[4] the cross-sections for such processes in detail and have found that an experiment with doubly tagged e^+e^- beams looking for the process $e^+e^- \to e^+e^-$ + "*missing energy/momentum*" gets an essentially negligible contribution from the standard model but has quite a large cross-section in supergravity theories due to the mechanism described above. In fact, the basic photon-photon cross-section reaches

around 100 pb at a center-of-mass energy of 100 GeVfor a gravitino mass as light as allowed by Fayet's bound and a photino mass of around 50 GeV. This process then is expected to be measurable in the laboratory (*e.g.* LEP 1) and can be used to better the existing lower bound on the gravitino mass.

Such large cross-sections also indicate a potential inconsistency in these theories. Essentially because gravity is non-renormalizable, these theories make sense only as effective low energy theories at the lowest level of perturbation theory. Nevertheless, one does expect that in the exact theory, the higher orders in the expansion will not become important at a scale lower than the Planck scale, the inverse of which, (κ) appears as the expansion parameter in the theory. The largeness of these cross-sections mentioned above however arises from another large ratio in the theory, namely the ratio between the matter (or gauge) sector supermultiplet mass splits and the gravitino mass. In principle, this large ratio can come into the perturbative expansion and make the higher terms important at energies much lower than the Plank scale.

To study this situation, we calculated[5] the amplitude for another process in the theory, namely the fusion of two nonabelian gauge bosons g, *e.g.* gluons (carrying gauge indices a,b), to produce the corresponding gaugino $\tilde{g}$ (carrying gauge index c) and a gravitino. By the same argument as before, we expect this process to be proportional to $\kappa\omega m_{\tilde{g}} m_{3/2}^{-1}$. We explicitly calculate this amplitude and study its partial wave projections. We find that, due to chirality and symmetry properties of this amplitude, the leading helicity amplitude turns out to be the one in which the initial gauge bosons have opposite helicities whereas the final gaugino and the gravitino have the same helicity. The lowest partial wave in this channel obviously corresponds to an angular momentum two channel and is found to be

$$|\mathcal{T}^{2,abc}_{1-1;\frac{1}{2}\frac{1}{2}}| = f^{abc}\frac{g\kappa\omega(m_{\tilde{g}} - 2m_{3/2})}{48\pi m_{3/2}},$$

where g is the gauge coupling constant, f^{abc} is the structure constant of the gauge group and $\mathcal{T}^{J,abc}_{\lambda_1\lambda_2\lambda_3;\lambda_4}$ refers to the $J^{\rm th}$ partial wave projection in the channel in which the initial gauge boson helicities are λ_1 and λ_2, the helicity of the final gaugino is λ_3 and that of the gravitino is λ_4.One finds that this expressions becomes larger than unity, thus explicitly violating unitarity, at an energy scale $\omega_{cr.}$ defined by

$$f^{abc}\omega_{cr.} = \frac{48\pi}{g\kappa}\frac{m_{3/2}}{(m_{\tilde{g}} - 2m_{3/2})}.$$

The point to be noted above is that, as pointed out earlier, if $m_{\tilde{g}}$ is not of the same order as $m_{3/2}$, $\omega_{cr.}$ can be much smaller than the Planck scale.

The above process depended heavily on the fact that we started with a non-abelian gauge theory. However, this fact was not necessary in our general argument at all. To study this better, and indeed to understand the role of the super-Higgs phenomenon in generating all the supersymmetry-breaking mass terms better, we studied[6] the process $gg \to \tilde{G}\tilde{G}$. This is possible even in abelian gauge theories at the tree-level and does not see the typically non-abelian interaction terms in the lagrangian. More interestingly, an application of the reduction formalism to this process can be easily seen to lead to an amplitude for the scattering of gauge bosons to longitudinally polarized nearly massless gravitinos proportional to $\frac{2}{3}\kappa^2 m_{3/2}^{-2}\bar{\chi}\partial^x_\mu\partial^y_\nu \langle\Omega|\,{\rm T}S^\mu_x\bar{S}^\nu_y\,|gg\rangle\,\chi$, where χ, $\bar{\chi}$ are some spinors coming from the wavefunction of the gravitino, T is the time ordering symbol, S^μ,S^ν are the supercurrents which stand as the sources for the gravitinos, and $\langle\Omega|$ and $|gg\rangle$ are the asymptotic vacuum and the incoming two-gauge boson states respectively. Taking the derivatives in through the time ordering symbol produces two non-zero terms in general: a 'σ-commutator' term containing $m_{3/2}^{-2}\{\partial_\mu S^\mu, S^0\}\delta(x^0 - y^0)$ and a normal term going as $m_{3/2}^{-2}{\rm T}\partial_\mu S^\mu\partial_\nu S^\nu$. Knowing that the divergence of the supercurrent is proportional to the supersymmetry breaking masses present in the problem, we find on dimensional grounds that the normal term grows with energy as $\kappa^2\omega^2$ as expected,

but the 'σ-commutator' term grows faster, as $\kappa^2\omega^3$. This latter, if present, would lead to unitarity violations at scales much below the Planck scale even in the absence of a hierarchy between the gaugino and gravitino masses.

On the other hand, if supersymmetry is broken not by explicit terms in the Lagrangian, but by a super-Higgs mechanism, this conclusion cannot hold. In fact, in such theories we can proceed to do the calculation in the so-called 'R-gauge' and make use of a Ward-identity there relating the derivative of the matrix element of the time ordered product of supercurrents (as used above) to the amplitude for the emission of two goldstinos. In the absence of large mass hierarchies, the couplings of the goldstino are the usual couplings of matter to supergravity and hence violate unitarity only at the Planck scale. In fact, to achieve this, the super-Higgs mechanism demands that one should have certain scalar low-mass particles in the theory which interact in such a way as to cancel the offending amplitude quoted above. These particles, normally called axions, are thus shown to be the analogues of the Higgs boson in ordinary gauge theories and are required to exist by the very structure of spontaneously broken local supersymmetry theories.

With these put in, we have calculated the partial wave amplitudes for this process. The most stringent bounds come from the helicity channel in which the gauge bosons have opposite helicities and so do the gravitinos. The lowest partial wave in this channel is the second one, given by

$$|\mathcal{T}^2_{1-1;\frac{1}{2}-\frac{1}{2}}| = \frac{\kappa^2\omega^2}{72\pi}\left(\frac{m^2_{\tilde{g}}}{m^2_{3/2}} - \frac{6}{5}\right).$$

Thus partial wave unitarity is seen to break down at a scale given by

$$\omega_{cr.} = 6\sqrt{2\pi}\left(\frac{m^2_{\tilde{g}}}{m^2_{3/2}} - \frac{6}{5}\right)^{-\frac{1}{2}} M_{Pl}.$$

Exactly as in the non-abelian case, this critical energy can be much smaller than the

Planck scale in the presence of a large ratio between the gaugino and the gravitino masses.

Apart from these theoretical considerations, an attempt is also made to put direct laboratory limits on the mass of the gravitino. Essentially, the cross-section for gluon-gluon scattering to a gluino and a gravitino rises with the energy and should be detectable at the Tevatron in the form of a substantially large mono-jet signature coming from gluino decaying into a gluon and a gravitino.

3. List of Papers

1. "Probing a superlight gravitino in photon-photon collisions"

   Tanmoy Bhattacharya and Probir Roy, *Phys. Rev. Letters* **59**(1987), 1517.

2. "Unitarity limit on the gaugino-gravitino mass ratio"

   Tanmoy Bhattacharya and Probir Roy, *Physics Letters* **B206**(1988), 655.

3. "Role of chiral scalar and pseudoscalar in two-photon production of a superlight gravitino"

   Tanmoy Bhattacharya and Probir Roy, *Phys. Rev.* **D38**(1988), 2284.

4. "Tree unitarity in broken supergravity I: single gravitino amplitude"

   Tanmoy Bhattacharya and Probir Roy, TIFR/TH/88-38

   (submitted to Nucl. Phys. B)

5. "Tree unitarity in broken supergravity II: double gravitino amplitude"

   Tanmoy Bhattacharya and Probir Roy, TIFR/TH/88-41

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Tree Unitarity Breakdown in Spontaneously Broken N=1 Supergravity Theories and Phenomenology of a Superlight Gravitino

4. Introduction

It has long been the hope of theoretical physics to put *all* the known forces and interactions into a unified framework. Despite some tentative steps, the details of such a scheme have eluded physicists so far. Even a qualitative understanding of the nature of such a unification remains problematic. One of the main hurdles in formulating any consistent unified field theory of nature has been the multitude of hierarchically different scales exhibited. More specifically, quantum corrections tend to mix up all the available scales in the theory unless such mixings are prohibited by some symmetry respected by the Lagrangian. Efforts have been made, therefore, to qualitatively understand how a scale as small as the weak scale of about a 100 GeV could survive in the presence of other scales necessarily present in the problem, *e.g.* the scale of gravitational interactions of around 10^{19} GeV or the grand-unification scale in the range of 10^{15} - 10^{16} GeV.

It has been noted for some time that the problem exists essentially because the scale of the electroweak symmetry is controlled by a bosonic mass parameter in the theory. Specifically, electroweak symmetry is supposed to get broken due to a scalar field which gets a vacuum expectation value due to its self interactions. The exact scale at which this takes place is controlled by the mass term for this field in the Lagrangian, provided that the self-couplings of the field are of order unity. Because the full theory has to possess much larger scales in it, as mentioned above, this immediately poses two related problems in all realistic models: naturalness and hierarchy.

First of all, it is aesthetically unpleasant to have many different scales in the theory, because a different physical origin is needed for every dimensionless parameter in the theory that is very different from the 'natural value' of unity, *e.g.* the ratio of hierarchically different scales. Furthermore, in the absence of any symmetry restricting it, quantum corrections tend to dilute all herarchies present in the classical action. This

difficulty is especially severe for the scalar mass terms alluded to above. Quantum corrections to this term, in contrast with those to the fermionic mass terms or the scalar self interaction term, are additive and are not proportional to the mass originally present in the classical theory. This happens because of an apparent quadratic divergence present in the one-loop corrections to the scalar mass; renormalization removes the divergence and the highest scale present in the problem (*e.g.* gravitational scale M_{Pl}) may make everything finite, but then the squared mass tends to run linearly to this highest scale.

The above contrasts strongly with a fermion mass term. In any chiral gauge theory, this is controlled by the chiral symmetry. Quantum corrections to this quantity vanish so long as this symmetry is exact. So, if the scale of symmetry breaking is small for some reason, the fermion masses will remain small, thus circumventing the naturalness problem. Now a theory, endowed with a symmetry linking its bosonic and fermionic sectors, might succeed in solving the naturalness problem mentioned above. In such a case, that symmetry might demand the equality of masses between the bosonic and fermionic particles linked by it. The latter cannot develop large masses, on account of protection by the chiral symmetry, and therefore nor can the former. The exact mechanism by which this comes about is a somewhat drastic cancellation in the apparently divergent terms (between contributions from bosonic loops and fermionic ones) in the perturbative expansion of the physical mass parameter.

In a theory containing at least one mass scale and a chiral fermion, such a symmetry is essentially unique[7,8] and is called N=1 supersymmetry. Supersymmetry thus is a symmetry which has both fermions and bosons in the same irreducible representation. Now, whereas the commutators of bosonic creation operators form a local algebra, only anticommutators of fermionic operators are local. Hence, supersymmetry is realized in field theory by a graded Lie algebra containing some anticommuting operators on

the Hilbert space in addition to the usual commuting ones. There are extra generators linking the bosonic and fermionic particle creation operators, which, because of the spin-statistics theorem, must differ in spin by a half integral amount. Since angular momentum operators form a part of the Poincaré algebra, supersymmetry generators cannot commute with the Poincaré algebra. Furthermore, there is a theorem due to Haag, Łopuzanski and Sohnius[7] which prohibits supersymmetry from being in a semi-direct sum along with the Poincaré algebra. In fact, one of the anticommutators of two supersymmetry generators can be shown to yield a four-momentum translation generator which again belongs to the Poincaré algebra. In other words, the supersymmetry algebra forms a superalgebra of the Poincaré algebra in an essential way. This means that supersymmetry is a spacetime symmetry instead of being an internal symmetry.

The big hurdle in these types of theories is the requirement of breaking supersymmetry. Experiments clearly show that at low energies supersymmetry is not exact in the particle spectrum. In fact, examining the properties of the known particles, one finds that no two of them can be superpartners of each other. Thus supersymmetry must be broken, *i.e.* it must not be present in the spectrum of the theory. Of course, one could add arbitrary explicitly supersymmetry-breaking terms to the Lagrangian, but that would be quite ad hoc. A more attractive proposition is a supersymmetric Lagrangian but a non-supersymmetric spectrum, *i.e.* a "spontaneous breakdown" of supersymmetry. This, however, is problematic because of the very cancellations that give supersymmetry its desirable properties. In fact, there exist non-renormalization theorems which state that many of the classical attributes of a supersymmetric theory are not affected by quantum corrections. In particular, there exist arguments[9] to show that in a non-gauged version of supersymmetry, quantum corrections alone are unable to produce any asymmetry in the spectrum. On the other hand, a spontaneously broken

supersymmetric theory, at the classical level itself leads to phenomenological difficulties[10]. So, on phenomenological as well as theoretical grounds we are forced to deal with the gauged version of supersymmetry.

To put across the point more clearly, let us make a didactic digression. Any (super-) symmetry is realized by a (graded-) Lie algebra of operators acting on the Hilbert space of states of the theory. In fact, in this space there is a unique vacuum state cyclic with respect to a set of operator valued distributions loosely called the 'creation operators'. The Lie algebra elements (called the 'generators' of the symmetry) mentioned above are realized on the space of the Hilbert space operators by commutation or anticommutation relations. Take, for example, the translation generators P_μ belonging to the Poincaré algebra. These are realized on all operators as

$$i[P_\mu, a^\dagger(x)] = \partial_\mu a^\dagger(x),$$

where $a^\dagger$ is any creation operator. In contrast, supersymmetry is realized through its spinorial generator Q_α (α being a spinor index) by

$$[Q_\alpha, a^\dagger(x)] = b^\dagger_\alpha(x),$$

where $a^\dagger$ and $b^\dagger$ are creation operators for some bosonic and fermionic states respectively in the same supermultiplet.

On gauging the same symmetry, we introduce an algebra of the operators multiplied by arbitrary functions of spacetime. For spacetime symmetries, an essentially different algebra is produced because the multiplicative *functions of spacetime* are not left invariant by the operators belonging to the algebra. Thus, typically, if the Poincaré group is gauged, the new algebra of the 'momentum generators' should be written as

$$[f(x)P_\mu, g(x)P_\nu] = (f(x)\partial_\mu g(x))\, P_\nu - (g(x)\partial_\nu f(x))\, P_\mu,$$

This contrasts with the globally symmetric case which had a trivial commutator for

the translation generators. In fact, physically, the gauged theory defines a completely different situation in which there exists a symmetry under independent changes (*e.g.* translation) at each spacetime point. This leads to a much larger invariance and in the particular case of the Poincaré symmetry exhibited above, generates general coordinate invariance — leading thereby to a theory of gravity.

As mentioned above, supersymmetry sits nontrivially in the Poincaré algebra. For example, let us consider the anticommutator of two supersymmetry transformations. Since the supersymmetry generators form a spin-half representation of the rotation group, the anticommutator of two of these will produce a spin-one generator. However, the Coleman-Mandula theorem states that the only such object which can be a symmetry of a non-trivial theory is the translation generator. Thus we find that *repeated supersymmetry transformations produce a translation*, *i.e.*

$$\{Q_\alpha, Q_\beta\} = \gamma^\mu_{\alpha\beta} P_\mu.$$

Hence, the enlargement of global supersymmetry to a gauged supersymmetry has to be accompanied by the gauging of the Poincaré group. This can be readily seen. Multiply the supersymmetry generators on the left hand side of the equality by spinorial spacetime dependent functions $\bar{\lambda}_\alpha(x)$ and $\chi_\beta(x)$ and contract α, β. This leads on the right hand side to $\bar{\lambda}_\alpha(x)\gamma^{\alpha\beta}\chi_\beta(x)$, *i.e.* a spacetime function times the translation generator, *which is an element of the gauged Poincaré algebra.* Hence, in trying to construct a phenomenologically acceptable theory of supersymmetry, we are invariably led to a theory of supergravity.

The quantum theory of gravity is beset with its own problems which are in no way linked directly to the presence or absence of supersymmetry. These problems are essentially of two kinds: interpretative questions dealing with the global structure of

any gravity theory[11] (stemming from the fact that gravity allows the asymptotic regions to be non-flat) and the difficulty[12] in dealing with the theory perturbatively at small distances* (*i.e.* nonrenormalizablity). Though the first still remains a formidable conceptual problem and will perhaps need a non-perturbative understanding, supersymmetry does improve upon the second difficulty. In fact, due to the remarkable cancellations already mentioned above, the short-distance divergences in supergravity are much weaker. Nonetheless, these divergences are far too severe to be renormalized into a finite number of renormalization constants. This situation is markedly different from the case of most other known interacting field theories in four dimensions. In these theories, at least in certain domains, all physical quantities have an asymptotic expansion in a perturbation series upto a sufficiently high order. On the other hand, supergravity today makes sense only at the lowest level of perturbation theory.

In contrast to the above, consistency requirements on the quantum theory (like the conservation of probability) are statements about the full theory and need not hold order by order in general. These requirements thus need to be checked out by explicit calculations. Consequently, the validity of the perturbation expansion to any given order (or indeed to all orders) can get restricted. Thus, in the supergravity models described above, one has to check whether the theory satisfies these requirements if truncated right at the tree level. One of these requirements is unitarity. The truncated theory should remain unitary for the entire range of energies at which this theory is expected to be valid as an effective theory. In other words, tree diagrams computed in this theory should satisfy tests of unitarity until the center-of-mass energy increases beyond this range. This requirement must hold even though the theory is non-renormalizable and higher order loop calculations cannot be done consistently in this theory.

$\star$ To date, no satisfactory non-perturbative quantum approach is known to this problem

Normally, perturbation theory is expected to be valid in the domain where the effective coupling constant is less than unity. However in gravity the coupling constant turns out to be $\kappa = \sqrt{8\pi G_{\rm Newton}}(= 4.11 \times 10^{-19}\ {\rm GeV}^{-1})$ which is dimensional. Hence, the effective expansion parameter depends on the energy scale of interest. In fact, one would expect the failure of perturbation theory to take place at energies near the inverse of this coupling constant, *i.e.* near the Planck scale ($M_{Pl} = \kappa^{-1}$). This can also be seen as the scale where the quantum length scale controlled by the Compton wavelength of a massive particle becomes comparable to the classical length scale set by its Schwarzschild radius. The validity of perturbation theory upto the Plack scale, however, depends on the absence of any large (dimensionless) parameters in the theory. If such a large parameter is present, the non-perturbative nature can manifest itself earlier than the above scale. In particular, the matrix elements can contain the product of the effective coupling constant with this dimensionless parameter; that quantity can become large much before the scale at which the coupling constant reaches unity. In this thesis we intend to show that such large dimensionless numbers do make their appearance in a class of supergravity theories: no-scale theories which contain a relatively light gravitino. Hence, it follows that *these theories fail to exist perturbatively at relatively low energies* due to internal inconsistencies (namely, failure of tree unitarity).

Our calculation proceeds by considering a typical amplitude present in any realistic supergravity theory: the scattering of two gauge bosons into gravitinos and possibly gauginos. In these reactions, when the emitted gravitino is of helicity one-half, one finds that the matrix elements are very large. This result can be understood easily by noting that the massless limit of any theory containing a massive particle of spin greater than half is singular. The main reason behind this curious fact this is that the number of physical states for a particle of mass zero in these cases is always two, whereas

that of the massive particle is larger. Hence, when taking the limit of vanishing mass, a readjustment of the degrees of freedom takes place among the various fields in the theory. Thus for example, a massive spin 3/2 field the gravitino $\widetilde{G}$ has two more degrees of freedom than the massless one. These are the longitudinal degrees of freedom. In the massless limit they appear as a distinct spin half particle. However, for very small gravitino masses, these longitudinal polarization states gather large kinematic factors in their polarization part. In fact, if u_μ represents a longitudinal polarization state for the gravitino,[3]

$$\mathrm{u}_\mu \overset{m_{3/2}\to 0}{\Longrightarrow} \sqrt{\frac{2}{3}}\frac{p_\mu}{m_{3/2}}\mathrm{u}$$

where, $m_{3/2}$ is the gravitino mass and p_μ is its momentum. Here u is an ordinary spin 1/2 spinor. It is apparent that in the absence of precise relationships between the interactions of these fields, the theory then becomes singular in the massless limit. Indeed, the requirement of positivity of the norm of the physical states in the Hilbert space along with the indefinite nature of the Lorentz metric together imply that particles with spin greater than half have to be constrained by auxiliary conditions in the Lagrangian. In the case of Higgs type theories, gauge invariance provides these additional constraints. *At the same time gauge invariance demands a relationship between the different symmetry breaking masses and the interaction parameters in the theory.*

In particular, a gauge symmetry demands that the gauge particle in the theory (*i.e.* the particle corresponding to the gauge field, which transforms inhomogeneously under the gauge transformation) must be massless if the vacuum state is left invariant by the gauge (super-) symmetry. The same condition also implies that the symmetry partners of any particle must be degenerate in mass with it. A similar argument is valid for supersymmetry. On the other hand, if the vacuum state is not left invariant by the symmetry generators, the gauge (super-) symmetry is realized in the (super-)

Higgs phase. In such a case, the gauge particle gets a mass proportional to the (super-) gauge coupling and to the (super-) symmetry breaking vacuum expectation value of some operator O in the theory. The additional degrees of freedom that are necessary for the gauge particle (nescessarily of spin greater than or equal to unity) are provided by the 'absorption' of the goldstone (goldstino), *i.e.* states created by the (super-) partner of the operator O. In other words, the (super-) symmetry partners of this operator disappear from the spectrum and the corresponding degrees of freedom reappear as the additional helicity states of the gauge particle.

Another consequence of the gauge (super-) symmetry being realized in the Higgs phase is that the mass-splits in any (super-) multiplet ($\mathcal{S}$) are proportional to the vacuum expectation values of the operator O and to the strength of the coupling between the (super-) multiplet $\mathcal{S}$ and that which contains O. Hence, any large ratio in these masses can come only from large interaction parameters in the theory. This becomes most clear in the so-called 'R-gauge.' Though not very useful for looking at the spectrum of the theory, it is very convenient for understanding the (super-) Higgs phenomenon. In this gauge the interactions involving the gravitinos in the helicity state one-half get linked to matrix elements of the 'goldstino' field whose interaction strength is naturally given by the ratio of some supersymmetry breaking mass scale in the process to the mass of the gravitino. In our example, we therefore find that the ratio that becomes relevant is the ratio of the gaugino mass to the gravitino mass, or alternatively, the dimensionless coupling of the goldstino superfield to the gauge supermultiplet.

The above is exactly analogous to calculations in the electroweak theory that limit the mass of the heaviest fermion (*e.g.* top quark) on unitarity grounds.[13] There again essentially the same argument applies. In trying to bound the top quark mass in terms of a given $W-$mass, one actually bounds an interaction strength in the theory. There

are however a few interesting differences between that case and the present one. First, the electro-weak theory is a renormalizable theory and hence the failure of tree-level unitarity does not imply the breakdown of that theory, though the interactions becoming strong would have interesting phenomenological consequences. Second, one can calculate the higher loop corrections, but the Higgs sector couplings in that theory not being asymptotically free, they only make the bound stronger. In fact, in lattice calculations, where for very different reasons (triviality of $\lambda\phi^4$ theories) the theory must be treated as an effective theory, the bounds reappear as consistency bounds in the theory. Finally, since the spin 3/2 gravitino has a longitudinal mode which couples through its inverse mass, and since some of the gaugino masses are experimentally constrained[14] to be above around 80 GeV, the amplitude for emitting a gravitino much lighter than this in processes mediated by gauge bosons is very large. We carry out the partial wave unitarity of such processes. Consequently, the gravitino mass is constrained, the nature of the constraint depending on one's theoretical prejudices about the scale at which supergravity should become strong.

On the phenomenological front, the same inconsistency shows up as a blowing up of the cross-sections. Hence, far below the point where the theory becomes inconsistent by violating tree unitarity and needs to be modified, these cross-sections become sufficiently large to be experimentally observable. This fact can be used to put limits on the gravitino mass in the theory. In particular, at colliders like the Tevatron, the cross-section for gravitino production by two gluons fusing into a gluino and a gravitino is already large enough to improve the existing lower bound on the gravitino mass.

This thesis will be divided into six chapters. In the first chapter, we shall describe the supersymmetry algebra in general and then specialize to the case of N=1 supersymmetry algebra. The next chapter will describe the construction of the terms in the Lagrangian

relevant to our later discussion. The third chapter contains the analysis of a particular simple process involving one external gravitino. The fourth chapter tries to understand the results in terms of a qualitative analysis. With this understanding, we treat the case of multigravitino processes (with an example of a double gravitino process) in the fifth chapter. The sixth chapter focuses on the experimental limits and the general phenomenology of superlight gravitinos. Finally, we state our conclusions.

1. Supersymmetry Algebra and Transformations

Much of the physical essence of a theory can be determined from the symmetry properties of the theory alone. Attempts have been made to classify all possible symmetries of various theories of interest. Specifically, symmetries of local quantum field theories have been studied in great detail. Any of these theories is defined on a Hilbert space with a unique vacuum state and contains a complete set of operator valued distributions ('creation and destruction operators') whose commutators or anticommutators are local, *i.e.* proportional to the Dirac delta function. Coleman and Mandula[15] determined the most general Lie-algebra which could be realized on this operator algebra as commutators. They found that spacetime symmetries along with the requirements of analyticity often constrain the interaction matrix to be identically zero. Their pioneering result states that the largest Lie-symmetry of a local quantum field theory with a non-trivial S-matrix is a direct product of the conformal group in four dimensional Minkowskian spacetime with an 'internal symmetry group', *i.e.* one which connects operators at the same space-time point. This space-time symmetry group contains a scale transformation and four special conformal transformations apart from the usual translations and Lorentz boosts which form the Poincaré subalgebra. Since, in four dimensions, no scale-free theory is known (due, amongst other problems, to severe infrared divergences) scale-transformation cannot be a symmetry of such physical theories. Since the largest sub-algebra of the conformal algebra not containing scale transformations is the Poincaré algebra, the Coleman-Mandula theorem effectively states that the largest 'spacetime' Lie-group in these theories is the Poincaré subgroup of the conformal group.

