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Matthew Pritsker

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Abstract

The growing share of financial assets that are held and managed by large institutional investors whose desired trades move asset prices is at odds with the traditional competitive assumption that investors are small and take prices as given. This paper relaxes the traditional price-taking assumption and instead presents a dynamic multiple asset model of imperfect competition in asset markets among large investors who differ in their risk aversion. The model is used to study asset price dynamics during an LTCM-like scenario in which market rumors of distressed asset sales are followed at a later date by the sales themselves. Using the model, it is shown that large investors front-run distressed sales; asset prices overshoot their long-run fundamentals; and asset pricing models experience temporary breakdown. During the period of model breakdown assets equilibrium returns are explained by the market portfolio and by transient liquidity factors.

Keywords: Strategic Investors, Contagion, Cournot Competition
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*Board of Governors of the Federal Reserve System. The views expressed in this paper are those of the author but not necessarily those of the Board of Governors of the Federal Reserve System, or other members of its staff. The author thanks conference participants and discussants at the Bank for International Settlements, and the Washington Area Finance Association; as well as seminar participants at the University of Houston, the University of Texas at Austin, and The Federal Reserve Board. Address correspondence to Matt Pritsker, The Federal Reserve Board, Mail Stop 91, Washington DC 20551. Matt may be reached by telephone at (202) 452-3534, or Fax: (202) 452-3819, or by email at mpritsker@frb.gov.

1 Introduction

An increasingly large share of financial assets are owned or managed by large institutional investors whose desired orderflow can be large enough to move prices – and who account for their price impact when trading [Chan and Lakonishok (1995)]. Because large investors cannot buy or sell all of the assets that they desire at prevailing prices, markets are not completely liquid from their perspective. In this paper, I theoretically study how the liquidity problems associated with large investors affect equilibrium asset prices, market liquidity, shock absorption, and the transmission of shocks across asset markets. These topics are especially relevant for understanding market function during times when or more large investors are financially distressed and have to quickly liquidate some or all of their positions to meet margin calls, or for other cash needs. The most recent well known stress liquidation occurred during the period surrounding the Fall of 1998 when rumors that the financially distressed hedge fund Long Term Capital Management (LTCM) would be forced to liquidate its positions, were followed sometime later by the liquidations themselves. During the period of LTCM's troubles, liquidity declined in a number of markets, and the performance of standard asset pricing models deteriorated. One form of this deterioration was the emergence of additional factors which helped to price assets during the times of the LTCM crisis.[1] In addition to the asset pricing anomalies, there were also widespread rumors that LTCM's trades were being front-run by other market participants. Empirical support that LTCM's trades may have been front-run is provided by Cai (2003).[2]

To analyze how large investors affect market functioning, I build a multi-asset, multi-participant, dynamic model of imperfect competition in asset markets. All participants in the model are risk averse, and fully rational. Using the model I characterize how large investors affect asset pricing relationships in general, and I study an LTCM-like scenario in which rumors about future distressed sales are followed sometime later by the sales themselves. The resulting pattern of asset prices and trades are broadly consistent with the stylized facts during the period of LTCM's troubles. The anticipation of future distressed sales is accompanied by front-running like behavior in which some investors sell ahead of the future sales. Additionally, prices overshoot their long-run fundamental values, and asset pricing models (in this model the CAPM) temporarily breakdown. During the period of the breakdown, asset returns satisfy a multi-factor model in which one factor is the market portfolio, and the other factors emerge as a result of how imperfect liquidity conditions affect the pattern of dynamic risk sharing in the aftermath of a large shock.

[1] For example, in the interest rate swaptions market, Longstaff, Santa Clara, and Schwartz (2000) found that an additional "event-related" pricing factor was required to explain the behavior of swaption prices during periods of severe market stress such as in the Fall of 1998.

[2] Part of LTCM's positions reportedly involved a short position in U.S. treasuries, and a long position in Danish mortgage backed securities and other high-yield fixed income instruments (Edwards, 1999). When LTCM was in financial distress, it presumably (their trades are private information) liquidated its positions by purchasing U.S. treasuries and selling high-yield debt. Cai's data can only be used to make inference on LTCM's trades in the Treasury bond futures markets. Her findings suggest that LTCM's purchases of U.S. Treasury futures were front-run.

Investors risk aversion is an important determinant of market liquidity and the pattern of equilibrium risk sharing. In a one period setting with perfect liquidity, investors risk aversions' measure the certainty equivalent wealth that they are willing to pay to eliminate risk from their portfolio. In a dynamic setting, the certainty equivalents translate into the liquidity costs that investors are willing to pay to eliminate or take on additional risks; and differences in investors risk aversion dictate how risks are shared among investors through time.

The importance of market illiquidity for asset pricing depends on investors demands to trade. During normal circumstances, if large investors asset holdings are nearly consistent with optimal risk sharing, and large shocks are rare, then investors have little need for trade, and liquidity considerations do not affect prices even though markets are illiquid. However, liquidity is priced following large shocks because market liquidity determines the resulting pattern of equilibrium risk-sharing. Because market liquidity is priced *following* large shocks, this mechanism for liquidity to affect asset returns is distinct from liquidity affecting asset returns through hedging against future shocks as in the ICAPM.

The analysis in this paper is related to the voluminous literature on market liquidity and to the literature on strategic trading. Asset market illiquidity is usually modeled as deriving from some combination of the following three sources: exogenous transaction costs, asymmetric information, or imperfect competition in asset markets.[3] This paper is most closely related to the literature on imperfect competition in asset markets when information is symmetric.[4] Lindenberg (1979) models the behavior of many large investors trading many assets in a single period mean-variance setting. Basak(1997) and Kihlstrom(2001) expand the analysis to allow for multiple time periods, but they only consider a setting in which there is a single large investor. The model presented here can be understood as an extension of Lindenberg, Basak, and Kihlstrom, which allows for multiple time periods, multiple risky assets, and multiple large investors who vary in their risk aversion. Although I did not do so, the model could also have been derived by extending the dynamic models of Urosevic (2002a) or Vayanos (2001) to allow for multiple assets and multiple large investors who vary in their risk aversion while removing the moral hazard elements of Urosevic's model, or by

[3]The literature is far too large to cite all the relevant papers. Illiquidity resulting from exogenous transaction costs is studied by Constantinides (1986), Heaton and Lucas (1996), Vayanos (1998), Vayanos and Vila (1999), Huang (1999), and Milne and Neave (2003). One type of transaction costs is search costs; this is studied by Duffie et. al. (2001). Illiquidity resulting from asymmetric information about asset payoffs is studied in many papers including Glosten and Milgrom (1985), Kyle (1985), Kyle (1989), Eisfeldt (2001). Asymmetric information about market participants asset holdings is studied by Cao and Lyons (1999), Vayanos (1999), and Vayanos (2001). Finally, illiquidity resulting from imperfect competition in asset markets is studied by Lindenberg(1979), Kyle (1985), Kyle (1989), Basak (1997), Cao and Lyons (1999), Vayanos (1999), DeMarzo and Urosevic (2000), Urosevic(2002a), Urosevic(2002b), Vayanos (2001), and Kihlstrom (2001). Although less common, illiquidity has also been modeled as resulting from Knightian uncertainty [Cherubini and Della Lunga (2001, forthcoming), Routledge and Zin (2001)]; or as the outcome of optimal security design [Boudoukh and Whitelaw (1993), DeMarzo and Duffie (1999)].

[4]One strand of this literature which is not pursued here follows Grossman and Miller (1988) and models liquidity as a function of the number of non-strategic market makers, which is itself endogenous. Fernando (2003) and Fernando and Herring (2003) make the number of market makers an endogenous function of the relative importance of idiosyncratic and systematic valuation shocks.

removing the noise traders and information asymmetry that are present in Vayanos' model.[5]

The distressed investor analysis is related to models in which strategic investors take advantage of the constraints faced by other "distressed" investors. For example, in a futures market corner and squeeze, distressed investors need to buy in order to cover their short positions, and strategic investors corner the market to drive up the prices paid by distressed investors [Cooper and Donaldson (1998), Chatterjea and Jarrow, (1998)]. This paper is more closely related to recent theoretical work on distressed sellers. Brunnermeier and Pedersen (2002), hereafter BP, show that investors who are forced to sell off their positions are vulnerable to a predatory trading strategy in which large non-distressed investors profit by selling ahead of, or at the same time as the distressed seller, and then purchase back assets after prices have fallen. Attari, Mello, and Ruckes (2002), hereafter AMR, model a situation in which a risk neutral arbitrageur who is also a large investor faces financing constraints. The constraints provide other investors with opportunities to drive prices against the arbitrageur. The distinguishing feature of AMR is their very careful analysis of the issues raised by financing constraints in a strategic setting.

The model presented here differs from both BP and AMR in two key modeling assumptions. First, all investors in their models are risk neutral, while all investors here are risk averse. Second, and more importantly, some of the investors in BP and AMR not fully rational. More specifically, both BP and AMR assume that there is a long-term investor with an exogenously given downward sloping demand curve that takes the other side of strategic investors trades. This is equivalent to assuming that there is a substantial basis for trade between the strategic investor and the long-term investor. This assumption is questionable because the long-term investor buys high before the distressed sales and then sell low afterwards. Presumably, the long-term investor would behave differently if aware of the distressed sales.

In contrast with BP and AMR, in my model, all investors are fully rational, risk averse, and fully aware of the distressed sales. Even in my model, when large investors differ in their risk aversion, it turns out that the distressed sales generate a basis for trade among the non-distressed investors—and front-running and predatory trading sometimes result. However, when all investors agree there are distressed sales and account for it in their choices, the basis for trade among other investors during or before the distressed sales is weak, and the amount of front-running and predatory trading that arise in equilibrium is very small relative to the volume of distressed sales. More importantly, the bulk of the distressed sellers losses are a result of how imperfect competition affects the prices he receives, and is not a result of front-running or predatory trading. In light of the differences between my results and those reported in BP, I interpret my results as applicable to situations where most

[5] Urosevic's (2002a) extends DeMarzo and Urosevic's (2000) dynamic model of a single large investor to a setting in which several large shareholders choose to trade the assets of one or more firms while simultaneously choosing how carefully to monitor the behavior of the firms' management. Urosevic (2002a) does not examine a setting with many large investors and many risky assets. He instead examines a setting with one large investor and many risky assets, or one risky asset and many large investors. He also only considers the case in which all large investors have the same risk aversion. Urosevic (2002b) empirically tests the theoretical model in Urosevic (2002a).

investors are aware of the distressed sales, and I interpret their results as applicable when some participants are aware of the distressed sales but many others are not.[6]

The results on distressed sales are also related to the sunshine trading literature which studied the relationship between preannouncement of trades and the price impact of trading. For example, Admati and Pfleiderer (1991) model a noisy competitive setting with asymmetric information; in this setting they show that uninformed traders can reduce their own price impact by preannouncing their trades and signalling that they are uninformed. In my model, the distressed sellers can be viewed as sunshine traders because their trades are announced ahead of time and everyone knows they do not have private information. However, my setting differs from that of Admati and Pfleiderer because the setting is strategic. This raises the question of whether distressed investors benefit from preannouncing their trades in a strategic setting. I analyze this question as part of the distressed investor analysis below.

The remainder of the paper consists of six sections. The next section presents the general model of large investors. Section 3 studies the implications of the model for asset pricing and shock transmission and absorption; section 4 examines how large investors affect equilibrium trades, asset prices, and market liquidity when there is a distressed seller; section 5 studies the relationship between liquidity and the distribution of risk-sharing capacity across large investors; a final section concludes.

2 The Model

In this section I construct a model of the dynamic interaction of large and small investors in financial markets. The analysis builds on Kihlstrom's (2001) model of a dynamic monopolist that sells a single risky asset over two time periods to a price-taking investor.[7] The model extends Kihlstrom's dynamic monopoly framework to a dynamic oligopoly setting that has multiple investors, assets, and time periods. The economy contains M infinitely-lived market participants, $m = 1, \ldots M$, who consume and trade N risky assets and a single risk-free asset over a large but finite number of time periods $t = 1, \ldots T - 1$. In periods T and after,

[6]The model presented here is very different from AMR, but is more related to BP. Other differences of this model from BP occur because BP require restrictive assumptions to generate an equilibrium with risk neutral large investors: Large investors are constrained in the size of the long or short positions that they can take in asset markets; and large investors face a complicated exogenous transaction costs technology which takes the form that when investors net purchases are above a certain threshold, then each investor experiences investor-specific transaction costs which depend on the marginal contribution of that investors trades to the distance from the threshold. By contrast, in my model with risk averse investors, there are no constraints on position size, nor are there exogenous assumptions about market liquidity nor about a transaction costs technology.

[7]Kihlstrom shows that traded financial assets are analogous to durable goods; and therefore as in Coase (1972) the monopolist's inability to commit from selling additional assets in period 2 erodes his monopoly power by lowering the price he can charge in period 1.

participants no longer trade risky assets, but they continue to receive dividends, and borrow and lend at the riskfree rate in order to finance their consumption.[8]

Participant 1 represents the net demands of a continuum of infinitesimal investors, indexed by s, that are uniformly distributed on the unit interval. Each small investors take asset prices and the state of the economy (including other investors trades) as given. In analogy with the literature on industrial organization, participant 1 is sometimes referred to as the competitive fringe. Participants 2 through M are large investors whose desired orderflow moves prices. Each participant m has CARA utility of per period consumption with coefficient of absolute risk aversion A_m:

$$U_m(C_m(t)) = -e^{-A_m C_m(t)}. \tag{1}$$

Participants choose their asset holdings to maximize their discounted expected future utility of consumption:

$$U_m(C_m(t), \ldots, C_m(\infty)) = \mathrm{E}_t \left\{ \sum_{i=0}^{\infty} \delta^i U_m(C_m(t+i)) \right\}. \tag{2}$$

Participants risk tolerances $1/A_m$ play an important role in the analysis. When markets are competitive, the risk premium for bearing market risk is inversely proportional to the sum total of investors risk tolerances, $\sum_{m=1}^{M} 1/A_m$. Define this sum as the economy's total risk bearing capacity; and define RBC_m, the risk bearing capacity of investor m, as investor m's share of the economy's total risk bearing capacity:

$$RBC_m = \frac{1/A_m}{\sum_{m=1}^{m}(1/A_m)}.$$

In a competitive economy, the share of risk bearing capacity held by each investor does not affect return dynamics. However, when markets are imperfectly competitive, whether the risk bearing capacity is concentrated among a small number of investors, or is more diffusely held is important. I will refer to the distribution of risk bearing capacity among investors as the economy's market structure. For the purposes of this paper, the market structure is exogenous. My preferred interpretation for the origins of the market structure is that all investors in the economy are small; and the market structure reflects how the small investors have chosen to have their assets managed. Those investors who manage their own portfolios are the competitive fringe and are collectively represented by investor 1. In the appendix I show that investor 1's risk tolerance is the sum total of the risk tolerances of all investors who manage their own portfolios. Investors that have similar risk preferences can economize on portfolio management costs by having their portfolios collectively managed by a financial intermediary such as an institutional investor.[9] Because each institutional

[8]The device of allowing for T trading periods, but having infinitely lived investors and assets was introduced in DeMarzo and Urosevic (2000). I use this device because it makes the model much more parsimonious. In earlier versions of this paper, investors and assets were both T-period lived.