Haag, Łopuzanski and Sohnius (HLS) continued with this line of work and relaxed the requirement of the Lie-algebra nature of the symmetry. They considered the generalization to graded Lie algebras, *i.e.* those which can be expressed in terms of a finite

Table 1

	Description	grading	symbol	number
i	The Momentum generators	even	P_μ	4
ii	The Angular Momentum generators	even	$M_{\mu\nu}$	6
iii	The Supersymmetry generators	odd	Q_i^α	4N
iv	The 'Outer Automorphisms'	even	R_n	N(2N − 1)
v	The Central Charges	even	$Z_{ij}^{R,I}$	N(N − 1)

The largest space-time symmetry consistent with the non-triviality of the S-matrix.

number of generators which close under a graded commutation. Specifically, they considered two classes of generators: the 'even' and 'odd' generators. The generalized Lie-product is defined to be the commutator of an even generator and any other generator or the anticommutator of two odd generators. This graded Lie agebra is realized on the space of creation and destruction operators (which is, of course, naturally graded into the even 'bosonic' operators and the odd 'fermionic' operators) by commutators or anticommutators as needed by the grading requirements. HLS found[7]that in this case the symmetry allowed by the requirement of nontrivial S-matrix à la Coleman-Mandula is much larger. Discarding, as before, the possibility of exact scale invariance in the theory, they found the largest allowed space-time symmetry algebra. It is characterized by a positive integer N and consist of the generators given in Table 1. We note that apart from the Poincaré generators, and the fermionic supersymmetry generators, this algebra contains additional bosonic generators. These elements of the algebra fall into two categories. The first of these classes consists of the 'outer automorphisms.' These generators express a symmetry between the various supersymmetry generators themselves. Sitting as a semi-direct sum with the supersymmetry algebra, these need not

Table 2

$$
\begin{aligned}
\{Q_i^\alpha, Q_j^\beta\} &= 2i(\gamma^\mu P_\mu C^{-1})^{\alpha\beta}\delta_{ij} + C^{-1}(Z_{ij}^R + i\gamma^5 Z_{ij}^I \\
[P_\mu, P_\nu] &= 0 \\
[Q_i^\alpha, M_{\mu\nu}] &= -i\sigma_{\mu\nu}{}^\alpha{}_\beta Q_\beta \\
[P_\mu, M_{\kappa\lambda}] &= -i(\eta_{\mu\kappa}P_\lambda - \eta_{\mu\lambda}P_\kappa) \\
[M_{\mu\nu}, M_{\kappa\lambda}] &= \eta_{\mu\lambda}M_{\nu\kappa} + \eta_{\nu\kappa}M_{\mu\lambda} - \eta_{\mu\kappa}M_{\nu\lambda} - \eta_{\nu\lambda}M_{\mu\kappa}, \\
[Z_{ij}^{R,I}, P_\mu] &= [Z_{ij}^{R,I}, M_{\mu\nu}] = [Z_{ij}^{R,I}, Q_k^\alpha] = 0 \\
[R_{ij}, Q_k^\alpha] &= i(U_{ij})_{kl}Q_l^\alpha
\end{aligned}
$$

The supersymmetry algebra in four dimensions. C refers to the charge conjugation matrix. U_{ij} are hermitian matrices forming the generators of an automorphism group (either SU(N) or a subgroup of it)

be realised as symmetries of any supersymmetric theory. Hence, in the table, only the maximum possible numbers[16] of these generators are tabulated. The second category of extra bosonic generators appear only in extended supersymmetries. In this case, it may not be possible to realize a closed algebra of the above generators in a physical theory. However, a central extension of the algebra (*i.e.* one containing additional generators called the 'central charges') often solves the problem.[16] The exact extension needed varies according to the physical requirements. In the the table, the maximal central extension to the algebra is mentioned. It is to be noted, however, that super-Jacobi identities restrict the number of outer automorphisms to be less than its maximal value in theories realized as central extensions.

The (anti-)commutation relations of these generators are shown in Table 2, where,

for any two operators A and B, $[A, B]$ represents their commutator, $\{A, B\}$, their anticommutator and $[A, B\}$, their graded commutator (*i.e.* $\{A, B\}$ if both are odd and $[A, B]$ otherwise). All the even generators are represented by Hermitian operators in the Hilbert space of states of the quantum theory, while the Q's are represented by Majorana spinor operators.[⋆]

Whereas the action of Poincaré generators on the single particle states is a pure spacetime operation (like translating or rotating their wavefunctions), supersymmetry generators change the nature of these particles by altering their spins by half. In fact, each irreducible representation of the supersymmetry algebra contains both fermionic and bosonic states. Thus, for example, the ordinary gauge bosons, matter fermions, Higgs scalars and the graviton, sit in 'supermultiplets'. In the simplest case of N = 1 supersymmetry these are:

$$\begin{pmatrix} \text{fermion} \\ \text{sfermion} \end{pmatrix}, \begin{pmatrix} \text{higgsino} \\ \text{higgs} \end{pmatrix}, \begin{pmatrix} \text{gauge boson} \\ \text{gaugino} \end{pmatrix}, \begin{pmatrix} \text{graviton} \\ \text{gravitino} \end{pmatrix}.$$

Here each of the first two supermultiplets consists of spin half and spin zero partners. The third one comprises spin one and spin half partners while the gravity supermultiplet has a spin two and a spin three-half member.

Theoretically, N — the number of distinct supersymmetries in the theory — is a free parameter which must be a power of 2, *i.e.* of the form 2^n for all positive integers n. However, from the supersymmetry algebra, it is easy to see that sequential applications of the supersymmetry generators on a state with spin zero will build up states with higher spins.[†] However, whenever any generator is applied twice, we can use the

⋆ We use the metric convention $\eta = diag(1, -1, -1, -1)$, and the γ-matrices are defined as in Bjorken and Drell.[17] The matrix σ is then defined by $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/4$. The Dirac conjugate of a spinor ψ is defined to be $\bar{\psi} = \psi^\dagger \gamma^0$ and its Majorana conjugate is defined as $\psi^c = C^{-1}\bar{\psi}^T$, where the charge conjugation matrix C is numerically equal to $i\gamma^0\gamma^2$. A spinor ψ is said to be Majorana if $\psi = -i\psi^c$, and then its components are all real (or hermitian).

† Needless to say, the square of the supersymmetry generator being equal to the Poincaré generator, a non-vacuum state cannot be inert under supersymmetry

anticommutation relations to express the reult in terms of a smaller number of supersymmetry generators and some Poincaré generators. Since the latter do not change the spins of the states (the magnitude of the angular momentum being a Casimir for the Poincaré subalgebra), there is a limit to the maximum spin which one can achieve by this process. The actual number of spin states required for a given number of supersymmetry generators is therefore easy to calculate. It turns out that supersymmetry with $n > 2$ needs particle states with spins greater than 1. Such particles do not seem to have consistent interactions (except in supergravity theories). Rigid supersymmetric theories in four dimensions are therefore restricted to have either N = 1, 2, or 4 supersymmetries. Even with the inclusion of supergravity, only N = 8 is allowed in addition.

Empirically, we know that the gauge group relevant to low energies is chiral. This, in essence, means that the interactions of gauge bosons with fermions do not conserve parity in general, *i.e.* they behave differently towards the left and right chiral fermions. On the other hand, extended supersymmetries always contain the left and the right chiral spin half fermions in the same extended supermultiplet (provided we do not allow particles other than gravitons and gravitinos to have a spin more than one). Hence, if supersymmetry stays unbroken, the interactions cannot be chiral. This leads to a vectorlike gauge theory in disgreement with the observed chiral nature of the electroweak theory. Hence, hereafter, we consider only the case of N=1 supersymmetry[8].

Phenomenologically, however, even global $N = 1$ supersymmetry poses problems. The absence of mass-degenerate bosons and fermions implies that supersymmetry is not expressed in the particle spectrum. On the other hand, a study of global supersymmetry breaking[10]establishes mass relations that imply that a supersymmetric theory not

involving an extra U(1) factor in the gauge group[‡] and spontaneously broken at the tree level always possesses at least one superparticle lighter than the corresponding particle. Experimentally, such a relation is not tenable as none of the superpartners have yet been seen. This forces any spontaneous supersymmetry breaking to be a loop effect, arising from quantum corrections to the classically supersymmetric configuration.

Further, due to the presence of odd generators in the supersymmetry algebra, supersymmetric theories contain an equal number of fermionic and bosonic states in any except possibly the lowest zero energy state.[§] In addition, the same symmetry imposes severe constraints on its interactions. Because of these constraints, perturbative corrections to many processes are highly suppressed, the contributions from bosons being almost cancelled (due to the similarity of fermionic and bosonic interactions and yet the antisymmetry of the fermions) by those arising from fermions. As a result, a number of important non-renormalization theorems[10] follow. These theorems show that a number of interaction terms are not renormalized at all by the perturbative loop corrections to the lowest order result. Among them an important class consists of the non-supersymmetric vacuum expectation values in the theory. Thus, perturbative quantum corrections in a rigid supersymmetric theory can never break a 'classically' unbroken global supersymmetry.[9] These arguments, however, do not hold in locally supersymmetric or supergravity theories. There one could have a spontaneous symmetry breaking in a hidden sector of the theory inert under all observable gauge interactions (so that ordinary phenomenology is not adversely affected) either at the tree level or induced by quantum loops. In the $\kappa \to 0$ limit of the theory (when the 'gauge paricles' of supergravity, *i.e.* the gravitino and the graviton decouple and one is left with a glob-

‡ Such factors in the gauge group usually make the theory anomalous and hence are not considered.

§ This is obvious because the supersymmetry algebra implies $\{Q, Q^\dagger\} \propto H$, where H is the Hamiltonian and $Q^\dagger$ is the hermitian conjugate of Q. Hence, no nonzero eigenvector of H can be annihilated by Q.

ally supersymmetric theory), the effect of the breaking will linger in terms of 'soft' (*i.e.* scale dimensionality < 4) explicit supersymmetry breaking terms in the Lagrangian.

We can now construct the most useful representations of this algebra. For us that will be the representation on the gauge multiplet which consists of two particles: a massless vector particle corresponding to the field A_μ and a Majorana spin half fermion given by the field λ. The infinitesimal supersymmetry transformations for the noninteracting theory are given by[16]

$$\begin{aligned} \delta A_\mu &= -i\bar{\epsilon}\gamma_\mu\lambda, \\ \delta\lambda &= -F_{\mu\nu}\sigma^{\mu\nu}\epsilon, \end{aligned} \tag{1.1}$$

where $F_{\mu\nu}$ is the structure tensor corresponding to the gauge field A_μ. Further, δA on any field A is defined to be $i[\bar{\epsilon}Q, A]$ where ϵ is the 'anticommuting spinorial parameter' for supersymmetry transformations. The use of anticommuting variables is mainly for the purpose of notational convenience. It is defined as a Majorana four component spinor whose entries commute with all complex numbers and bosonic operators and anticommute with all anticommuting parameters and fermionic operators. We will often consider this parameter to be a 'function' of spacetime coordinate and define a formal spacetime derivative of this object. Essentially, we enlarge our space of scalars from the complex numbers to the space of Grassmann numbers,[18] with a complex conjugation operation defined. The calculus of these objects is also defined in the usual way.[18] Furthermore, it is to be noted that the above algebra closes only on shell, *i.e.* only when the fields satisfy their (massless, non-interacting) equations of motion and, then too, only upto gauge transformations.

The above form is, however, not very useful in trying to construct Lagrangians in general. This is mainly because these transformations form an algebra when the equations of motion are satisfied. However, the latter are known only a posteriori from

the Lagrangian. Hence, we need the so-called off-shell version of the algebra, which is valid without the equations of motion. This requires the introduction of auxiliary fields which get eliminated by algebraic equations of motion in the final Lagrangian. The origin of these auxiliary fields is easy to identify. Essentially, supersymmetry connects bosonic and fermionic states and hence the number of bosonic states in a supersymmetric theory is always equal to the number of fermionic states (except possibly for a vacuum state). This rule is already explicit for all the on-shell multiplets illustrated above. However, particles of different spins have different number of components off shell. Thus, for example, a spin-half fermion has a first order equation of motion and hence is a constrained system. Therefore off-shell it has four complex degrees of freedom whereas on-shell it has only four *real* degrees of freedom. On the other hand, a scalar field has the same number of degrees of freedom off-shell as on-shell. So the only way that fermionic and bosonic degrees of freedom can match is when off-shell there are additional bosonic *fields* which disppear on-shell, *i.e.* if there are auxiliary fields in the Lagrangian.

The determination of the exact set of fields required to close the algebra off-shell is quite complicated. Fortunately, there is often enough gauge freedom in the problem to make useful gauge choices. A particular class of gauges called the Wess-Zumino gauges is found especially useful. In these gauges, the Lagrangian of the pure gauge sector of the theory turns out to be[19] a polynomial in the fields and a number of auxilliary fields disappear from the Lagrangian. For example, in this gauge, the gauge multiplet mentioned above needs only one scalar (or pseudo-scalar) field D to close the algebra off-shell (upto a gauge transformation). This gauge is not supersymmetric; the commutator of two supersymmetry variations gives rise to the required space time translation only after a gauge transformation. That, however, is known *a priori* and hence does not prevent the construction of the Lagrangian. The Lagrangian in any case is always writ-

ten in a gauge-fixed form. We find that in such a case only the fermion transformation laws pick up extra contributions which on elimination of auxiliary variables become at least quadratic in the fields.

The transformation laws for the gravitational multiplet can also be discussed in a similar spirit. This multiplet, being the 'gauge multiplet' for supergravity, contains inhomogeneous (affine) transformations. The multiplet consists of two on-shell fields: a massless spin 2 graviton $g_{\mu\nu}$ and the spin 3/2 gravitino $\widetilde{G}_\mu$. It is easier to write the transformations in terms of the vierbein fields (e^a_μ) which are the square roots of the graviton field: $g_{\mu\nu} = e^a_\mu e_{a\nu}$, a being a local tangent frame index while μ stands for a global one. The on shell infinitesimal transformation laws are:

$$\begin{aligned}
\delta e^a_\mu &= i\kappa\bar{\epsilon}\gamma^a\widetilde{G}_\mu, \\
\delta\widetilde{G}_\mu &= \frac{2}{\kappa}(\partial_\mu + \omega_{\mu ab}\sigma^{ab})\epsilon,
\end{aligned}$$

where $\sigma^{ab} = [\gamma^a, \gamma^b]/4$ and the 'spin connection' ω is given by:

$$\begin{aligned}
\omega_{\mu ab} = {} & e^\nu_a \partial_{[\mu} e_{\nu]b} - e^\nu_b \partial_{[\mu} e_{\nu]a} - e^\rho_a e^\sigma_b e^c_\mu \partial_{[\rho} e_{\sigma]c} \\
& + \frac{i\kappa^2}{4}(2\overline{\widetilde{G}}_\mu \gamma_{[a}\widetilde{G}_{b]} + \overline{\widetilde{G}}_a \gamma_\mu \widetilde{G}_b),
\end{aligned}$$

with the square (round) brackets in the subscripts or superscripts denoting (anti-) symmetrization in the indices. The minimal off-shell formulation of this multiplet requires the introduction of a complex scalar u and a vector A_μ auxiliary field.[20]

We shall also briefly consider later the chiral supermultiplet in an example of the super-Higgs mechanism. The transformation laws for this multiplet will be given in the relevant place. This multiplet consists of two on-shell fields, a complex scalar field z and a spin-half Majorana fermion field χ. The transformation laws of this multiplet are chiral (or complex) in the sense that the transformation laws of z involves only one (conventionally left) chiral component of χ. Likewise, the supersymmetry transforma-

tion law for the left chiral part of χ is linear in z and does not involve z^*. Its on-shell transformation laws are given as

$$\delta z = \sqrt{2}\bar{\epsilon}\chi_L$$

$$\delta\chi_L = -\sqrt{2}i\partial\!\!\!/\epsilon_R z$$

We shall often look upon the complex field z as being composed of two real fields: a scalar S and a pseudoscalar P. Off-shell this multiplet contains one complex scalar auxiliary field F.

To construct a Dirac fermion we start with two of the above chiral multiplets and combine them in a complex combination. The field content is then doubled. Hence, the multiplet then contains a Dirac fermion, *two* complex scalar fields and two complex scalar auxiliary fields. The known Dirac fermions may be realized in supersymmetry by these multiplets.

2. Lagrangian of Local Supersymmetry

The Lagrangian for any supersymmetric theory can be constructed in a systematic fashion in more ways than one. Probably, the simplest method relies on the principle of Noether coupling. Due to the celebrated Noether's theorem,[21] we know that every global symmetry of the Lagrangian density provides us with a conserved current. In fact, if $\epsilon_g \delta_g \phi_i$ represents the variation of a field ϕ_i under a given symmetry element g by an infinitesimal amount ϵ_g, then the total variation of the lagrangian density under this symmetry:

$$\epsilon_g \delta_g \mathcal{L} = \epsilon_g \left\{ \frac{\delta \mathcal{L}}{\delta \phi_i} \delta_g \phi_i + \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi_i} \partial_\mu \delta_g \phi_i \right\}, \tag{2.1}$$

has to be a total divergence. Let us denote this total divergence by $\epsilon_g \partial_\mu \Omega^\mu$. In that case, the current defined by

$$J^\mu = \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \delta_g \phi - \Omega^\mu \tag{2.2}$$

is conserved as a result of the equations of motion.

When this particular symmetry is made local, so that the parameter ϵ_g becomes a spacetime function, the original Lagrangian is no longer invariant. In fact, looking at (2.1) one finds that the extra contribution to the variation is

$$\frac{\delta \mathcal{L}}{\delta \partial_\mu \phi_i} \delta_g \phi_i \partial_\mu \epsilon_g. \tag{2.3}$$

In addition, $\epsilon_g \partial_\mu \Omega^\mu$ is no longer a total divergence, and it is less than the true divergence $\partial_\mu (\epsilon_g \Omega^\mu)$ by an amount given by

$$\Omega^\mu \partial_\mu \epsilon_g. \tag{2.4}$$

Hence, if the variation of the Lagrangian was reduced by the amount (2.3) − (2.4), then the new Lagrangian would be symmetric. But, this difference is nothing but $J^\mu \partial_\mu \epsilon_g$ as

can be seen from (2.2). This reduction is achieved by introducing a gauge field A_μ and a gauge coupling constant g (dimensionless in 4D), adding a term to the Lagrangian of the form $gA_\mu J^\mu$ and prescribing the variation of the gauge field to be

$$\delta_g A_\mu = \frac{1}{g}\partial_\mu \epsilon_g. \tag{2.5}$$

It is easy to see that this makes the new Lagrangian* invariant to the lowest order in g. One can now proceed from this point and calculate the new variation of the Lagrangian, the new Noether current and change the Lagrangian by higher order terms in the gauge coupling constant. If this process terminates, we get a Lagrangian which is symmetric under the local symmetry.

In cases where the symmetry is known, *a priori,* only on shell, that is, if the closure of the symmetry algebra involves the use of equations of motion, the same process can still be applied. However, in these cases, the variation of the Lagrangian at any nonzero† order of the coupling constant can produce terms proportional to the equation of motion in addition to a total divergence. Nevertheless, it can be shown[21] that in such cases, an adjustment of the symmetry variations $\delta_g \phi_i$ by terms of higher order in the coupling constant can solve the problem.

In the case of supersymmetric theories, we have an additional problem stemming from the fact that these theories invariably involve fermions. Furthermore, in carrying out the Noether technique, we are always dealing with the Lagrangian at a classical level. It is easy to see that if the fermion field operators are treated as ordinary func-

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$\star$ It is to be noticed that this is equivalent to the minimal coupling scheme of replacing ∂_μ by $(\partial_\mu - igA_\mu)$ everywhere in the Lagrangian in the simple cases where $\Omega^\mu = 0$

$\dagger$ As the onshell algebra is required to be a symmetry of the Lagrangian, the variation of the global Lagrangian still produces a total divergence. At higher orders, as subsequent symmetry algebra operations produce terms proportional to the equation of motion in addition to another algebra operation, this is not true anymore.

tions of spacetime in the classical limit,[‡] we run into contradictions.[§] To avoid this embarrassment, we extend the space of our classical variables from ordinary complex numbers to elements of a Grassman algebra, the properties of which have already been described in the preceding chapter. With the supersymmetry parameter now defined to be a Grassman object as well, the entire procedure outlined above goes through. After a lot of tedious algebra, one can derive[21] the entire Lagrangian and transformation laws for local supersymmetry.

The other method for deriving the Lagrangian can be appleied to any gauge theory. In particular, supergravity[22] starts off by defining a fiber bundle (involving Grassmann objects, the resulting structure being called a 'superspace') over spacetime with the gauge group[¶] as its structure group. The (super-)multiplets of the matter fields are then viewed as sections of this bundle, whereas the gauge field is seen to define a connection on this bundle. In this language, writing down the invariant Lagrangian involves writing down an appropriate structure out of the fields and their covariant derivatives defined with the connection specified above. The final Lagrangian can then be expanded and written in terms of the component fields.

The major difficulty with this approach is that because of its very structure, the Lagrangian is defined later than the gauge group. As a result, this requires the symmetry algebra to be closed *without reference to the Lagrangian*. In other words, the symmetry must be known off-shell. The Noether technique however works and produces a unique result as long as an off-shell version of the symmetry exists, *whether known or*

‡ Truly speaking, a fermion field operator *does not have* a classical limit. This is permissible since only fermion bilinears are observables and hence need to have classical limits.

§ For example, the fermion kinetic energy written in a form that is real, turns out to be a total divergence.

¶ In case of supersymmetry, we define the group from the algebra in the 'usual way' by taking the exponential maps with Grassman parameters.

unknown. Fortunately, for N=1 supergravity theory, the off-shell formulation exists, and the Lagrangian is fully derived.[23]. We shall often use the Lagrangian as quoted there.

The Lagrangian thus constructed is explicitly supersymmetric. In actual practice however, we want supersymmetry to be spontaneously broken. This can happen when some of the fields (or their condensates) which have non-trivial supersymmetry transformations have a non-zero vacuum expectation value. In such a case, though the Lagrangian is supersymmetric, some physical fields develop inhomogeneous terms in their supersymmetry transformation laws. Since Poincaré invariance is observed to be an exact symmetry, the object that gets the vacuum expectation value must be a scalar under Poincaré transformations. However, if this quantity is the lowest component (as determined by its mass dimension) of a super-multiplet, then the supersymmetry transformations (which involve the lowest components only through *derivatives*) leave it invariant. Hence this vacuum expectation value fails to break supersymmetry. Thus for example, the gauge multiplet transformation laws (off-shell, after gauge fixing to the so-called Wess-Zumino gauge[19]) can be written in terms of the gauge field A^a_μ, the Majorana gaugino field λ and the auxiliary field D^a as

$$\begin{aligned} \delta_g A^a_\mu &= -\frac{i}{2}\bar{\epsilon_g}\gamma_\mu\lambda^a \\ \delta_g\lambda &= -\frac{1}{2}(\sigma^{\mu\nu}F^a_{\mu\nu} + D^a)\epsilon_g \\ \delta_g D^a &= \frac{1}{2}\bar{\epsilon_g}\not{\partial}\lambda \end{aligned} \tag{2.6}$$

In this if the auxiliary field D^a gets a vacuum expectation value, then in the vacuum state, $\delta_g\lambda \neq 0$. Hence under a supersymmetry transformation the vacuum does not remain invariant. Thus, to break supersymmetry, we need either the higher components of a supermultiplet (which are normally auxiliary fields and hence composites in the on-shell representation) or some other composites to develop vacuum expectation values. The exact quantity which develops a vacuum expectation value is unimportant as far

as our results are concerned. However, for illustrative purposes, and often to motivate our results, we shall consider a chiral multiplet whose auxiliary component acquires a vacuum expectation value, breaking supersymmetry. Further, since we require the supersymmetry breaking to be unaccompanied by any gauge-symmetry breaking, we demand this chiral multiplet to be invariant under the observed gauge interactions, and for simplicity, under all gauge interactions.

A chiral supermultiplet consists of a complex chiral scalar field z and a chiral fermion (conventionally taken to be left handed) χ_L^α as its on shell degrees of freedom. The algebra is trivially realized off-shell also with the addition of an extra auxiliary complex scalar field F. In terms of these, the infinitesimal transformations are

$$\begin{aligned} \delta_g z &= \sqrt{2}\bar{\epsilon}_g \chi_L^\alpha \\ \delta_g \chi_L^\alpha &= -\sqrt{2}\left(i\partial\!\!\!/ z + \delta \mathrm{Re} F + i\gamma^5 \mathrm{Im} F\right)^{\alpha\beta} \epsilon_{g\,L} \\ \delta_g F &= i\bar{\epsilon}_g \partial\!\!\!/^{\alpha\beta} \chi^\beta \end{aligned} \tag{2.7}$$

When the auxiliary field F acquires a vacuum expectation value, as expected, χ acquires a constant term in its supersymmetry transformation.

$$\delta_g \chi_L^\alpha \propto \langle F \rangle \epsilon_g + \text{field dependent parts},$$

where $\langle F \rangle$ refers to the vacuum expectation value of F. In addition, the Lagrangian, in general, develops non-diagonal bilinear terms of the form[24]

$$\mathcal{L} = \frac{i}{2}\bar{\chi}\partial\!\!\!/\chi + \frac{i}{\langle F \rangle}\bar{\chi}\gamma^\mu \widetilde{G}_\mu + \cdots .$$

These mix the gravitino and other fields, in particular, the field χ which is the chiral fermionic component of this multiplet. The field combination that mixes with the gravitino is called the 'goldstino.' A rediagonalization of the bilinear terms in the Lagrangian is then needed to carry out the usual perturbative analysis. Since the

supervariation of the goldstino contains a chiral piece, this can always be achieved by a 'field dependent' supersymmetry transformation that transforms the spinor χ identically to zero. This can also be looked upon as a redefinition of field variables which eliminates the goldstino field χ. Of course, as the Lagrangian was supersymmetric to start with, the new Lagrangian after this transformation, turns out to be precisely the same as the old one except that all goldstino field gets replaced by zero. In other words, the goldstino field gets transformed away into the gravitino. The physical superpartner of the goldstino, the complex scalar field z, however, stays in the spectrum. We shall usually represent it in terms of its real components, $S = \mathrm{Re}z/\sqrt{2}$ and $P = \mathrm{Im}z/\sqrt{2}$ which turn out to be a scalar and pseudoscalar respectively. These are analogous to the Higgs particle in ordinary gauge theory and are responsible for the consistency of a spontaneously broken theory as opposed to an ordinary (asymmetric) theory with current conservation. In a later chapter we shall see that they provide cancellations essential to maintaining tree-level unitarity in the low energy theory. On the other hand, this breaking now changes the cosmological constant in the theory. This can be readily understood by noting that a supersymmetry-noninvariant vacuum cannot have zero energy. Hence, starting from a supersymmetric theory in a Minkowskian (flat) background, supergravity breaking produces a de Sitter universe. A massless particle in a de Sitter universe, as opposed to one in a flat universe, must have a non-derivative bilinear 'mass-like' term in the Lagrangian.[25] However, since the broken Lagrangian has the same form as the unbroken one, there can be no such term in the theory under consideration, and one can conclude that the gravitino becomes massive on breakdown of local supersymmetry. Alternatively, if we start from supergravity in an anti-de Sitter* universe (in which case, the super-invariant Lagrangian contains bilinears in the gravitino field as is required of a massless spin-3/2 field here), the vacuum energy

$\star$ Supersymmetry is not possible in a de-Sitter universe.

can be tuned to be just enough to make the background space-time Minkowskian after supergravity breaking. In such a situation, the bilinear term persists on supersymmetry breaking and in a flat background appears as a mass for the gravitino. Even though the details are thus different, this result is exactly analogous to the Higgs mechanism where a gauge non-singlet scalar field gets a constant vacuum expectation value leading to a massive gauge boson. There, the Lagrangian written in terms of properly defined fields (which represent fluctuations about the vacuum), turns out to have terms which mix the gauge boson (analogous to the gravitino here) with the gauge partner of the field getting a vacuum expectation value. A formal gauge transformation, however, rediagonalises the kinetic terms leaving no trace of this field and producing a mass for the gauge boson. Because of this formal similarity with the Higgs mechanism, the mechanism here[26] is called the super-Higgs mechanism.