[9]Other reasons why small investors might turn their portfolios over to institutional investors include better management of individual investors liquidity needs (Nanda and Singh, 1998), and reduction in the costs of implementing dynamic trading strategies (Mamaysky and Spiegel, 2002).

investor trades on behalf of a positive mass of small investors, the instituional investor is large in asset markets. In the appendix I show that if each large institutional investor makes optimal portfolio decisions on behalf of a mass μ_m of identical small investors that each have risk tolerance $1/A_{m,s}$, then the large investor behaves as if his mass is 1 and his risk tolerance is $1/A_m = \mu_m/A_{m,s}$. Therefore, each large investor's risk bearing capacity depends on the risk tolerance of the base of small investors that he represents, and on the number (measure) of investors who make up that base.[10]

2.1 The Assets

The economy contains N risky assets that are in fixed supply X. The risky assets pay perfectly liquid cash dividends that are distributed i.i.d. through time:[11]

$$D(t) \sim \text{ i.i.d. } \mathcal{N}(\bar{D}, \Omega) \tag{3}$$

Participant m's holdings of risky assets at the beginning of time period t is denoted $Q_m(t)$; the stacked $NM \times 1$ vector of all participants risky assets holdings is denoted $Q(t)$. Similarly, the changes in participant m's and all investors risky asset holdings during period t are denoted by $\Delta Q_m(t)$ and $\Delta Q(t)$ respectively.[12]

The economy also contains a single riskfree asset that is in perfectly elastic supply. Within each time period the holdings of riskfree assets are exactly equivalent to cash and to consumption goods, but between periods the assets in the grow at rate r. Investor m's holdings of riskfree assets at the beginning of period t is denoted by $q_m(t)$. During period t, $\Delta q_m(t)$ denotes the change in his holdings of the riskfree assets. Hence his end of period holdings are $q_m(t) + \Delta q_m(t)$, and his holdings at the beginning of the next period are:

$$q_m(t+1) = r[q_m(t) + \Delta q_m(t)]. \tag{4}$$

[10]If a large investor takes on independent and actuarily fair risk ϵ, and then spreads it among μ_m identical investors that each have risk aversion $A_{m,s}$, then the Arrow-Pratt certainty equivalent that each would require to take on his share of the risk is approximately $.5A_{m,s}\sigma_\epsilon^2/\mu_m^2$. Hence the total risk compensation required by the mass μ_m of investors is $.5A_{m,s}\sigma_\epsilon^2/\mu_m$. This shows absolute risk aversion of the large investor is equal to that of each small investor divided by μ_m; or equivalently that the large investors risk tolerance is equal to $\mu/A_{m,s}$.

[11]When dividends are i.i.d. asset prices are a deterministic function of time and investors asset holdings. In ongoing work, I instead allow dividends to follow a multivariate AR(1) process with correlated shocks. Prices are stochastic in the modified framework; nevertheless all results on equilibrium expected asset returns continue to hold. I have not yet considered if non-i.i.d. dividends alter the dynamics of the model when there is a distressed investor.

[12]Therefore

$$Q_m(t+1) = Q_m(t) + \Delta Q_m(t), \quad \text{and} \quad Q(t+1) = Q(t) + \Delta Q(t).$$

2.2 The Trading Process

At time 1, each investor m is endowed with risky and riskless asset holdings $Q_m(1)$ and $q_m(1)$ respectively. For each time $t < T$ each investor m enters the period with risky and riskless asset holdings $Q_m(t)$, and $q_m(t)$ respectively. Investors then receive dividends $Q_m(t)'D(t)$ on their beginning of period risky asset holdings, and then they trade and choose their consumption.

The process of trade is modeled as a dynamic Cournot-Stackelberg game of full information. In period $t < T$, the strategic environment is described by the state variable $(Q(t), t)$. Given the strategic environment, small investors form a demand schedule which describes the market clearing prices at which the competitive fringe is willing to absorb all possible quantities of large investors orderflow. Given this demand schedule, large investors play a standard Cournot game in which they choose their equilibrium trades to maximize their value functions subject to the budget constraint:

$$C_m(t) + \Delta Q_m(t)'P(t) + \Delta q_m(t) \leq Q_m(t)'D(t), \tag{5}$$

where $P(t)$ denotes the price of risky assets at time t. Unless stated otherwise there are no constraints on shortselling or borrowing.[13] After investors trade and consume, the period ends; and then the same process repeats itself in all future time periods $t < T$.

During time periods T and after, investors do not trade risky assets, but in each period they do make consumption (and hence savings) choices. The relevant budget constraint for periods T and after takes the form:

$$C_m(t) + \Delta q_m(t) \leq Q_m(t)'D(t). \tag{6}$$

2.3 Investors' Information

One goal in writing this paper was to model imperfect asset liquidity in a rational framework without asymmetric information. To rule out asymmetric information, I make the strong assumption that all market participants have exactly the same information set: that which is known by one participant is known by all. This means that all investors fully rational, know the structure of the model (investor's utility functions, distribution of asset payoffs, etc.), as well as investors asset holdings and trades at the beginning of each time period, and all of this is common knowledge.

[13]The budget constraint is standard: it requires that expenditure on period t consumption must be financed by period t dividend income, sales of risky assets, or by running down cash holdings.

2.4 Solving the Model

The complete solution for the model is provided in the appendix. The solution technique uses dynamic programming to solve for investors' value functions of entering period T with given holdings of risky and riskfree assets. Given the value functions at time T, investors' value functions in earlier periods are solved through backwards induction. To perform the induction, I conjecture that the relevant state variable for each investor's value function at each time period is the investor's own holdings of riskfree assets and $Q(t)$, the entire $NM \times 1$ vector of all investors' risky asset holdings. Given this state vector, there are four main steps in the induction. First, for a given state vector, and value function at time $t+1$, I solve for the competitive fringe's (investor 1) demand function to trade risky assets in period t. Inverting this demand function produces a schedule relating large investors' trades to time t equilibrium asset prices. Second, given this price schedule, I solve for large participants equilibrium trades when they take the price schedule and each others' trades as given. Third, given the equilibrium trades, I solve for the participants' optimal consumption choices, and then use the resulting choices to solve for each investors value function of entering period t for a given set of state variables. Given the value functions at time t, the same steps are repeated to solve for the value functions in all earlier periods.

The Price Schedule Faced by Large Investors

To illustrate the price schedule's derivation, imagine that investors enter time period t with risky asset holdings $Q(t)$ and then the large investors submit risky-asset orderflow $\Delta Q_m(t)$, $m = 2, \dots M$. Based on this orderflow, there exists a market clearing risky asset price $P(., t)$, for which the risky asset demand, $\Delta Q_s(t)$, of each infinitesimal investor s, $s \in [0, 1]$, solves the maximization problem:

$$\max_{C_s(t), \Delta Q_s(t), \Delta q_s(t)} -e^{-A_s C_s(t)} + \delta E_t\{V_s(r(q_s(t) + \Delta q_s(t)), Q_s(t) + \Delta Q_s(t); Q(t) + \Delta Q(t), t+1)\}, \tag{7}$$

subject to the budget constraint,

$$C_s(t) + \Delta Q_s(t)' P(., t) + \Delta q_s(t) \leq Q_s(t)' D(t),$$

where $V_s(., t+1)$ is investor s's value of entering period $t+1$. The first three arguments of the value function correspond to the investors time $t+1$ holdings of riskfree and risky assets, and to the economy's time $t+1$ state-vector of risky asset holdings.

For the price schedule $P(., t)$ to be market clearing, small investors demands must satisfy equation (7) and prices must be set so that the net orderflow of the small and large investors sums to 0.

$$\int_0^1 \Delta Q_s(t)\ ds + \sum_{m=2}^{M} \Delta Q_m(t) = 0$$

The price schedule must also be consistent with an additional internal consistency condition for small investors orderflow. Recall that small investors take the orderflow of other small investors as given and treat it as a state-variable. For small investors beliefs about the state variable to be internally consistent, $\Delta Q_1(t)$, their beliefs about the net trades of all small investors in equation (7), must be consistent with the optimal behavior of small investors conditional on their beliefs; i.e. internal consistency requires that[14]:

$$\Delta Q_1(t) = \int_0^1 \Delta Q_s(t) ds \tag{8}$$

For any given set of trades by the large investors, I solve for equilibrium prices which satisfy the market clearing and internal consistency conditions. Each such price $P(., t) = P(\Delta Q(t), Q(t), t)$ is one point on the price schedule which is faced by the large investors. The full price schedule is found by solving the above problem for all possible $Q(t)$ and all possible $\Delta Q(t)$. The resulting price schedule turns out to a linear function of the elements of $Q_m(t)$ and $\Delta Q_m(t)$, $m = 2, \ldots M$:[15]

$$P(., t) = \frac{1}{r}\left(\beta_0(t) - \beta_Q(t)Q(t) - \sum_{m=2}^{M} \beta_m(t)\Delta Q_m(t)\right), \tag{9}$$

where $(1/r)\beta_m(t)$ is the slope of the price schedule with respect to ΔQ_m, large investor $m's$ orderflow at time t. The coefficients $\beta_m(t)$ are formally derived in the appendix and not in the text.[16]

Large Investors Portfolio Problem

The large investors choose their optimal portfolios by solving a maximization problem which is similar to the one that the fringe faces in equation (7) with the difference that each large investor plays a Cournot game in which it solves its maximization problem while using the price schedule in equation (9) to explicitly account for the effect that its trades have on prices. The large investors orderflow in period t is a Stackelberg-Cournot-Nash equilibrium if the large investors take the price function as given, if each large investors' orderflow is optimal given the price function and given the orderflow of the other large investors, and if the total orderflow is market clearing. To solve for the equilibrium trades of the large investors, I first solve for each investors reaction function. Investor m's reaction function is an equation which specifies her optimal risky asset trades given the trades of other large investors. A set of large investors trades which simultaneously satisfies all of the reaction function equations is a Stackelberg-Cournot-Nash equilibrium.[17] As noted above, given large

[14] $\Delta Q_1(t)$ corresponds to the first N rows of the $Q(t) + \Delta Q(t)$ argument of the small investors value function in equation (7).

[15] In the appendix, see the derivation of equation (A11).

[16] In the appendix, I refer to the matrix vech$[\beta_2(t), \beta_3(t), \ldots, \beta_M(t)]$ as $\beta_{Q_B}(t)$.

[17] When the investors trades are chosen they take the price schedule as given; thus criteria 1 for a Stackelberg Cournot Nash equilibrium is satisfied. Any set of trades which jointly satisfies all reaction functions is

investors trades, it is possible to solve for optimal consumption and then to use the result to solve for investors value functions at time $t-1$.

The resulting value functions and equilibrium trades are subgame perfect because they are solved for by backwards induction. The backwards induction begins from period T, the first period in which investors cannot trade. Since there is no more trade in risky assets after period T, investors value functions for periods $t \geq T$ depend on the their own risky asset holdings, but not on the risky asset holdings of other investors. Moreover, because investors have CARA utility, and because dividends are normally distributed, each investor's value function at times $t \geq T$ is exponential linear quadratic in the investor's own asset holdings:

Proposition 1 *Let m index small or large investors in periods $t \geq T$. Then, for all investors m with CARA utility and risk aversion A_m, the value of entering period $t \geq T$ with riskfree asset holdings $q_m(t)$ and risky asset holdings $Q_m(t)$ is given by:*

$$V_m(Q_m(t), q_m, t) = -k_m(t) \exp^{-A_m(t)q_m(t) - A_m(t)Q_m(t)'\bar{V}(t) + .5A_m(t)^2 Q_m(t)'\Omega(t)Q_m(t)} \tag{10}$$

where,

$$\begin{aligned} k_m(T) &= \left(\frac{r}{r-1}\right) \times (\delta r)^{[1/(r-1)]} \\ A_m(t) &= A_m[1-(1/r)] \\ \bar{V}(t) &= \frac{\bar{D}}{[1-(1/r)]} \\ \Omega(t) &= \frac{\Omega}{[1-(1/r)]} \end{aligned}$$

Proof: See section B.2 of the appendix.

In earlier periods of the model, the investors' value functions are also exponential linear quadratic, but the value functions are much more complicated since the value function depend on future trading opportunities, which in turn depends on the allocation of risky asset holdings among all investors. In the appendix, I show that for periods $t < T$, investors value functions have the following functional forms:

Proposition 2 *Small investors value functions for entering period $t < T$ with asset holdings Q_s and q_s when the economy's vector of risky asset holdings at time t is Q is given by:*

optimal given the other investor's trades. Thus, criteria 2 is satisfied. Finally market clearing is guaranteed since the price function was constructed under the condition that the fringe absorbs the large investors net orderflow.

$$
\begin{aligned}
V_s(q_s, Q_s, Q, t) &= -K_s(t)\ F(Q,t)\ e^{-A_s(t)q_s(t)-A_s(t)Q_s'(\bar{D}+P(Q,t))+.5A_s(t)^2Q_s'\Omega(t)Q_s}, \\
where\ F(Q,t) &= e^{-Q'\bar{v}_s(t)-\frac{1}{2}Q'\theta_s(t)Q} \\
P(Q,t) &= \tfrac{1}{r}(\alpha(t)-\Gamma(t)Q).
\end{aligned} \tag{11}
$$

where $P(Q,t)$ is the equilibrium price for risky assets that is realized when investors enter period t when the state-vector of risky asset holdings is given by Q.

For large investors m, $m = 2, \ldots M$, the value function for entering period $t < T$ when the state variable is Q and m's holdings of riskfree assets are q_m is given by:

$$
V_m(q_m, Q, t) = -K_m(t)e^{-A_m(t)q_m(t)-A_m(t)Q_m'\bar{v}_m(t)+.5A_m(t)^2Q'\theta_m(t)Q}. \tag{12}
$$

Investors value functions are a high dimensional function of the state variables, which in this case are $NM+1$ dimensional. Typically, a dynamic model with a high dimensional state space would be difficult to solve unless there are simplifying assumptions. In this case, the simplifying assumptions are that investors maximize discounted expected time-separable CARA utility of consumption, and assets' dividends are normally distributed, and i.i.d. through time. Because of these assumptions, the value functions have a simple exponential linear quadratic form in Q and q_m. Additionally, the only dynamic state variable that the parameters of the value functions depend upon is time. The time-varying parameters are the solution to a system of nonlinear Riccati difference equations. Because of the simplicity of numerically solving the Riccati equations, it is possible to solve for the behavior of asset prices and trades in the dynamic model even when the number of investors, assets, and time periods is large. The main properties of the model for asset returns and trades are reviewed in the next section.

3 Properties of the General Model

To analyze the effects that large investors have on financial market equilibrium, it is useful to first study market equilibrium for a competitive benchmark economy which contains the same market participants and assets, but where are all participants take prices as given.

3.1 Competitive Benchmark

The properties of the competitive economy are formally derived in section B.9 of the appendix. The properties of the competitive economy are presented in the following proposition:

Proposition 3 *When all participants in model described in section 2 take asset prices as given, then assets expected excess returns over the risk free rate satisfy the Capital Asset Pricing Model and are given by the equation:*

$$P(t+1) + \bar{D} - rP(t) = \lambda_x \Omega X \tag{13}$$

where λ_x, the price of market risk, is given by:

$$\lambda_x = \frac{1 - (1/r)}{\sum_{m=1}^{M} 1/A_m}. \tag{14}$$

Additionally, risky asset prices are constant for all times t, and equal to:

$$P(t) = \frac{\bar{D}}{r-1} - \frac{\Omega X}{r \sum_{m=1}^{M} (1/A_m)}. \tag{15}$$

The vector of investors optimal risky asset holdings is also constant through time and denoted by Q^W. The risky asset holdings of investor m are denoted by Q_m^W and given by:

$$Q_m^W = \frac{(1/A_m)X}{\sum_{m=1}^{M} (1/A_m)}. \tag{16}$$

Proof: See section B.9 of the appendix.