The physical effects of the super-Higgs mechanism are very different from those of the Higgs mechanism. The essential features of any pointlike quantum field theory of gravity is that it is non-renormalizable. Hence, there is no more any physical motivation for a polynomial Lagrangian. Most of the supergravity Lagrangians considered are therefore non-polynomial in nature. Now, in a theory based upon a polynomial Lagrangian, we expect the formal manipulations alluded to above not to introduce any drastically new scales in the theory: the scales in the physical theory would be more or less those present in the original Lagrangian. This expectation breaks down in the non-polynomial theory. The vacuum expectation values and the resulting physical scales now can be very different from the original one. For example, in the class of Lagrangians of our interest — namely the so-called no-scale models — the tree-level potential turns out to be flat in certain directions. The exact magnitude of the vacuum expectation value is then controlled by loop corrections, and can therefore be at a scale which is very different from

whatever appears in the original Lagrangian. This seed vacuum expectation value then controls the various masses finally present in the spectrum of the theory. However, since these masses are not linked to that vacuum expectation value by polynomial relations, the physical masses can again be very different in magnitude from the latter. In this arena, the consistency requirements of perurbative unitarity imposing bounds on various mass ratios are therefore non-trivial.

To amplify the above point, let us consider supergravity theories in general. A supergravity theory, being nonrenormalizable, has so much freedom in its coupling parameters that it is more succinctly defined by two arbitrary *coupling functions* rather than by coupling constants.[23] The first of these is a real-valued function $\mathcal{G}(z, z^*)$ of the chiral scalar fields z present in the theory. It is related to Kähler potential $d(z, z^*)$ and the superpotential $f(z)$ by $\mathcal{G}(z, z^*) = \kappa^2 d(z, z^*) - ln|\kappa^3 f(z)|^2$. The other is a complex analytic function $f^{\alpha\beta}(z)$ of the same variables, which takes values in the symmetric

tensor product of two adjoint representations of the gauge group in the theory. The portion of the Lagrangian[23] relevant for our discussion here is summarized below:

$$\mathcal{L} = -\frac{1}{4}\langle f_{ab}\rangle F^a_{\mu\nu}F^{b\mu\nu} + \frac{1}{2\kappa^2}R + \frac{1}{2}e^{\mu\nu\rho\sigma}\overline{\widetilde{G}}_\mu\gamma_5\gamma_\nu D_\rho\widetilde{G}_\sigma + \frac{i}{2}\langle f_{ab}\rangle\bar{\lambda}^a\not{D}\lambda^b + \frac{i}{6}\langle u\rangle\overline{\widetilde{G}}_\mu\sigma^{\mu\nu}\widetilde{G}_\nu - \frac{1}{2\kappa}\langle F_i\rangle\left\langle\frac{\partial f_{ab}}{\partial z_i^*}\right\rangle\lambda^a\lambda^b$$

where u and F_i refer to the vacuum expectation values of the scalar auxiliary field in the gravitational and chiral multiplets respectively. If no fermion condensate gets formed, the symmetry transformation laws imply relations among the various auxiliary field vacuum expectation values. Using, in addition, D_a to represent the scalar auxiliary field in the gauge multiplet, one has[23]

$$\begin{aligned}\langle u\rangle &= 3\left\langle e^{-\mathcal{G}/2}\right\rangle\\ \langle D_a\rangle &= \mathrm{Re}\left\langle f^{-1}_{ab}g\frac{\partial\mathcal{G}}{\partial z}T^b z\right\rangle\\ \langle F_i\rangle &= -\frac{1}{2}\left\langle e^{\frac{-\mathcal{G}}{2}}\right\rangle\left\langle\frac{\partial^2\mathcal{G}}{\partial z_i\partial z_j^*}\right\rangle^{-1}\left\langle\frac{\partial\mathcal{G}}{\partial z_j}\right\rangle\end{aligned}$$

where T^a are the representation matrices of the gauge group in the representation to which z belongs and angular brackets signify vacuum expectation values.

The point to be emphasized in these relations is that the gravitino mass $m_{3/2}$ is given by the expression $\kappa^{-1}\exp\{-\mathcal{G}/2\}$. On the other hand, the supersymmetry breaking mass parameter Δm_S, characterizing the low-lying mass-splits in the supermultiplets, is set by the vacuum expectation values of the auxiliary fields which contain the derivatives $\partial\mathcal{G}/\partial z$ amongst others. It is of course obvious from its expression that $m_{3/2}$ cannot be made zero so long as the vacuum expectation values mentioned above are non-zero. However, whereas for any polynomial $\mathcal{G}$, the magnitude of the mass term can be written down as κ times the vacuum expectation values and the coefficients of

the polynomial, the same is not true for a non-polynomial $\mathcal{G}$. In fact in a general case, there is no simple relation between the gravitino mass and the other mass scales in the theory, *e.g.* the gaugino mass. Hence, if $\mathcal{G}$ is non-polynomial, *naturalness, a priori, does not constrain the gravitino mass to be of the same order of magnitude as κ times the seed vacuum expectation value or the mass-splits in the supermultiplets.* Nevertheless, the super-Higgs mechanism does imply that supergravity cannot be broken without at the same time giving a mass to the gravitino.

The full Lagrangian shows that in addition to giving the gravitino a mass, the super-Higgs mechanism also affects other supermultiplets. In particular, the members of any supermultiplet do not remain mass-degenerate any more. Thus, as was expected, the super-Higgs mechanism implies a broken-supersymmetric spectrum of physical states. An additional point which shall be relevant to our later analysis is obvious from the above discussion. Since the mass-splits in the various supermultiplets essentially stem from the vacuum expectation values of auxiliary scalars of some supermultiplet (call it the 'super-Higgs multiplet'), *a supermultiplet which does not couple to this super-Higgs multiplet remains mass degenerate at the tree level.*

Finally, except for the generation of masses and the consequential interaction with gravity (which couples to the total energy including mass), the interactions in the theory are not changed by the super-Higgs mechanism. In this sense, we describe the super-Higgs symmetry breakdown as 'soft.' This knowledge enables us to explicitly calculate any matrix element perturbatively without actually working out the details of the super-Higgs effect. We shall use only this much in our subsequent analysis without making any assumption about the details of the supersymmetry breaking. Therefore, our results will be completely general.

3. Single Gravitino Amplitude

3.1 *Introduction*

As seen in the Chapter 2, the minimal N=1 supergravity theory contains couplings of chiral matter and gauge supermultiplets to the fundamental supergravity multiplet. The physical fields in the latter are the spin 2 graviton $h_{\mu\nu}$ and the Majorana spin $\frac{3}{2}$ gravitino $\widetilde{G}_\mu$. The gauge supermultiplets generically consist of gauge bosons A^a_μ and Majorana spin $\frac{1}{2}$ gauginos λ^a, a being a gauge index. Their couplings are determined by the requirement of local supersymmetry (which needs to break spontaneously to allow for mass terms). In this chapter we study a particular helicity amplitude in the theory, namely, the scattering of two non-abelian gauge bosons (A) to produce the corresponding gaugino (λ) and a gravitino ($\widetilde{G}$), *i.e.* $A + A \to \lambda + \widetilde{G}$.

The terms in the Lagrangian relevant to us in that case then turn out to be precisely those in the local version of super-Yang-Mills theory except that we also have terms reflecting the gaugino mass $m_{\tilde{g}}$ and the gravitino masse $m_{3/2}$. This Lagrangian can be written[23] in Bjorken-Drell[17] notation (introducing slanted capitals for tangent-frame indices) as

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\widetilde{G}}_\mu\gamma_5\gamma_\nu(\partial_\rho - \frac{i}{4}\omega_{\rho AB}\sigma^{AB})\widetilde{G}_\sigma - \frac{e}{4}F^a_{\mu\nu}F^{a\mu\nu} + \\ & \frac{ie}{2}\overline{\lambda}^a[(\not\partial - \frac{i}{4}\not\omega_{AB}\sigma^{AB})\delta_{ab} - g\not A^c f_{cab}]\lambda^b + \frac{ie}{2}m_{3/2}\overline{\widetilde{G}}_\mu\sigma^{\mu\nu}\widetilde{G}_\nu - \frac{e}{2}m_{\tilde{g}}\overline{\lambda}^a\lambda^a + \\ & \frac{e\kappa}{4}\overline{\lambda}^a\gamma^\rho\sigma^{\mu\nu}\widetilde{G}_\rho F^a_{\mu\nu}. \end{aligned} \tag{3.1}$$

In (3.1) $e = det\ e_\mu{}^A$ and

$$\begin{aligned}\omega_{\mu AB} = & e^\nu_A\partial_{[\mu}e_{B\nu]} - e^\nu_B\partial_{[\nu}e_{A\nu]} - e^\rho_A e^\sigma_B e^C_\mu\partial_{[\rho}e_{\sigma]C} \\ & + i\frac{\kappa^2}{4}(2\overline{\widetilde{G}}_\mu\gamma_{[A}\widetilde{G}_{B]} + \overline{\widetilde{G}}_A\gamma_\mu\widetilde{G}_B),\end{aligned}$$

e^μ_A is the inverse to the vierbein $e_\mu{}^A$. Further, $F^a_{\mu\nu} = 2\partial_{[\mu}A^a_{\nu]} + gf^{abc}A^b_\mu A^c_\nu$ with g the gauge coupling constant and f^{abc} the structure constants of the gauge group.

Figure 1

Vertices relevant to our process. The solid wavy lines represent gauge bosons. A solid line through the middle indicates the corresponding superpartners. A solid line through a dashed wavy line represents the gravitino.

The tree level vertex tensors in position space are easily derived from (3.1) by taking functional derivatives of the action $S = \int d^4x\mathcal{L}$ with respect to the fields which correspond to the legs of the vertex. For the vertices depicted in Fig. 1, we can therefore write

$$\begin{aligned}\left(V_{\mu\nu}^{A\lambda\widetilde{G}}\right)^{ab} &= \left.\frac{\delta(iS)}{\delta A^{\mu a}(x_1)\delta\lambda^b(x_2)\delta\overline{\widetilde{G^\nu}}(x_3)}\right|_0 \\ &= \frac{i\kappa}{2}\sigma_{\alpha\mu}\gamma_\nu\int d^4x\delta^{(4)}(x-x_2)\delta^{(4)}(x-x_3)\partial^\alpha\delta^{(4)}(x-x_1)\delta^{ab},\end{aligned} \tag{32a}$$

$$\begin{aligned}\left(V_{\mu}^{A\lambda\lambda}\right)^{abc} &= \left.\frac{\delta(iS)}{\delta A^{\mu a}(x_1)\delta\lambda^b(x_2)\delta\overline{\lambda}^c(x_3)}\right|_0 \\ &= -gf^{abc}\gamma_\mu\int d^4x\delta^{(4)}(x-x_1)\delta^{(4)}(x-x_2)\delta^{(4)}(x-x_3),\end{aligned} \tag{32b}$$

$$\begin{aligned}\left(V_{\mu\nu\lambda}^{AAA}\right)^{abc} &= \left.\frac{\delta(iS)}{\delta A^{\mu a}(x_1)\delta A^{\nu b}(x_2)\delta A^{\lambda c}(x_3)}\right|_0 \\ &= -igf^{abc}\int d^4x\{\eta_{\lambda\mu}\delta^{(4)}(x-x_2)[\delta^{(4)}(x-x_3)\partial_\nu\delta^{(4)}(x-x_1)\end{aligned}$$

$$
\begin{aligned}
& - \delta^{(4)}(x-x_1)\partial_\nu\delta^{(4)}(x-x_3)] \\
+ \; & \eta_{\mu\nu}\delta^{(4)}(x-x_3)[\delta^{(4)}(x-x_1)\partial_\lambda\delta^{(4)}(x-x_2) \\
& - \delta^{(4)}(x-x_2)\partial_\lambda\delta^{(4)}(x-x_1)] \\
+ \; & \eta_{\nu\lambda}\delta^{(4)}(x-x_1)[\delta^{(4)}(x-x_2)\partial_\mu\delta^{(4)}(x-x_3) \\
& - \delta^{(4)}(x-x_3)\partial_\mu\delta^{(4)}(x-x_1)]\}, \qquad (32c)
\end{aligned}
$$

$$
\begin{aligned}
\left(V^{AA\lambda\widetilde{G}}_{\mu\nu\lambda}\right)^{abc} &= \left.\frac{\delta(iS)}{\delta A^{\mu a}(x_1)\delta A^{\nu b}(x_2)\delta\lambda^c(x_3)\delta\overline{\widetilde{G^\lambda}}(x_4)}\right|_0 \\
&= \frac{i}{2}\kappa g f^{abc}\sigma_{\mu\nu}\gamma_\lambda \int d^4x\delta^{(4)}(x-x_1)\delta^{(4)}(x-x_2)\delta^{(4)}(x-x_3)\delta^{(4)}(x-x_4)\ . \qquad (32d)
\end{aligned}
$$

In (32) the subscript zero in the functional derivative implies that all fields are to be replaced by their vacuum expectation values after differentiation. The momentum space vertex tensors which appear in the Feynmann diagrams follow from the Fourier transforms of the above. They are given in Table 3.

3.2 *Helicity Amplitudes For* $A + A \to \lambda + \widetilde{G}$

The matrix element for our process is best evaluated by going to the CM frame of the incident gauge bosons. We choose the frame in such a way that the momenta p_1,p_2,p_3 and p_4 of the two gauge bosons, the gravitino and the gaugino respectively (see Figure 2), are given by

$$
\begin{aligned}
p_1 =&(\omega, 0, 0, -\omega), \qquad & p_2 =&(\omega, 0, 0, \omega), \\
p_3 =&(E, 0, k\sin\theta, k\cos\theta), & p_4 =&(E', 0, -k\sin\theta, -k\cos\theta),
\end{aligned} \qquad (33a)
$$

where

Table 3

Description	Name	Expression
$A(p_1,\epsilon_1^\mu)^a\lambda(p_2,\mathrm{v})^b\widetilde{G}(p_3,\bar{\mathrm{u}}^\nu)$	$(V_{\mu\nu}^{A\lambda\widetilde{G}})^{ab}$	$\frac{\kappa}{2}\sigma_{\alpha\mu}\gamma_\nu p_1^\alpha\delta^{ab}$
$A(p_1,\epsilon_1^\mu)^a\lambda(p_2,\mathrm{v})^b\lambda(p_3,\bar{\mathrm{u}})^c$	$(V_\mu^{A\lambda\lambda})^{abc}$	$-gf^{abc}\gamma_\mu$
$A(p_1,\epsilon_1^\mu)^aA(p_2,\epsilon_2^\nu)^bA(p_3,\epsilon_3^\lambda)^c$	$(V_{\mu\nu\lambda}^{AAA})^{abc}$	$gf^{abc}[\eta_{\mu\lambda}(p_3-p_1)_\nu$ $+\eta_{\nu\lambda}(p_2-p_3)_\mu$ $+\eta_{\nu\mu}(p_1-p_2)_\lambda]$
$A(p_1,\epsilon_1^\mu)^aA(p_2,\epsilon_2^\nu)^b\lambda(p_3,\mathrm{v})^c\widetilde{G}(p_4,\mathrm{u}^\lambda)$	$(V_{\mu\nu\lambda}^{AA\lambda\widetilde{G}})^{abc}$	$\frac{i}{2}\kappa gf^{abc}\sigma_{\mu\nu}\gamma_\lambda$

The relevant vertex expressions in the theory. The particle labels are explained in the text. The quantities within parentheses following the particle labels denote the momenta and polarization/spin state of the particle. The momenta are all taken towards the vertex.

$$\begin{aligned} E =&\omega+\frac{m_{\tilde{g}}^2-m_{3/2}^2}{4\omega}\ ,\\ E' =&\omega-\frac{m_{\tilde{g}}^2-m_{3/2}^2}{4\omega}\ ,\\ k =&\sqrt{E^2-m_{\tilde{g}}^2}\ . \end{aligned} \tag{33b}$$

Here ω is the CM energy of each incident gauge boson, E and E' are the energies of the final state gaugino and gravitino respectively and θ is the scattering angle. The gauge

Figure 2

Kinematics for our process.

boson polarization vectors $\epsilon^{\pm}$ (corresponding to helicities ± 1) and the gaugino spinors[⋆] $\mathrm{v}^{\pm}$ (corresponding to helicities $\pm\frac{1}{2}$) can now be constructed. Our explicit choice is[†]

$$\epsilon_1^{\pm} = \epsilon_2^{\mp} = \frac{1}{\sqrt{2}}(0, 1, \mp i, 0), \tag{34a}$$

$$\mathrm{v}^{+} = \begin{pmatrix} -i\sin\frac{\theta}{2}\sqrt{E - m_{\tilde{g}}} \\ -\cos\frac{\theta}{2}\sqrt{E - m_{\tilde{g}}} \\ i\sin\frac{\theta}{2}\sqrt{E + m_{\tilde{g}}} \\ \cos\frac{\theta}{2}\sqrt{E + m_{\tilde{g}}} \end{pmatrix}, \qquad \mathrm{v}^{-} = \begin{pmatrix} i\cos\frac{\theta}{2}\sqrt{E - m_{\tilde{g}}} \\ -\sin\frac{\theta}{2}\sqrt{E - m_{\tilde{g}}} \\ i\cos\frac{\theta}{2}\sqrt{E + m_{\tilde{g}}} \\ -\sin\frac{\theta}{2}\sqrt{E + m_{\tilde{g}}} \end{pmatrix}. \tag{34b}$$

To construct the gravitino spinors we project out the spin $\frac{3}{2}$ part in the direct product of the spin 1 and spin $\frac{1}{2}$ representations using the Clebsch-Gordan decomposition. In the helicity basis, we can choose the spin 1 polarization vectors $\zeta^{\pm}$ (corresponding to helicities ± 1) and ζ^{L} (corresponding to zero helicity) as well as the spin $\frac{1}{2}$ spinors $\mathrm{u}^{\pm}$

⋆ We choose v for the gaugino and u for the gravitino. Since the particles are Majorana, the reverse choice is equivalent.

† This choice of phase ensures the simplest transformation of the polarizations and spins under CP-transformations.

Figure 3

Feynman diagrams contributing to our process at tree-level.

(corresponding to helicities $\pm\frac{1}{2}$) to be given by

$$\zeta^{\pm} = \frac{1}{\sqrt{2}}(0, 1, \mp i\cos\theta, \pm i\sin\theta), \qquad \zeta^{L} = \frac{1}{m_{3/2}}(k, 0, -E\sin\theta, -E\cos\theta),$$

$$\mathrm{u}^{+} = \begin{pmatrix} \sin\frac{\theta}{2}\sqrt{E'+m_{3/2}} \\ -i\cos\frac{\theta}{2}\sqrt{E'+m_{3/2}} \\ \sin\frac{\theta}{2}\sqrt{E'-m_{3/2}} \\ -i\cos\frac{\theta}{2}\sqrt{E'-m_{3/2}} \end{pmatrix}, \qquad \mathrm{u}^{-} = \begin{pmatrix} \cos\frac{\theta}{2}\sqrt{E'+m_{3/2}} \\ i\sin\frac{\theta}{2}\sqrt{E'+m_{3/2}} \\ -\cos\frac{\theta}{2}\sqrt{E'-m_{3/2}} \\ -i\sin\frac{\theta}{2}\sqrt{E'-m_{3/2}} \end{pmatrix}. \tag{3.5}$$

In terms of these, the gravitino spinors $\mathrm{u}^{\lambda}_{3/2}$ of helicity $\lambda(=\pm\frac{1}{2}, \pm\frac{3}{2})$ are given by

$$\begin{aligned} \left(\mathrm{u}^{\pm\frac{3}{2}}_{3/2}\right)_{\mu} &= \zeta^{\pm}_{\mu}\mathrm{u}^{\pm}, \\ \left(\mathrm{u}^{\pm\frac{1}{2}}_{3/2}\right)_{\mu} &= \pm\left(\frac{1}{\sqrt{3}}\zeta^{\pm}_{\mu}\mathrm{u}^{\mp} + i\sqrt{\frac{2}{3}}\zeta^{L}_{\mu}\mathrm{u}^{\pm}\right). \end{aligned} \tag{3.6}$$

The Feynman diagrams contributing to our process are shown in Figure 3. Using

Table 3, we can construct the invariant amplitude:

$$
\begin{aligned}
\mathcal{M}^{abc} = & -2i\kappa\bar{\mathrm{u}}_{3/2\,\alpha}\Big\{ i\left(\eta^{\alpha[\mu}\gamma^{\nu]}p_{2\mu}\epsilon_{2\nu}\right)(\not{p}_1-\not{p}_3-m_{\tilde{g}})^{-1}\left(-g\not{\epsilon}_1 f^{abc}\right) \\
& \quad + i\left(\eta^{\alpha[\mu}\gamma^{\nu]}p_{1\mu}\epsilon_{1\nu}\right)(\not{p}_4-\not{p}_1-m_{\tilde{g}})^{-1}\left(-g\not{\epsilon}_2 f^{cba}\right)\Big\}\mathrm{v} \\
& + gf^{acb}\Big\{\eta^{\lambda\nu}\left(-2p_1-p_2\right)^{\mu}+\eta^{\mu\nu}\left(p_1+2p_2\right)^{\lambda}+\eta^{\mu\lambda}\left(p_1-p_2\right)^{\nu}\Big\}\epsilon_{1\lambda}\epsilon_{2\mu} \\
& \quad \times(-i)\left\{\eta_{\nu\tau}-(p_1+p_2)_{\nu}(p_1+p_2)_{\tau}(p_1+p_2)^{-2}\right\}(p_1+p_2)^{-2} \\
& \quad \times\bar{\mathrm{u}}_{3/2\,\alpha}\Big\{-2i\kappa\eta^{\alpha[\beta}\gamma^{\tau]}(p_1+p_2)_{\beta}\Big\}\mathrm{v} \\
& + \bar{\mathrm{u}}_{3/2\,\alpha}\left\{\kappa g f^{bac}(\eta^{\alpha\beta}\gamma^{\tau}-\eta^{\alpha\tau}\gamma^{\beta})\epsilon_{1\beta}\epsilon_{2\tau}\right\}\mathrm{v}.
\end{aligned}
\tag{3.7}
$$

The tree-level helicity amplitudes can be calculated from (3.7) by substitution of the spinors and vectors in the centre-of-mass frame. Let the invariant amplitude with the two gauge bosons having helicities $\lambda_{1,2}$ and the gaugino, gravitino having μ_1, μ_2 respectively be denoted by $\mathcal{M}^{abc}_{\lambda_1\lambda_2;\mu_1\mu_2}$, where a, b and c are the gauge indices of the two gauge bosons and the gaugino respectively. Due to CP invariance and the amplitude being proportional to f^{abc}, $\left|\mathcal{M}^{abc}_{\lambda_1\lambda_2;\mu_1\mu_2}(\theta)\right| = \left|\mathcal{M}^{abc}_{-\lambda_1\,-\lambda_2;-\mu_1\,-\mu_2}(\theta)\right|$. Further, the identical nature of the incident gauge bosons implies that $\left|\mathcal{M}^{abc}_{\lambda_1\lambda_2;\mu_1\mu_2}(\theta)\right| = \left|\mathcal{M}^{bac}_{\lambda_2\lambda_1;\mu_1\mu_2}(\theta+\pi)\right|$. Hence, there are only twelve independent helicity amplitudes. We select those in which the helicity of the first gauge boson is $+1$. Since the values of the CM energy at which tree-unitarity gets violated are large compared to $m_{3/2}$ and $m_{\tilde{g}}$, we can justifiably work in the limit $\omega \gg m_{3/2}, m_{\tilde{g}}$. The expressions for the twelve helicity amplitudes in this limit are enumerated below. The REDUCE 3 program used for evaluating these is

displayed in Appendix I.

$$\left|\mathcal{M}_{1\,1;\,\frac{1}{2}\,-\frac{3}{2}}\right| = 0, \tag{38a}$$

$$\left|\mathcal{M}_{1\,1;\,\frac{1}{2}\,-\frac{1}{2}}\right| = 2\sqrt{\frac{2}{3}}\left|f_{abc}g\kappa m_{\tilde{g}}(m_{\tilde{g}}-2m_{3/2})m_{3/2}^{-1}(\sin\theta)^{-1}\right|, \tag{38b}$$

$$\left|\mathcal{M}_{1\,1;\,\frac{1}{2}\,\frac{1}{2}}\right| = 4\sqrt{\frac{2}{3}}\left|f_{abc}g\kappa m_{\tilde{g}}m_{3/2}\omega^{-1}\cos\theta\right|(\sin\theta)^{-2}, \tag{38c}$$

$$\left|\mathcal{M}_{1\,1;\,\frac{1}{2}\,\frac{3}{2}}\right| = 2\sqrt{2}\left|f_{abc}g\kappa m_{3/2}(\sin\theta)^{-1}\right|, \tag{38d}$$

$$\left|\mathcal{M}_{1\,1;\,-\frac{1}{2}}-\frac{3}{2}\right| = \frac{1}{2\sqrt{2}}\left|f_{abc}g\kappa m_{\tilde{g}}(2m_{\tilde{g}}^2-m_{3/2}^2)\omega^{-2}(\sin\theta)^{-1}\right|, \tag{38e}$$

$$\left|\mathcal{M}_{1\,1;\,-\frac{1}{2}}-\frac{1}{2}\right| = 2\sqrt{\frac{2}{3}}\left|f_{abc}g\kappa m_{\tilde{g}}^2m_{3/2}^2\omega^{-3}\cos\theta\right|(\sin\theta)^{-2}, \tag{38f}$$

$$\left|\mathcal{M}_{1\,1;\,-\frac{1}{2}\,\frac{1}{2}}\right| = \frac{1}{\sqrt{6}}\left|f_{abc}g\kappa(m_{\tilde{g}}^2+m_{3/2}^2)(m_{\tilde{g}}-2m_{3/2})\omega^{-2}(\sin\theta)^{-1}\right|, \tag{38g}$$

$$\left|\mathcal{M}_{1\,1;\,-\frac{1}{2}\,\frac{3}{2}}\right| = 0, \tag{38h}$$

$$\left|\mathcal{M}_{1\,-;\,1\,\frac{1}{2}}-\frac{3}{2}\right| = \sqrt{2}\left|f_{abc}g\kappa\omega(1-\cos\theta)\right|, \tag{38i}$$

$$\left|\mathcal{M}_{1\,-;\,1\,\frac{1}{2}}-\frac{1}{2}\right| = \frac{1}{\sqrt{6}}m_{3/2}^{-1}|(1+\cos\theta)^{-1}f_{abc}g\kappa\sin\theta[\{(m_{\tilde{g}}-m_{3/2})^2-2m_{3/2}^2\}$$
$$+\{(m_{\tilde{g}}-m_{3/2})^2+2m_{3/2}^2\}\cos\theta]|, \tag{38j}$$

$$\left|\mathcal{M}_{1\,-;\,1\,\frac{1}{2}\,\frac{1}{2}}\right| = \sqrt{\frac{2}{3}}\left|f_{abc}g\kappa\omega m_{3/2}^{-1}(m_{\tilde{g}}-2m_{3/2})(1-\cos\theta)\right|, \tag{38k}$$

$$\left|\mathcal{M}_{1\,-;\,1\,\frac{1}{2}\,\frac{3}{2}}\right| = \sqrt{2}\left|f_{abc}g\kappa m_{\tilde{g}}\sin\theta\right|. \tag{38l}$$

From the helicity amplitudes calculated in the previous section we can project out the corresponding partial wave helicity amplitudes. For simplicity, we consider only the lowest nonvanishing partial wave amplitude since this yields the generic high energy dependence. We need consider only $\mathcal{M}_{1\,-1;\,\frac{1}{2}\,\frac{1}{2}}$ which is the sole helicity amplitude for which unitarity requirement yields a nontrivial constraint. In this case, the lowest non-vanishing partial wave has J=2. Denoting the Jth partial wave of $\mathcal{M}_{\lambda_1\,\lambda_2;\,\mu_1\,\mu_2}$ by

$\mathcal{T}^{\rm J}_{\lambda_1\lambda_2;\mu_1\mu_2}$, we have

$$\mathcal{T}^{J}_{\lambda_1\lambda_2;\mu_1\mu_2} = \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta \mathcal{M}_{\lambda_1\,\lambda_2;\,\mu_1\,\mu_2} d^{J}_{\lambda_1-\lambda_2\,\mu_1-\mu_2}(\theta)\ . \tag{3.9}$$

It follows from $(38k)$ that

$$\left|\mathcal{T}^{2abc}_{1\,-1;\frac{1}{2}\frac{1}{2}}\right| \sim f^{abc}\frac{g\kappa\omega(m_{\tilde{g}}-2m_{3/2})}{48\pi m_{3/2}}\ . \tag{3.10}$$

The RHS of (3.10) overshoots unity when ω exceeds $\omega_{cr.}$ where

$$\omega_{cr.} = \frac{48\pi}{g\kappa}\frac{m_{3/2}}{(m_{\tilde{g}}-2m_{3/2})}\ . \tag{3.11}$$

We see that $\omega_{cr.}$ can be quite small compared to the Planck scale if $m_{3/2} \ll m_{\tilde{g}}$.