The results in proposition 3 are well known and are a special case of the analysis in Stapleton and Subrahmanyam (1978). Note that in the current setting, when market are competitive, they are effectively complete and hence equilibrium asset holdings are pareto optimal, and risk sharing is efficient. When risk sharing is efficient the percentage of risky assets that each investor owns is equal to his risk bearing capacity; and the equilibrium asset prices and expected returns only depend on the risk bearing capacity of the economy, and not on the market structure as measured by the distribution of risk bearing capacity across investors.

3.2 Imperfect Competition and Asset Pricing

Given the features of the competitive benchmark model, I now turn to the properties of the imperfect competition model. Because the utility functions and assets in both economies are the same, the imperfect competition model inherits many of the the properties of the perfect competition benchmark model, as detailed in the next proposition:

Proposition 4 *When asset markets are imperfectly competitive as specified in section 2 of the text, then if market participants initial asset holdings are Q^W, then investors will hold Q^W forever, and asset prices and expected returns will be the same as when there is perfect competition.*

Proof: When investors risky asset holdings are Q^W, then investors asset holdings are pareto optimal in all time periods. Hence there is no basis for trade among the investors and their asset holdings will remain at Q^W. Because Q^W is the vector of asset holdings from a competitive equilibrium, the resulting prices and expected returns which support Q^W are the same as in the competitive equilibrium. □

The practical consequence of proposition 4 is that it establishes that the imperfectly competitive model nests CAPM pricing in the special case that asset holdings are pareto optimal. If investors initial asset holdings are not Q^W, then there is a basis for trade among the investors. When there is a basis for trade, the appendix shows that the equilibrium path of trades and prices is a deterministic function of time and the holdings of risky assets. The deterministic dynamics follow from the CARA utility assumption, and the assumptions which guarantee that the investment opportunity set does not change through time.

The main consequences of the model for risky asset pricing appear when investors initial asset holdings are not pareto optimal. In such circumstances, investors trade until they reach a pareto optimal allocation of risky assets; but the process for reaching that allocation depends on market liquidity. Large investors can reduce their liquidity costs by trading at a slower rate and breaking up their trades through time. The speed with which large investors trade toward a pareto optimal allocation is also influenced by their risk aversions. Investors that have high coefficients of absolute risk aversion are expected to be willing to pay a higher liquidity cost in order to quickly eliminate undesirable risk from their portfolio. This reasoning suggests that when markets are illiquid, the allocation of risky assets holdings among investors who differ in their risk aversion will influence asset prices because it will influence how risks are shared among investors. This intuition is confirmed below:

Proposition 5 *When investors asset holdings are not Pareto Optimal, equilibrium expected asset returns satisfy a linear factor model in which one factor is the market portfolio, and the other factors correspond to the deviation of large investors asset holdings from pareto optimal asset holdings.*

$$P(t+1) + \bar{D} - rP(t) = \lambda_X \Omega X + \sum_{j=2}^{M} \lambda(m,t)\Omega(Q_m(t) - Q_m^W) \tag{17}$$

Proof: See section B.4 of the appendix.

Proposition 5 shows that when markets are illiquid 1-period returns have a factor-like structure in which the market portfolio and deviation of large investors asset holdings from their pareto optimal asset holdings are priced factors. The corresponding risk prices $\lambda_m(t)$ differ by large investors risk aversion because investors with different risk aversion trade back towards a pareto optimum at different rates. Whenever two or more large investors have the same risk aversion, their $\lambda_m(t)$ will be identical. In the special case where all large investors have the same risk aversion, because their $\lambda_m(t)$ are identical, two-factor pricing will result.[18]

The factor model representation of asset returns shows that additional factors appear that capture the effect of illiquidity on expected returns. Asset prices have a similar representation. The proof of the representation relies on the fact that investors risky asset holdings converge to Q^W as $T \to \infty$ (which implies risky asset prices converge to their competitive equilibrium value P^W). Although I do not yet have a formal proof of this convergence, it appears to be true in simulations.[19] Assuming this convergence takes place, then risky asset prices when there is imperfect competition are similar to risky asset prices with perfect competition but contain additional premia for suboptimal risk sharing due to illiquidity as shown in the following proposition.

Proposition 6 *Let $P(Q(t), t, T)$ denote the equilibrium risky asset price at time t when risky asset holdings are $Q(t)$, and when there are $T-1$ periods in which the risky asset is traded. Then, in the imperfect competition model of section 2, if $\lim_{T\to\infty} \lim_{t\to T-1} Q(t) = Q^W$, then,*

$$\lim_{T\to\infty} P(Q(t), t, T) = \frac{\bar{D}}{r-1} - \frac{\Omega X}{r\sum_{m=1}^{M}(1/A_m)} + \sum_{i=0}^{\infty}\sum_{m=2}^{M}\left(\frac{1}{r}\right)^{i+1}\gamma_m(t+i)(Q_m(t+i) - Q_m^W). \tag{18}$$

Proof: The proof is straightforward by solving equation (17) forward for $P(Q(t), t, T)$. See the appendix for details. □

Propositions 5 and 6 are related to the empirical asset pricing literature that treats returns on portfolios of assets as factors—the Fama-French 3-factor model being one prominent example. An issue within that literature is identifying why the returns on the portfolios are priced. Proposition 5 points towards one explanation: the priced portfolios proxy for the magnitude of imperfect risk sharing when asset markets are illiquid. It is the imperfect risk sharing that is being priced.

Because risky asset holdings do adjust back to Q^W, the identified risk factors are transitory; and their importance depends on their persistence. One measure of the persistence is the effect that a deviation from pareto optimal risk holdings today has on future expected one-period excess returns. Because asset prices and trades in the model are deterministic, at any time t, assets τ period ahead one-period excess returns can be expressed as a function of deviations from pareto optimal asset holdings at time t as shown in the following formula:[20]

$$P(t+\tau+1) + \bar{D} - rP(t+\tau) = \lambda_X \Omega X + \sum_{m=2}^{M}\lambda_m(t,\tau)\Omega(Q_m(t) - Q_m^W), \tag{19}$$

[18] The factors are the market portfolio and $\sum_{j=2}^{M}(Q_m(t) - Q_m^W)$.

[19] Urosevic (2002a) has a proof for convergence when all large investors have the same risk aversion. To be the best of my knowledge he does not have a proof for the more general case.

[20] For a derivation, see corollary 5 in the appendix.

where $\lambda_m(t, \tau)$ are τ period ahead risk factor prices. When these factor prices remain large for high τ, then the corresponding risk factor has a substantial effect on returns at a τ period horizon.

Simulation Analysis

To illustrate the importance that deviations from optimal risk sharing could have on asset returns, I solved the model for risk prices and for forward risk prices when there are 6 investors who strongly vary in their risk aversion. The large investors risk aversion increases with investor number so that investor 2 is the least risk averse and investor 6 is the most risk averse. Additional details on this example are provided in section C of the appendix. To solve for the risk prices, I solved the model while allowing for 2000 trading periods. The length of each trading period is assumed equal to 1 day.

The risk prices from equation (17) measure the contribution that a deviation from optimal risk sharing by investor m has on the next period's asset returns. The risk prices vary through time (Figure 1). The price of market risk is positive, as expected. The other prices of risk are negative, which reflects the fact that if a large investor begins with more risky asset than is pareto optimal, he will sell the assets back slowly to avoid liquidity costs, and hence the marginal investor (the competitive fringe) bears less of the risk of holding those assets. Consistent with my reasoning on the rates at which investors trade out of positions, the market prices of risk are the more negative for large investors with the greater risk tolerances because they prefer to sell more slowly to avoid liquidity costs. One puzzling aspect of the risk prices is that they increase through time and eventually converge as the number of remaining trading periods becomes small. The convergence occurs because the risk prices measure how changes in current risky asset allocations affect the competitive fringe's future holdings. When there are very few, or in the limit no trading periods remaining, then the risk prices become the same because there is no time over which the deviations from pareto optimal asset holdings can be reversed. Similarly, the risk prices grow because investors require more risk compensation when deviations from pareto optimal holdings cannot be reversed.

To study the persistence of deviations from optimal risk sharing, I solved for forward risk prices as of period $1,000$. The analysis confirms the intuition that the risk aversion of the investors whose positions deviate from pareto optimal asset holdings strongly influences the pattern of asset prices. If the positions of investors 3-6 deviate the effect on one-period expected returns is significant for a period of at most 5 days (Figure 2, Panel A). By contrast, a deviation by investor 2, the most risk tolerant large investor, has an effect on equilibrium excess returns that persists for more than 500 days.

This analysis implies that the number of factors that appear to explain asset returns depend on whether investors asset holdings were for some reason shocked away from a pareto optimum. If asset holdings were not shocked away, then assets are only priced with a single risk factor, the market portfolio. If instead all large investors asset holdings are shocked far enough away from pareto optimal holdings, then for short periods of time asset returns will appear to have a multi-factor representation; in the case of the example there will be 5 factors. Because these factors vanish at different rates, eventually the assets in the example will be priced as if there are 2-factors, and then after a very long time vanish back to 1 factor pricing.

3.3 Contagion

In addition to using this analysis to better understand how asset pricing factors can appear to be important at some times but not at others, the analysis also can be used to analyze contagion. For example, a standard way of thinking about shock propagation in the contagion literature is that some shocks originate within one asset class or market and then spread to others. To model this phenomenon, suppose asset holdings are pareto optimal and then a shock occurs for asset j which causes the reallocation of investors holdings of that asset only. Any such reallocation can be expressed as a linear combination of basis shocks that cause each large investors asset holdings to differ from his pareto optimal asset holdings. Because the trajectory of asset holdings and asset price dynamics are linear in the basis shocks (see the appendix), the propagation of shocks can be studied through basis shocks alone. Additionally, because of linearity, it is sufficient to study the first derivative of assets' excess return response to the basis shocks. Other assets price responses to basis shocks for asset j are presented below:

Proposition 7 *When investors asset holdings are initially pareto optimal, a perturbation which drives large investor m's holdings of asset j away from a pareto optimum in period t while other large investors holdings are held fixed alters time $t+1$ asset returns by*

$$\frac{\partial(P(t+1)+\bar{D}-rP(t))}{\partial(Q_m(j,t)-Q_m(j)^W)} = \lambda_{m,t}\Omega[.,j], \tag{20}$$

and alters time $t+\tau$ asset returns by

$$\frac{\partial P(t+\tau+1)+\bar{D}-rP(t+\tau)}{\partial(Q_m(j,t)-Q_m(j)^W)} = \lambda_m(t,\tau)\Omega[.,j], \tag{21}$$

where $Q_m(j,t)$ is the j'th element of $Q_m(t)$ and $\Omega[.,j]$ is the j'th column of Ω.

Proof: Straightforward by differentiating equations (17) and (19).

Because the effects of the shocks are proportional to the j'th column of Ω, the model suggests that the price effects of the shocks have a "beta" representation. This is confirmed below:

Corollary 1 *Let* $\beta[i,j] = \Omega[i,j]/\Omega[j,j]$*, and let* $\Delta Z[i,t+\tau]$ *and* $\Delta Z[j,t+\tau]$ *denote the expected excess return effects for assets* i *and* j *that result from a shock or shocks which disrupt pareto optimal risk sharing for asset* j*. Then, for all* $0 < \tau < T - \tau$*,*

$$\Delta Z[i,t+\tau] = \beta[i,j]\Delta Z[j,t+\tau] \tag{22}$$

Proof: Straightforward from proposition 7.

Proposition 7 shows that the effect of basis shocks to investor m in market j has a proportional effect on all other markets in future time periods. Therefore when a shock to an investor has a long-lived effect on asset returns in market j it also has a long-lived effect for all other assets. Because the persistence of shocks varies by investor, the model shows that one reason that contagious shocks vary in the persistence of their effects on prices and returns is differences in the risk preferences of the investors who were initially affected by the shock.

The model makes strong predictions about the price and trading effects of shocks to investors holdings of asset j. Corollary 1 shows that such shocks only affect the returns of assets with correlated dividends. The trading volume implications are even stronger, and in fact unrealistic. In the appendix I show that the equilibrium trades for each asset only depend on whether holdings for that asset are pareto optimal, irrespective of the holdings of other assets. This implies that a deviation from pareto optimal asset holdings for asset j has no effect on trading volume for asset i.

Before closing, it is important to note that the contagion analysis focuses on restrictive shocks that take the form of reshuffling investors' asset holdings. If instead a cashflow shock occurs that requires an investor to optimally sell assets in order to raise cash, then price and trade effects will occur for many assets irrespective of dividend correlations. I hope to study the effects of cashflow shocks in future work. The next section focuses on a simplified variant of cashflow shocks in which a distressed investor sells his holdings in a single risky asset. The contagious effects of such shocks are straightforward from corollary 1, therefore the next section only focuses on the single asset case.

4 Distressed Asset Sales

This section studies the dynamics of asset prices and trades when market participants anticipate future distressed sales by one participant. For simplicity, there is a single risky asset and the setup is otherwise essentially the same as in the simulation analysis in section 3.2 except that there is a 7'th large investor in the model that serves as a device to introduce distressed sales; his risk tolerance initially plays no role in the analysis, and is not explicitly specified.

Investors in the model trade for 2200 periods starting from time -200. All investors initial asset holdings are pareto optimal. At time 0, all investors learn a rumor that investor 7 will be forced to sell out his position at a constant rate during time periods 390 to 400, after which he exits the market forever. For simplicity, it is assumed that investor 7's position is locked in and that he cannot trade until the specified time periods. For simplicity it is also assumed that the rumor is true.[21] The scenario of an investor who is forced to sell assets over several periods when there is imperfect competition among the buyers is essentially the scenario that is examined in Brunnermeier and Pedersen (2003), although as noted earlier there are important differences between the models. The distressed asset sales can alternatively be interpreted as the announcement of a seasoned equity issuance followed later by issuance at the time of the distressed sales. The resulting price dynamics are qualitatively similar to Newman and Rierson's (2003) study of how bond spreads respond to additional issuance in the European Telecommunications market.[22]

For purposes of contrast, I first analyze the effects of distressed sales when the asset markets are perfectly competitive.

4.1 Competitive Benchmark

In the competitive economy, investors 1-6 are all price-takers. Because they also have CARA utility, there is a representative investor, and equilibrium asset prices are set so that the representative investor is willing to hold his risky asset endowment. Before time 0 and during and after time 400, investors believe they will hold their risk asset allocations forever. In the appendix, I show that equilibrium prices under these circumstances are set so that:

$$P(t) = \frac{\bar{D}}{r-1} - \frac{\Omega X}{r\sum_{m=1}^{7} 1/A_m}, \qquad t \leq 0; \tag{23}$$

and after investor 7 sells all of his assets and exits the market, prices are set so that:

$$P(t) = \frac{\bar{D}}{r-1} - \frac{\Omega X}{r\sum_{m=1}^{6} 1/A_m}, \qquad t \geq 400 < T, \tag{24}$$

where X is the outstanding supply of risky assets.

The only part of the price path which remains undetermined is the period between times 0 and time 399. During this period investors 1-6 are the only investors actively trading in financial markets. Therefore, asset prices must provide them sufficient excess return per period to compensate them for the risky assets they hold during the period, subject to the

[21] The long span of time between when investors learn of the rumor and when the sales occur is made for the purposes of clarity of the figures. Price and trade dynamics are qualitatively similar when the time span is shortened.