The different factors arising in the final expression in (3.10) are quite easy to trace. The longitudinal part of the gravitino polarization pulls down an $m_{3/2}$ factor to the denominator, whereas its interaction vertex brings in a factor of κ in the numerator. Similarly, the nonabelian gauge boson or gaugino vertices (including the quartic $AA\lambda\widetilde{G}$ vertex) are proportional to g. The "longitudinal" polarization of the gravitino is, however, proportional to its momentum p_4 (upto terms of order $m_{3/2}$). This is because $\zeta^L_\mu = \frac{1}{m_{3/2}}(p_{4\mu} + o(m^2_{3/2}))$. The replacement of the gravitino polarization vector in the Feynman diagram by this momentum vector is equivalent to taking the divergence of the supercurrent that is the source of the gravitino. This supercurrent, in turn, is conserved (divergenceless) except for the gaugino and gravitino mass terms. Dimensional considerations then force the remaining factor of ω.

A similar analysis can be presented for the other partial wave helicity amplitudes also. However, the helicity $\pm\frac{3}{2}$ polarization tensors do not contain the $m_{3/2}$ factor in the denominator and hence are unimportant. Further, none of the vertices contains explicit

factors of $m_{\tilde{g}}$. Hence a single gaugino mass factor can arise only from the gaugino propagator or from the mismatch of chirality and helicity at the external gaugino leg. However, all fermion vertices involved are chirality flipping as also is the part of the gaugino propagator that is independent of the gaugino mass. Therefore, in all the diagrams in question, for the case when the chiralities of the gravitino and the gaugino are the same, the propagator has to contribute a factor of $m_{\tilde{g}}$. Conversely, if those chiralities are opposite, there can be no odd power of $m_{\tilde{g}}$ arising from the propagator.

If the chirality of a gravitino, represented by a v-spinor, equals its helicity, there will be a factor of $m_{3/2}$. For a gaugino represented by a u-spinor, on the other hand, a mismatch of chirality and helicity produces a factor of $m_{\tilde{g}}$. Hence, if the helicities of the gaugino and the gravitino are opposite, only an even number of fermion mass factors can appear. This number is at least two because of supercurrent conservation, as mentioned earlier. Dimensional analysis now makes the leading term independent of energy and hence uninteresting for unitarity violation unless $m_{\tilde{g}} \gtrsim \kappa^{-1} m_{3/2}/m_{\tilde{g}}$. The leading terms in the only other potentially interesting amplitudes $\mathcal{M}_{1\,1;\,\frac{1}{2}\,\frac{1}{2}}$ and $\mathcal{M}_{1\,1;\,-\frac{1}{2}\,-\frac{1}{2}}$, are found to cancel on explicit evaluation. Finally, only $\mathcal{M}_{1\,-1;\,\frac{1}{2}\,\frac{1}{2}}$ yields the sought after high energy behaviour.

3.3 *Discussion*

In this chapter, a particular process involving two non-abelian gauge bosons and the corresponding gaugino plus a superlight gravitino has been studied as an example of an amplitude that violates tree level unitarity at a fairly low energy. Normally, tree unitarity violation in gravitational theories is expected at or beyond the Planck scale. The present example, however, illustrates that if the gaugino mass is hierarchically larger than the gravitino mass, the violation will come down to much lower energies. On the

other hand, if tree-unitarity is to be maintained all the way upto the Planck scale, the gravitino mass must be larger than or of the same order of magnitude as the gaugino mass. This result can be modified in perturbation theory only if the gauginos mix with other particles of very different masses present in the theory, which must necessarily belong to the adjoint representation of the gauge group to preserve the unbroken gauge symmetry. In a later chapter, a different example will be constructed to show that the non-abelian nature of the gauge interaction here was just incidental. Tree-level unitarity violation at lower energies takes place even in theories with only abelian gauge interactions.

4. R-Gauge Analysis of The Super-Higgs Mechanism

In broken supergravity theories with a large mass hierarchy between the gravitino and the gaugino, it is possible for tree unitarity to be violated at a relatively low scale. This we saw in Chapter 3 by studying a particular process, *viz.* the scattering of two non-abelian gauge boson into a gravitino and a gaugino at the tree level. We have explained this result in terms of a kinematic factor of the gravitino mass arising in the denominator of the (longitudinally polarized) gravitino 'polarization spinor' and a factor of gaugino mass in the numerator originating from supercurrent nonconservation. However, unitarity being a fundamental requirement of the full quantum theory, tree-level unitarity violation normally results from large coupling constants in the theory. This would render any perturbative calculation invalid. In this chapter, we proceed to show that even though large coupling constants are not visible in the supergravity theories that we consider, *these theories must actually contain such large coupling constants as invalidate perturbative analysis.*

We notice that we are dealing with a local symmetry which, after breakdown, is realized in the 'Higgs phase'. It is well known[27] that in such a theory, the inherent symmetry of the Lagrangian still manifests itself in two ways: (i) the physical matrix elements calculated with different gauge choices turn out to be the same, and (ii) different matrix elements in the same gauge get related by the symmetry. The latter relations are often known as Ward identities and play a very significant role in these theories.

We can exemplify the situation by considering the case of the ordinary Higgs mechanism in a renormalizable gauge theory. For definiteness, let us consider an abelian gauge theory containing a gauge boson A_μ and a single complex scalar field ϕ. The

fields have the usual gauge freedom under the gauge transformations

$$\begin{aligned} A_\mu &\to A_\mu + i\partial_\mu \epsilon, \\ \phi &\to e^{i\epsilon}\phi. \end{aligned} \tag{4.1}$$

When ϕ develops a vacuum expectation value, conventionally chosen to be real, the gauge symmetry is said to be realized in the 'Higgs phase'. In this case, the true field variable that measures the 'field oscillations' about the vacuum value and is relevant to perturbation theory is the 'shifted field' $\phi' = \phi - v$ where v refers to the vacuum expectation value of ϕ. If we rewite the original symmetric Lagrangian in terms of this field, it turns out to have terms like $A^\mu \partial_\mu \mathrm{Im}\phi'$ where $\mathrm{Im}\phi'$ is defined by $\phi' = (\mathrm{Re}\phi' + i\mathrm{Im}\phi')/\sqrt{2}$) which are bilinear in A_μ and $\mathrm{Im}\phi'$. In fact, the quadratic part of the Lagrangian turns out to be

$$\begin{aligned} \mathcal{L}_{quadr.} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(\partial_\mu \mathrm{Re}\phi' \partial^\mu \mathrm{Re}\phi' + \\ \partial_\mu \mathrm{Im}\phi' \partial^\mu \mathrm{Im}\phi') - \frac{\mu^2}{2}(\mathrm{Re}\phi')^2 + \\ \frac{m^2}{2}A_\mu A^\mu + mA^\mu \partial_\mu \mathrm{Im}\phi'. \end{aligned} \tag{4.2}$$

In (4.2) μ is a mass term for the field $\mathrm{Re}\phi'$ and m is the boson mass that gets generated by this 'symmetry breaking' and is related to the vacuum expectation value v as $m = gv$, g being the gauge coupling constant. This implies that neither of these fields is an asymptotic particle state of the system and the Lagrangian needs to be rediagonalized.

In the so-called U-gauge, where the particle content of the theory is evident, this is achieved by defining the amplitude $\phi_{amp.}$ and phase ϕ_{phase} oscillators by

$$\phi' = \phi_{amp.} \exp(i\phi_{phase}). \tag{4.3}$$

After a trivial field redefinition

$$A'_\mu = A_\mu + \frac{1}{m}\partial_\mu \phi_{phase}, \tag{4.4}$$

it turns out that the Lagrangian is expressible in terms of only two fields A'_μ and $\phi_{amp.}$.

Moreover, it is diagonal in terms of these. In this chosen gauge, the vector boson A'_μ is massive and there is one additional physical field, the real scalar field $\phi_{amp.}$.

It was the gauge freedom in the original that Lagrangian allowed us to fix the value of $\partial_\mu A^\mu$ arbitrarily. Exploiting this feature we could subtract the gauge fixing term $m^2/2\left(\partial_\mu A^\mu - \frac{1}{m}\mathrm{Im}\phi'\right)^2$ from the Lagrangian so as to cancel (upto total divergences) the offending off-diagonal terms mixing A_μ and ϕ'. In such a gauge we have a massless vector field and a complex scalar field as independent fields. This gauge thus contains more 'fields' than the U-gauge. The number of physical degrees of freedom is, however, the same in both the gauges. The longitudinal degree of polarization of the gauge boson (which is now massless) disppears in this gauge only to reappear as the phase of the scalar field. Moreover, as shown by Lee and Zinn-Zustin[27], matrix elements for any process involving only gauge bosons and the physical ϕ' excitations as external states and evaluated in this so-called R-gauge and the U-gauge are related by simple field redefinition constants. Thus physical matrix elements can be conveniently evaluated in the R-gauge even though in this gauge the particle content of the theory is unphysical.

Furthermore, the same gauge invariance implies the existence in the R-gauge of a Ward identity relating the matrix elements of gauge bosons to those of the Goldstone field ϕ_{phase}. In fact the relation implies

$$\frac{1}{m}\partial_\mu \mathcal{M}(A_\mu, \ldots) = \mathcal{M}(\phi_{phase}, \ldots). \tag{4.5}$$

In momentum space it thus describes an equivalence of the momentum-contracted matrix element of a gauge boson with the matrix element of a Goldstone field.

An exactly similar analysis can be carried over to the super-Higgs effect. As mentioned in an earlier chapter, in order to get a flat space broken supergravity theory, we have to start with an unbroken supergravity theory in an anti-de Sitter space. Let us

consider such a theory. As shown in reference 24, the part of the Lagrangian bilinear in the gravitino ($\widetilde{G}$) and the goldstino (χ) fields after supersymmetry breaking can be written as

$$\mathcal{L} = \frac{ie}{2}\bar{\chi}\partial\!\!\!/\chi - me\bar{\chi}\chi + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\widetilde{G}}_\mu\gamma_5\gamma_\nu D_\rho\widetilde{G}_\sigma - em_{3/2}\overline{\widetilde{G}}_\mu\sigma^{\mu\nu}\widetilde{G}_\nu - e\sqrt{\frac{3}{2}}m_{3/2}\bar{\chi}\gamma^\mu\widetilde{G}_\mu + \cdots$$

where the vacuum expectation value of the auxiliary field has been adjusted to make the final theory appear in flat space-time. A trivial field redefinition of

$$\widetilde{G}_\mu \to \widetilde{G}_\mu + \frac{1}{2}\sqrt{\frac{2}{3}}\gamma_\mu\chi$$

leaves a Lagrangian with only the mixing between the gravitino and the now massless goldstino in the kinetic sector of the theory. One can now proceed to fix the gauge in the theory and remove this mixing. As in the case of the Higgs mechanism discussed above, a Ward identity can now be derived to relate the gravitino matrix elements to the corresponding ones involving the goldstino.

In this respect, the broken supergravity theory behaves simlar to the ordinary gauge theories incorporating the Higgs mechanism. The only difference now is that the gravitino mass (the analogue of the gauge boson mass m in the above discussion) is no longer related to the vacuum expectation values of the auxiliary fields that break supersymmetry quite so simply as in the Higgs case. This fact was, however, irrelevant in the above discussion. In particular, we still get an R-gauge Ward identity which links the derivative of the gravitino matrix elements to those of the goldstino.

We now recall the origin of the 'singularity' of the massless limit of a theory involving particles of spin greater than half. This arises from the polarization tensor of such a particle carrying — as an external state — a factor of its mass in the denominator. Thus, for example, gravitino S-matrix elements become singular because in the massless limit,

they contain

$$\sqrt{\frac{2}{3}}\frac{p_\mu}{m_{3/2}}\bar{\chi}\mathcal{M}(\overline{\widetilde{G}}_\mu,\ldots)\times\text{other polarization factors} \tag{4.6}$$

where χ is the goldstino spinor and $\mathcal{M}(\overline{\widetilde{G}}_\mu,\ldots)$ stands for the rest of the amplitude. As already mentioned, we can look upon the same amplitude in the R-gauge. In this gauge the Ward identity mentioned above can be used to reexpress this amplitude in terms of

$$\sqrt{\frac{2}{3}}\bar{\chi}\mathcal{M}(\bar{\chi},\ldots)\times\text{other polarization factors,}$$

where $\mathcal{M}(\bar{\chi},\ldots)$ represents the corresponding matrix element of the goldstino. In this form, the singular factor of $m_{3/2}$ in the denominator does not appear!

The reason for this apparent contradiction in the Ward identity is that in a spontaneously broken theory, even in the U-gauge the S-matrix element is not really singular. In addition to providing us with equivalent descriptions (gauge choices) and Ward identities, gauge invariance also provides for the conservation of the current that is the source of the gauge particle in the limit that the symmetry is unbroken. Furthermore, in any broken phase, the Higgs mechanism provides for the gauge particle to have a mass. In other words the massless limit for the gauge particle is also the current conservation limit of the theory. In this limit therefore, the numerator which is proportional to the current non-conserving mass also vanishes and the apparent singularity is resolved.

The R-gauge analysis highlights a few interesting aspects of the theory. We now know that a large ratio between the symmetry non-conservation scale and the scale of the gauge particle mass would imply large coupling strengths of the Goldstone (or goldstino as the case may be) as well. This leads to an interesting observation. As the symmetry partners of the Goldstone (or the goldstino) stay back as a physical particle called the Higgs (Reϕ' in our example or scalar or pseudoscalar S,P in the super-Higgs case), there is a possiblity of observing the large couplings of this particle directly from

experiment. We shall exploit this in our phenomenological discussion of the gravitino mass later. Furthermore, in the super-Higgs case, the ratio of the gaugino and gravitino physical masses is proportional to some couplings of the goldstino in the R-gauge. This will be elaborated in Chapter 5. We only remark here that the failure of perturbation theory through the violation of tree unitarity at a sub-Planckian scale for a large value of this mass ratio is due to the largeness of one of those couplings.

In the current context of supergravity theories we can now argue that the high energy behaviour of the S-matrix element can never be super-Planckian. In other words, at large energies, the invariant matrix-elements can grow at most as a polynomial in $\kappa\omega$ where ω is the center-of-mass energy for the process. This follows trivially in the R-gauge goldstino matrix elements, as dimensional arguments dictate that in the absence of any extraneous mass factors in the denominator,[⋆] the dimensionless invariant matrix element can only be a function of $\kappa\omega$ mentioned above. The number of κ factors that appear in the matrix element can however be trivially calculated in perturbation theory as the number of supergravity vertices in the diagram. *Hence, the R-gauge analysis gives a bound on the high energy behaviour of all matrix elements in the theory.* Of course, since supergravity is not a renormalizable theory, this argument can only be given for tree diagrams.

In Chapter 3, we derived the same bound on the matrix element that we had from considerations of current conservation. In this particular case, therefore, the extra analysis of this chapter does not lead to any new physical results. In fact, the calculations of Chapter 3 apply to any softly broken supergravity theory which has current conservation

⋆ Mass factors cannot appear in the denominator from any couplings for we do not have any coupling other than κ with inverse mass dimension; nor can they do so from the propagators which at large energies fall off with energy. The only places where they can arise are the polarization tensors of longitudinally polarized particles with spin more than half as external states and these do not apply to the goldstino.

in it. This is a property of single gravitino amplitudes and does not hold when more than one gravitino participates in the process. In the next chapter we shall therefore study in detail a case of double gravitino emission in which current conservation alone will not be able to ensure the correct high energy growth of the amplitude.

In fact, consider the scattering of two gauge bosons into two gravitinos. In this case the application of the methods of Chapter (4.2) leads us to conclude that the matrix element should be proportional to

$$\frac{1}{m_{3/2}^2}\partial_\mu^x\partial_\nu^y \langle\Omega| TS^\mu(x)S^\nu(y) |AA\rangle$$

where T refers to the time-ordering involved and S^μ is the supercurrent which is the source of the gravitino field. Taking the derivative operations through the time-ordering symbol produces the usual $\partial \cdot S$ term, but in addition† *gives a σ-anticommutator term involving* $\{S^0, \partial_\mu S^\mu\}$. This term has only a single factor of the supersymmetry breaking scale in the numerator and hence dimensional analysis would seem to imply a super-Planckian $\kappa^2\omega^3 m_{\tilde{g}}/m_{3/2}^2$ growth at high energies.

As we have just argued, such a growth is incompatible with the super-Higgs mechanism.Though, we can construct theories in which such a growth is actually realized even after super-current conservation is ensured, these will not be compatible with the super-Higgs mechanism. The latter has a stronger constraint which rules out any such theory. In fact, as will be shown by direct evaluation in the next chapter in a simplified case of supergravity breaking, the offending term is cancelled by contributions arising from the exchange of the Higgs-like particles — the residual superpartners S and P of the goldstino. Nevertheless, as will be seen, the remaining, Planckian terms turn out to provide a stronger limit on the gaugino-gravitino mass ratio.

† A third term which comes both in the single and the double gravitino cases involves the anticommutator between S^0's. Being proportional to the Hamiltonian, it vanishes identically between the non-vacuum and the vacuum state.

5. The Double Gravitino Process

5.1 *Introduction*

In this chapter we consider certain amplitudes with two gravitinos. Specifically, we direct our attention to the process gauge boson (A) + gauge boson (A) $\rightarrow$ gravitino ($\widetilde{G}$) + gravitino ($\widetilde{G}$) for abelian as well as nonabelian gauge interactions. We recall that the process discussed in Chapter 3 was allowed only with non-abelian gauge bosons. The reason why the process considered now goes both for abelian and nonabelian gauge interactions is that the final state is gauge invariant while the initial state always contains a gauge invariant part. We look for conditions which ensure the validity of tree unitarity in this process for all energies upto the natural scale in the theory which is the Planck mass $M_{Pl} \equiv \kappa^{-1}$.

In Chapter 3 the single gravitino process gauge boson (A) + gauge boson (A) $\rightarrow$ gaugino (λ) + gravitino ($\widetilde{G}$) for nonabelian gauge interactions was considered. There supercurrent conservation was shown to be responsible for the cancellation of the leading bad high-energy growth of the tree-level helicity amplitudes. The next-to-leading growth was found to be present and could be made consistent with unitarity upto the Planck scale in the absence of any major mass-hierarchy between the gaugino and the gravitino. In fact, the latter requirement yielded a nontrivial upper bound on the ratio of their masses. However, supercurrent conservation alone does not cancel all the bad high energy growths in the double gravitino case. There are additional complications, which we analyze here.

For the sake of definiteness, we take a single unbroken gauge multiplet and couple its members plus their superpartners to a spontaneously broken N = 1 supergravity system. We do allow for additional chiral matter supermultiplets in connection with

the super-Higgs effect. However, we need not consider their members as external legs in helicity amplitudes whose leading energy growths we study to look for the violation of tree unitarity. This is because energy growths leading to such violations arise only from interactions between the gauge and the gravitino supermultiplets. In the helicity amplitudes for the process $AA \to \widetilde{G}\widetilde{G}$, the leading energy growth, permitted by dimensional analysis, is again (as in Chapter 3) cancelled by supercurrent conservation. However, the next-to-leading term, if present, would also have violated tree unitarity much below M_{Pl}; that gets cancelled by a different mechanism connected with the super-Higgs effect.

Consider those helicity amplitudes of the process $AA \to \widetilde{G}\widetilde{G}$ which correspond to each of the two final-state gravitinos having helicity $\pm 1/2$. Their polarization vectors then bring down two factors of the gravitino mass $m_{3/2}$ in the denominator. On the other hand, the leading term in the numerator, contributed by each of these polarization vectors, is proportional to the gravitino momentum. The accompanying factor in the amplitude can be viewed as the matrix element (between the two gauge bosons and the vacuum) of the (anti-) time-ordered product of two supercurrents. (The origin of this matrix element is easy to understand once the interaction associated with each gravitino field $\widetilde{G}_\mu$ is written as proportional to $\overline{\widetilde{G^\mu}} S_\mu$, S_μ being the supercurrent). The divergence of each of these supercurrents yields the low-energy supersymmetry-breaking mass-scale ΔM_S in the theory, $\partial \cdot S \propto \Delta M_S$. The contractions with the gravitino momenta lead to* two terms in the amplitude, one proportional to $(\Delta M_S)^2 \kappa^2 m_{3/2}^{-2}$ and the other to $\Delta M_S \kappa^2 m_{3/2}^{-2}$. By dimensional arguments, the former grows as E_{CM}^2 which, being the standard energy growth of gravitationally mediated second order tree amplitudes, can be made to respect unitarity upto M_{Pl}. The other term, an analogue of the sigma-commutator in current algebra theories, grows as E_{CM}^3 - violating tree unitarity at an

* $\partial_\mu^x \partial_\nu^y \langle \Omega | \bar{T} \bar{S}^\mu(x) S^\nu(y) | AA \rangle = \langle \Omega | \bar{T} \partial \cdot \bar{S}(x) \partial \cdot S(y) + \delta(x_0 - y_0)[\{\bar{S}^0(x), \partial \cdot S(y)\} - \{\partial \cdot \bar{S}(x), S^0(y)\}] | AA \rangle$, where S^μ is the spinorial supercurrent.

energy scale $\sim (m_{3/2}^2 M_{Pl}^2 m_{\tilde{g}}^{-1})^{1/3}$. In many broken supergravity theories this would be much below M_{Pl}.

However, in the spectrum of a spontaneously broken N = 1 supergravity theory, there exist a certain scalar and a pseudoscalar with masses $\lesssim \Delta M_S$, belonging to the hidden sector. These are the superpartners of the goldstino that is absorbed by the gravitino in the process of becoming massive through the super-Higgs mechanism. They have only gravitational couplings to gauge bosons of the observable sector. Nevertheless, they contribute to our process through interactions that are totally fixed by the super-Higgs mechanism. Their contributions cancel the offending term mentioned earlier.

As mentioned in Chapter 4, this last cancellation can be understood most clearly by viewing the process in the so-called R-gauge. In this gauge the goldstino appears as a virtual propagating mode without being present in the initial or final state. Since the gauge condition requires the longitudinal component of the gravitino to be the divergence of this goldstino field, Ward identities can be derived linking the amplitude for our process to that of double goldstino emission. The latter, however, does not involve coupling with any massive particle with spin greater than half. It is expected to yield an energy growth only upto E_{CM}^2 and to preserve tree unitarity below M_{Pl}. Thus all unitarity-violating E_{CM}^3 terms in our physical amplitude have to cancel.

The above analysis assumes, of course, that all dimensionless constants appearing in the Lagrangian are of order unity. As explained in Chapter 3, this may not be the case in some supergravity models – notably in no-scale theories.[2] For such models, severe constraints are put on the parameters by our analysis. In particular, we recover a stronger version of the principal result of that chapter, that the gaugino mass cannot exceed the gravitino mass by a large factor. We also find a weak upper bound on the RMS mass of the scalar and pseudoscalar particles S, P mentioned above.

5.2 *Lagrangian and Vertices*

The minimal N = 1 supergravity theory[23] contains couplings of chiral matter supermultiplets S_i and gauge multiplets V^a (the corresponding spinorial superfields being W^a) with the gravity supermultiplet embodied in the supervierbein E^μ_A. In terms of these the Lagrangian density is

$$\mathcal{L} = \int d^8z\ E[\Phi(S, \bar{S}e^{2V^aT^a}) + Re\ R^{-1}\{g(S) + f_{ab}(S)W^aW^b\}]. \tag{5.1}$$

Here the superspace measure is $d^8z = d^4x\ d^2\theta\ d^2\bar{\theta}$, E is the supervierbein determinant and R the scalar curvature superfield. Φ is a real function and the superpotential g an analytic function of their respective arguments. T^a are the generators of the gauge group in the representation to which S belongs and f_{ab} is an analytic function symmetric in the adjoint representation indices a and b.

A super-Weyl rescaling of the supervierbein $E \to E' = |g(S)|^{2/3}E$ can be used to simplify the Lagrangian of (5.1) to[23]

$$\mathcal{L} = \int d^8z\ E'exp\{-\frac{1}{3}\mathcal{G}(S, \bar{S}e^{2V^aT^a})\} + Re\int d^8z\ E'(R')^{-1}[1 + f_{ab}(S)W^aW^b], \tag{5.2}$$

where R' is the scalar curvature superfield calculated from E' and

$$\mathcal{G}(S, \bar{S}e^{2V^aT^a}) = 3\ln\Phi(S, \bar{S}e^{2V^aT^a}) - 2\ \ln|g(S)|.$$

A redefinition of fields, to make the kinetic energy terms diagonal, yields the following expression for the part of the Lagrangian in (5.2) that is relevant to us:

$$\begin{aligned}\mathcal{L}_0 =& \frac{e}{2\kappa^2}R + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\widetilde{G}}_\mu\gamma_5\gamma_\nu(\partial_\rho - \frac{i}{4}\omega_{\rho AB}\sigma^{AB})\widetilde{G}_\sigma - \frac{e}{4}\mathrm{Re}f_{ab}\ F^a_{\mu\nu}F^{\mu\nu b}\\ &+ \frac{i}{4}\mathrm{Im}f_{ab}\ F^a_{\mu\nu}F^b_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma} + \frac{ie}{2}\mathrm{Re}f_{ab}\bar{\lambda}^a(\not{\partial} - \frac{i}{4}\not{\omega}_{AB}\sigma^{AB})\lambda^b + e\mathcal{G}''_{j\bar{i}}D_\mu\bar{z}_jD^\mu z_i\\ &+ \frac{ie}{2\kappa}e^{\mathcal{G}/2}\overline{\widetilde{G}}_\mu\sigma^{\mu\nu}\widetilde{G}_\nu + \frac{e}{4\kappa}(e^{\mathcal{G}/2}\mathcal{G}'_{\bar{l}}\mathcal{G}''^{-1}_{l\bar{k}}f^{*\prime}_{abk}\bar{\lambda}^a_L\lambda^b_R + h.c.)\\ &+ \frac{e\kappa}{4}\mathrm{Re}f_{ab}\ \bar{\lambda}^a\gamma^\mu\sigma^{\rho\sigma}\widetilde{G}_\mu F^b_{\rho\sigma} + \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\overline{\widetilde{G}}_\mu\gamma_\nu\widetilde{G}_\rho\mathrm{Re}(\mathcal{G}'_{\bar{i}}D_\sigma z_i)\\ &+ e^{\mathcal{G}}(3 - \mathcal{G}'_{\bar{l}}\mathcal{G}''^{-1}_{l\bar{k}}\mathcal{G}'_k).\end{aligned} \tag{5.3}$$

In (5.3) we have denoted the vierbein determinant by e, the scalar curvature by R, the gravitino field by $\widetilde{G}_\mu$, the gauge field strength by $F^a_{\mu\nu}$, the gaugino field by λ and the scalar components of the chiral superfields by z. The spin connection is

$$\begin{aligned}\omega_{\rho AB} =& e^\nu_A \partial_{[\mu} e_{B\nu]} - e^\nu_B \partial_{[\mu} e_{A\nu]} - e^\mu_A e^\nu_B e^C_\rho \partial_{[\mu} e_{\nu]C} \\ &+ i\frac{\kappa^2}{4}(2\overline{\widetilde{G}}_\mu \gamma_{[A} \widetilde{G}_{B]} + \overline{\widetilde{G}}_A \gamma_\mu \widetilde{G}_B).\end{aligned}$$

For any object O, $O'_{\bar{i}}$ means $\partial O/\partial \bar{z}_{\bar{i}}$ and O'_i means $\partial O/\partial z_i$. The covariant derivative D_μ refers to gauge covariance and the γ-matrix and spinorial conventions are those of Bjorken and Drell, except that $\bar{u}u$ equals twice the mass.