[22] Newman and Rierson focus on new issuance of telecom bonds that are close substitutes to outstanding telecom bonds. The price dynamics reported below are similar to those for the outstanding telecom bonds when a new issue occurs. This is because a new issue is a sufficiently close substitute to existing bonds, that it is almost like a seasoned issue of the existing bonds from the standpoint of an investor.

boundary condition that prices at time 400 are given by the expression in equation (24). This requires that asset prices between periods t and t+1 must satisfy the difference equation:

$$P(t+1) + \bar{D} - rP(t) = \frac{[1 - (1/r)]\Omega X[1:6,t]}{\sum_{m=1}^{6}(1/A_m)}, \qquad 0 \leq t \leq 399, \tag{25}$$

subject to the boundary condition (24), where $X[1:6,t]$ is the net risky asset holdings that investors 1-6 hold from time t to time $t+1$. These risky asset holdings change through time because, for comparability with BP, the distressed seller is assumed to sell over multiple time periods.

Solving for the competitive price path shows that the effect of the rumor causes competitive asset prices to jump down to a new trajectory, and then to slowly decline until they stabilize at a lower level (Figure 3). The rate of price decline is so slow that it is barely discernible in the figure. Because markets are perfectly liquid in the competitive equilibrium, the future distressed sales do not create a basis for trade among investors 1-6, therefore trading only occurs during those times when the distressed seller is selling.

4.2 Imperfect Competition

When asset market are imperfectly competitive the pattern of asset prices and trades depart from those observed in the competitive case. Whether there is perfect or imperfect competition prices drop sharply on the rumor of future distressed sales, but prices drop by more when there is imperfect competition, overshooting the competitive price path. Prices then drift down until the asset sales have been completed, and then slowly recover back towards the competitive price path (Figure 3). Intuition for price overshooting comes from static models of imperfect competition in which nonprice taking agents reduce the price they pay per unit by cutting back slightly on purchases. In a dynamic setting, investor 2, the most risk tolerant large investor, contributes to lower asset prices by choosing to purchase tiny amounts of distressed sales during the time they occur. Instead other investors absorb the sales initially, and then sell to investor 2 over a long period of time (Figure 4).

The implications of the distressed sales for asset pricing come from from proposition 5. Prior to the distressed sales, and after the distressed sales occur, asset prices satisfy the CAPM when markets are perfectly competitive; additionally, during the period of distressed sales returns satisfy a CAPM pricing relationship provided that asset supply is measured as the float of outstanding assets held by investors 1-6. When markets are instead imperfectly competitive, asset prices satisfy the CAPM before the distressed sales; but after the rumor, liquidity considerations affect the price dynamics and CAPM-pricing breaks down. Between the time of the rumor and the completion of the distressed sales, it is not clear whether any factor model describes returns. After the distressed sales are completed, proposition 5, shows that returns are temporarily priced by a multi-factor model. After investors trade back to efficient asset holdings, assets will again be priced by the CAPM.

When there is imperfect competition, distressed asset sales generate far more trading volume than would occur with perfect competition. When markets are perfectly competitive, and hence perfectly liquid, the distressed asset sales in each period are distributed immediately to those investors who will ultimately bear the risk. Hence, the number of time periods of trade is equal to the number of periods of distressed sales. When markets are instead imperfectly competitive, the amount of trading volume at the time of the distressed sales is the same as in the competitive case, but unlike the competitive case, because risk-sharing is not efficient, there are thousands of periods of retrade long after the distressed sales are over (Figure 4). Thus, imperfect competition generates significant trading volume and persistence in trading volume.

Part of the trading activity that is generated by the distressed sales occurs before the sales, and appears to be front-running. If front-running is defined as an investor choosing to sell ahead of anticipated future asset sales, then investors 1, 2, and 4-6 front-run the distressed sales once they learn about them at time 0 (Figure 4). In addition to front-running, Brunnermeier and Pedersen discuss predatory trading, which occurs when large investors sell at the same time as distressed investors. During the time over which the distressed sales occur in figure 4, all other investors purchase from the distressed seller, so predatory trading is not present although front-running is present. It is important to distinguish between the definition of front-running that is used in this paper from the more standard understanding of front-running. Traditionally, front-running occurs when a dealer or the dealers friends uses the dealer's private knowledge of his own customers sales to sell ahead of that orderflow to uninformed buyers at high prices. By doing so, the front-runners not only take advantage of uninformed buyers, but they also depress the prices that the distressed seller receives. Front-running is unusual in this paper because there are no investors that have an informational advantage. Instead, all investors are perfectly informed about the distressed sales before they occur. I suspect that because investors are perfectly informed about the distressed sales, the distressed sales do not create much of a basis for trade among the nondistressed investors. Hence, the front-running trades are small relative to the amount of distressed sales. If instead the front runners traded with uninformed buyers, then I expect the basis for trade to be much greater because the front runners know they are getting a good deal, and the uninformed buyers believe they are getting a good deal as well.

Because the front-running sales have a nontraditional interpretation, it is not ex ante clear whether the front-running is simply the result of investors sharing the risks associated with the future distressed sales (since the sharing of risks must involve one party selling to another) or if the trades have the effect of reducing the revenue received by the distressed seller. To examine this question, I solved for the price received by the distressed seller as a function of the amount of warning time that other investors have that distressed sales are coming. Presumably, if the trades are designed to take advantage of the distressed seller, then more warning time should translate into lower prices when the distressed sales occurs. For simplicity I assume that there is only one distressed trade at time 400. I found that advance knowledge of the distressed sales does lead to a lower price for the distressed seller. If the advance knowledge is interpreted as preannouncement of the distressed sales, as in the Sunshine Trading literature, then this analysis shows that when trading is strategic, and all investors are rational, then preannouncement can harm the party that announces the trades even if those trades are known to be uninformed.

Although prior knowledge of the distressed sales is detrimental to the interests of the distressed seller in the example, the losses due to preannouncement are slight in the example considered here. The advance warning of 400 time periods costs the distressed seller only 4 cents per share; and shorter warning times would cost the distressed seller even less. Therefore, the costs imposed on the distressed seller from front-running are very small when all investors have symmetric information about the distressed sales. The more important cost to the distressed seller occurs because he has to sell into an illiquid market. The costs of doing so are measured by the difference in the prices paid by the distressed seller versus the prices he would have paid if markets were competitive. Examination of figure 3 shows that this cost is about 85 cents per share.

4.3 Endowment Shocks

One of the unrealistic aspects of the distressed sales analysis is that the behavior of the distressed seller is mechanical; the distressed seller does not choose an optimal trading strategy given the liquidity that other investors in the market make available. A simple method to examine distressed sales when distressed investors follow an optimal trading strategy is to consider how one of the investors whose behavior is formally modelled respond to an endowment shock that increases their holdings of risky assets. Following such a shock, the investor will follow an optimal trading strategy in transferring part of their position to the other investors in the model. For purposes of comparison, the size of the endowment shock is equal to the quantity of distressed sales used to generate figures 3 and 4; and the shocks are applied to large investor 2, the most risk tolerant large investor, and investor 6, the least risk tolerant large investor. A comparison of price and trade dynamics reveals very significant differences based on the identity of the investor who receives the shock. When investor 6 is shocked, because he is very risk averse, he very rapidly sells off the risky assets to other investors, eliminating most of his holdings within 4 trading periods (Figure 5, panel F). When instead investor 2 is shocked, because he is less risk averse than investor 6, he instead sells slowly through time to minimize the price impact of his trades (Figure 7, panel B). Because investor 6 sells rapidly, prices overshoot their competitive equilibrium values (not shown). By contrast, because investor 2 only transfers risk to other investors slowly through time, prices *undershoot* the competitive price path (Figure 6).

Whether or not front-running like behavior is present also depends on the identity of the investor who receives the endowment shock. Because investors have no notice of the endowment shocks, any sales that are made by large investors occur simultaneously with the distressed sales of the investor who receives the shock. When other investors sell at the same time as the distressed seller, then Brunnermeier and Pedersen define the trading behavior as predatory. Based on their definition, predatory trading occurs when investor 2 is shocked, but not when investor 6 is shocked. I don't have strong intuition for why this behavior occurs at some times but not at others. However, differences in the price dynamics help to provide some intuition. When there is an endowment shock to investor 2, the risky assets equilibrium expected excess returns are actually lower than they would be with perfect competition, and the one period expected excess returns decline over time. When excess returns decline over time, one would expect price-taking investors to slowly sell through time, as they do, and it not unreasonable to believe that other investors might as well. By contrast, when investor 6 is shocked, risky assets expected excess returns are higher than those along the competitive price path, which should encourage other investors to buy risky assets instead of selling them.[23]

4.4 Optimal Liquidations

An alternative method for modelling distressed sales is to assume that investor 7 (the distressed seller) follows an optimal trading strategy when liquidating his position. To do so, I assume that all investors at time 0 learn that one large investor must liquidate his risky asset holdings by time 400 and then exit the market forever. For purposes of comparability with other distressed investor analysis in the model, the distressed investor has CARA utility of consumption and is infinitely lived. Additionally, the distressed investors risk tolerance is chosen so that other investors asset holdings at time 0 are pareto optimal and equal to that in the earlier distressed investor analysis, and so that the distressed investors risky asset endowment is equal to the total distressed sales in my earlier analysis.

The distressed investors' problem of maximizing utility subject to liquidating his position by a specified date is the subject of the optimal liquidation literature [Bertsimas and Lo (1998), Almgren and Chriss (2000), Subramanian and Jarrow (2001), Subramanian (2000), and others]. To the best of my knowledge, all papers in that literature model liquidations in a partial equilibrium setting in which the price impact function for the liquidating investors trades is exogenously specified. The unique aspect of my analysis of optimal liquidations is that the price impact function is endogenously determined by the behavior of other investors in the market, and their knowledge that optimal liquidations will take place. I do not yet have theorems about how the model behaves when there are optimal distressed sales; all results are based on simulations.

In the baseline distressed investor example, when the distressed investor can optimize his sales, the resulting path of prices and trades is qualitatively different than when his sales are concentrated at time 400. The main difference is that prices overshoot the competitive price-path by less when distressed sales are optimal than when they are concentrated (Figure 8). Furthermore, the minimal front-running that was present when distressed trades were concentrated vanishes when distressed sales are optimal; instead, the distressed seller begins selling immediately upon news that he must liquidate and all of the other investors purchase assets immediately and along nearly the entire price path (Figure 10).[24]

[23] Graphs of the excess returns are not included in the paper.

[24] For reasons that are not yet clear, the distressed seller purchases a small quantity of risky asset just before liquidating his position. I am investigating this further.

The trades of the liquidating investor are of independent interest. When there is a distressed investor and markets are illiquid, intuition suggests that the distressed investor should break up his sales to minimize price impact, but doesn't specify how to do so; for example should liquidations occur through a large number of small trades, or a small number of large trades? In the example, the optimal liquidation strategy involves selling large amounts of risky assets at times 0 and 400 and dribbling out small additional amounts of risky asset during the periods in between (Figure 9). Additional simulations reveal that the optimal liquidation strategy depends on market structure as measured by how the risk bearing capacity of the other investors in the economy is distributed among the remaining investors. Intuitively, when the economy's risk bearing capacity is evenly dispersed among many large investors the market will be more competitive than when risk bearing capacity is concentrated among a smaller number of investors. I find that the concentration of risk bearing capacity among the other investors affects the path of optimal liquiations. At the extreme in which markets are very competitive, the liquidating investor holds all of his assets until time 400 and then liquidates it all at once (not shown). In the other extreme, when markets are highly uncompetitive, the liquidating investor sells a lot at time 0, and the rest at time 400, but sells little or nothing in between (not shown). Much more analysis can be done on optimal liquidations in this framework; I hope to purse this topic in future work.

To close this section, I would like to compare the results in this section with those in Brunnermeier and Pedersen (BP). Recall that BP's model generates both front-running as well as predatory trading, where predatory trading involves some large investors selling at the same time as the distressed investor. My principal criticisms of the BP model is that it contains an investor whose demand curve for risky assets is not derived from first principles; and the model makes strong assumptions about constraints on long and short positions, on the transaction costs technology, and on the feasible trading strategies of the liquidating investor. These assumptions raise the question of whether their results on front-running and predatory trading are consistent with a framework in which all investors optimize. The basic answer provided by the analysis here is a qualified yes. The main qualification is that front-running and predatory trading have small effects on trades and prices when other participants are aware of the distressed sales and optimize to account for the them. I suspect the reason that front-running and predatory trading have small price effects is because there is little basis for investors to engage in these activities when all investors anticipate the future distressed sales.

Based on the above, my interpretation of the results in BP is that their model describes equilibrium when knowledge of the distressed sales is not widely known, and in particular is not known by the buyers who take the other side of the front-runners' trades. My results also suggest that the primary reason that distressed sellers do not want knowledge of their distress to be widely known is not because they fear front-running or predatory trading, but rather that they fear losing the opportunity to sell at a high price to buyers who are unaware of their distress.

When all investors are aware of the distressed seller, the losses suffered by the distressed seller are related to the severity of the imperfect competition in asset markets, which in turn depends on market structure, as measured by the distribution of the market's risk bearing capacity across the investors in the market, and on market liquidity. These subjects are addressed in the next section.

5 Liquidity and Market Structure

In section 3.2, I established that when markets are imperfectly competitive, the distribution of risk tolerances across investors influences equilibrium asset prices and risk sharing. In this section, I study how the distribution of risk bearing capacity across investors influences market liquidity. Recall that in this CARA economy, the eonomy's total risk bearing capacity is the sum total of investors risk tolerances, $\sum_{m=1}^{M}(1/A_m)$. I interpret the distribution of risk tolerances across investors as the economy's market structure. This notion of market structure captures differences in the ideal size of investors positions as measured by their asset holdings when risk sharing is pareto optimal. For example, if an investor has x percent of the economy's risk bearing capacity, then it should optimally hold x percent of the economy's risk. If large investors are interpreted as agents for smaller investors, then the large investors risk bearing capacity is representative of the number of investors who placed their assets with the large investor; i.e. abstracting from differences in small investors risk preferences a large mutual fund family is likely to have more risk bearing capacity than a small mutual fund.

There are many possible ways to define and measure liquidity. I consider three simple measures. The first measure is the price discount that a distressed seller pays by selling into an imperfectly competitive (illiquid) market instead of into a perfectly competitive liquid market. The second is the liquidity that large investors receive from the competitive fringe of small investors. This second measure is the slope of the price schedule that large investors face when choosing their risky asset holdings, as given in equation (9). It is in some sense comparable to the λ coefficient measure of liquidity in Kyle (1985), or to to the bid-ask spread. A deficiency of slope as a liquidity measure is that is only based on the fringe's willingness to absorb orderflow from a large investor: it holds the behavior of other large investors as fixed. The third measure remedies this deficiency by measuring the immediate price impact that occurs when one large investor has to (for unmodeled exogenous reasons) immediately sell 1 share of his asset holdings to the competitive fringe and other large investors. This price impact measure is most sensible when investors have no other basis for trade that might confound the price impact computations. Therefore, I compute the third measure when investors asset holdings are pareto optimal.[25]

To study the importance of market structure in determining the liquidity received by a distressed seller, I normalized the economy's annualized total risk bearing capacity to 1, and then studied the effects that different market structures have on the prices received by the distressed seller in the example used in section 4, but for simplicity I assumed that all asset

[25]See section B.8 of the appendix for details.

sales occur at time 400.[26] In all simulations, the competitive fringe was allocated 10 percent of the risk bearing capacity and the 5 large investors received 90 percent. The risk bearing capacity of the large investors was chosen to be geometically declining in investor number, i.e. $(1/A_{m+1}) = \rho(1/A_m)$ where $\rho \in (0, 1)$ subject to the constraint that large investors total risk bearing capacity was 90 percent. This parameterization admits all of the large investors risk bearing capacity held by one large investor and the risk bearing capacity shared equally by large investors as limiting cases.

In the above parameterization of market structure, the fraction of risk bearing capacity held by the most risk tolerant large investor is a sufficient statistic for the market structure. As expected differences in market structure have a very significant effect on the liquidity cost to the distressed investor. When 60% of the large investors risk bearing capacity is held by investor 2, as it is in the analysis in figure 3, then the liquidity cost to the the distressed investor is about 90 cents per share. If instead 90% of the large investors risk bearing capacity were held by investor 2, then the liquidity cost would have been dramatically higher, at \$7.75 per share, which is 150% greater than the total price impact of the distressed trades in a competitive environment. On the other hand, if investor 2 only held 50% of the large investors risk bearing capacity, then the liquidity cost is reduced to 0.23 cents, and grows smaller yet as the distribution of risk bearing capacity grows more even. The risk bearing capacity of the competitive fringe also affects the liquidity received by the distressed investor. If the risk bearing capacity of the fringe expands, and the total capacity of the large investors shrinks, then the market becomes more competitive and more liquid. The risk-free rate of interest also affects equilibrium liquidity. Simulations show that when the riskfree real rate is fixed at 4 percent instead of 2 percent, then the price discount for illiquidity shrinks. Intuition for this result comes from noting that large investors excercise their market power over a distressed seller by holding back on their purchases at the time of the distressed sale so that they can later acquire the assets from other investors at depressed prices. At higher levels of interest rates, the revenues from following this strategy are discounted at a greater rate, which reduces the benefit from delaying purchases, and erodes large investors market power, causing the price discount to shrink.

The second and third liquidity measures are investor-specific: they measure the amount of liquidity that is available to each large investor. A surprising features of the model is that liquidity varies by large investor. For both liquidity measures, in almost all time periods of the model, liquidity is monotone decreasing in investors risk tolerance: the more risk tolerant a large investor the less liquidity that he receives (Figure 11 for the measure 2, not shown for measure 3). Investor 2, the most risk tolerant of the large investors receives far less liquidity than the other investors. Partial intuition for the slope result comes from noting that the slope at a point in time measures the change in the competitive fringe's valuation of the risky asset when some of that asset is transferred from the fringe to a large investor. In a dynamic setting, the change in the fringe's marginal valuation depends on the future trading strategy of the large investor who buys the asset. If the large investor is very risk averse,

[26] The normalization of aggregate risk tolerances to 1 scales equilibrium risk premia, which are homogeneous of degree -1 in investors risk tolerances, but has no effect on equilibrium trades which are homogeneous of degree 0 in investors risk tolerances.

one might expect the large investor to quickly sell the asset back to the fringe again. In this case a point in time purchase by a very risk tolerant investor has only a small effect on the fringe's marginal valuation. Conversely, if the investor is very risk tolerant, then he will hold the asset for a longer time, which means a sale to a very risk tolerant large investor has a greater effect on the fringe's marginal valuation.

Partial intuition for results using the third measure comes from noting that the price impact is measured when risk is transferred from one investor to the other investors in the model. When the most risk tolerant large investor transfers risk to other less risk tolerant large investors, one should expect a bigger price move because the less risk tolerant investors should require a bigger premium to temporarily take on the additional risk.

It must be emphasized that the intuition for the results on liquidity for individual investors is incomplete because the monotone relationship between risk tolerances and large investors' liquidity breaks down after period 1600. I suspect that the breakdown occurs because the opportunities to avoid liquidity costs by breaking up trades diminish when the last period of trade is approached.

I have also investigated the relationship between liquidity for large investors and uncertainty over the quantity of distressed sales (see section B.6 of the appendix). In the proof of proposition 10, I show that the slope measure of liquidity for each large investor has form:

$$\beta_m(t) = f_m(A_1, \ldots, A_m, t) \times \Omega,$$

where $f_m(.)$ is a scalar function of market structure and time. The expression for slope suggests that price impact per share depends on both market structure and uncertainty about dividends. This suggests that if there is price uncertainty caused by uncertainty about the quantity of distressed sales, then it should increase slope and thus reduce liquidity per share. Simulations of the model show that price uncertainty does reduce liquidity for a period of time before the distressed sales; however, for some model parameterizations I find that uncertainty about future distressed sales can reduce the slope coefficient in earlier periods of the model; i.e. future uncertainty can sometimes increase current liquidity. This result is contrary to intuition and suggests that the relationship between liquidity and market conditions is an extremely complicated topic, even within this stylized model.

To conclude this section, I want to emphasize that the rich patterns of liquidity that are generated in the model are especially intriguing because they are not dependent on differences in investors information, or on noise trading. Instead, they are based upon strategic considerations and differences in investors risk preferences. I would also like to emphasize that fully characterizing and comprehending the patterns of liquidity in the model remain a subject of ongoing research.

6 Summary and Conclusions

A large share of financial assets are held and traded by large institutional investors whose desired orderflow is large enough that they account for the price impact of the orderflow when trading. This paper studied the effects of large investors on asset pricing, contagion, liquidity, and market function during times of asset sales by distressed market participants. Our primary results show that when large investors asset holdings are pareto efficient, then asset prices and asset returns are the same as they would be in a competitive economy, but that if there are shocks that push investors holdings away from a pareto optimum, then competitive asset pricing relationships break down and additional risk factors that proxy for imperfect risk sharing due to illiquidity are needed to price all assets. An important feature of the model is that the length of time that pricing models break down is related to the identities of the investors that are shocked, as well as market structure where market structure is measured by cross-sectional differences in large investors capacity for bearing risk. Although market structure is an important determinant of shock absorption and transmission, this paper has only examined a very limited set of market structures. In future work, I plan to examine the implications of more complicated market structures for asset returns.

Appendix

A Notation

There are M investors and N risky assets. $Q(t)$ denote the $NM \times 1$ vector of all investors risky asset holdings at time t where

$$Q(t) = \begin{pmatrix} Q_1(t) \\ \vdots \\ Q_M(t) \end{pmatrix}.$$

$Q_1(t)$ represents the net asset holdings of a continuum of infinitesimal small investors indexed by s:

$$Q_1(t) = \int_0^1 Q_s(t)\mu(s)ds.$$

The small investors are often collectively referred to as the competitive fringe. $Q_2(t)$ through $Q_M(t)$ denotes the net asset holdings of large investors, and is denoted by the $N \times (M-1)$ vector $Q_B(t)$. The change investors risky asset holdings from the beginning of time period t to the beginning of time period $t+1$ is denoted by the $NM \times 1$ vector $\Delta Q(t)$. Similarly, $\Delta Q_1(t)$ and $\Delta Q_B(t)$ denote the change in the competitive fringe's asset holdings, and the change in the asset holdings of the large investors.

The algebra which follows requires many summations. Rather than write summations explicitly, I use the matrix $S = \iota_M' \otimes I_N$ to perform summations where ι_M is an M by 1 vector of ones, and I_N is the $N \times N$ identity matrix.[27] In some cases, the matrix S may have different dimensions to conform to the vector whose elements are being added. In all such cases, S will always have N rows. The matrix S_i is used for selecting submatrices of a larger matrix. S_i has form

$$S_i = \iota_{i,M}' \otimes I_N,$$

where $\iota_{i,M}$ is an M vector has a 1 in its i'th element, and has zeros elsewhere.[28] As above S_i will sometimes have different dimensions to conform with the matrices being summed, but it will always have N rows.

In the rest of the exposition, I will occasionally suppress time subscripts to save space.

[27] For example, $SQ(t) = \sum_{m=1}^{M} Q_m(t)$

[28] To illustrate the use of the selection matrix, $Q_m(t) = S_m Q(t)$.

B Proofs

Let $P(Q,t)$ denote the equilibrium function for prices during time t when investors enter period t with risky asset vector Q. The distribution of dividends in time period t is known for all t and has form:

$$D(t) \sim \mathcal{N}(\bar{D}, \Omega) \tag{A1}$$

To establish backwards induction in the proofs, I will use the more general form:

$$D(t) \sim \mathcal{N}[\bar{D}, \Omega(t)] \tag{A2}$$

where, $\Omega(t)$ is a deterministic function of time. In periods $t = 1, \ldots T-1$, $\Omega(t) = \Omega$. In period T and after, there is no more trade in risky assets, and $\Omega(T) \neq \Omega$. However, this distinction is immaterial since the induction only requires that the value function be expressible in the form in the proposition below. The proposition is given in section 2.4 of the text, and is restated below:

Proposition 2: *Small investors value functions for entering period t with asset holdings Q_s and q_s when the economy's vector of risky asset holdings at time t is Q is given by:*

$$\begin{aligned} V_s(q_s, Q_s, Q, t) &= -K_s(t)\ F(Q,t)\ e^{-A_s(t)q_s(t) - A_s(t)Q_s'(\bar{D}+P(Q,t)) + .5A_s(t)^2 Q_s'\Omega(t)Q_s}, \\ \text{where } F(Q,t) &= e^{-Q'\bar{v}_1(t) - \frac{1}{2}Q'\theta_1(t)Q} \\ P(Q,t) &= \tfrac{1}{r}(\alpha(t) - \Gamma(t)Q). \end{aligned} \tag{11}$$

Additionally, large investor m's value function for entering period t when the state variable is Q and his holdings of riskfree assets are q_m is given by:

$$V_m(q_m, Q, t) = -K_m(t) e^{-A_m(t)q_m(t) - A_m(t)Q_m'\bar{v}_m(t) + .5A_m(t)^2 Q'\theta_m(t)Q}. \tag{12}$$

Proof: The proof is by induction. Part I of the proof establishes that if the value function has this form at time t, then it has the same form at time $t-1$. Part II of the proof establishes the result for time T, the first period in which trade cannot occur.

B.1 Part I:

Suppose the form of the value function is correct for time t. Then, to establish the form of the value function at time $t-1$, I first solve for the competitive fringe's demand curve for absorbing the net order flow of the large investors. I then solve the large investors and competitive fringe's equilibrium portfolio and consumption choices, and then solve for the value function at time $t-1$.

The competitive fringe's demand curve

The competitive fringe represents a continuum of infinitesimal investors that are distributed uniformly on the unit interval with total measure 1, i.e. $\mu(s) = 1$ for $s \in [0, 1]$. At time $t-1$, each participant s of the competitive fringe solves:

$$\max_{\substack{C_s(t-1),\\ \Delta Q_s(t-1),\\ \Delta q_s(t-1)}} \quad -e^{-A_s C_s(t-1)} - \delta V_s(q_s(t), Q_s(t), Q(t), t) \tag{A3}$$

subject to the budget constraint:

$$C_s(t-1) + \Delta Q_s' P(., t-1) + \Delta q_s(t-1) = Q_s(t-1)' D(t-1) \tag{A4}$$

where,

$$\begin{aligned} q_s(t) &= r(q_s(t-1) + \Delta q_s(t-1)), & \text{(A5)}\\ Q_s(t) &= Q_s(t-1) + \Delta Q_s(t-1), & \text{(A6)}\\ Q(t) &= Q(t-1) + \Delta Q(t-1), & \text{(A7)} \end{aligned}$$

and $P(., t-1)$ represents the equilibrium price vector for the risky assets at time $t-1$. Using the budget constraint to solve for $\Delta q_s(t-1)$, and then plugging the result into equation (A5) to solve for $q_s(t)$, and then plugging the solution for $q_s(t)$ into the value function transforms the small investors problem into the following unconstrained problem:

$$\begin{aligned} \max_{\substack{C_s(t-1),\\ \Delta Q_s(t-1)}} \quad & -e^{-A_s C_s(t-1)} - \Big(\delta K_1(t)\ F(Q,t)\ e^{-A_s(t) r [q_s(t-1) + Q_s(t-1)'(D(t-1)+P(.,t-1)) - C_s(t-1)]} \\ & \times e^{A_s(t) r [Q_s(t-1)+\Delta Q_s(t-1)]' P(.,t-1)} \\ & \times\ e^{-A_s(t)(Q_s(t-1)+\Delta Q_s(t-1))'(\bar{D}+P(Q,t)) + .5 A_s(t)^2 (Q_s(t-1)+\Delta Q_s(t-1))' \Omega(t) (Q_s(t-1)+\Delta Q_s(t-1))'}\Big) \end{aligned} \tag{A8}$$

Since the first line of the unconstained maxization problem depends on $C_s(t-1)$ but not $\Delta Q_s(t-1)$, and since the later lines don't depend on $C_s(t-1)$, consumption and portfolio trades $\Delta Q_s(t-1)$ can be chosen separately. Solving for $\Delta Q_s(t-1)$ shows:

$$\Delta Q_s(t-1) = \frac{1}{A_s(t)} \Omega(t)^{-1} \left(\bar{D} + P(Q,t) - rP(., t-1)\right) - Q_s(t-1) \tag{A9}$$

Integrating both side of the above equation with respect to $\mu(s)$ generates the net demand of the competitive fringe:

$$
\begin{aligned}
\Delta Q_s(t-1) &= \int_0^1 \Delta Q_s(t-1)\mu(s)ds \\
&= \tfrac{1}{A_1(t)}\Omega(t)^{-1}\left(\bar{D} + P(Q,t) - rP(.,t-1)\right) - Q_1(t-1)
\end{aligned} \tag{A10}
$$

where,

$$
\frac{1}{A_1(t)} = \int_0^1 \frac{1}{A_s(t)}\mu(s)ds
$$

and

$$
Q_1(t-1) = \int_0^1 Q_s(t-1)\mu(s)ds.
$$

The Price Schedule Faced by Large Investors

The price schedule faced by large investors maps the desired orderflow of large investors into the prices at which the competitive fringe is willing to absorb the large investors net orderflow. To solve for the price schedule, I solve for prices $P(.,t-1)$ in equation (A10) such that when the large investors choose trade $\Delta Q_B(t-1)$ at time t-1, then the competitive fringe chooses trade $-S\Delta Q_B(t-1)$.

Rearranging, equation (A10) shows:

$$
\begin{aligned}
P(.,t-1) &= \frac{1}{r}\left(\bar{D} + P(t,Q(t)) - A_1(t)\Omega(t)[Q_1(t-1) + \Delta Q_1(t-1)]\right) \\
&= \frac{1}{r}\left(\bar{D} + P(t,Q(t-1) + \Delta Q(t-1)) - A_1(t)\Omega(t)[Q_1(t-1) + \Delta Q_1(t-1)]\right)
\end{aligned}
$$

Substituting $-S\Delta Q_B(t-1)$ for $\Delta Q_1(t-1)$, and noting that

$$
P(t,Q(t)) = \frac{1}{r}\left(\alpha(t) - \Gamma(t)Q(t)\right)
$$

then shows that the price schedule has form:

$$
P(.,t-1) = \frac{1}{r}\left(\beta_0(t-1) - \beta_Q(t-1)Q(t-1) - \beta_{Q_B}(t-1)\Delta Q_B(t-1)\right), \tag{A11}
$$

where,

$$
\begin{aligned}
\beta_0(t-1) &= \bar{D} + (1/r)\alpha(t) && \text{(A12)} \\
\beta_Q(t-1) &= (1/r)(\Gamma(t) + rA_1(t)\Omega(t)S_1) && \text{(A13)} \\
\beta_{Q_B}(t-1) &= (1/r)\Gamma(t)\begin{pmatrix} -S \\ I \end{pmatrix} - A_1(t)\Omega(t)S && \text{(A14)}
\end{aligned}
$$

Given the price schedule in equation (A11), large investors at time $t-1$ solve the maximization problem:

Large Investors Maximization Problem

$$\max_{\substack{C_m(t-1),\\ \Delta Q_m(t-1),\\ \Delta q_m(t-1)}} \quad -e^{-A_m C_m(t-1)} - \delta V_m(q_m(t), Q(t), t) \tag{A15}$$

subject to the budget constraint:

$$C_m(t-1) + \Delta Q_m(t-1)' P(.,t-1) + \Delta q_m(t-1) = Q_m(t-1)' D(t-1) \tag{A16}$$

where $P(.,t-1)$ in the budget constraint is the price schedule from equation (A11).