The functions $\mathcal{G}$ and f_{ab} of (5.2) acquire vacuum expectation values (VEVs) on account of the super-Higgs mechanism in the spontaneously broken version of the theory. In consequence, the physical fields have to be rescaled to maintain their canonical normalization. Any vacuum expectation value developed by $\mathrm{Im} f_{ab}$ will generate a CP-violating $F\tilde{F}$ term in the Lagrangian. Since CP-violation, as observed, is small, we demand reality for the VEV of f_{ab}, as denoted by $\langle f^R_{ab} \rangle$. Being symmetric in a, b, this matrix can be diagonalized. We choose our basis in the adjoint representation in such a way that $\langle f^R_{ab} \rangle$ is diagonal. Now the normalized gauge boson and gaugino fields are $\hat{A}^a_\mu = \sqrt{f^R_{aa}} A^a_\mu$ and $\hat{\lambda}_a = \sqrt{f^R_{aa}} \lambda_a$. Simultaneously, the gauge coupling constant g gets rescaled to $\hat{g} = g/\sqrt{f^R_{aa}}$. Similarly, a basis can be chosen in the Kähler manifold (in which the z's take values) where the (real) VEV of $\mathcal{G}''_{i\bar{j}}$, i.e. $\left\langle \mathcal{G}''_{i\bar{j}} \right\rangle$ is diagonal. In that basis the scalar fields get normalized to $\hat{z}_i = \kappa^{-1}\sqrt{\mathcal{G}''_{ii}} z_i$. We use normalized fields henceforth and drop the hats for convenience.

The Lagrangian of (5.3) can now be written as

$$\begin{aligned}\mathcal{L} =& \frac{e}{2\kappa^2} R + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} \overline{\widetilde{G}}_\mu \gamma_5 \gamma_\nu (\partial_\rho - \frac{i}{4}\omega_{\rho AB}\sigma^{AB}) \widetilde{G}_\sigma - \frac{e}{4} F^a_{\mu\nu} F^{\mu\nu a} \\ &+ \frac{ie}{2}\bar{\lambda}^a (\not{\partial} - \frac{i}{4}\not{\omega}_{AB}\sigma^{AB})\lambda^a + \frac{e}{4}\sum_i (\partial_\mu S^i \partial^\mu S^i + \partial_\mu P^i \partial^\mu P^i) + \frac{ie}{2} m_{3/2} \overline{\widetilde{G}}_\mu \sigma^{\mu\nu} \widetilde{G}_\nu\end{aligned}$$

$$
\begin{aligned}
&-\frac{e}{2}(m_{\tilde{g}})_{ab}\bar{\lambda}^a\lambda^b+\frac{e}{2}\sum_i(m_{S_i}^2S_i^2+m_{P_i}^2P_i^2)+\frac{e\kappa}{4}\bar{\lambda}^a\gamma^\rho\sigma^{\mu\nu}\widetilde{G}_\rho F^{\mu\nu a}\\
&+\frac{e\kappa}{4}\sum_i c_{iab}S_iF^a_{\mu\nu}F^{\mu\nu b}+\frac{\kappa}{8}\sum_i c_{iab}P_iF^a_{\mu\nu}F^b_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}\\
&+\frac{i\kappa}{4}\sum_i d_i\epsilon^{\mu\nu\alpha\beta}\overline{\widetilde{G}}_\mu\gamma_\nu\widetilde{G}_\alpha\partial_\beta P_i+\frac{ie\kappa}{2}\sum_i d_im_{3/2}\overline{\widetilde{G}}_\mu\sigma^{\mu\nu}\widetilde{G}_\nu S_i. \qquad (5.4)
\end{aligned}
$$

In (5.4) $S_i=(z^i+\bar{z}^{\bar{i}})/\sqrt{2}$ and $P_i=(z^i-\bar{z}^{\bar{i}})/(\sqrt{2}i)$. Moreover,

$$
c_{iab}=-\frac{1}{\sqrt{2}}\left\langle f'_{abi}\right\rangle(\left\langle\mathcal{G}''_{i\bar{i}}\right\rangle\left\langle f^R_{aa}\right\rangle\left\langle f^R_{bb}\right\rangle)^{-1/2}, \qquad (55a)
$$

$$
d_i=\frac{1}{\sqrt{2}}\left\langle\mathcal{G}'_i\right\rangle(\left\langle\mathcal{G}''_{i\bar{i}}\right\rangle)^{-1/2}. \qquad (55b)
$$

By comparison of (5.3)and (5.4), we see that the gravitino and the gaugino masses are given respectively by

$$
m_{3/2}=\kappa^{-1}e^{\langle\mathcal{G}\rangle/2}, \qquad (56a)
$$

$$
\begin{aligned}
(m_{\tilde{g}})_{ab}=&-\frac{1}{2\kappa}\sum_i e^{\langle\mathcal{G}\rangle/2}\left\langle\mathcal{G}'_{\bar{i}}\right\rangle\left\langle\mathcal{G}''^{-1}_{i\bar{i}}\right\rangle\\
&\left\langle f'_{abi}\right\rangle(\left\langle f^R_{aa}\right\rangle\left\langle f^R_{bb}\right\rangle)^{1/2}. \qquad (56b)
\end{aligned}
$$

It follows from (55) and (56) that

$$
\sum_i c_{iab}d_i=m_{3/2}^{-1}(m_{\tilde{g}})_{ab}. \qquad (5.7)
$$

(5.7) is our central relation linking the super-Higgs mechanism to the cancellation of the bad subleading high-energy (E^3_{CM}) behaviour mentioned earlier.

This relation can also be seen to be the basis of the unitarity violation as is clear from the R-gauge analysis on the lines of Chapter 4. In fact, if χ is the goldstino field the coupling constants c_{iab} and d_i also make their appearance in the couplings of χ to the gauge-gaugino system and the graviton-gravitino system:

$$
\mathcal{L}_{goldstino}=\frac{e\kappa}{\sqrt{2}}c_{iab}\bar{\chi}_{iL}\sigma_{\mu\nu}F^{\mu\nu}a\lambda^b_L+ie\kappa m_{3/2}d_ih^\mu_a\overline{\widetilde{G}}_{R\mu}\gamma^a\chi_{Li}+h.c. \qquad (5.8)
$$

where h is the graviton field. Since the product of these two quantities c_{iab} and d_i, summed over i, is the ratio of the gravitino and gaugino masses in the theory, in case

the latter ratio is large, one or both of these couplings must be large. The tree unitarity breakdown that we shall see is just a manifestation of this large coupling in the R-gauge Lagrangian.

To proceed further, let us assume for simplicity that the gauge-group under consideration admits, as for every simple or U(1) factor of the gauge group, a single invariant tensor of rank two, δ_{ab}. Since only gauge invariant fields are allowed to have VEVs, it follows that $\langle f_{ab}\rangle = \langle f\rangle\,\delta_{ab}$, $\left\langle f'_{iab}\right\rangle = \langle f'_i\rangle\,\delta_{ab}$. So we have

$$(m_{\tilde{g}})_{ab} = m_{\tilde{g}}\delta_{ab}, \tag{59a}$$

$$c_{iab} = c_i\delta_{ab}, \tag{59b}$$

$$\sum_i c_i d_i = m_{3/2}^{-1} m_{\tilde{g}}. \tag{59c}$$

Expressions for the scalar and pseudoscalar masses m_{S_i} and m_{P_i} can be obtained by expansion of the scalar potential

$$V(z,\bar{z}) = e^{\mathcal{G}}(3 - \mathcal{G}'_{\bar{l}}\mathcal{G}''^{-1}_{l\bar{k}}\mathcal{G}'_k)$$

around the stable vacuum with zero cosmological constant. The coefficients of the terms that are quadratic in the scalar and pseudoscalar shifted fields can then be isolated and yield m_{S_i}, m_{P_i}. We omit these detailed expressions since they serve no purpose here.

The relevant vertex tensors can now be written down via functional derivatives of the action $S = \int d^4x\mathcal{L}$. We first define the graviton (h) tensor field in terms of the vierbein

$$h_{\mu\nu} = (\sqrt{2}\kappa)^{-1}(e^A_\mu e_{\nu A} - \eta_{\mu\nu}),$$

where $\eta_{\mu\nu}$ is the flat (+ - - -) metric. The vertex tensors then are

$$(V^{A\lambda\widetilde{G}}_{\mu\nu})^{ab} = \frac{\delta(iS)}{\delta A^{\mu a}(x_1)\delta\lambda^b(x_2)\delta\widetilde{\widetilde{G}}^{\nu}(x_3)}$$

$$=\frac{i\kappa}{2}\sigma_{\alpha\mu}\gamma_\nu\int d^4x\ \delta^{(4)}(x-x_2)\delta^{(4)}(x-x_3)\partial^\alpha\delta^{(4)}(x-x_1)\delta^{ab}. \tag{510a}$$

$$\begin{aligned}(V^{GAA}_{\mu\nu\alpha\beta})^{ab}=&\frac{\delta(iS)}{\delta A^{\mu a}(x_1)\delta A^{\nu b}(x_2)\delta h^{\alpha\beta}(x_3)}\\ =&4\sqrt{2}i\kappa(\delta^\rho_{(\alpha}\delta^\sigma_{\beta)}-\frac{1}{4}\eta^{\rho\sigma}\eta_{\alpha\beta})\eta^{\gamma\delta}\delta^{ab}\\ &\int d^4x\ \delta^{(4)}(x-x_3)\eta_{\mu[\gamma}\partial_{\rho]}\delta^{(4)}(x-x_1)\eta_{\nu[\delta}\partial_{\sigma]}\delta^{(4)}(x-x_2).\end{aligned} \tag{510b}$$

$$\begin{aligned}(V^{\widetilde{G}\widetilde{G}G}_{\lambda\rho\mu\nu})^{ab}=&\frac{\delta(iS)}{\delta h^{\lambda\rho}(x_1)\delta\widetilde{G^\nu}(x_2)\delta\overline{\widetilde{G^\mu}}(x_3)}\\ =&\frac{\kappa}{2\sqrt{2}}\delta^{ab}\int d^4x[-2im_{3/2}(2\eta_{\mu(\lambda}\eta_{\rho)\nu}\\ &-\eta_{\mu\nu}\eta_{\lambda\rho})\delta^{(4)}(x-x_1)\delta^{(4)}(x-x_2)\delta^{(4)}(x-x_3)\\ &-\frac{1}{2}\delta^{(4)}(x-x_2)\delta^{(4)}(x-x_3)\epsilon_{\mu\sigma\nu(\lambda}\gamma^5[\gamma^\sigma,\sigma_{\rho)\tau}]_+\partial^\tau\delta^{(4)}(x-x_1)\\ &+i\delta^{(4)}(x-x_1)\epsilon_{\mu\sigma\nu(\lambda}\gamma^5\gamma_{\rho)}\{\delta^{(4)}(x-x_3)\partial^\sigma\delta^{(4)}(x-x_2)\\ &-\delta^{(4)}(x-x_2)\partial^\sigma\delta^{(4)}(x-x_3)\}].\end{aligned} \tag{510c}$$

$$\begin{aligned}(V^{S_iAA}_{\alpha\beta})^{ab}=&\frac{\delta(iS)}{\delta A^{\alpha a}(x_1)\delta A^{\beta b}(x_2)\delta S_i(x_3)}\\ =&2i\kappa c_i\delta^{ab}\int d^4x\ (x-x_3)\partial_{[\mu}\eta_{\nu]\alpha}\delta^{(4)}(x-x_1)\partial^{[\mu}\delta^{\nu]}_\beta\delta^{(4)}(x-x_2).\end{aligned} \tag{510d}$$

$$\begin{aligned}(V^{P_iAA}_{\alpha\beta})^{ab}=&\frac{\delta(iS)}{\delta A^{\alpha a}(x_1)\delta A^{\beta b}(x_2)\delta P_i(x_3)}\\ =&-i\kappa c_i\epsilon_{\mu\nu\alpha\beta}\delta^{ab}\int d^4x\ \delta^{(4)}(x-x_3)\partial^\mu\delta^{(4)}(x-x_1)\partial^\nu\delta^{(4)}(x-x_2).\end{aligned} \tag{510c}$$

$$
\begin{aligned}
V_{\mu\nu}^{\widetilde{G}\widetilde{G}S_i} =& \frac{\delta(iS)}{\delta S_i(x_1)\delta\widetilde{G^\nu}(x_2)\delta\overline{\widetilde{G^\mu}}(x_3)} \\
=& -\kappa d_i m_{3/2}\sigma_{\mu\nu}\int d^4x\ \delta^{(4)}(x-x_1)\delta^{(4)}(x-x_2)\delta^{(4)}(x-x_3)
\end{aligned}
\tag{510f}
$$

$$
\begin{aligned}
V_{\mu\nu}^{\widetilde{G}\widetilde{G}P_i} =& \frac{\delta(iS)}{\delta P_i(x_1)\delta\widetilde{G^\nu}(x_2)\delta\overline{\widetilde{G^\mu}}(x_3)} \\
=& \frac{\kappa}{2}d_i\epsilon_{\mu\nu\alpha\beta}\gamma^\alpha\int d^4x\ \delta^{(4)}(x-x_2)\delta^{(4)}(x-x_3)\partial^\beta\delta^{(4)}(x-x_1).
\end{aligned}
\tag{510g}
$$

The usual momentum space rules are obtained as Fourier transforms of these and are exhibited in Table 4. The corresponding vertices are shown in Fig. 4.

Table 4

Description	Designation	Expression
$A(p_1,\epsilon_1^\mu)A(p_2,\epsilon_2^\nu)G(P,\epsilon^{\alpha\beta})$	$V_{\mu\nu\alpha\beta}^{AAG}$	$-4\sqrt{2}i\kappa p_{1[\rho}\eta_{\gamma]\mu}p_{2[\sigma}\eta_{\delta]\nu}\eta^{\gamma\delta}[\delta^\rho_{(\alpha}\delta^\sigma_{\beta)}-(1/4)\eta^{\rho\sigma}\eta_{\alpha\beta}]$
$A(p,\epsilon^\mu)\widetilde{G}(k,\overline{\mathrm{u}}^\lambda)\lambda(q,\mathrm{v})$	$V_{\mu\lambda}^{A\lambda G}$	$-(\kappa/2)\sigma_{\mu\nu}p^\nu\gamma_\lambda$
$G(P,\epsilon^{\rho\lambda})\widetilde{G}(q,\overline{\mathrm{u}}^\mu)\widetilde{G}(k,\mathrm{v}^\nu)$	$V_{\lambda\rho\mu\nu}^{G\widetilde{G}\widetilde{G}}$	$(\kappa/2\sqrt{2})\{\epsilon_{\mu\sigma\nu(\lambda}\gamma^5\gamma_{\rho)}(k-q)^\sigma+(1/2)\epsilon_{\mu\sigma\nu(\lambda}\gamma^5[\gamma^\sigma,\sigma_{\rho)\tau}]_+P^\tau$ $-2im_{3/2}(2\eta_{\mu(\lambda}\eta_{\rho)\nu}-\eta_{\mu\nu}\eta_{\lambda\rho})\}$
$A(p_1,\epsilon_1^\alpha)A(p_2,\epsilon_2^\beta)S_i(P)$	$V_{\alpha\beta}^{AAS_i}$	$-2i\kappa c_i p_{1[\mu}\eta_{\nu]\alpha}p_2^{[\mu}\delta_\beta^{\nu]}$
$A(p_1,\epsilon_1^\alpha)A(p_2,\epsilon_2^\beta)P_i(P)$	$V_{\alpha\beta}^{AAP_i}$	$i\kappa c_i\epsilon_{\mu\nu\alpha\beta}p_1^\mu p_2^\nu$
$\widetilde{G}(q,\overline{\mathrm{u}}^\mu)\tilde{G}(k,\mathrm{v}^\nu)S_i(P)$	$V_{\mu\nu}^{\widetilde{G}\widetilde{G}S_i}$	$i\kappa d_i m_{3/2}\eta_{\mu\nu}$
$\widetilde{G}(q,\overline{\mathrm{u}}^\mu)\widetilde{G}(k,\mathrm{v}^\nu)P_i(P)$	$V_{\mu\nu}^{\widetilde{G}\widetilde{G}P_i}$	$(i\kappa/2)d_i\epsilon^{\mu\alpha\nu\beta}\gamma_\alpha P_\beta$

The vertex expressions. Here A indicates a gauge boson, G a graviton and S_i and P_i are the scalar and pseudoscalar particles described in the text. λ and $\widetilde{G}$ represent the gaugino and the gravitino repectively. The quantities within parentheses following the particle label specify the momenta and polarization vectors/spinors, if any. All momenta are taken towards the vertex.

Figure 4

Depiction of vertices described in Table 4. The solid and dashed wavy lines stand for gauge bosons and gravitons respectively. A solid line through the middle indicates the corresponding superpartners. The dashed and dotted lines represent the scalar and pseudoscalar particles respectively.

5.3 *Helicity and Mass Factor Rules*

The dimensional parameters in our helicity amplitudes are κ, $m_{3/2}$ and $m_{\tilde{g}}$. Since this is a second-order gravitational process, κ^2 factors out. Coming to overall factors of the gravitino mass $m_{3/2}$ in the amplitude, let us consider the light gravitino limit. The gravitino is always an external leg here. Being Majorana, it can be described by either a u-spinor or a v-spinor. (Of course, a two gravitino final state has to be described in terms of a $\bar{\mathrm{u}}$ and a v). When the gravitino is described by a u-spinor, the leading term in it with helicity $\frac{1}{2}$ $(-\frac{1}{2})$ would carry positive (negative) chirality and an accompanying

factor of $m_{3/2}^{-1}$ although there would be a nonleading term of opposite chirality which is of zeroth order in $m_{3/2}$. For the case when a v-spinor is chosen, the helicity and chirality sign correlations for the leading and nonleading terms are exactly opposite. Thus, for each gravitino here, one can take an overall factor of $m_{3/2}^{-1}$ provided that any helicity-chirality mismatch for the gravitino is taken care of by a corresponding factor of $m_{3/2}$. Finally, there is no naturally occurring overall factor of $m_{\tilde{g}}$. The origin of $m_{\tilde{g}}$-terms through gaugino-exchange diagrams is discussed later.

From the vertices listed in Table 4 and depicted in Fig. 4, we can derive some additional consequences regarding mass-factors in specific diagrams, given the helicities of the external particles in our process. These follow either from the γ-structure of any of the fermionic vertices or from the nature of the boson couplings. We enumerate them below.

Rule 1 The AAG vertex $V^{AAG}_{\mu\nu\alpha\beta}\epsilon_1^\mu\epsilon_2^\nu$, defined with all momenta going inwards and the gauge boson polarizations contracted, can be explicitly seen to vanish whenever the gauge bosons have identical helicities.[⋆] This is equivalent (since helicity flips sign under crossing) to the statement that an incident gauge boson, scattered in a gravitational field, does not change its helicity. The latter fact is a direct consequence of the geometric origin of the gravitational bending of light: parallel transport along a null geodesic does not change the polarization of a wave. This means, however, that the exchange of the graviton in the incident channel can contribute to the process $AA \to \widetilde{G}\widetilde{G}$ only when the initial gauge bosons have opposite helicity.

⋆ For instance, take the case when each gauge boson has a positive helicity with polarization vector $\epsilon^+_{1,2}$. Now $\epsilon_1^{+\mu}\epsilon_2^{+\nu}V^{AAG}_{\mu\nu\alpha\beta} = \epsilon_1^+\cdot\epsilon_2^+ p_{1(\alpha}p_{2\beta)} - \epsilon_1^+\cdot p_2\epsilon^+_{2(\alpha}p_{1\beta)} - \epsilon_2^+\cdot p_1\epsilon^+_{1(\alpha}p_{2\beta)} + p_1\cdot p_2\epsilon^+_{1(\alpha}\epsilon^+_{2\beta)} - \frac{1}{2}p_1\cdot p_2\epsilon_1^+\cdot\epsilon_2^+\eta_{\alpha\beta} + \frac{1}{2}p_1\cdot\epsilon_2^+ p_2\cdot\epsilon_1^+\eta_{\alpha\beta}$. In the centre of mass one may use $\epsilon_1^+\cdot\epsilon_2^+ = -1, \epsilon_1^+\cdot p_2 = 0 = \epsilon_2^+.p_1$ and $p^\mu_{1,2} = \omega(1,0,0,\mp 1)$, $\epsilon^{+\mu}_{1,2} = \frac{1}{\sqrt{2}}(0,1,\pm i,0)$ to show that this vanishes. The same is the case when each gauge boson has negative helicity.

Figure 5

Tree diagrams for $A + A \to \widetilde{G} + \widetilde{G}$.

Rule 2 Consider the $A\lambda\widetilde{G}$ vertex $V_{\mu\lambda}^{A\lambda\widetilde{G}}$ of Table 4 with the momenta and polarizations as defined in the leftmost column. This vertex is always proportional to $\sigma_{\mu\nu}p^\mu\epsilon^\nu = \chi$. Consider the case when the gravitino is in the final state. If the Majorana gravitino spinor is represented by $\bar{\mathrm{u}}^\lambda$, say, then χ acts on $\bar{\mathrm{u}}^\lambda$ to the left. Alternatively, if it is described by v^λ, χ acts on v^λ to the right. However, in the frame in which $p^\mu = \omega(1,0,0,1)$, the polarization vector for a gauge boson of helicity ± 1 is $\epsilon^{\mu\pm} = \frac{1}{\sqrt{2}}(0,1,\pm i,0)$. Thus $\chi^\pm = \epsilon^{\mu\pm}p^\nu\sigma_{\mu\nu} = \mp i\omega(1 \pm \gamma_5)\begin{pmatrix} \sigma_\pm & 0 \\ 0 & \sigma_\pm \end{pmatrix}$ and evidently a gauge boson of helicity ± 1 picks out a gravitino of chirality $\mp$. For a very light gravitino, with spin projection along the line of motion being 1/2, this corresponds (to zeroth order in $m_{3/2}$) to helicity $\mp\frac{1}{2}$ while for a massive gravitino the sign of the helicity could be the reverse but at the cost of an $m_{3/2}$ factor in the numerator. In other words, a matching (mismatch) of helicity signs between the gauge boson and the gravitino in the $A\lambda\widetilde{G}$ vertex will generate an odd (even, possibly zero) number of $m_{3/2}$ factors.

Let us take now the gaugino-exchange diagrams for our process (Fig. 5). The

gaugino mass $m_{\tilde{g}}$ in the numerator occurs through the chirality-conserving part of the gaugino propagator. The gaugino internal line, however, has an $A\lambda\widetilde{G}$ vertex on either side. Either chirality has to be conserved or at least an additional power of $m_{3/2}$ has to be accommodated through chirality-violation. It follows from the above reasoning that the gaugino exchange diagram contains an odd (even, possibly zero) number of $m_{\tilde{g}},\ m_{3/2}$ linear combination factors whenever the gauge boson helicities are equal (opposite).

Rule 3 The $G\widetilde{G}\widetilde{G}$ vertex $V^{G\widetilde{G}\widetilde{G}}_{\lambda\rho\mu\nu}$ in Table 4 may be seen — to zeroth order in $m_{3/2}$ — to contain an odd number of γ-matrices times a γ_5. In other words, between a $\overline{spinor}$ and a *spinor*, it conserves chirality. However, two Majorana fermions (in this case gravitinos) in the same (initial or final) state have opposite helicities for the same chirality. Thus, ignoring $m_{3/2}$ terms, the helicities of the two gravitinos in the same state participating in the $G\widetilde{G}\widetilde{G}$ vertex must be opposite. This means that for such gravitinos having the same (opposite) helicity, one must have an odd (even, possibly zero) number of $m_{3/2}$ factors.

Rule 4 Take now the $S_i\widetilde{G}\widetilde{G}$ and $P_i\widetilde{G}\widetilde{G}$ vertices. The former flips chirality (like a mass-insertion) but has an accompanying $m_{3/2}$ factor from the super-Higgs mechanism. The latter conserves chirality like a current vertex but would need a factor of $m_{3/2}$ for helicity conservation since two final state gravitinos have opposite chiralities for the same helicity. Thus, if the two gravitinos participating in the $S_i\widetilde{G}\widetilde{G}$ or $P_i\widetilde{G}\widetilde{G}$ vertices have identical (nonidentical) helicities, there will be an odd (even, at least two) number of accompanying $m_{3/2}$ factors. More generally, diagrams involving such vertices have S_i - and P_i -exchanges only in the incident channel provided that the net spin component along the line of motion in the centre of mass is zero both for the initial and the final states. Hence these

contributions vanish whenever either the two incident gauge bosons or the two final state gravitinos have nonidentical helicities.

Rule 5 All the previous rules are such as to make each helicity amplitude either totally odd or totally even in the fermion (gaugino, gravitino or any linear combination of these) masses. Once the κ^2 factor is taken out, these are the only masses present in the problem. Hence, in case the leading energy growth in some helicity amplitude happens to get cancelled, dimensional considerations dictate that the next-to-leading energy growth of that amplitude should be suppressed by two powers of the CM energy.

5.4 *Helicity Amplitudes and Partial Wave Unitarity*

The tree amplitude for the process $A(p_1,\epsilon_1)+A(p_2,\epsilon_2)\to\widetilde{G}(p_3)+\widetilde{G}(p_4)$ can be written down from the vertices given in Table 4 by use of the gaugino, graviton* and scalar (pseudoscalar) propagators as follows:

$$
\begin{aligned}
M = \kappa^2\bar{u}_\alpha(p_4)[&-4i\ \eta^{\alpha[\mu}\gamma^{\nu]}p_{2\mu}\epsilon_{2\nu}(\not{p}_1-\not{p}_3-m_{\tilde{g}})^{-1}\eta^{\beta[\rho}\gamma^{\sigma]}p_{1\rho}\epsilon_{1\sigma}\\
&-4i\ \eta^{\alpha[\mu}\gamma^{\nu]}p_{1\mu}\epsilon_{1\nu}(\not{p}_4-\not{p}_1-m_{\tilde{g}})^{-1}\eta^{\beta[\rho}\gamma^{\sigma]}p_{2\rho}\epsilon_{2\sigma}\\
&+4p_{1[\rho}\epsilon_{1\gamma]}p_{2[\sigma}\epsilon_{2\delta]}\eta^{\gamma\delta}(\delta^\rho_\mu\delta^\sigma_\nu-\frac{1}{4}\eta^{\rho\sigma}\eta_{\mu\nu})(p_1+p_2)^{-2}\\
&(\delta^\mu_{(\tau}\delta^\nu_{\lambda)}-\frac{1}{2}\eta^{\mu\nu}\eta_{\tau\lambda}).\\
&.\{\epsilon^{\alpha\xi\beta(\lambda}\gamma_5\gamma^{\tau)}(p_4-p_3)_\xi-\epsilon^{\alpha\xi\beta(\lambda}\gamma_5[\gamma_\xi,\sigma^{\tau)\xi}]_+(p_3+p_4)_\xi\\
&+im_{3/2}(-4\eta^{\beta(\tau}\eta^{\lambda)\alpha}+2\eta^{\beta\alpha}\eta^{\tau\lambda})\\
&+m_{3/2}\eta^{\alpha\beta}\sum_i c_id_i\{2ip_{1[\mu}\epsilon_{1\nu]}p_2^{[\mu}\epsilon_2^{\nu]}((p_1+p_2)^2-m^2_{S_i})^{-1}\\
&+\epsilon_{\mu\nu\rho\sigma}p_1^\mu\epsilon_1^\nu p_{2\rho}\epsilon_2^\sigma((p_1+p_2)^2-m^2_{P_i})^{-1}\gamma_5\}]v_\beta(p_3)\,.
\end{aligned}
\tag{5.11}
$$

In writing (5.11) we have chosen to use a v-spinor for a gravitino with four momentum

$\star$ A discussion of the graviton propagator used in (5.11) may be found in Ref. 28.

Figure 6

Kinematics in the CM frame.

p_3 and a u-spinor for a gravitino with four momentum p_4. The reverse choice would lead to the same final result.

In order to evaluate this matrix element explicitly, we go to the CM frame (Fig. 6) with CM energy E and scattering angle θ. The momenta are then given by

$$\begin{aligned} p_1 &= (\omega, 0, 0, -\omega), \quad p_2 = (\omega, 0, 0, \omega), \\ p_3 &= (\omega, 0, k\sin\theta, k\cos\theta), \quad p_4 = (\omega, 0, -k\sin\theta, -k\cos\theta), \end{aligned} \tag{5.12}$$

where $\omega = \frac{1}{2}E$ and $k = (E^2/4 - m_{3/2}^2)^{1/2}$. The polarization vectors of the gauge bosons (positive and negative superscripts stand for helicity +1 and -1 respectively) are given by

$$\epsilon_1^{\pm} = \epsilon_2^{\mp} = \frac{1}{\sqrt{2}}(0, 1, \mp i, 0). \tag{5.13}$$

We describe the spin 3/2 gravitino spinors as direct products of spin $\frac{1}{2}$ and spin 1 representations. Thus first define the additional spin 1 polarization vectors $\zeta_\mu^{\pm}$ and ζ_μ^{L}

(corresponding to helicities ± 1 and 0 respectively):

$$
\begin{aligned}
\zeta^{\pm}_{3\mu} &= \zeta^{\mp}_{4\mu} = \frac{1}{\sqrt{2}}(0, 1, \pm i\cos\theta, \mp i\sin\theta), \\
\zeta^{L}_{3\mu} &= m^{-1}_{3/2}(k, 0, \omega\sin\theta, \omega\cos\theta), \\
\zeta^{L}_{4\mu} &= m^{-1}_{3/2}(k, 0, -\omega\sin\theta, -\omega\cos\theta).
\end{aligned}
\tag{5.14}
$$

Then introduce the following $\mathrm{u}^{\pm}-$ and $\mathrm{v}^{\pm}-$spinors with spin $\frac{1}{2}$ and helicities $\pm\frac{1}{2}$:

$$
\begin{aligned}
\mathrm{v}^{+}_{3} &= \begin{pmatrix} -i\sin\frac{\theta}{2}\sqrt{\omega - m_{3/2}} \\ -\cos\frac{\theta}{2}\sqrt{\omega - m_{3/2}} \\ i\sin\frac{\theta}{2}\sqrt{\omega + m_{3/2}} \\ \cos\frac{\theta}{2}\sqrt{\omega + m_{3/2}} \end{pmatrix}, \\
\mathrm{v}^{-}_{3} &= \begin{pmatrix} i\cos\frac{\theta}{2}\sqrt{\omega - m_{3/2}} \\ -\sin\frac{\theta}{2}\sqrt{\omega - m_{3/2}} \\ i\cos\frac{\theta}{2}\sqrt{\omega + m_{3/2}} \\ -\sin\frac{\theta}{2}\sqrt{\omega + m_{3/2}} \end{pmatrix}, \\
\mathrm{u}^{+}_{4} &= \begin{pmatrix} \sin\frac{\theta}{2}\sqrt{\omega + m_{3/2}} \\ -i\cos\frac{\theta}{2}\sqrt{\omega + m_{3/2}} \\ \sin\frac{\theta}{2}\sqrt{\omega - m_{3/2}} \\ -i\cos\frac{\theta}{2}\sqrt{\omega - m_{3/2}} \end{pmatrix}, \\
\mathrm{u}^{-}_{4} &= \begin{pmatrix} -\cos\frac{\theta}{2}\sqrt{\omega + m_{3/2}} \\ -i\sin\frac{\theta}{2}\sqrt{\omega + m_{3/2}} \\ \cos\frac{\theta}{2}\sqrt{\omega - m_{3/2}} \\ i\sin\frac{\theta}{2}\sqrt{\omega - m_{3/2}} \end{pmatrix}.
\end{aligned}
\tag{5.15}
$$

Finally, the gravitino spinors $\mathrm{u}^{\lambda}_{\mu}$, $\mathrm{v}^{\lambda}_{\mu}$ of helicity λ $(= \pm\frac{3}{2}, \pm\frac{1}{2})$ can be written as

$$
\begin{aligned}
\mathrm{v}_\mu^{\pm\frac{3}{2}}(p_3) &= \mp\zeta_{3\mu}^{\mp}\mathrm{v}_3^{\pm}, \\
\mathrm{v}_\mu^{\pm\frac{1}{2}}(p_3) &= \mp(\frac{1}{\sqrt{3}}\zeta_{3\mu}^{\mp}\mathrm{v}_1^{\pm} + i\sqrt{\frac{2}{3}}\zeta_{3\mu}^{L}\mathrm{v}_1^{\pm}), \\
\mathrm{u}_\mu^{\pm\frac{3}{2}}(p_4) &= \pm\zeta_{4\mu}^{\pm}\mathrm{u}_4^{\pm}, \\
\mathrm{u}_\mu^{\pm\frac{1}{2}}(p_4) &= \pm(\frac{1}{\sqrt{3}}\zeta_{4\mu}^{\pm}\mathrm{u}_4^{\mp} + i\sqrt{\frac{2}{3}}\zeta_{4\mu}^{L}\mathrm{u}_4^{\pm}).
\end{aligned}
\tag{5.16}
$$

From eqs. (5.12)-(5.16) one can compute the helicity amplitudes $M_{\lambda_1\lambda_2;\mu_1\mu_2}$ (using the notation of I) in the different channels. The presence of two identical particles in the initial state as well as in the final state lead to the relations

$$
|M_{\lambda_1\lambda_2;\mu_1\mu_2}(\theta)| = |M_{\lambda_2\lambda_1;\mu_1\mu_2}(\theta+\pi)| = |M_{\lambda_1\lambda_2;\mu_2\mu_1}(\theta+\pi)| = |M_{\lambda_2\lambda_1;\mu_2\mu_1}(\theta)|.
$$

Furthermore, due to CP invariance in the theory,

$$
|M_{\lambda_1\lambda_2;\mu_1\mu_2}(\theta)| = |M_{-\lambda_1-\lambda_2;-\mu_1-\mu_2}(\theta)|.
$$

Hence there are only sixteen independent helicity amplitudes. We choose these from among those with helicity +1 for the first gauge boson. The energy scale at which tree unitarity gets violated is much larger than the values of $m_{3/2}$ and $m_{\tilde{g}}$ in nearly all models of low energy supersymmetry. Thus it makes sense to keep only the leading terms in powers of ω by taking $\omega \gg m_{3/2}, m_{\tilde{g}}$. Using the REDUCE 3 program displayed in Appendix II, we can evaluate the expressions for the sixteen tree helicity amplitudes. They are then:

$$
|M_{1\,1;\frac{3}{2}\,\frac{3}{2}}| = \frac{1}{2}\kappa^2\omega^{-1}|m_{\tilde{g}}^3 + 4m_{3/2}^2 m_{\tilde{g}}\cot^2\theta + \sum_i \frac{1}{2}c_i d_i m_{3/2}(m_{S_i}^2 - m_{P_i}^2)|, \tag{517a}
$$

$$
|M_{1\,1;\frac{3}{2}\,\frac{1}{2}}| = \frac{4}{\sqrt{3}}\kappa^2 m_{\tilde{g}} m_{3/2}|\cot\theta|, \tag{517b}
$$

$$
|M_{1\,1;\frac{3}{2}\,-\frac{1}{2}}| = \frac{2}{\sqrt{3}}\kappa^2 m_{\tilde{g}}\omega, \tag{517c}
$$

$$
|M_{1\,1;\frac{3}{2}\,-\frac{3}{2}}| = 0, \tag{517d}
$$

$$|M_{1\,1;-\frac{3}{2}\,\frac{1}{2}}| = \frac{1}{2\sqrt{3}}\kappa^2 m_{\tilde{g}} m_{3/2}^2 \omega^{-1}, \tag{517e}$$

$$|M_{1\,1;-\frac{3}{2}\,-\frac{1}{2}}| = \frac{1}{\sqrt{3}}\kappa^2 m_{\tilde{g}}^3 m_{3/2} \omega^{-2} |\cot\theta|, \tag{517f}$$

$$|M_{1\,1;-\frac{3}{2}\,-\frac{3}{2}}| = \kappa^2\omega^{-1}|m_{\tilde{g}}^3 - m_{3/2}^2 m_{\tilde{g}} + \frac{1}{4}\sum_i m_{3/2} c_i d_i (m_{S_i}^2 + m_{P_i}^2)|, \tag{517g}$$

$$|M_{1\,1;\frac{1}{2}\,\frac{1}{2}}| = \frac{\kappa^2}{3}\omega|4m_{\tilde{g}} + \sum_i c_i d_i (m_{S_i}^2 - m_{P_i}^2) m_{3/2}^{-1}|, \tag{517h}$$

$$|M_{1\,1;\frac{1}{2}\,-\frac{1}{2}}| = \frac{4}{3}\kappa^2 m_{\tilde{g}}^3 m_{3/2}^{-1} |\cot\theta|, \tag{517i}$$

$$|M_{1\,1;-\frac{1}{2}\,-\frac{1}{2}}| = \frac{4}{3}\kappa^2\omega|m_{\tilde{g}}^2 m_{3/2}^{-2} - 1) m_{\tilde{g}} + \frac{1}{4} m_{3/2}^{-1}\sum_i c_i d_i (m_{S_i}^2 + m_{P_i}^2)|, \tag{517j}$$

$$|M_{1\,-1;\frac{3}{2}\,\frac{3}{2}}| = \frac{1}{2}\kappa^2 m_{3/2}\omega^{-1}|m_{\tilde{g}}^2 + m_{3/2}^2(1 - 2\sin^2\theta)|, \tag{517k}$$

$$|M_{1\,-1;\frac{3}{2}\,\frac{1}{2}}| = \frac{1}{2\sqrt{3}}\kappa^2 \sin\theta |m_{\tilde{g}}^2 - 3m_{3/2}^2(1 - 2\cos\theta) + \frac{2m_{3/2}^2}{(1+\cos\theta)}| \tag{517l}$$

$$|M_{1\,-1;\frac{3}{2}\,-\frac{1}{2}}| = \frac{1}{2}\kappa^2 m_{3/2}\omega^{-1}(1 - \cos\theta)|m_{\tilde{g}}^2 + m_{3/2}^2\cos\theta|, \tag{517m}$$

$$|M_{1\,-1;\frac{3}{2}\,-\frac{3}{2}}| = \kappa^2\omega^2(1 - \cos\theta)|\sin\theta|, \tag{517n}$$

$$|M_{1\,-1;\frac{1}{2}\,\frac{1}{2}}| = \frac{4}{3}\kappa^2 m_{3/2}\omega |m_{3/2}^{-2} m_{\tilde{g}}^2 - \frac{3}{2}\sin^2\theta|, \tag{517o}$$

$$|M_{1\,-1;\frac{1}{2}\,-\frac{1}{2}}| = \frac{2}{3}\kappa^2\omega^2|\sin\theta[m_{3/2}^{-2} m_{\tilde{g}}^2 - \frac{3}{2}(1 - \cos\theta)]|. \tag{517p}$$

As explicitly seen from (517), helicity amplitudes with any of the gravitinos carrying helicity either $\frac{3}{2}$ or $-\frac{3}{2}$ do not contain the sought after term with $m_{3/2}^{-2}$ that will lead to an ω^2 growth. Thus we need not bother with these any further. On the other hand, not all of the five helicity amplitudes with each gravitino helicity being $+\frac{1}{2}$ or $-\frac{1}{2}$ contain that term, nor do they exhibit the same energy growth. Only the helicity amplitude of (517p) grows as ω^2, the rest are either constant or grow more slowly. This may be understood by employing the helicity and mass-factor rules of Section 5.3

Consider the helicity amplitude $M_{1\,1;\frac{1}{2}\,\frac{1}{2}}$ of (517h) first. By rule 1) the incident channel graviton exchange diagram cannot contribute here. Further, the gauge boson

and gravitino helicity signs match while the gauge boson helicities are identical. Rule 2) therefore generates two powers of $m_{3/2}$ from the two $A\lambda\widetilde{G}$ vertices and an accompanying factor which could be a linear combination of $m_{\tilde{g}}$ and $m_{3/2}$. Thus the gaugino exchange diagrams grow linearly with ω. On the other hand, for the S_i, P_i-exchange diagrams, Rule 4) can at most generate one $m_{3/2}$ factor and so these can grow as $\kappa^2 m_{3/2}^{-1}\omega^3$. We know, however, from our general discussion on the super-Higgs mechanism in the R-gauge (section 5.1) that such a growth is not allowed. Thus the S_i - and P_i -exchanges must mutually cancel the ω^3 terms in this helicity amplitude and generate a $m_{S_i}^2 - m_{P_i}^2$ factor (apart from $c_i d_i$ which comes from the vertices) from $(4\omega^2 - m_{S_i}^2)^{-1} - (4\omega^2 - m_{P_i}^2)^{-1}$ in the large ω limit. Employing Rule 5), we see then that the total amplitude exhibits a linear growth in ω — as explicitly shown above.

We next take up $M_{1\,1;\frac{1}{2}\,-\frac{1}{2}}$ as given in (517i). Once again, by Rule 1), there is no graviton exchange. Similarly, the gravitino helicities being nonidentical, by Rule 5), there is no S- or P-exchange. Coming to gaugino exchange diagrams, there is one matching and one mismatch between gauge boson and gravitino helicity signs with the gauge boson helicities being identical. Rule 2) now generates one power of $m_{3/2}$ and one factor of a linear combination of $m_{\tilde{g}}$ and $m_{3/2}$. As such, this would have meant a leading ω^2 growth which would go with the lowest partial wave of the $d^J(\theta)$ function. However, since the difference of the two helicities in the initial state is zero and that in the final state one, by the property of $d^J(\theta)$ functions, the lowest partial wave is J = 1. That is incompatible with the Bose symmetry of the initial state and the corresponding amplitude vanishes. The next partial wave contributes an energy-independent term (see Rule 5)).

Consider now $M_{1\,1;-\frac{1}{2}\,-\frac{1}{2}}$ of (517j). The lack of graviton exchange is evident from Rule 1). The gaugino exchange diagrams can only yield one mass factor from the identity

of gauge boson helicities so that an ω^3 growth is still allowed. The same is true of the S_i -,P_i -exchanges. The super-Higgs mechanism and the R-gauge analysis then dictate that all these growths must cancel. By Rule 5) only a linear growth in ω survives. The term corresponding to S_i -, P_i -exchanges yields, apart from $c_i d_i$, a factor $m_{S_i}^2 + m_{P_i}^2$ in the numerator coming from $(4\omega^2)^{-1} - (4\omega^2 - m_{S_i}^2)^{-1} - (4\omega^2 - m_{P_i}^2)^{-1}$ in the large ω limit.

Coming to $M_{1\,-1;\frac{1}{2}\,\frac{1}{2}}$ of (517o), the gauge-boson helicities being unequal, S_i - and P_i -exchanges are excluded by Rule 5). The incident channel graviton exchange is possible but by Rule 3) it contributes extra an factor of $m_{3/2}$ in the numerator. Similarly, in the gaugino exchange diagrams, one matching of gauge boson and gravitino helicity signs generates an $m_{3/2}$ factor by Rule 2). Thus the amplitude would seem to be able to grow with energy like $\kappa^2 m_{3/2}^{-1}\omega^3$. We know, however, from our discussion on the R-gauge in Section 1 that this is not permitted. In fact, supercurrent conservation effects a cancellation of all the ω^3 terms from the s-, t- and u-channel exchanges leaving, by Rule 5), only a linear growth in ω.

Finally, the operative helicity amplitude is $M_{1\,-1;\frac{1}{2}\,-\frac{1}{2}}$. Rule 4) implies the absence of s-channel scalar and pseudoscalar exchange. However, the application of Rules 2) and 3) to the remaining diagrams can only generate an even number (including zero) of $m_{3/2}$ factors. Thus a leading energy growth proportional to $\kappa^2 m_{3/2}^{-2}\omega^4$ would seem to be allowed in violation of the R-gauge argument in the Introduction. Once again, supercurrent conservation comes to the rescue cancelling the ω^4 terms from all the three exchange diagrams. By Rule 5) we see that the residual growth in ω has to be quadratic.

An inspection of all the equations in (517) makes it evident that the helicity amplitude $M_{1\,-1;\frac{1}{2}\,-\frac{1}{2}}$ in the last equation has the fastest energy growth. Since the difference of the two helicities in the initial state here is 2, the lowest partial wave, by the property

of $d^J(\theta)$ functions is J = 2. The corresponding projected amplitude (changing from ω to E) is

$$\begin{aligned}\mathcal{T}^2_{1\,-1;\frac{1}{2}\,-\frac{1}{2}} &= \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta\ d^2_{21}(\theta)\ M_{1-1;\frac{1}{2}-\frac{1}{2}}(\theta) \\ &= \frac{\kappa^2 E^2}{288\pi}(m^2_{\tilde{g}} m^{-2}_{3/2} - \frac{6}{5}).\end{aligned} \tag{5.18}$$

Thus the partial wave unitarity requirement

$$|\mathcal{T}^2_{1\,-1;\frac{1}{2}\,-\frac{1}{2}}| < 1$$

is violated at a critical energy scale

$$E_{cr.} = 12\sqrt{2\pi}(m^2_{\tilde{g}} m^{-2}_{3/2} - \frac{6}{5})^{-1/2} M_{Pl}. \tag{5.19}$$

Alternatively, the requirement of the validity of tree unitarity of E upto κ^{-1} implies the inequality

$$m_{\tilde{g}} m^{-1}_{3/2} < (288\pi + \frac{6}{5})^{1/2} \sim 30. \tag{5.20}$$

The same requirement as above, imposed on (517j), implies upper bounds on the masses m_{S_i}, m_{P_i} of the hidden sector chiral scalars and pseudoscalars. In this case the lowest partial wave projection is

$$\mathcal{T}^0_{1\,1;-\frac{1}{2}\,-\frac{1}{2}} = \frac{\kappa^2 E}{24\pi}[(m^2_{\tilde{g}} m^{-2}_{3/2} - 1)m_{\tilde{g}} + \frac{1}{4m_{3/2}}\sum_i c_i d_i (m^2_{S_i} + m^2_{P_i})] \tag{5.21}$$

and this has to obey

$$|\mathcal{T}^0_{1\,1;-\frac{1}{2}\,-\frac{1}{2}}| < 1.$$

Suppose that there is only one light scalar, pseudoscalar pair (S,P) which matters practically for both (59c) and (5.21). Then the requirement of the maintenance of tree unitarity all the way upto M_{Pl} dictates the upper bound on the RMS scalar, pseudoscalar mass

$$(m^2_S + m^2_P)^{1/2} < [96\pi\kappa^{-1} m^2_{3/2} m^{-1}_{\tilde{g}}]^{1/2}. \tag{5.22}$$

5.5 *Discussion*

In this Chapter, we have extended the tree unitarity analysis of Chapter 3 to the double gravitino amplitude gauge boson + gauge boson $\rightarrow$ gravitino + gravitino. This process has the advantage of being allowed for abelian gauge interactions also. Such was not the case for the single gravitino amplitude where nonabelian gauge interactions were required for the reaction to take place. Furthermore, the role of the chiral gauge-singlet scalar S and pseudoscalar P (superpartners of the goldstino) is quite important here whereas they were nonexistent in the previous process.

Gravitationally mediated tree amplitudes are generally characterized by Planckian energy growths where, for high enough energies, the powers of κ and ω match. If for some amplitude the power of ω happens to exceed that of κ, the growth is super-Planckian. This is bad since it brings down the scale of violation of tree unitarity much below the Planck mass. On the other hand, if the power of ω is lower than that of κ, the energy growth is sub-Planckian and harmless so far as unitarity is considered. The process considered here is of second order in κ. However, two powers of $m_{3/2}$ in the denominator are generated for two longitudinally polarized gravitinos. This suggests a leading super-Planckian energy growth proportional to $\kappa^2 m_{3/2}^{-2}\omega^4$ which has been shown to cancel through supercurrent conservation. This is somewhat analogous to the case in Chapter 3. However, a next-to-leading non-Planckian energy growth proportional to $\kappa^2 m_{3/2}^{-2}\Delta m_S \omega^3$ was also possible. That has been shown to cancel in consequence of the super-Higgs mechanism through the additional exchanges of S and P. This verifies through an explicit example the conclusions drawn in Chapter 4: namely that the R-gauge analysis of the super-Higgs mechanism shows the impossibility of super-Planckian growth of any tree amplitude in a spontaneously broken N = 1 supergravity theory. The residual energy growth is Planckian, being proportional at high energies

to $\kappa^2 m_{\tilde{g}}^2 m_{3/2}^{-2} \omega^2$, as contrasted with $g\kappa m_{\tilde{g}} m_{3/2}^{-1} \omega$ in Chapter 3. Numerically, the upper bound on the gaugino-gravitino mass ratio $m_{\tilde{g}} m_{3/2}^{-1}$, imposed by the requirement of tree unitarity being valid upto the Planck scale, is stronger here by nearly an order of magnitude as compared with that derived in Chapter 3. In addition, an upper bound on the RMS mass of the S and the P has emerged.*

$\star$ Recently, unitarity analysis of the elastic scattering of two gravitinos has been carried out. This yields bounds on the ratio of the masses of the scalar and the pseudoscalar to that of the gravitino.[29] These bounds are stronger than the ones obtained here.

6. Phenomenology of a Superlight Gravitino

6.1 *Introduction*

In the previous chapters we have derived theoretical bounds relating the scale of tree unitarity violation to the ratio of the gaugino and the gravitino masses in a broken supergravity theory. While a large value for the latter number and the consequent breakdown of perturbation theory much below the Planck scale are certainly unæsthetic features, it is interesting to study the direct experimental consequences of such a theory. In particular, we note that the scale of unitarity violation can often be much larger than the laboratory energies for a wide range of values of the gravitino mass relative to the gaugino mass. In such a case, the question of consistency of the theory will not arise vis-a-vis laboratory experiments. On the other hand, there will be spectacular predictions for the latter to verify. In this chapter, we shall indicate a few possible means of bounding the mass of the gravitino from below by use of direct experimental considerations. These bounds rely on the following fact. Long before the point of tree unitarity violation is reached, the perturbative matrix element for any process with a longitudinally polarized gravitino already grows significantly to be possibly detected in the laboratory.

Initial estimates of this kind were made by Fayet[30] by studying the electron-positron annihilation processes giving rise to a photino and a gravitino and also to a photon, a photino and a gravitino. For a photino sufficiently light to be produced in the experiment, he derived a lower bound of 2.3 μeV on the mass of the gravitino from existing data. We extend this analysis to consider an amplitude in which two photons from e^+e^- beams collide and produce two gravitinos in the final state. The gravitinos escape detection so that the process studied in the laboratory becomes $e^-e^+ \to e^-e^+ +$ "nothing"

considered in the double-effective-photon approximation. Here the e^-, e^+ are "tagged" at very near forward angles so that the two colliding photons are almost real and "nothing" is characterized by missing energy and transverse momentum ($\not{E}$, $\not{p}_T$). The lack of any experimental signal for this process studied at 100 – 200 GeV can be used to supply an improved lower bound on the mass of the gravitino. This would be an interesting programme to pursue at the LEP and SLC machines. Additionally, we look at the process of fusion of two gluons into a gluino and a gravitino. At Tevatron and future hadron accelerators, one can look for this process in mono-jet signatures. The new preliminary Tevatron runs at a center-of-mass energy of 1.6 TeV suggest that for modest gluino masses, a lower limit of 3.5 μeV can already be derived on the gravitino mass.

6.2 *Photon-photon Collision*

The coupling of a superlight gravitino to the gauge-matter sector is controlled by $\kappa\overline{\widetilde{G}^\mu}S_\mu$ where S_μ is the spinorial source supercurrent and

$$\kappa = (8\pi G_{Newton})^{1/2} = 4.11 \times 10^{-19}\text{GeV}^{-1}$$

Since, a longitudinally polarized gravitino field is $\widetilde{G}_{\mu L} \propto m_{3/2}^{-1}\partial_\mu\chi$, the coupling can be written as $\kappa m_{3/2}^{-1}\Delta m_S$, where $\Delta m_S \equiv \partial^\mu S_\mu$ is the supersymmetry-breaking mass in that sector. Hence, for $\Delta m_S \sim M_W$ and $m_{3/2} \sim 10^{-6}$ eV, the coupling is $\sim 10^{15}\kappa$ so that at an energy scale of 10^2 GeV or so it becomes measurable. We are specifically interested in the possiblity of measuring sizable cross-sections for the pair production of such gravitinos in photon-photon collisions (*i.e.* $\gamma + \gamma \to \widetilde{G} + \widetilde{G}$) around $\sqrt{s} = 100$

GeV at the SLAC Linear Collider SLC or the CERN collider LEP through the process $e\bar{e} \to e\bar{e}$ + "nothing" considered in the double-effective-photon approximation.*

A characteristic feature of this process is the following. In addition to the photon (A_μ), photino (λ), graviton ($h_{\mu\nu}$) and the gravitino ($\widetilde{G}_\mu$) fields, one has to consider certain chiral scalar (S_i) and pseudoscalar (P_i) fields. These are the superpartners of certain chiral fermions one of which is the Goldstino absorbed by the gravitino in the process of becoming massive. Their interactions with the gravitino are controlled by the super-Higgs mechanism. This is analogous to the case of the Higgs particle — in the standard electroweak theory — whose couplings to the gauge bosons become determined by the Higgs mechanism. In particular, we shall show that, though these particles belong to the "hidden sector," their interactions become large enough to cancel the leading high-energy behaviour of the amplitude in question. Nevertheless, their interactions with ordinary matter are still sufficiently small for them to be extremely long lived. Thus their pair production also contributes to the reaction $e\bar{e} \to e\bar{e}$ + "nothing". The fundamental processes contributing to this reaction then are $\gamma\gamma \to \widetilde{G}\widetilde{G}$, $\gamma\gamma \to S_iS_i$, $\gamma\gamma \to P_iP_i$, and $\gamma\gamma \to S_iP_i$.