After substituting the budget constraint into the value function, and writing $Q(t)$, and $Q_m(t-1)$ as $Q + \Delta Q$, and Q_m, the m'th large investors maximization problem becomes:

$$\begin{aligned}\max_{C_m(t-1),\Delta Q_m} & -e^{-A_m C_m(t-1)} - \delta \left\{ k_m(t) \times \exp\left(-rA_m(t)[q_m(t-1) + Q_m' D(t-1) - C_m(t-1)]\right) \right. \\ & \left. \times \ \exp\left(-A_m(t)(Q+\Delta Q)' \bar{v}_m(t) + .5 A_m(t)^2 (Q+\Delta Q)' \theta_m(t)(Q+\Delta Q) + rA_m(t)\Delta Q_m' P(.,t-1)\right)\right\}.\end{aligned} \tag{A17}$$

Examination of the maximand shows that the the large investors portfolio choices can be solved for before choosing optimal consumption. Each large investor chooses ΔQ_m while taking the choices of the other large investors as given. Each large investor in choosing ΔQ_m accounts for the effect of his choices on price, and on the asset holdings of investor 1 since by construction $\Delta Q_1 = -S\Delta Q_m$. The first order condition for large investor m has form:

$$\begin{aligned} 0 = -A_m(t)[(-S_1 + S_m)\bar{v}_m(t)] + A_m(t)^2(-S_1 + S_m)[(\theta_m(t) + \theta_m(t)')/2](Q + \Delta Q) \\ + A_m(t)\left[rP(.,t-1) - S_m \beta_{Q_B}(t-1)' S_m \Delta Q_B\right], \end{aligned} \tag{A18}$$

After substituting for $P(.,t-1)$ from equation (A11), writing $Q+\Delta Q$ as $Q + \begin{pmatrix} -S\Delta Q_B \\ \Delta Q_B \end{pmatrix}$ and simplifying, this produces the following reaction function for large investor m:

$$\pi_m(t-1)\Delta Q_B = \chi_m(t-1) + \xi_m(t-1)Q, \tag{A19}$$

where,

$$\begin{aligned} \pi_m(t-1) = & A_m(t)(-S_1 + S_m)[(\theta_m(t) + \theta_m(t)')/2] \begin{pmatrix} -S \\ I \end{pmatrix} \\ & - \beta_{Q_B}(t-1) - S_m \beta_{Q_B}(t-1)' S_m \end{aligned} \tag{A20}$$

$$\chi_m(t-1) = (-S_1 + S_m)\bar{v}_m(t) - \beta_0(t-1) \quad \text{(A21)}$$
$$\xi_m(t-1) = \beta_Q(t-1) - A_m(t)(-S_1 + S_m)[(\theta_m(t) + \theta_m(t)')/2] \quad \text{(A22)}$$

Stacking the (M-1) reaction functions produces a system of $(M-1)N$ linear equations in $(M-1)N$ unknowns:

$$\Pi(t-1)\Delta Q_B(t-1) = \chi(t-1) + \xi(t-1)Q(t-1) \quad \text{(A23)}$$

Assume that $\Pi(t-1)$ is invertible. Then the solution for $\Delta Q_B(t-1)$ is unique, and given by

$$\Delta Q_B(t-1) = \Pi(t-1)^{-1}\chi(t-1) + \Pi(t-1)^{-1}\xi(t-1)Q(t-1) \quad \text{(A24)}$$

Equilibrium Asset Holdings

The solution for $\Delta Q_1(t-1)$ is $-S\Delta Q_B(t-1)$. Therefore, the solution for $\Delta Q(t-1) = (\Delta Q_1(t-1)', \Delta Q_B(t-1)')'$ can be written as:

$$\Delta Q(t-1) = H_0(t-1) + H_1(t-1)Q(t-1). \quad \text{(A25)}$$

where,

$$H_0(t-1) = \begin{pmatrix} -S\Pi(t-1)^{-1}\chi(t-1) \\ \Pi(t-1)^{-1}\chi(t-1) \end{pmatrix}, \quad \text{and} \quad H_1(t-1) = \begin{pmatrix} -S\Pi(t-1)^{-1}\xi(t-1) \\ \Pi(t-1)^{-1}\xi(t-1) \end{pmatrix}. \quad \text{(A26)}$$

With the above notation, the equilibrium purchases by large participant m in period $t-1$ are given by

$$\Delta Q_m(t-1) = S_m[H_0(t-1) + H_1(t-1)Q(t-1)] \quad \text{(A27)}$$

Additionally, the equilibrium transition dynamics for beginning of period risky asset holdings are given by:

$$Q(t) = G_0(t-1) + G_1(t-1)Q(t-1) \quad \text{(A28)}$$

where $G_0(t-1) = H_0(t-1)$ and $G_1(t-1) = H_1(t-1) + I$.

Equilibrium Price Function

Recall that the equilibrium price function in each time period maps investors beginning of period holdings of risky assets to an equilibrium price after trade. The equilibrium price function for period $t-1$ is found by plugging the solution for large investors equilibrium trades from equation (A24) into the price schedule faced by large investors (equation (A11)). The resulting price function for period $t-1$ has form:

$$P(t-1,Q) = \frac{1}{r}\left(\alpha(t-1) - \Gamma(t-1)Q\right) \tag{A29}$$

where,

$$\alpha(t-1) = \beta_0(t-1) - \beta_{Q_B}(t-1)\pi(t-1)^{-1}\chi(t-1) \tag{A30}$$
$$\Gamma(t-1) = \beta_Q(t-1) + \beta_{Q_B}(t-1)\pi(t-1)^{-1}\xi(t-1) \tag{A31}$$

Large Investors Consumption

Large investors optimal time $t-1$ consumption depends on optimal time $t-1$ trades. After plugging the expressions for equilibrium prices, and equilibrium trades [equations (A28), (A29), and (A25)] into equation (A17), large investors consumption choice problem has form:

$$\max_{C_m(t-1)} -e^{-A_m C_m(t-1)} - \delta k_m(t) e^{rA_m(t)C_m(t-1)} \times \psi_m(Q(t-1), q_m(t-1), D(t-1), t-1), \tag{A32}$$

where

$$\begin{aligned}\psi_m(Q, q_m, D(t-1),&t-1) = \\ &e^{-A_m(t)r(q_m(t-1)+Q_m(t-1)'D(t-1))} \\ &\times e^{-A_m(t)r[S_m(H_0(t)+H_1(t)Q)(t-1)]'(\alpha(t-1)-\Gamma(t-1)Q(t-1))/r} \\ &\times e^{-A_m(t)(G_0(t)+G_1(t)Q(t-1))'\bar{v}_m(t)+.5A_m(t)^2(G_0(t)+G_1(t)Q(t-1))'\theta_m(t)(G_0(t)+G_1(t)Q(t-1))}\end{aligned} \tag{A33}$$

The first order condition for choice of optimal consumption implies that optimal consumption is given by:

$$C_m(t-1) = \frac{-1}{A_m(t)r + A_m}\ln\left(\frac{\delta k_m(t)A_m(t)r\psi_m(Q(t-1), q_m(t-1), D(t-1), t-1)}{A_m}\right) \tag{A34}$$

Large investors value function at time $t-1$

Define $V_m(t-1, D(t-1), Q, q_m)$ as the value function to large investor m from entering period $t-1$ when the vector of risky asset holdings is Q, and his riskless asset holdings are q_m, and the dividend realization at time $t-1$ is $D(t-1)$. After substituting the optimal consumption choice in (A34) into equation (A32), this value function is given by:

$$V_m(q_m, Q, t-1, D(t-1)) = -\left[\frac{1+r_m^*(t)}{r_m^*(t)}\right]\left[\delta k_m(t) r_m^*(t)\psi_m(Q, q_m, D(t-1), t-1)\right]^{\frac{1}{1+r_m^*(t)}} \tag{A35}$$

where,

$$r_m^*(t) = A_m(t)r/A_m \tag{A36}$$

Taking the expectation of the value function in equation (A35) with respect to the distribution of $D(t-1)$ then produces $V_m(q_m, Q, t-1)$, which is the value to large investors $2, \ldots M$ from entering period $t-1$ when the distribution of risky asset holdings is Q and investor m's holdings of riskfree assets are q_m:

$$V_m(q_m, Q, t-1) = -k_m(t-1) \times e^{-A_m(t-1)q_m - A_m(t-1)Q'\bar{v}_m(t-1) + .5A_m(t-1)^2 Q'\theta_m(t-1)Q} \tag{A37}$$

where the parameters of the value function at time $t-1$ are given by the following Riccati difference equations.

$$A_m(t-1) = A_m(t)r/(1 + r_m^*(t)) \tag{A38}$$

$$\begin{aligned} k_m(t-1) = & \left[\frac{r_m^*(t)+1}{r_m^*(t)}\right] [\delta k_m(t) r_m^*(t)]^{\frac{1}{1+r_m^*(t)}} \\ & \times e^{A_m(t-1)H_0(t-1)'S_m'\alpha(t-1)/r - A_m(t-1)G_0(t-1)'\bar{v}_m(t)/r + .5A_m(t-1)^2((1+r_m^*(t))/r^2)(G_0(t-1)'\theta_m(t)G_0(t-1))} \end{aligned} \tag{A39}$$

$$\begin{aligned} \bar{v}_m(t-1) = & S_m'\bar{D} - H_1(t-1)'S_m'\alpha(t-1)/r + \Gamma(t-1)'S_m H_0(t-1)/r + G_1(t-1)'\bar{v}_m(t)/r \\ & - A_m(t-1)(1 + r_m^*(t))G_1(t-1)'\left(\frac{\theta_m(t) + \theta_m(t)'}{2}\right) G_0(t-1)/r^2 \end{aligned} \tag{A40}$$

$$\theta_m(t-1) = \frac{-2H_1(t-1)'S_m'\Gamma(t-1)}{rA_m(t-1)} + (1+r_m^*(t))G_1(t-1)'\theta_m(t)G_1(t-1)/r^2 + S_m'\Omega(t-1)S_m \tag{A41}$$

Small investors optimal consumption

The solution for each small investors consumption depends on small investors optimal trades (equation (A9)) after substituting in the equilibrium price function $P(Q, t-1)$ for $P(., t-1)$. The other determinants of the small investors consumption is the post-trade vector of risky asset holdings, $Q(t-1) + \Delta Q(t-1)$, and equilibrium prices [equations (A28) and (A29)]. Substituting these expressions into equation (A8), small investors consumption choice problem has form:

$$\max_{C_s(t-1)} -e^{-A_s C_s(t-1)} - \delta k_s(t) e^{rA_s(t)C_s(t-1)} \times \psi_s(Q(t-1), Q_s(t-1), q_s(t-1), D(t-1), t-1), \tag{A42}$$

where,

$$\psi_s(Q(t-1),Q_s(t-1),q_s(t-1),D(t-1),t-1) = e^{-A_s(t)r\{q_s(t-1)+Q_s(t-1)'[P(t-1,Q(t-1))+D(t-1)]\}} \times e^{-.5\{\bar{D}+P[t,Q(t-1)+\Delta Q(t-1)]-rP[t-1,Q(t-1)]\}'\Omega(t)^{-1}\{\bar{D}+P[t,Q(t-1)+\Delta Q(t-1)]-rP[t-1,Q(t-1)]\}} \times e^{-[G_0(t-1)+G_1(t-1)Q(t-1)]'\bar{v}_s(t)-.5[G_0(t-1)+G_1(t-1)Q(t-1)]'\theta_s(t)[G_0(t-1)+G_1(t-1)Q(t-1)]} \quad \text{(A43)}$$

The first order condition for choice of optimal consumption implies that optimal consumption is given by:

$$C_s(t-1) = \frac{-1}{A_s(t)r + A_s} \ln\left(\frac{\delta k_s(t)A_s(t)r\psi_s(Q(t-1),Q_s(t-1),q_s,D(t-1),t-1)}{A_s}\right) \quad \text{(A44)}$$

Small investors value function at time $t-1$

Define $V_s(q_s(t-1),Q_s(t-1),t-1,D(t-1))$ as the value function to small investor s from entering period $t-1$ when the vector of risky asset holdings is $Q(t-1)$, and his risky and riskless asset holdings are $Q_s(t-1)$ and $q_s(t-1)$, and the dividend realization at time $t-1$ is $D(t-1)$. After substituting the optimal consumption choice in (A44) into equation (A42), this value function is given by:

$$V_s(q_s(t-1),Q_s(t-1),Q(t-1),t-1,D(t-1)) = -\left[\frac{1+r_s^*(t)}{r_s^*(t)}\right][\delta k_s(t)r_s^*(t)\psi_s(Q(t-1),Q_s(t-1),q_s(t-1),D(t-1),t-1)]^{\frac{1}{1+r_s^*(t)}} \quad \text{(A45)}$$

where,

$$r_s^*(t) = A_s(t)r/A_s \quad \text{(A46)}$$

Taking the expectation of the value function function with respect to the distribution of $D(t-1)$ produce the value function to a small investor from entering period $t-1$ with asset holdings q_s and Q_s when the vector of all investors asset holdings is given by Q:

$$\begin{aligned} V_s(q_s,Q_s,Q,t-1) &= -K_s(t-1)\ F(Q,t-1)\ e^{-A_s(t-1)q_s(t-1)-A_s(t-1)Q_s'(\bar{D}+P(Q,t-1))+.5A_s(t-1)^2Q_s'\Omega(t-1)Q_s}, \\ \text{where } F(Q,t-1) &= e^{-Q'\bar{v}_s(t-1)-\frac{1}{2}Q'\theta_s(t-1)Q} \\ P(Q,t-1) &= \tfrac{1}{r}(\alpha(t-1)-\Gamma(t-1)Q). \end{aligned} \quad \text{(A47)}$$

The parameters in the small investors value functions at time $t-1$ are a function of time t parameters as expressed in the following Riccati equations:

$$A_s(t-1) = \frac{rA_s(t)}{1+r_s^*(t)} \quad \text{(A48)}$$

$$k_s(t-1) = \left[\frac{r_s^*(t)+1}{r_s^*(t)}\right]\left[\delta k_s(t-1) r_s^*(t) e^{-.5a_0(t-1)'\Omega(t)^{-1}a_0(t-1) - G_0(t-1)'\bar{v}_s(t) - .5G_0(t-1)'\theta_s(t)G_0(t-1)}\right]^{\frac{1}{1+r_s^*(t)}}, \tag{A49}$$

where,

$$a_0(t-1) = \bar{D} + \frac{1}{r}\left[\alpha(t) - \Gamma(t)G_0(t-1)\right] - \alpha(t-1) \tag{A50}$$

$$\bar{v}_s(t-1) = \frac{a_1(t-1)'\Omega(t)^{-1}a_0(t-1) + G_1(t-1)'\bar{v}_s(t) + G_1(t-1)'[(\theta_s(t)+\theta_s(t)')/2]G_0(t-1)}{1+r_s^*(t)}, \tag{A51}$$

where,

$$a_1(t-1) = \Gamma(t-1) - \frac{1}{r}\Gamma(t)G_1(t-1). \tag{A52}$$

$$\theta_s(t-1) = \frac{a_1(t-1)'\Omega(t)^{-1}a_1(t-1) + G_1(t-1)'\theta_s(t)G_1(t-1)}{1+r_s^*(t)} \tag{A53}$$

This completes part I of the proof because equations (A37) and (A47) verify that the value functions at time $t-1$ have the same form as at time t.

B.2 Part II.

To establish part II of the proof, I need to show that investors value functions for entering entering period T has the same functional form as given in the proposition. The functional form is given below.

Investors Value Functions at Time T

Recall that investors are infinitely lived but that from time T onwards they cannot alter their holdings of risky assets, but they can continue to alter their consumption, and their holdings of riskless assets. Because investors cannot trade after period T, the distinction between small and large investors after this period is irrelevant. Hence, the index m used below could be for either a large or small investor. Using the Bellman principle, the value function of entering period T with risky asset holdings Q_m and risk-free holdings q_m, conditional on time T dividends $D(T)$ satisfies the system of equations:

$$V_m(Q_m(T), q_m(T), t|D(T)) = \max_{C_m(T)} -\exp^{-A_m C_m(T)} + \delta \operatorname{E} V_m(Q_m(t+1), q_m(t+1), t+1), \tag{A54}$$

subject to the constraints:

$$q_m(T+1) = r[q_m(T) + Q_m(T)'D(T) - C(T)] \tag{A55}$$

$V_m(Q_m(T), q_m(T), T)$, the unconditional value of entering period T with holdings $Q_m(T)$ and $q_m(T)$, is given by:

$$V_m(Q_m(T), q_m(T), T) = \int_{D(T)} V_m(Q_m(T), q_m(T), T|D(T)) f(D(T)) dD(T) \tag{A56}$$

where, $f(D(T))$ is the probability density function for $D(T)$.

Equation (A56) is the Bellman equation for this optimization problem. Inspection shows that the function,

$$V_m(Q_m(T), q_m, T) = -k_m(T) \exp^{-A_m(1-(1/r))q_m(T) - A_m(1-(1/r))\frac{Q_m(T)'\bar{D}}{1-(1/r)} + .5A_m^2(1-(1/r))^2\frac{Q_m(T)'\Omega Q_m(T)}{1-(1/r)}} \tag{A57}$$

solves the Bellman equation, where

$$k_m(T) = \left(\frac{r}{r-1}\right) \times (\delta r)^{\frac{1}{r-1}} \tag{A58}$$

For large investors, this equation can be rewritten as:

$$V_m(Q_m(T), q_m, T) = -k_m(T) \exp^{-A_m(T)q_m(T) - A_m(T)Q'\bar{v}_m(T) + .5A_m(T)^2 Q'\theta_m(T)Q} \tag{A59}$$

where,

$$A_m(T) = A_m[1 - (1/r)] \tag{A60}$$

$$\bar{v}_m(T) = \frac{S_m'\bar{D}}{1-(1/r)} \tag{A61}$$

$$\theta_m(T) = \frac{S_m'\Omega S_m}{1-(1/r)} \tag{A62}$$

For small investors, the value function in equation (A57) can be rewritten in the form:

$$V_s(Q_s(T), q_s, T) = -k_s(T)F(Q,T) \exp^{-A_s(T)q_s(T) - A_s(T)Q_s(T)'[\bar{D}+P(Q,T)] + .5A_s(T)^2 Q_s(T)'\Omega(T)Q_s(T)} \tag{A63}$$

solves the Bellman equation, where

$$\begin{aligned}
A_s(T) &= A_s[1-(1/r)] && \text{(A64)}\\
\Gamma(T) &= 0 && \text{(A65)}\\
k_s(T) &= \left(\frac{r}{r-1}\right)\times(\delta r)^{\frac{1}{r-1}} && \text{(A66)}\\
F(Q,T) &= -e^{Q'\bar{v}_s(T)-.5Q'\theta_s(T)Q} && \text{(A67)}\\
\bar{v}_s(T) &= 0 && \text{(A68)}\\
\theta_s(T) &= 0 && \text{(A69)}\\
P(Q,t) &= \frac{1}{r}(\alpha(T)-\Gamma(T)Q) && \text{(A70)}\\
\alpha(T) &= \bar{D}/[1-(1/r)] && \text{(A71)}\\
\Gamma(T) &= 0 && \text{(A72)}\\
\Omega(T) &= \Omega/[1-(1/r)] && \text{(A73)}
\end{aligned}$$

Since these value functions have the same form as given in the proposition, this completes the induction and the proof of proposition 2. □

Interpretation of Large Investors Risk Aversion

In this subsection, the portfolio problem of large investors is recast into a related problem in which the demands of each large investor represent the portfolio choices of a mass of identical small investors that pool their investment management decisions to economize on the costs of monitoring the market. Specifically, the demands and consumption choices of each large investor m, represents the demands of a mass μ_m of small investors that have identical risk aversion $A_{m,s}$, identical endowments of risky and risk free assets ($Q_{m,s}(T-1)$ and $q_{m,s}(T-1)$), and choose identical consumption $C_{m,s}(T-1)$. Given investors time T value functions (equation (A57)), at time $T-1$ each large investor maximizes the utility of a representative small investor by solving the maximization problem:

$$\max_{C_{m,s},\Delta Q_{m,s}} -e^{-A_{m,s}C_{m,s}} - \delta V_{m,s}(Q_{m,s}(T), q_{m,s}(T), T) \quad \text{(A74)}$$

subject to the standard budget constraint (A16) and subject to the constraint that the large investors total trading activity and consumption choices are split evenly among the mass μ_m of small investors that he represents. For example, if he wants to buy 1 share of stock for each small investor, and there are 100 small investors then he has to buy 100 shares of stock. More generally, the large investors choices are related to the small investors choices as follows:

$$\begin{aligned}
Q_m + \Delta Q_m &= (Q_{m,s} + \Delta Q_{m,s}) \times \mu_m\\
q_m + \Delta q_m &= (q_{m,s} + \Delta q_{m,s}) \times \mu_m\\
C_m &= C_{m,s} \times \mu_m
\end{aligned} \quad \text{(A75)}$$

Using these to subsitute for $C_{m,s}$, $Q_{m,s}+\Delta Q_{m,s}$ and $q_{m,s}+\Delta q_{m,s}$ in the objective function then shows that the maximization problem solved for the representative small investor has the exact same form as the maximization problem for a large investor with risk aversion $A_{m,s}/\mu_m$. If each large investor is interpreted as a mutual fund, this means that the risk tolerance of the fund is equal to the risk tolerance of the typical investor in the fund times the number of investors in the fund as measured by μ_m. Given this relationship is true in the last period, it is straightforward to establish it in earlier periods.

B.3 Solutions for Value Function Parameters

This section provides further information on the parameters of investors value functions. I first present solutions when there are no distressed sales.

No Distressed Sales Case

The backwards induction analysis expresses the parameters of the value functions as resulting from a series of Riccati difference equations. The purpose of this subsection is to more fully characterize the parameters of the value functions. The next proposition provides more information about the parameters of the value functions for the large investors:

Proposition 8 *For all time periods $t = 1, \ldots, T-1$, and for large investors $m = 2, \ldots M$:*

$$\bar{v}_m(t) = \frac{S_m' \bar{D}}{1-(1/r)} \tag{A76}$$

$$\alpha(t) = \bar{D}/[1-(1/r)] \tag{A77}$$

$$A_m(t) = A_m[1-(1/r)] \tag{A78}$$

$$r^*(t) = r-1 \tag{A79}$$

$$k_m(t) = \left(\frac{r}{r-1}\right) \times (\delta r)^{\frac{1}{r-1}} \tag{A80}$$

Proof:

For $\bar{v}_m(t)$ and $\alpha(t)$:

The proof is by induction. First, suppose that the results for $\bar{v}_m(t)$ and $\alpha(t)$ are true at time t. Then, since $\alpha(t) = \bar{D}/[1-(1/r)]$, then from equation (A12), $\beta_0(t-1) = \bar{D}/[1-(1/r)]$. This implies that from equation (A21) that$(-S_1 + S_m)\bar{v}_m(t) - \beta_0(t-1) = 0$. As a result $\chi(t-1) = 0$, which implies from equation (A30) that $\alpha(t-1) = \beta_0(t-1) = \bar{D}/[1-(1/r)]$ and from equations (A26) and (A28) that $H_0(t-1) = G_0(t-1) = 0$. Substituting for $H_0(t-1)$ and $G_0(t-1)$ in equation (A40) and simplifying then shows:

$$\bar{v}_m(t-1) = S_m' \bar{D} - H_1(t-1)' S_m' \alpha(t-1)/r + G_1(t-1)' \bar{v}_m(t)/r \tag{A81}$$

Recalling that $G_1(t-1) = H_1(t-1) + I$, then plugging in the expressions for $\alpha(t-1)$ and $\bar{v}_m(t)$ into the expression for $\bar{v}_m(t-1)$ confirms the result for time $T-1$. To complete the induction, it suffices to note that the results hold at time T as shown in equations (A71) and (A61). This completes the proof for $\bar{v}_m(t)$ and $\alpha_m(t)$.

For $A_m(t)$ and $r^*(t)$:

The proof is by induction. Suppose the results are true at time t, then applying the solutions for $A_m(t)$ and $r^*_m(t)$ in equation (A38) produces the hypothesized expression for $A_m(t-1)$, and applying the solution for $A_m(t-1)$ in equation (A36) produces the hypothesized expression for $r^*_m(t-1)$. To complete the induction note that the result is true for $A_m(T)$ and $r^*(T)$.

For $k_m(t)$:

The proof is by induction. Assume it is true for time t. Then plugging the hypothesized solution into equation (A39) and plugging in the solutions for $r^*_m(t)$ and plugging in $H_0(t-1) = G_0(t-1) = 0$ (established in the proof for $\bar{v}_m(t)$) and then simplifying shows the result holds for $k_m(t-1)$. To complete the induction note that $k_m(T)$ has the hypothesized form. □

The next proposition provides information on the value functions of the small investors:

Proposition 9 *For all time periods $t = 1, \ldots, T-1$, and for each small investor s*

$$a_0(t) = 0 \tag{A82}$$
$$\bar{v}_s(t) = 0 \tag{A83}$$
$$A_s(t) = A_s[1 - (1/r)] \tag{A84}$$
$$r^*_s(t) = r - 1 \tag{A85}$$
$$k_s(t) = \left(\frac{r}{r-1}\right) \times (\delta r)^{\frac{1}{r-1}} \tag{A86}$$

Proof:

For $a_0(t)$ and $\bar{v}_s(t)$: Plugging the solutions for $\alpha(t)$ and $G_0(t-1)$ from proposition 8 into equation (A50) shows that $a_0(t) = 0$ for all times t. Since $G_0(t-1) = 0$ for all times t, it then follows from equation (A51) that if $\bar{v}_s(t) = 0$, then so does $\bar{v}_s(t-1)$. To complete the induction, note that $\bar{v}_s(T) = 0$ (equation (A68)).

For $A_s(t)$, $r^*_s(t)$, and $k_s(t)$:

The form of the proof is identical to that given in proposition 8.□

Proposition 10 *Assume that for $t < T$, conditional on state variable $Q(t)$ the Nash Equilibrium trades of the large investors exists and is unique. Then for all $m = 2, \ldots, M$ and*

$t = 1, \ldots, T$, $\theta_m(t)$ *has form:*

$$\vartheta_m(t) \otimes \Omega, \tag{A87}$$

where, $\vartheta_m(t)$ *is* $M \times M$*; and*

$$\Gamma(t) = \gamma(t) \otimes \Omega, \tag{A88}$$

where, $\gamma(t)$ *is* $1 \times M$.

Proof: The proof is by induction. First, assume that the theorem is true at time t. Then, from equations (A14) and (A13) $\beta_{Q_B}(t-1) = B_{Q_B}(t-1) \otimes \Omega$, and $\beta_Q(t-1) = B_Q(t-1) \otimes \Omega$, where $B_{Q_B}(t-1)$ is $1 \times M - 1$ and $\beta_Q(t-1)$ is $1 \times M$. Applying these substitutions in large investors reaction functions and then stacking the results reveals that in equation (A23), $\pi(t-1) = \mathcal{P}(t-1) \otimes \Omega$ and $\xi(t-1) = Z(t-1) \otimes \Omega$. The assumption that the Nash Equilibrium trades in each period are unique implies that $\mathcal{P}(t-1)$ is invertible. Solving for $H_0(t-1)$ and $H_1(t-1)$ then shows that $H_0(t-1) = 0$ and

$$\begin{aligned} H_1(t-1) &= \begin{pmatrix} -S[P(t-1)^{-1}Z(t-1)] \otimes I_N \\ (P(t-1)^{-1}Z(t-1)) \otimes I_N \end{pmatrix} && \text{(A89)} \\ &= \begin{pmatrix} [-\iota'_M \mathcal{P}(t-1)^{-1}Z(t-1)] \otimes I_N \\ (\mathcal{P}(t-1)^{-1}Z(t-1)) \otimes I_N \end{pmatrix} && \text{(A90)} \\ &= \mathcal{H}_1(t-1) \otimes I_N && \text{(A91)} \end{aligned}$$

where ι_M is a $1 \times M$ vector of ones, and $\mathcal{H}_1(t-1)$ is $M \times M$. Since $G_1(t-1) = H_1(t-1) + I_{NM}$, it follows that $G_1(t-1) = \mathcal{G}_1(t-1) \otimes I_N$ for $\mathcal{G}_1(t-1) = \mathcal{H}_1(t-1) + I_M$. From here, substitution in equation (A31) shows that $\Gamma(t-1) = \gamma(t-1) \otimes \Omega$ and substitution in equation (A41) shows that $\theta_m(t-1) = \vartheta_m(t-1) \otimes \Omega$. To complete the induction, note that the conditions of the proposition are satisfied at time T. $\Box$.

Corollary 2 *For each small investors, and for each time period* $t = 1, \ldots T$,

$$\theta_s(t) = \vartheta_s \otimes \Omega,$$

where ϑ_s *is* $M \times M$.

Proof: Straightforward induction involving application of the results from proposition 10.

Corollary 3 *Let* $\Delta Q_{.,n}(t)$ *and* $Q_{.,n}(t)$ *denote the* $M \times 1$ *vector of time* t *asset holdings and trades of asset* n *by investors 1 through* M. *Then,under the assumptions of proposition 10 for all* $n = 1, \ldots N$, $\Delta Q_{.,n}(t) = \mathcal{H}_1(t) Q_{.,n}(t)$.

Proof: By definition, $\Delta Q_{.,n}(t) = (I_M \otimes e'_n)\Delta Q(t)$, where e_n is an $N \times 1$ vector which has 1 for its n'th element and zeroes elsewhere. Since $H_0(t) = 0$, we also know

$$\begin{aligned} \Delta Q(t) &= H_1(t)Q(t) && \text{(A92)} \\ &= [\mathcal{H}_1(t) \otimes I_N]Q(t). && \text{(A93)} \end{aligned}$$

Multiplying both sides of equation (A93) by $(I_M \otimes e'_n)$ and then simplifying establishes the result $\square$.

Corollary 3 shows that the equilbrium trades for each asset n only depend on market participants holdings of that asset. They do not depend on market participants holdings of other assets. Clearly, the result in the corollary is very special, not general.