On imposing naturalness and CP conservation in the observable sector, the relevant part of the Lagrangian becomes[23]

$$
\begin{aligned}
e^{-1}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2\kappa^2}R + \frac{1}{2}e^{\mu\nu\rho\sigma}\overline{\widetilde{G}}_\mu\gamma_5\gamma_\nu D_\rho\widetilde{G}_\sigma + \frac{1}{4}\kappa\bar{\lambda}\gamma^\rho\sigma^{\mu\nu}\widetilde{G}_\rho F_{\mu\nu} + \frac{i}{2}\bar{\lambda}\not{D}\lambda + \\
&\frac{1}{2}\sum_i\left(\partial_\mu S_i\partial^\mu S_i + \partial_\mu P_i\partial^\mu P_i\right) + \\
&+\frac{i}{2}m_{3/2}\overline{\widetilde{G}}_\mu\sigma^{\mu\nu}\widetilde{G}_\nu - \frac{1}{2}m_{\widetilde{\gamma}}\bar{\lambda}\lambda + \frac{1}{2}\sum_i\left(m_{S_i}^2S_i^2 + m_{P_i}^2P_i^2\right) +
\end{aligned}
$$

* Certain objections to the use of the DEPA with low mass, more than spin one particles in the final state have been raised by Jayaraman et al.[31] Their objection, however, does not apply to our case as explained in Ref. 32

$$\frac{\kappa}{4}\sum_i c_i\left(S_iF_{\mu\nu}F^{\mu\nu}+\frac{1}{2}P_ie^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right)$$
$$+\frac{i\kappa}{2}\sum_i d_i\left(m_{3/2}\overline{\widetilde{G}}_m u\sigma^{\mu\nu}\widetilde{G}_\nu S_i+\frac{1}{2}e^{\mu\nu\alpha\beta}\overline{\widetilde{G}}_\mu\gamma_\nu\widetilde{G}_\alpha\partial_b etaP_i\right) \tag{6.1}$$

in the notation of Bjorken and Drell. We have normalized the fields so as to set the kinetic-energy terms in the canonical form. In (6.1),

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu,\quad D_\rho\widetilde{G}_\sigma=\left(\partial_\rho-\frac{i}{4}\omega_{\mu ab}\sigma^{ab}\right)\widetilde{G}_\sigma,$$

where $\omega_{\mu ab}$ is the spin connection given by

$$\omega_{\mu ab}=e_a{}^\nu\partial_{[\mu}e_{b\nu]}-e_b{}^\nu\partial_{[\mu}e_{a\nu}-e_a{}^\rho e_b{}^\sigma e^c{}_\mu\partial_{[\rho}e_{\sigma]c}+O(\kappa^2).$$

Furthermore, $e_a{}^\rho$ and $e^c{}_\mu$ are the vierbein and its inverse, respectively, e is the determinant of the vierbein, $e^{\mu\nu\rho\sigma}$ is the unit antisymmetric tensor with $e^{0123}=-e^{-1}$ and R is the scalar curvature in the sign convention of Weinberg.[33] Here $m_{\widetilde{\gamma}}$, m_{S_i}, and m_{P_i} are the masses of the photino, the scalars and the pseudoscalars, respectively.⋆ Finally, the signature of our metric is $(+---)$ and moreover our square brackets and parentheses in the superscripts or subscripts denote antisymmetrization and symmetrization, respectively, with unit weight.

The parameters c_i and d_i that appear in the above Lagrangian are related[34] to the functions $\mathcal{G}(z,\bar{z})$ and $f_{\alpha\beta}(z)$ of Ref. 23 by

$$c_i=-\frac{1}{\sqrt{2}}\left\langle\partial_i f\right\rangle\left\langle\mathcal{G}_i''\right\rangle^{-1/2}\left\langle f\right\rangle^{-1}, \tag{62a}$$
$$d_i=\frac{1}{\sqrt{2}}\left\langle\partial_i f\right\rangle\left\langle\mathcal{G}_i''\right\rangle^{-1/2} \tag{62b}$$

We choose our basis in the Kähler manifold such that the matrix $\left\langle(\partial/\partial z_i)(\partial/\partial\bar{z}_{\bar{j}})\mathcal{G}\right\rangle$ diagonal with $\langle\mathcal{G}_i''\rangle$ as the corresponding diagonal elements. We further take the vacuum

⋆ In no-scale supergravity theories the scalar and pseudoscalar masses are zero at the tree level. In some explicit models (like those considered in Ref. 2), nonperturbative effects give them a finite but small mass. We neglect these masses in the rest of the calculation

expectation values (VEV's) not to break the gauge symmetries so that $\langle f_{\alpha\beta}\rangle = \langle f\rangle\,\delta_{\alpha\beta}$ and $\langle(\partial/\partial z_i)f_{\alpha\beta}\rangle = \langle\partial_i f\rangle\,\delta_{\alpha\beta}$ within each simple and U(1) factor of the gauge group. As discussed in Chapter 5 the super-Higgs mechanism[23] implies,

$$m_{\tilde{\gamma}} = -\frac{1}{2\kappa}\sum_i \left\langle e^{\mathcal{G}/2}(\partial_i\mathcal{G})\mathcal{G}_i''^{-1}\right\rangle \langle\partial_i f\rangle\,\langle f\rangle^{-1}\,, \tag{63a}$$

$$m_{3/2} = \frac{1}{\kappa}\left\langle e^{\mathcal{G}/2}\right\rangle. \tag{63b}$$

Thus,

$$m_{\tilde{\gamma}} = m_{3/2}\sum_i c_i d_i. \tag{6.4}$$

Equation (6.4) implies that a large hierarchy between the photino and gravitino masses[2] will make the interactions of S_i and P_i, which are characterized by c_i and d_i, non-negligible. We can eliminate $\langle\partial_i f\rangle\,\langle f\rangle^{-1}$ between (62) and (63). Moreover, we assume for simplicity that it suffices to take the contribution from only one scalar-pseudoscalar pair to the VEV's, those from observable fields, if any, being small in comparison. This leads via the substitution[23] $\mathcal{G} = -3\ln(2\kappa\mathrm{Re}z + \text{observable-sector fields})$ in the case of no-scale theories, to

$$c = -\left(\frac{2}{3}\right)^{1/2}\frac{m_{\tilde{\gamma}}}{m_{3/2}}, \quad d = -\left(\frac{3}{2}\right)^{1/2}. \tag{6.5}$$

The seven basic interaction vertices relevant to us are the following: $\gamma\tilde{\gamma}\tilde{G}$, $\gamma\gamma G$, $G\tilde{G}\tilde{G}$, $\gamma\gamma S$, $\gamma\gamma P$, $\tilde{G}\tilde{G}S$, and $\tilde{G}\tilde{G}P$. The corresponding vertex tensors are defined in Fig. 7

while their expressions in terms of momenta (all incoming) are given in Table 5. There are five Feynman diagrams contributing to the process $\gamma\gamma \to \tilde{G}\tilde{G}$ and two of each for the processes $\gamma\gamma \to SS$, $\gamma\gamma \to PP$, and $\gamma\gamma \to SP$ as shown in Figs.8(a), 8(b), 8(c),

Figure 7

The relevant vertices in the theory. The solid and dashed wavy lines represent the photon and the graviton, respectively. A solid line through the center indicates the corresponding superpartner. The dashed and dotted straight lines represent the scalar and pseudoscalar particles respectively.

and 8(d), respectively.† For $m_{3/2}$, much smaller than all other scales in the problem, the gravitino spinors can be approximated, as in Chapter 3, by

$$\begin{aligned} \mathrm{u}_\mu^{\pm 1/2} &\simeq \pm i(\frac{2}{3})^{1/2}\frac{p_\mu}{m_{3/2}}\mathrm{u}^\pm(p), \\ \mathrm{v}_\mu^{\pm 1/2} &\simeq \mp i(\frac{2}{3})^{1/2}\frac{p_\mu}{m_{3/2}}\mathrm{v}^\pm(p). \end{aligned} \tag{6.6}$$

In (6.6) u and v are the usual massless spinors of spin half. The $m_{3/2}$ factor in the denominator makes the contribution of the $\pm\frac{3}{2}$ helicities of the gravitino negligible.

† S-channel graviton exchange in the SS and PP final state processes need not be considered from our approximation since they do not produce any $m_{3/2}^{-1}$ factors in the amplitude.

TABLE 5.

Designation	Expression
$V^{\gamma\gamma G}_{\mu\nu\alpha\beta}$	$-4\sqrt{2}i\kappa q_{1[\rho}\eta_{\gamma]\alpha}q_{2[\sigma}\eta_{\delta]\beta}\eta^{\gamma\delta}(\delta^{\rho}_{(\mu}\delta^{\sigma}_{\nu)}-\frac{1}{4}\eta^{\rho\sigma}\eta_{\mu\nu})$
$V^{\gamma\tilde{\gamma}\tilde{G}}_{\mu\lambda}$	$-\frac{\kappa}{2}\sigma_{\mu\nu}q^{\nu}\gamma_{\lambda}$
$V^{G\tilde{G}\tilde{G}}_{\lambda\rho\mu\nu}$	$\frac{\kappa}{2\sqrt{2}}[\epsilon_{\mu\sigma\nu(\lambda}\gamma^5\gamma_{\rho)}(r_1-r_2)^{\sigma}+\frac{i}{2}\epsilon_{\mu\sigma\nu(\lambda}\gamma^5[\gamma^{\rho},\sigma_{\rho)\tau}]_+P^{\tau}$
	$-2im_{3/2}(2\eta_{\mu(\lambda}\eta_{\rho)\nu}-\eta_{\mu\nu}\eta_{\lambda\rho})]$
$V^{\gamma\gamma S_i}_{\alpha\beta}$	$-2i\kappa c_i q_{1[\mu}\eta_{\nu]\alpha}q_2^{[\mu}\delta^{\nu]}_{\beta}$
$V^{\gamma\gamma P_i}_{\alpha\beta}$	$i\kappa c_i\epsilon_{\mu\nu\alpha\beta}q_1^{\mu}q_2^{\nu}$
$V^{\tilde{G}\tilde{G}S_i}_{\mu\nu}$	$i\kappa d_i m_{3/2}\eta_{\mu\nu}$
$V^{\tilde{G}\tilde{G}P_i}_{\mu\nu}$	$\frac{i\kappa}{2}d_i\epsilon_{\mu\alpha\nu\beta}\gamma^{\alpha}P^{\beta}$

The relevant vertex expressions in the theory. The vertices are shown in Fig. 7.

Figure 8

The tree level Feynman diagrams contributing to our process. The legend is given in the caption of Fig. 7.

Using (6.6), all nonzero helicity amplitudes can be constructed as follows:

$$|\mathcal{M}^{\widetilde{G}\widetilde{G}}_{1\,1;-\frac{1}{2}\,-\frac{1}{2}}(\theta)| = |\mathcal{M}^{\widetilde{G}\widetilde{G}}_{-1\,-1;\frac{1}{2}\,\frac{1}{2}}(\theta)| = 2\kappa^2 m_{\widetilde{\gamma}}^3 \omega 3 m_{3/2}^2 \left(\frac{\cos^2\theta/2}{y+\cos^2\theta/2} + \frac{\sin^2\theta/2}{y+\sin^2\theta/2} \right), \tag{67a}$$

$$\begin{aligned} |\mathcal{M}^{\widetilde{G}\widetilde{G}}_{1\,-1;-\frac{1}{2}\,\frac{1}{2}}(\theta)| &= |\mathcal{M}^{\widetilde{G}\widetilde{G}}_{-1\,1;\frac{1}{2}\,-\frac{1}{2}}(\theta)| = |\mathcal{M}^{\widetilde{G}\widetilde{G}}_{1\,-1;\frac{1}{2}\,-\frac{1}{2}}(\pi-\theta)| = |\mathcal{M}^{\widetilde{G}\widetilde{G}}_{-1\,1;-\frac{1}{2}\,\frac{1}{2}}(\pi-\theta)| \\ &= \frac{2\kappa^2 m_{\widetilde{\gamma}}^2 \omega^2}{3m_{3/2}^2} |\sin\theta| \frac{\cos^2\theta/2}{y+\cos^2\theta/2}, \end{aligned} \tag{67b}$$

$$\begin{aligned} |\mathcal{M}^{SS}_{1\,1}(\theta)| &= |\mathcal{M}^{SS}_{-1\,-1}(\theta)| = |\mathcal{M}^{PP}_{1\,1}(\theta)| = |\mathcal{M}^{PP}_{-1\,-1}(\theta)| = |\mathcal{M}^{SP}_{1\,1}(\theta)| = |\mathcal{M}^{SP}_{-1\,-1}(\theta)| \\ &= \frac{4\kappa^2 m_{\widetilde{\gamma}}^2 \omega^2}{3m_{3/2}^2}, \end{aligned} \tag{67c}$$

$$\begin{aligned} |\mathcal{M}^{SS}_{1\,-1}(\theta)| &= |\mathcal{M}^{SS}_{-1\,1}(\theta)| = |\mathcal{M}^{PP}_{1\,-1}(\theta)| = |\mathcal{M}^{PP}_{-1\,1}(\theta)| \\ &= \frac{2\kappa 2 m_{\widetilde{\gamma}}^2 \omega^2}{3m_{3/2}^2}, \end{aligned} \tag{67d}$$

$$|\mathcal{M}^{SP}_{1\,-1}(\theta)| = |\mathcal{M}^{SP}_{-1\,1}(\theta)| = \frac{2\kappa 2 m_{\widetilde{\gamma}}^2 \omega^2}{3m_{3/2}^2} |\cos\theta|. \tag{67e}$$

All other possible helicity amplitudes in the $\widetilde{G}\widetilde{G}$ sector vanish in the present approximation. In Eqs. (67) the subscripts refer to the helicities, ω is the c.m. energy of one of the photons, $y = m_{\widetilde{\gamma}}^2/4\omega^2$, and θ is the scattering angle in the center-of-mass frame.

From these helicity amplitudes, the cross-sections in the center-of-mass frame can be computed to be (with $s = 4\omega^2$)

$$\begin{aligned} \sigma(\gamma\gamma \to \widetilde{G}\widetilde{G}) = \frac{\kappa^4 m_{\widetilde{\gamma}}^4 s}{864\pi m_{3/2}^4} \Bigg\{ \frac{1}{2} + 3y - 12y^2 \left[1 - \frac{1}{4(1+y)} \right] - \\ 3\ln\left[1+\frac{1}{y}\right] \left[y^2 - 4y^3 \left(1 + \frac{1}{2(1+2y)} \right) \right] \Bigg\}, \end{aligned} \tag{68a}$$

$$\sigma(\gamma\gamma \to SS) = \sigma(\gamma\gamma \to PP) = \frac{5\kappa^4 m_{\widetilde{\gamma}}^4 s}{2304\pi m_{3/2}^4}, \tag{68b}$$

$$\sigma(\gamma\gamma \to SP) = \frac{13\kappa^4 m_{\widetilde{\gamma}}^4 s}{3456\pi m_{3/2}^4}. \tag{68c}$$

Figure 9

Plot of $\sigma(\gamma\gamma) \to$ "nothing$''$) vs the center-of-mass energy of the photon-photon system for three photino masses of interest. The gravitino mass has been taken to be 2.3 μeV.

Adding these, we obtain the total cross-section for the experimentally observable process $\gamma\gamma \to$ "nothing$''$ to be

$$\begin{aligned}\sigma_{\rm tot} =& \frac{\kappa^4 m_{\tilde{\gamma}}^4 s}{576\pi m_{3/2}^4}\{5 + 2y - 8y^2\left[1 - \frac{1}{4(1+y)}\right] \\ & - 2\ln\left[1 + \frac{1}{y}\right]\left[y^3 - 4y^4\left(1 + \frac{1}{4(1+2y)}\right)\right]\}.\end{aligned} \tag{6.9}$$

Plots of the cross-section against the c.m. energy $\sqrt{s}$ are shown in Fig. 9 for various values of the photino mass and a gravitino mass 2.3×10^{-6}eV which is the present experimental lower bound.[30] Note that the cross-section rises sharply and is quite large for $\sqrt{s} \gtrsim 100$GeV.

The main background to our signal comes from the reaction $\gamma\gamma \to \nu\bar{\nu}$ which goes in lowest order via a box diagram. Various combinations of internal lines for the box are possible. One can have a box with three charged leptons plus one W boson or with three W's plus one charged lepton, or with three charged Higgs scalars (in the

minimal supersymmetric theory) plus one charged lepton or with three charged leptons plus one charged scalar as well as their supersymmetric counterparts. Since we consider the external momenta to be comparable to the W-mass and assume that the relevant superparticle masses are in the same range, there is only one dimensional scale in the problem. Thus each box diagram contributes a cross-section $O(\alpha^2 G_F^2 s) \sim 10^{-2}$picobarn. With three species of neutrinos, there can at the very most be an enhancement factor of 3×8^2 yielding a total background cross-section in the picobarn range. As is clear from the graph, the major region of investigation is thus almost free from any competitive background.

6.3 *Gluon-gluon collisions*

In the previous section we showed how a study of the double gravitino process $\gamma + \gamma \to \widetilde{G} + \widetilde{G}$ could lead to a lower bound on the mass of the gravitino. The main advantage of that process is that it does not need to produce any heavy supersymmetric particles and therefore, can be applied at energies lower than a couple of hundreds of GeV — the expected mass scale of the matter-gauge sector superparticles. On the other hand, this particular experiment is hard, and even in the absence of any sizeable background, is difficult to perform since the parent e^+, e^- of the two photons have to be doubly tagged.

In this section we shall study a different process which can be studied at hadronic colliders. As these cross-sections are typically gravitational, the higher energies that these machines provide would be expected to give better limits than the leptonic data. On the other hand, because of the hadronic debris, it is not possible to use the the kind of tests proposed in the previous section in these machines.[35] Hence we study the monojet production in these theories. We note that at sufficiently high energies (above the mass

thresholds for all the known particles) the standard model predictions for monojet rates are extremely small. Furthermore, because of R-symmetry, in standard supersymmetric theories ordinary superparticles are produced in pairs. Hence except for jet merging and experimental cuts, no 'theoretical monojet' events can ever be achieved.

On the contrary, in theories involving a superlight gravitino, one can produce any of the ordinary supersymmetric particles in conjunction with the gravitino without violating R-parity. The interactions of the gravitino being extremely weak, this particle is not observable. Yet, for a gravitino as light as 2.3 μeV and for superparticle mass-splits larger than 80 GeV as reported by TEVATRON,[14] the effective coupling $\kappa\omega\Delta m_S/m_{3/2}$ is already larger than the strong coupling $\alpha_S \sim 0.1$ at these energies. Hence, any other supersymmetric particle that may be produced chooses to decay into the corresponding ordinary particle and a gravitino rather than decaying into other lighter superparticles as assumed in the globally supersymmetric theory. The net result of this is that in theories containing a superlight gravitino, the sole signal of supersymmetry will be a large monojet cross-section.

To illustrate this process let us consider proton-antiproton collisions at 1.8 TeV. The possible monojet processes include the scattering of two gluons into a gluino and a gravitino, the scattering of a quark-antiquark pair into a gluino and a gravitino, the scattering of a quark and a gluon into a squark and a gravitino or processes involving the scalars and pseudoscalars mentioned in the last section. In each of the above cases we can assume that the produced superparticle decays into the corresponding particle and a gravitino. As all these processes add up incoherently, an estimate of the order of magnitude of the predicted cross-sections can be made on the basis of any one of these cross-sections. The bound that this will put on the gravitino will be weaker than the exact bound that can be attained with a complete calculation.

To illustrate, let us concentrate on the process of gluon-gluon fusion leading to the production of a gluino and a gravitino. The matrix elements for this process can be evaluated on the lines of Chapter 3. In the limit that the gravitino mass is much smaller than the other energy scales in the problem, they turn out to be

$$|\mathcal{M}_{1\,1;\frac{1}{2}\,\frac{1}{2}}(\theta)| = |\mathcal{M}_{-1\,-1;-\frac{1}{2}\,-\frac{1}{2}}(\theta)| = 0, \tag{610a}$$

$$\begin{aligned}|\mathcal{M}_{1\,1;\frac{1}{2}\,-\frac{1}{2}}(\theta)| &= |\mathcal{M}_{-1\,-1;-\frac{1}{2}\,\frac{1}{2}}(\theta)| \\ &= 2\sqrt{\frac{2}{3}}\frac{(1-y)^{3/2}(1+y)\,f_{abc}g\kappa m_{\widetilde{g}}^2}{m_{3/2}}\frac{|\sin\theta|}{(1+y)^2-(1-y)^2\cos^2\theta},\end{aligned} \tag{610b}$$

$$|\mathcal{M}_{1\,1;-\frac{1}{2}\,\frac{1}{2}}(\theta)| = |\mathcal{M}_{-1\,-1;\frac{1}{2}\,-\frac{1}{2}}(\theta)| = 0, \tag{610c}$$

$$|\mathcal{M}_{1\,1;-\frac{1}{2}\,-\frac{1}{2}}(\theta)| = |\mathcal{M}_{-1\,-1;\frac{1}{2}\,\frac{1}{2}}(\theta)| = 0, \tag{610d}$$

$$\begin{aligned}|\mathcal{M}_{1\,-1;\frac{1}{2}\,\frac{1}{2}}(\theta)| &= |\mathcal{M}_{1\,-1;-\frac{1}{2}\,-\frac{1}{2}}(\pi-\theta)| = |\mathcal{M}_{-1\,1;\frac{1}{2}\,\frac{1}{2}}(\theta)| = |\mathcal{M}_{-1\,1;-\frac{1}{2}\,-\frac{1}{2}}(\pi-\theta)| \\ &= \sqrt{\frac{2}{3}}\frac{(1-y)^{3/2}f_{abc}g\kappa\omega m_{\widetilde{g}}}{m_{3/2}}\frac{\sin^2\theta}{(1+y)+(1-y)\cos\theta},\end{aligned} \tag{610e}$$

$$\begin{aligned}|\mathcal{M}_{1\,-1;\frac{1}{2}\,-\frac{1}{2}}(\theta)| &= |\mathcal{M}_{1\,-1;-\frac{1}{2}\,\frac{1}{2}}(\pi-\theta)| = |\mathcal{M}_{-1\,1;\frac{1}{2}\,-\frac{1}{2}}(\theta)| = |\mathcal{M}_{-1\,1;-\frac{1}{2}\,\frac{1}{2}}(\pi-\theta)| \\ &= \sqrt{\frac{1}{6}}\frac{(1-y)^{3/2}f_{abc}g\kappa m_{\widetilde{g}}^2}{m_{3/2}}\frac{|\sin\theta|(1-\cos\theta)}{(1+y)-(1-y)\cos\theta}.\end{aligned} \tag{610f}$$

The diffential cross-section for the process can then be obtained:

$$\frac{d\sigma}{d\Omega} = \frac{\overline{\sum|\mathcal{M}|^2}}{64\pi^2 s}, \tag{6.11}$$

where $\overline{\sum|\mathcal{M}|^2}$ refers to the sum of the matrix elements squared averaged over the initial spin and colour states. We fold this distribution with the proton structure functions[36] and select out monojet events in the rapidity range $|\eta| < 1.0$ and missing transverse energy $\not{E}_T > 40$ GeV using a typical Monte-Carlo algorithm. We thus obtain the cross-section for monojet production in proton-antiproton collisions due to a superlight gravitino. Experimentally, this cross-section is[14] known to be less than 0.14 nb at 90% confidence level. We can then plot an excluded region in the gluino gravitino mass plot

Figure 10

The excluded region in the gluino gravitino mass plot is shaded.

as shown in Fig. 10. We find that effectively, for a gluino of about a 100 GeV, gravitinos of masses less than 3.5 μ eV are ruled out.

6.4 *Discussion*

In this chapter, we have studied possible experimental limits on the ratio of masses of any gaugino and the gravitino. In particular, we have noted that photon-photon collision experiments at electron-positron colliders can find absolute lower limits on the ratio of the gravitino mass to the photino mass independent of the mass of the photino. This contrasts with the existing lower bounds on this ratio coming from the process considered by Fayet[30] which had to produce the photino in the final state and hence

had to assume a light photino. Our result is important because the photino is no more believed to be such a light particle.

In addition to the above, we have studied the possibility of putting limits on the gravitino mass from hadronic collisions. In this case, a coloured superparticle, for example a gluino, is necessary in the final state of the elementary subprocess. This makes the limit dependent on the mass of this final state particle. Nevertheless, because of the higher energies at which these reactions have already been studied and the more stringent limit on the gluino mass, the limits that we derive on the gravitino mass from this calculation turn out to be of the same order of magnitude as those obtained by Fayet earlier.

7. Conclusions

In this thesis, some consistency issues relating to the presence of widely different mass scales in a spontaneously broken N = 1 supergravity theory have been discussed. The requirement of such consistency all the way upto the Planck scale has been shown to yield new bounds on the gaugino gravitino mass ratio. Furthermore, in case these bounds are violated in nature, it has been shown how prominent experimental signatures will appear much below the energy scale where the theory becomes inconsistent.

Because of the presence of the dimensional coupling constant κ, such theories are non-renormalizable and make sense only as effective low energy theories to be treated perturbatively at the tree level. An important question concerns the energy domain of validity of this effective theory. The general expectation would be for such an effective field theory to be a valid description of phenomena upto the scale set by its dimensional coupling, *i.e.* the Planck mass $\kappa^{-1} \equiv M_{Pl}$. It is only around (and not far below) it that tree-level perturbation theory would be expected to break down, showing the failure of the effective theory.

A large amount of freedom exists in defining an effective theory such as this. One can actually adjust tree-level parameters to assign masses of widely different magnitudes to the various particles in the theory. This would seem to be perfectly compatible with the symmetry requirements of the effective theory. On the other hand, what such a procedure would do to the energy domain of validity of this theory would not be clear without a unitarity analysis. In this thesis we have carried out precisely such an analysis. We demonstrate that large dimensionless ratios of gaugino and gravitino masses imply large effective coupling constants in the theory which destroy the argument of the preceding paragraph and cause inconsistencies to develop at scales much lower than the Planck scale. Furthermore, these large couplings imply characteristic experimental

signatures in such theories drastically below energy scales where they become inconsistent. Experimental searches for these new signatures can therefore put limits on such large ratios of masses.

The main results obtained in this thesis are as follows:

1. As explained in Chapters 3 and 5, a large ratio of the gaugino mass to the gravitino mass results in the violation of tree-unitarity and the consequent failure of perturbation theory at a comparatively low scale. In fact, in the latter of those two chapters, it was derived that if supergravity are to satisfy the tree-level unitarity constraints all the way upto the Planck scale, then one must have

$$m_{\tilde{g}} m_{3/2}^{-1} < (288\pi + \frac{6}{5})^{1/2} \sim 30. \tag{7.1}$$

   The same constraint of validity of perturbation theory also gives the following mass bounds on the axion-like scalar and pseudoscalar 'super-Higgs' particles in the theory.

$$(m_S^2 + m_P^2)^{1/2} < [96\pi\kappa^{-1} m_{3/2}^2 m_{\tilde{g}}^{-1}]^{1/2}. \tag{7.2}$$

2. Irrespective of theoretical prejudice, there exist experimental means for checking whether such superlight gravitinos as well as scalars and pseudoscalars exist. In Chapter 6, two of these methods have been presented and discussed. We have found that for a gluino as heavy as 80 GeV, a gravitino lighter than a few μeV would already have been observed in TEVATRON experiments probing monojet signatures in gluon-gluon collisions. In addition, the possiblity of obtaining an experimental lower limit on the ratio of the gravitino mass to the photino mass has also been discussed. This process involves the production of two gravitinos in the final state of a photon-photon collison and hence is not limited in its scope by the photino mass.

At this point, it is interesting to speculate on the nature of a theory which violates the tree-unitarity constraint discussed above. First, we note that a simple modification of the theory cannot remove the tree-unitarity violation. The reason is that we have considered the *most general* N = 1 supergravity theory. The extended (N > 1) supergravities do not fit low energy phenomenology. Could there be new physics at the Planck length which would leave residual effects (*e.g.* through contact terms) at sub-Planckian energies affecting our unitarity analysis? One modification of the theory which can be used to eliminate the tree-unitarity constraint would be to invoke higher-derivative kinetic energy terms and their associated contact interactions which may arise from a non-pointlike nature of particles in the theory. This, for example, is the case with an underlying string theory. A different possiblity is to invoke composite models which treat both the gauge boson and the gaugino or the gravitino as composites. In such a case, the description in terms of true pointlike gauge bosons may fail long before tree-unitarity breaks down. These theories, however, still have to obey the experimental constraints described above.

On the other hand, it is notionally possible to have a theory in which supergravity really becomes strong at some high scale and tree-unitarity does get violated there. Such theories are, of course, quite unæsthetic. In fact, with our present day understanding, we have very good agreement between the expectations of particle physics and cosmology of the early universe. We find, for example, that the grand unification scale in supersymmetric particle physics (assuming the validity of a perturbative analysis) is around 10^{16} GeV, while inflationary models also need a phase transition around the same scale. In all these calculations supergravity has been assumed to be too weakly interacting to contribute. This would no longer be true if the gravitino is very light and the coincidence of scales mentioned above would then have to be explained as fortuitous.

Furthermore, being a gauge theory of a spacetime symmetry (namely, supersymmetry), if supergravity becomes strongly interacting, the nature of spacetime itself could become very different. For example, if the interaction strengths become large enough for gravitino condensates to form, they can introduce macroscopic torsion in the background spacetime posing a problem for the observed isotropy of the universe.

Finally, we can make one definitive statement on spontaneously broken $N = 1$ supergravity theories as a consequence of the work reported in this thesis. Either our tree unitarity constraints on the gaugino gravitino mass ratio must be obeyed in nature, or there would have to be radical new physics at the Planck scale, generating residual effects and experimental signatures at lower energies.

8. Appendix I: REDUCE 3 Program for $A + A \to \lambda + \tilde{G}$

```
IN UTILRED$

COMMENT DECLARE$
VECTOR E1,E2$
MASS P1=0,P2=0,P3=M,P4=MM,E1P=0,E2P=0,E1M=0,E2M=0$
MSHELL P1,P2,P3,P4,E1P,E2P,E1M,E2M$
MATRIX UBAR,V,E1DG,E2DG,SDG,TDG,UDG,P1DG,P2DG,P3DG,P4DG,MID$

COMMENT KINEMATICS AND FRAME CHOICE$
MASS E1P=0,E2P=0,E1M=0,E2M=0$                       % P AND M AS SUFFIX STAND
                                                    % FOR +VE AND -VE
MSHELL E1P,E2P,E1M,E2M$                             % "HELICITY".
MATRIX UBARP,UBARM,VP,VM,E1PDG,E1MDG,E2PDG,E2MDG,G0,G1,G2,G3$
P3 #= P4-P1-P2$                                     % THE MOMENTUM CONVENTION
                                                    % TAKES ONLY THE
                                                    % GRAVITINO(4)
                                                    % AS OUTGOING. HENCE GLUINO
                                                    % MOMENTUM (3) IS
                                                    % UNPHYSICAL
EPRIME #= W - (M**2-MM**2)/4/W$                     % W IS THE CM ENERGY OF
                                                    % ONE GLUON AND EPRIME
                                                    % IS THE CM ENERGY OF THE
                                                    % GRAVITINO.
K #= SQRT(2*W+M-MM)*SQRT(2*W-M-MM)*SQRT(2*W+M+MM)*SQRT(2*W-M+MM)/4/W$
                                                    % K IS THE CM MOMENTA OF
                                                    % THE FINAL STATE GLUINO
                                                    % AND NEGATIVE THAT OF THE
                                                    % GRAVITINO.
LET CS**2+SN**2=1$                                  % CS AND SN STAND FOR
                                                    % COS (THETA/2) AND
```

```
                                                        % SIN(THETA/2)
                                                        % WHERE THETA IS THE AN-
GLE
                                                        % BETWEEN THE EMRGENT
                                                        % GLUINO AND GLUON
                                                        % NUMBER 1$
   P1.P2 #= 2*W**2$
   S #= (P1+P2).(P1+P2)$
   T! #= (P4-P2).(P4-P2)$
   U #= (P4-P1).(P4-P1)$
   P1.P4 #= W*EPRIME-W*K*(CS**2-SN**2)$
   P2.P4 #= W*EPRIME+W*K*(CS**2-SN**2)$
   P1.E2P #= P1.E2M #= P2.E1P #= P2.E1M #= P1.E1P #= P1.E1M #= P2.E2P #=
           P2.E2M #= E1P.E2M #= E1M.E2P #= 0$
   P4.E1P #= P4.E2M #= -I*SQRT(2)*K*SN*CS$
   P4.E1M #= P4.E2P #= I*SQRT(2)*K*SN*CS$
   E1P.E1M #= E2P.E2M #= E1P.E2P #= E1M.E2M #= -1$
   EE#= 2*W-EPRIME$                                     % ENERGY OF THE GLUINO
   REMINSM #= SQRT(2*W-M-MM)*SQRT(2*W-M+MM)/2/SQRT(W)$
   REPLUSM #= SQRT(2*W+M-MM)*SQRT(2*W+M+MM)/2/SQRT(W)$
   REPMINMM #= SQRT(2*W+M-MM)*SQRT(2*W-M-MM)/2/SQRT(W)$
   REPPLUMM #= SQRT(2*W+M+MM)*SQRT(2*W-M+MM)/2/SQRT(W)$
   VP#=MAT( (-I*SN*REMINSM),(-CS*REMINSM),(I*SN*REPLUSM),(CS*REPLUSM))$
   VM#=MAT((I*CS*REMINSM),(-SN*REMINSM),(I*CS*REPLUSM),(-SN*REPLUSM))$
   UBARP #= MAT((SN*REPPLUMM,I*CS*REPPLUMM,-SN*REPMINMM,-I*CS*REPMINMM))$
   UBARM #= MAT((CS*REPPLUMM,-I*SN*REPPLUMM,CS*REPMINMM,-I*SN*REPMINMM))$
   VECTOR ZETAPLUS,ZETAMINUS,ZETAL,ZETA;
   LET ZETAPLUS.P1 = 1/SQRT(2)*(I*W)*(2*SN*CS),
       ZETAPLUS.P2 = - ZETAPLUS.P1,
       ZETAMINUS.P1 = - ZETAPLUS.P1,
       ZETAMINUS.P2 = ZETAPLUS.P1,
       ZETAL.P1 = 1/MM *(K*W -EPRIME*W*(CS**2-SN**2)),
       ZETAL.P2 = 1/MM *(K*W +EPRIME*W*(CS**2-SN**2)),
```

```
    ZETAPLUS.E1P = 1/2*(1+CS**2-SN**2),
    ZETAPLUS.E1M = 1/2*(1-CS**2+SN**2) ,
    ZETAPLUS.E2P = ZETAPLUS.E1M,
    ZETAPLUS.E2M = ZETAPLUS.E1P,
    ZETAMINUS.E1P = ZETAPLUS.E1M,
    ZETAMINUS.E2P = ZETAPLUS.E2M,
    ZETAMINUS.E1M = ZETAPLUS.E1P,
    ZETAMINUS.E2M = ZETAPLUS.E2P,
    ZETAL.E1P = 1/MM/SQRT(2)*(-I*EPRIME*2*SN*CS),
    ZETAL.E1M = -ZETAL.E1P,
    ZETAL.E2P = -ZETAL.E1P,
    ZETAL.E2M = ZETAL.E1P;
MID #= M*MAT((1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1))$ %M IS GLUINO MASS
G0 #= MAT((1,0,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1))$  %G0 TO G3 ARE GAMMA
G1 #= MAT((0,0,0,1),(0,0,1,0),(0,-1,0,0),(-1,0,0,0))$  %MATRICES
G2 #= MAT((0,0,0,-I),(0,0,I,0),(0,I,0,0),(-I,0,0,0))$
G3 #= MAT((0,0,1,0),(0,0,0,-1),(-1,0,0,0),(0,1,0,0))$
E1PDG #= E2MDG #= -1/SQRT(2)*(G1-I*G2)$                %VDG IS THE GENERAL
E2PDG #= E1MDG #= -1/SQRT(2)*(G1+I*G2)$                %SYMBOL FOR V.G,  I.E.
P1DG #= W*(G0+G3)$                                     %V0*G0-V1*G1-V2*G2
P2DG #= W*(G0-G3)$                                     %-V3*G3
P4DG #= EPRIME*G0+K*(2*SN*CS*G2+(CS**2-SN**2)*G3)$
P3DG #= P4DG-P1DG-P2DG$
SDG #= P1DG+P2DG$
TDG #= P4DG-P2DG$
UDG #= P4DG-P1DG$

WRITE " SYMBOLS DEFINED "$
%PAUSE$
COMMENT DEFINE MATRIX ELEMENT$
MATRIX SM$
ALGEBRAIC PROCEDURE DOIT(A,B,C,D)$
 BEGIN
```

```
  COMMENT A,B,C,D CAN BE P OR M SIGNIFYING +VE OR -VE HELICITIES FOR THE
          TWO GLUONS,THE GLUINO AND THE GRAVITINO RESPECTIVELY.A AND B CAN
          ALSO BE 0 TO REPLACE THE CORRESPONDING POLARIZATION BY THE MOMENTUM$
  IF A = P THEN <<E1 #= E1P$ E1DG #= E1PDG >> ELSE
        IF A = 0 THEN <<E1 #= P1$ E1DG #= P1DG >> ELSE
        IF A = M THEN <<E1 #= E1M$ E1DG #= E1MDG>>
         ELSE REDERR 'A! NOT! P!,M! OR! 0$
  IF B = P THEN <<E2 #= E2P$ E2DG #= E2PDG >> ELSE
        IF B = 0 THEN <<E2 #= P2$ E2DG #= P2DG >> ELSE
        IF B = M THEN <<E2 #= E2M$ E2DG #= E2MDG>>
         ELSE REDERR 'B! NOT! P!,M! OR! 0$
  V    #= IF C = P THEN VP ELSE IF C =M THEN VM
         ELSE REDERR 'C! NOT! P! OR! M$
IF D = 3 THEN << ZETA#= ZETAPLUS$ UBAR#= UBARP >>
ELSE IF D = 15 THEN << ZETA#=1/SQRT(3)*ZETAPLUS$ UBAR #= UBARM >>
ELSE IF D = 5 THEN << ZETA#=I*SQRT(2/3)*ZETAL$ UBAR #= UBARP >>
ELSE IF D =-15 THEN << ZETA#=-1/SQRT(3)*ZETAMINUS$ UBAR #= UBARP >>
ELSE IF D =-5 THEN << ZETA#=-I*SQRT(2/3)*ZETAL$ UBAR #= UBARM >>
ELSE IF D =-3 THEN << ZETA#= ZETAMINUS$ UBAR#=UBARM >>
ELSE REDERR 'D! INVALID $
SM#= -KAPPA*G*FABC*UBAR*((!%U#=(ZETA.P1*E1DG*(UDG+MID)*E2DG
    -ZETA.E1*P1DG*(UDG+MID)*E2DG)/(U-M**2))+(!%T#=-(ZETA.P2*E2DG*(TDG+MID)
    *E1DG
-ZETA.E2*P2DG*(TDG+MID)*E1DG)/(T! -M**2))+(!%S#=(ZETA.(P1+P2)*
(-E1DG*(2*P1+P2).E2
+E2DG*(2*P2+P1).E1+E1.E2*(P1DG-P2DG)+(E1.(P1+P2)*(2*P1+P2).E2
-E2.(P2+P1)*(2*P2+P1).E1
-E1.E2*(P1-P2).(P1+P2) )*SDG/S )+(+E1.ZETA*(2*P1+P2).E2-E2.ZETA*(2*P2+P1).E1
-E1.E2*(P1-P2).ZETA+(-(P1+P2).E1*(2*P1+P2).E2+(P1+P2).E2*(2*P2+P1).E1
+E1.E2*(P1+P2).(P1-P2))
       *(P1+P2).ZETA/S )*SDG)/S)+(!%C#=-(-ZETA.E2*E1DG +ZETA.E1*E2DG)))*V $
  RETURN SM(1,1)$
 END$
```

```
WRITE " MATRIX ELEMENT DEFINED "$

END$
```

9. Appendix II: REDUCE 3 Program for $A + A \to \tilde{G} + \tilde{G}$

```
OFF ECHO$
IN UTILRED$
COMMENT DECLARE$
VECTOR E1,E1P,E1M,
       E2,E2P,E2M,
       ZETA1,ZETA1P,ZETA1M,ZETA1L,
       ZETA2C,ZETA2CM,ZETA2CP,ZETA2CL$  % C STANDS FOR COMPLEX CONJUGATION.
MATRIX U2BAR,U2BARP,U2BARM,             % P AND M AS SUFFIX STAND FOR +VE AND
       V1,V1P,V1M,                      % -VE "HELICITIES" RESPECTIVELY. THE
       E1DG,E1PDG,E1MDG,                % UNMARKED ONES ARE THE GENERIC NAMES
       E2DG,E2PDG,E2MDG,                % FOR THOSE AND WILL BE EQUATED TO
       ZETA1DG,ZETA1PDG,ZETA1MDG,       % THESE BEFORE EVALUATION.
                        ZETA1LDG,
       ZETA2CDG,ZETA2CPDG,ZETA2CMDG,
                        ZETA2CLDG,      %   V1,U2 : GRAVITINO(3 & 4)
       P1DG,P2DG,P3DG,P4DG,MID,         %   E1,E2 : GAUGE BOSON (1 & 2)
       SDG,TDG,UDG,                     % DG IS THE GENERAL SYMBOL FOR DOTTING
       G0,G1,G2,G3,G5$                  % WITH GAMMA.

COMMENT KINEMATICS AND FRAME CHOICE$
MASS P1=0,P2=0,P3=MM,P4=MM,             % P1,P2,UNPHYSICAL P3 ARE INCOMING.
     E1P=0,E2P=0,E1M=0,E2M=0,           % P4 IS OUTGOING. MM IS GRAVITINO MASS.
     ZETA1P=0,ZETA2CP=0,ZETA1M=0,ZETA2CM=0$
MSHELL P1,P2,P3,P4,
       E1P,E2P,E1M,E2M,
       ZETA1P,ZETA2CP,ZETA1M,ZETA2CM$
P3 #= P4-P1-P2$
 % W IS THE CM ENERGY OF ONE GLUON
K #= SQRT(W-MM)*SQRT(W+MM)$             % K IS THE CM MOMENTUM OF GRAVITINO 3
LET CS**2+SN**2=1$        % CS AND SN STAND FOR COS (THETA/2) AND SIN(THETA/2)
```

```
                          % WHERE THETA IS THE ANGLE BETWEEN AN EMERGENT
                          % GRAVITINO 3 AND GLUON NUMBER 1$
P1.P2 #= 2*W**2$
S #= (P1+P2).(P1+P2)$
T! #= (P4-P2).(P4-P2)$
U #= (P4-P1).(P4-P1)$
P1.P4 #= W*W-W*K*(CS**2-SN**2)$
P2.P4 #= W*W+W*K*(CS**2-SN**2)$
P1.E2P #= P1.E2M #= P2.E2P #= P2.E2M #= 0$
P4.E2M #= -I*SQRT(2)*K*SN*CS$
P4.E2P #= I*SQRT(2)*K*SN*CS$
E2P.E2M #= -1$
REMINSM #= SQRT(W-MM)$
REPM #= SQRT(W+MM)$
V1P#=MAT( (-I*SN*REMINSM),(-CS*REMINSM),(I*SN*REPM),(CS*REPM))$
V1M#=MAT((I*CS*REMINSM),(-SN*REMINSM),(I*CS*REPM),(-SN*REPM))$
U2BARP #= MAT((SN*REPM,I*CS*REPM,-SN*REMINSM,-I*CS*REMINSM))$
U2BARM #= -MAT((CS*REPM,-I*SN*REPM,CS*REMINSM,-I*SN*REMINSM))$
LET ZETA2CM.P1 = 1/SQRT(2)*(I*W)*(2*SN*CS),
    ZETA2CM.P2 = - ZETA2CM.P1,
    ZETA2CP.P1 = - ZETA2CM.P1,
    ZETA2CP.P2 = ZETA2CM.P1,
    ZETA2CM.P4 =0,
    ZETA2CP.P4 =0,
    ZETA2CM.E2M =-1/2*(1-CS**2+SN**2),
    ZETA2CM.E2P =-1/2*(1+CS**2-SN**2),
    ZETA2CP.E2P = ZETA2CM.E2M,
    ZETA2CP.E2M = ZETA2CM.E2P,
    ZETA2CP.ZETA2CM = -1,
    EPS(E2M,E2P,P1,P2)=2*I*W*W,       % ORDER IMPORTANT!!!!!!!!
    ZETA1L.P1 = 1/MM * (K*W+W**2*(CS**2-SN**2)),
    ZETA1L.P2 = 1/MM * (K*W-W**2*(CS**2-SN**2)),
    ZETA1L.P4 = 1/MM * 2*K*W,
```

```
    ZETA1L.E2P = 1/MM * (-I*W*2*SN*CS)/SQRT 2,
    ZETA1L.E2M = 1/MM * I*W*2*SN*CS/SQRT 2,
    ZETA1L.ZETA2CP = 0,
    ZETA1L.ZETA2CM = 0,
    ZETA2CL.P1 = 1/MM * (K*W-W**2*(CS**2-SN**2)),
    ZETA2CL.P2 = 1/MM * (K*W+W**2*(CS**2-SN**2)),
    ZETA2CL.P4 = 0,
    ZETA2CL.E2P = 1/MM * (I*W*2*SN*CS)/SQRT 2,
    ZETA2CL.E2M = 1/MM * (-I*W*2*SN*CS)/SQRT 2,
    ZETA2CL.ZETA2CP = 0,
    ZETA2CL.ZETA2CM = 0,
    ZETA1L.ZETA2CL =  1/MM**2 * (K*K+W*W),
    ZETA1P = ZETA2CP,
    ZETA1M = ZETA2CM,
    E1M = E2P,
    E1P = E2M$
MID #= M*MAT((1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1))$  % MM IS GAUGINO MASS
G0 #= MAT((1,0,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1))$   % G0 TO G3 ARE GAMMA
G1 #= MAT((0,0,0,1),(0,0,1,0),(0,-1,0,0),(-1,0,0,0))$   % MATRICES. G5 IS THE
G2 #= MAT((0,0,0,-I),(0,0,I,0),(0,I,0,0),(-I,0,0,0))$   % GAMMA5 MATRIX.
G3 #= MAT((0,0,1,0),(0,0,0,-1),(-1,0,0,0),(0,1,0,0))$
G5 #= I*G0*G1*G2*G3$
E1PDG #= E2MDG #= -1/SQRT(2)*(G1-I*G2)$                % VDG IS THE GENERAL
E2PDG #= E1MDG #= -1/SQRT(2)*(G1+I*G2)$                % SYMBOL FOR V.G,  I.E.
P1DG #= W*(G0+G3)$                                     % V0*G0-V1*G1-V2*G2-
P2DG #= W*(G0-G3)$                                     % V3*G3
P4DG #= W*G0+K*(2*SN*CS*G2+(CS**2-SN**2)*G3)$
P3DG #= P4DG-P1DG-P2DG$
SDG #= P1DG+P2DG$
ZETA1PDG #= -(G1+I*(CS**2-SN**2)*G2-2*I*CS*SN*G3)/SQRT 2$
ZETA2CMDG #= -(G1-I*(CS**2-SN**2)*G2+2*I*CS*SN*G3)/SQRT 2$
ZETA1MDG #= ZETA2CMDG$
ZETA2CPDG #= ZETA1PDG$
```

```
ZETA1LDG #= (K*G0-2*W*CS*SN*G2-W*(CS**2-SN**2)*G3)/MM$
ZETA2CLDG #= (K*G0+2*W*CS*SN*G2+W*(CS**2-SN**2)*G3)/MM$
TDG #= P4DG-P2DG$
UDG #= P4DG-P1DG$

WRITE " SYMBOLS DEFINED "$
PAUSE$
COMMENT DEFINE MATRIX ELEMENT$
MATRIX SM,!%G,!%S,!%P,!%T,!%U$
ALGEBRAIC PROCEDURE DOIT(A,B,C,D)$
 BEGIN
  COMMENT A,B,C,D CAN BE P OR M SIGNIFYING +VE OR -VE HELICITIES FOR THE
          TWO GLUONS,THE GLUINO AND THE GRAVITINO RESPECTIVELY.A AND B CAN
          ALSO BE 0 TO REPLACE THE CORRESPONDING POLARIZATION BY THE MOMENTUM$
  IF A = P THEN <<E1 #= E1P$ E1DG #= E1PDG >> ELSE
        IF A = 0 THEN <<E1 #= P1$ E1DG #= P1DG >> ELSE
        IF A = M THEN <<E1 #= E1M$ E1DG #= E1MDG>>
         ELSE REDERR 'A! NOT! P!,M! OR! 0$
  IF B = P THEN <<E2 #= E2P$ E2DG #= E2PDG >> ELSE
        IF B = 0 THEN <<E2 #= P2$ E2DG #= P2DG >> ELSE
        IF B = M THEN <<E2 #= E2M$ E2DG #= E2MDG>>
         ELSE REDERR 'B! NOT! P!,M! OR! 0$
  IF C = P3B2 THEN <<V1 #=-V1P$ ZETA1DG #= ZETA1MDG$ ZETA1 #= ZETA1M>>
   ELSE IF C = P1T THEN <<V1 #= -V1M$ ZETA1DG #= ZETA1MDG/SQRT(3)$
                              ZETA1 #= ZETA1M/SQRT(3)>>
   ELSE IF C = P1L THEN <<V1 #= -V1P$ ZETA1DG #= ZETA1LDG*SQRT(2/3)*I$
                              ZETA1 #= ZETA1L*SQRT(2/3)*I>>
   ELSE IF C = M1L THEN <<V1 #= V1M$ ZETA1DG #= ZETA1LDG*SQRT(2/3)*I$
                              ZETA1 #= ZETA1L*SQRT(2/3)*I>>
   ELSE IF C = M1T THEN <<V1 #= V1P$ ZETA1DG #= ZETA1PDG/SQRT(3)$
                              ZETA1 #= ZETA1P/SQRT(3)>>
   ELSE IF C = M3B2 THEN <<V1 #= V1M$ ZETA1DG #= ZETA1PDG$ ZETA1 #= ZETA1P>>
         ELSE REDERR 'C! NOT! P!,M! OR! 0$
```

```
  IF D = P3B2 THEN <<U2BAR #= U2BARP$ ZETA2CDG#=ZETA2CPDG$ ZETA2C #= ZETA2CP>>
   ELSE IF D = P1T THEN <<U2BAR #= U2BARM$ ZETA2CDG #= ZETA2CPDG/SQRT(3)$
                              ZETA2C #= ZETA2CP/SQRT(3)>>
   ELSE IF D = P1L THEN <<U2BAR #= U2BARP$ ZETA2CDG #=-ZETA2CLDG*SQRT(2/3)*I$
                              ZETA2C #=-ZETA2CL*SQRT(2/3)*I>>
   ELSE IF D = M1L THEN <<U2BAR #=-U2BARM$ ZETA2CDG #=-ZETA2CLDG*SQRT(2/3)*I$
                              ZETA2C #=-ZETA2CL*SQRT(2/3)*I>>
   ELSE IF D = M1T THEN <<U2BAR #=-U2BARP$ ZETA2CDG #= ZETA2CMDG/SQRT(3)$
                              ZETA2C #= ZETA2CM/SQRT(3)>>
   ELSE IF D = M3B2 THEN <<U2BAR #=-U2BARM$ ZETA2CDG#=ZETA2CMDG$
                              ZETA2C #= ZETA2CM>>
         ELSE REDERR 'D! NOT! P!,M! OR! 0$
    SM#= -I*KAPPA**2*
              ((!%T #= U2BAR*((P2DG*E2DG-E2DG*P2DG)/4*ZETA2CDG*(TDG+MID)*
                      ZETA1DG*(P1DG*E1DG-E1DG*P1DG)/4)*V1/(T! -M**2))
            + (!%U #= U2BAR*((P1DG*E1DG-E1DG*P1DG)/4*ZETA2CDG*(UDG+MID)*
                      ZETA1DG*(P2DG*E2DG-E2DG*P2DG)/4)*V1/(U -M**2))
            + (!%G #= IF A=B THEN MAT((0))
                            ELSE U2BAR*E1DG*V1 * (+ZETA2C.E1 * P4.ZETA1
                                                  +ZETA1.E1 * P3.ZETA2C
                                                  -(P4+P3).E1 * ZETA2C.ZETA1
                                                                        /2) )
            + (!%S #= -2*M*(P1.P2*E1.E2-P1.E2*E1.P2)/2/(S-MS**2)*
                      ZETA2C.ZETA1 * U2BAR*V1)
            + (!%P #= I*M*EPS(P1,E1,P2,E2)/(S-MP**2)*ZETA2C.ZETA1 *
                      U2BAR*G5*V1)
                            )$
    RETURN SM(1,1)$
 END$
ALGEBRAIC PROCEDURE MATELEM(A,B,C,D)$
IF C=P1B2 THEN
      IF D=P1B2 THEN
        DOIT(A,B,P1L,P1L)+DOIT(A,B,P1L,P1T)+DOIT(A,B,P1T,P1L)+
```

```
        DOIT(A,B,P1T,P1T)
      ELSE IF D=M1B2 THEN
        DOIT(A,B,P1L,M1L)+DOIT(A,B,P1L,M1T)+DOIT(A,B,P1T,M1L)+
        DOIT(A,B,P1T,M1T)
      ELSE
        DOIT(A,B,P1L,D)+DOIT(A,B,P1T,D)
ELSE IF C=M1B2 THEN
      IF D=P1B2 THEN
        DOIT(A,B,M1L,P1L)+DOIT(A,B,M1L,P1T)+DOIT(A,B,M1T,P1L)+
        DOIT(A,B,M1T,M1T)
      ELSE IF D=M1B2 THEN
        DOIT(A,B,M1L,M1L)+DOIT(A,B,M1L,M1T)+DOIT(A,B,M1T,M1L)+
        DOIT(A,B,M1T,M1T)
      ELSE
        DOIT(A,B,M1L,D)+DOIT(A,B,M1T,D)
ELSE
      IF D=P1B2 THEN
        DOIT(A,B,C,P1L)+DOIT(A,B,C,P1T)
      ELSE IF D=M1B2 THEN
        DOIT(A,B,C,M1L)+DOIT(A,B,C,M1T)
      ELSE
        DOIT(A,B,C,D);

WRITE " MATRIX ELEMENT DEFINED "$

END$
```

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