B.4 Proofs of Asset Pricing Propositions

Proposition 5: *When investors asset holdings are not Pareto Optimal, equilibrium expected asset returns satisfy a linear factor model in which one factor is the market portfolio, and the other factors correspond to the deviation of large investors asset holdings from pareto optimal asset holdings.*

Proof: Let Q^W denote the vector of pareto optimal holdings of risky assets. Manipulation of the equation for equilibrium prices given in proposition 2, and substitution of $G_0(t)+G(t)Q(t)$ for $Q(t+1)$ shows:

$$P(t+1)+\bar{D}-rP(t) = [\frac{1}{r}\alpha(t+1)+\bar{D}-\alpha(t)]-[\frac{1}{r}\Gamma(t+1)G_0(t)]+[\Gamma(t)-\frac{1}{r}\Gamma(t+1)G_1(t)]Q(t)$$

Plugging in the solution for $\alpha(t) = \alpha(t-1) = \bar{D}/[1-(1/r)]$ shows the first term in braces is zero. The second term in braces is zero since proposition 8 shows that $G_0(t) = 0$. Adding and subtracting Q^W to $Q(t)$, the above equation can be rewritten as:

$$P(t+1)+\bar{D}-rP(t) = [\Gamma(t)-\frac{1}{r}\Gamma(t+1)G_1(t)](Q(t)-Q^W)+[\Gamma(t)-\frac{1}{r}\Gamma(t+1)G_1(t)]Q^W \quad \text{(A94)}$$

Using the fact that $Q_1 = X - SQ_B$, the vector $Q(t) - Q^W$ can be expressed in terms of the deviations of large investors asset holdings from pareto optimal asset holdings:

$$\begin{aligned} Q(t) - Q_W &= \begin{bmatrix} (X - SQ_B) - (X - SQ_B^W) \\ Q_B - Q_B^W \end{bmatrix} \\ &= \begin{bmatrix} -S \\ I \end{bmatrix} (Q_B - Q_B^W) \end{aligned}$$

Additionally, an implication of proposition 4 and the expression forequilibrium $P(t)$ with perfect competition, is that

$$[\Gamma(t) - \frac{1}{r}\Gamma(t+1)G_1(t)]Q^W = \lambda_X \Omega X.$$

Making both of these substitutions in equation (A94) shows

$$P(t+1) + \bar{D} - rP(t) = \lambda_X \Omega X + [\Gamma(t) - \frac{1}{r}\Gamma(t+1)G_1(t)] \begin{pmatrix} -S \\ I \end{pmatrix} (Q_B(t) - Q_B^W)$$

Finally, applying the algebra in proposition 10 shows

$$[\Gamma(t) - \frac{1}{r}\Gamma(t+1)G_1(t)] \begin{pmatrix} -S \\ I \end{pmatrix} = \lambda(t) \otimes \Omega \tag{A95}$$

where $\lambda(t)$ is $1 \times M-1$. Making this substitution then shows:

$$\begin{aligned} P(t+1) + \bar{D} - rP(t) &= \lambda_X \Omega X + [\lambda(t) \otimes \Omega](Q_B(t) - Q_B^W) && \text{(A96)} \\ &= \lambda_X \Omega X + \sum_{j=2}^{M} \lambda(m,t)\Omega(Q_m(t) - Q_m^W) && \text{(A97)} \end{aligned}$$

where $\lambda(m,t) = \lambda(t)s'_{m-1}$. $\Box$.

Corollary 4 *When asset holdings are not pareto optimal, then asset returns between time t and time t + 1 have an alternative M-factor model representation in which one factor is the market portfolio, and the other factors are the returns on large investors risky asset holdings.*

Proof: From proposition 5, we know that:

$$P(t+1) + \bar{D} - rP(t) = \lambda_X \Omega X + \sum_{j=2}^{M} \lambda(m,t)\Omega(Q_m(t) - Q_m^W)$$

Making the substitution $Q_m^W = \frac{(1/A_m)X}{\sum_{m=1}^{M} 1/A_m}$, and then simplifying shows

$$P(t+1) + \bar{D} - rP(t) = \lambda_X(t)\Omega X + \sum_{j=2}^{M} \lambda(m,t)\Omega Q_m(t)$$

where

$$\lambda_X(t) = \lambda_X - \sum_{m=2}^{M} \left(\frac{\lambda(m,t)/A_m}{\sum_{j=1}^{M} 1/A_j} \right) \quad \Box.$$

Corollary 5 *When asset holdings at time t are not pareto-optimal, then asset returns at time t + τ follow a factor model in which the market portfolio and the deviation of large investors asset holdings from pareto-optimal asset holdings at time t are factors.*

Proof: Iterating equation (A94), by τ periods shows:

$$\begin{aligned} P(t+\tau+1) + \bar{D} - rP(t+\tau) = [\Gamma(t+\tau) - \frac{1}{r}\Gamma(t+1+\tau)G_1(t+\tau)](Q(t+\tau) - Q^W) \\ + [\Gamma(t+\tau) - \frac{1}{r}\Gamma(t+\tau+1)G_1(t+\tau)]Q^W, \end{aligned} \tag{A98}$$

which implies:

$$P(t+\tau+1)+\bar{D}-rP(t+\tau) = \lambda_X \Omega X + [\lambda(t,\tau)\otimes\Omega](Q(t)-Q^W) \quad \text{(A99)}$$

$$= \lambda_X \Omega X + \sum_{m=2}^{M} \lambda_m(t,\tau)\Omega(Q_m(t)-Q_m^W) \quad \text{(A100)}$$

where,

$$\lambda(t,\tau)\otimes\Omega = [\Gamma(t+\tau) - \frac{1}{r}\Gamma(t+1+\tau)G_1(t+\tau)]\prod_{j=0}^{\tau-1} G_1(t+j),$$

and $\lambda_m(t,\tau) = \lambda(t,\tau)S'_{m-1}$ □.

Assume that asset returns are generated by the large investor model, and that investors asset holdings are pareto optimal. In the absence of any shocks which perturb investors asset holdings, asset returns will be indistinguishable from those associated with the CAPM. If there is a small 1-time perturbation in asset holdings, then because the deviation in asset prices is proportional to the size of the perturbation, for small enough perturbations, the behavior of the model is statistically indistinguishable from the CAPM for a given span of data. This (obvious) point is made formally in the next proposition:

Proposition 11 *In a sample of τ 1-period returns, let ϕ denote the size of a Chi-Square test that Jensen's α is not equal to 0, and let ρ denote the power of the test. Then for every $\rho > \phi$ there are perturbations from pareto optimal asset holdings for which the power of the test is less than ρ.*

Proof: For simplicity, assume that λ_X, Ω, and X are known. Then an estimate of Jensen's α is given by

$$\hat{\alpha} = \frac{1}{\tau}\sum_{i=1}^{\tau} P(t+i) + D(t+i) - rP(t+i-1) - \lambda_X\Omega.$$

Under the null hypothesis that assets are priced by the CAPM,

$$\sqrt{\tau}\hat{\alpha} \sim \mathcal{N}(0,\Omega),$$

and

$$\tau\hat{\alpha}'\Omega^{-1}\hat{\alpha} \sim \chi^2(0,N),$$

where the non-centrality parameter is 0. Under the alternative, assets are not priced by the CAPM. If there is an initial perturbation of asset holdings under the alternative, without loss of generality parameterize the direction of the perturbation for each $m = 2, \ldots M$ by $\bigtriangledown_m$ and the size of the perturbation by scalar δ so that $(Q_m(t) - Q_m^W) = \delta\bigtriangledown_m$. Then, $\sqrt{\tau}\hat{\alpha} \sim N(\mu,\Omega)$ where, $\mu = \delta\frac{1}{\sqrt{\tau}}\sum_{m=2}^{M}\left(\sum_{i=1}^{\tau}\lambda_m(t,t+i)\bigtriangledown_m\right)$. Therefore, $\tau\hat{\alpha}'\Omega^{-1}\hat{\alpha} \sim \chi^2(\psi,N)$ where $\psi = \mu'\Omega^{-1}\mu = O(\delta^2)$. The power of the test is continuously increasing in the non-centrality parameter, and hence in δ; and when δ is 0, the power of the test is equal to its size. Therefore, by the continuity of the power function, for small enough δ (small enough perturbations) the power of the test is less than ϕ. □

B.5 Distressed Sales

Suppose one of the large investors is forced to sell their holdings of risky assets at time τ_s, and then exit the market. In this section, I model how such distressed sales affect equilibrium trades and prices when investors learn of the future sales at time τ_R, but he sales do not occur until time τ_S. For simplicity, it is assumed that the distressed seller is not allowed to trade between times τ_R and τ_S, and that no market participants (including the distressed seller) are aware of the distress before time τ_R.

Time τ_S

After investors enter time τ_S, and receive their dividend and interest payments, I assume they learn that one large investor will sell ΔQ_D units of risky assets during trade in time period τ_S. To solve for how this affects investors value functions, I first solve for how it affects the prices at which the competitive fringe is willing to absorb the large investors orderflow. I then solve for the large investors equilibrium orderflow, and then I solve for equilibrium risky asset trades, prices, and consumption at time τ_S. Finally, I solve for the equilibrium value function for entering time τ_S.

For all of the analysis in this section, I assume there are $M+1$ investors whose risky asset holdings before time τ_R are equal to Q^W. Without loss of generality, I assume that the $M+1$'st investor will have to sell ΔQ_D at time τ_S. I further assume that this investor cannot trade until time τ_S. With these assumptions, the $M+1$'st investor is a modelling device for distressed sales at time τ_S.

The primary focus is on how the other M investors behave as a result of the distressed sales. Let $Q(\tau_S)$ denote the risky asset holdings of investors 1 through M at time τ_S. At time τ_S, for any given purchases of risky assets ΔQ_B by large investors 2 through M, prices at time τ_S and τ_S+1 must equilibrate so that the competitive fringe fringe is willing to absorb $\Delta Q_D - S\Delta Q_B$, which is the distressed sales less the net purchases of the other large investors. From the equilibrium price function at time τ_S+1, we know that given the distressed sales and hypothesized large investors purchases, that

$$Q(\tau_S+1) = \begin{pmatrix} Q_1(\tau_S) + \Delta Q_D - S\Delta Q_B(\tau_S) \\ Q_B(\tau_S) + \Delta Q_B(\tau_S) \end{pmatrix},$$

and that

$$P(\tau_S+1, Q(\tau_S+1)) = \frac{1}{r}\left(\alpha(\tau_S+1) - \Gamma(\tau_S+1)Q(\tau_S+1)\right)$$

To solve for the price schedule faced by large investors, I follow the steps beginning from equation (A9) to derive a price schedule which is analogous to that in equation (A11):

$$P(., \tau_S) = \frac{1}{r}\left(\beta_0(\tau_S) - \beta_Q(\tau_S)Q(\tau_S) - \beta_{Q_B}(\tau_S)\Delta Q_B(\tau_S)\right), \tag{A101}$$

where,

$$\beta_0(\tau_S) = \bar{D} + (1/r)\alpha(\tau_S+1) - (1/r)\Gamma(\tau_S+1)S_1'\Delta Q_D - A_1(\tau_S+1)\Omega(\tau_S+1)\Delta Q_D \quad \text{(A102)}$$

$$\beta_Q(\tau_S) = (1/r)(\Gamma(\tau_S+1) + rA_1(\tau_S+1)\Omega(\tau_S+1)S_1) \quad \text{(A103)}$$

$$\beta_{Q_B}(\tau_S) = (1/r)\Gamma(\tau_S+1)\begin{pmatrix} -S \\ I \end{pmatrix} - A_1(\tau_S+1)\Omega(\tau_S+1)S \quad \text{(A104)}$$

Notice, that the price schedule faced by large investors in equation (A101) differs from that in equation (A11) because $\beta_0(\tau_S)$ contains additional terms that cause the price to change to compensate the fringe for absorbing a portion of the distressed asset sales.

Given the price schedule, large investors will choose their net purchases $\Delta Q_B(\tau_S)$ while accounting for the fact that for a given purchase of risky assets on their behalf, the amount of risky assets that will be held by the fringe has increased by ΔQ_D. The resulting reaction function for large investor m is given by:

$$\pi_m(\tau_S)\Delta Q_B = \chi_m(\tau_S) + \xi_m(\tau_S)Q, \quad \text{(A105)}$$

where,

$$\begin{aligned}\pi_m(\tau_S) = {} & A_m(\tau_S+1)(-S_1+S_m)[(\theta_m(\tau_S+1)+\theta_m(\tau_S+1)')/2]\begin{pmatrix} -S \\ I \end{pmatrix} \\ & - \beta_{Q_B}(\tau_S) - S_m\beta_{Q_B}(\tau_S)'S_m\end{aligned} \quad \text{(A106)}$$

$$\begin{aligned}\chi_m(\tau_S) = {} & (-S_1+S_m)\bar{v}_m(\tau_S+1) - \beta_0(\tau_S) \\ & - A_m(\tau_S+1)(-S_1+S_m)\left(\frac{\theta_m(\tau_S+1)+\theta_m(\tau_S+1)'}{2}\right)S_1'\Delta Q_D\end{aligned} \quad \text{(A107)}$$

$$\xi_m(\tau_S) = \beta_Q(\tau_S) - A_m(t)(-S_1+S_m)[(\theta_m(\tau_S+1)+\theta_m(\tau_S+1)')/2] \quad \text{(A108)}$$

The distressed sales cause the χ_m term of large investors reaction function to change through a price schedule effect because $\beta_0(\tau_S)$ changes, and through a risk-sharing effect because the distressed sales that are not absorbed by large investors will be absorbed by the competitive fringe.

Stacking the reaction functions together and solving for the equilibrium trades of the large investors, produces a solution for large investors trades which is analogous to that in equation (A24). The competitive fringes equilibrium trades are then given by $\Delta Q_D - S\Delta Q_B$. Following the development in equation (A25), we have

$$\Delta Q(\tau_S) = H_0(\tau_S) + H_1(\tau_S)Q(\tau_S). \tag{A109}$$

where,

$$H_0(\tau_S) = \begin{pmatrix} -S\Pi(\tau_S)^{-1}\chi(\tau_S) + \Delta Q_D \\ \Pi(\tau_S)^{-1}\chi(\tau_S) \end{pmatrix}, \quad \text{and} \quad H_1(\tau_S) = \begin{pmatrix} -S\Pi(\tau_S)^{-1}\xi(\tau_S) \\ \Pi(\tau_S)^{-1}\xi(\tau_S) \end{pmatrix}. \tag{A110}$$

From the proof of proposition 8, we know that $\chi_m(\tau_s) = 0$ when $\Delta Q_D = 0$. It then follows that $\chi(\tau_S)$ and $H_0(\tau_S)$ are linear functions of ΔQ_D. For convenience, I write these relationships as:

$$\chi(\tau_S) = \chi(\tau_S)^* \Delta Q_D, \tag{A111}$$

$$H_0(\tau_S) = H_0(\tau_S)^* \Delta Q_D. \tag{A112}$$

Then, substituting the solution for ΔQ_B into equation (A101)shows that solutions for the coefficients in the equilibrium price function in period τ_S take the form:

$$\alpha(\tau_S) = \frac{\bar{D}}{1-(1/r)} - \alpha(\tau_S)^*(\Delta Q_D) \tag{A113}$$

where

$$\alpha(\tau_S)^* = (1/r)\Gamma(\tau_S+1)S_1' + A_1(\tau_S+1)\Omega(\tau_S+1) + \beta_{Q_B}(\tau_S)\pi(\tau_S)^{-1}\chi(\tau_S)^*.$$

The expression for $\Gamma(\tau_S)$ is not changed by the distressed sales.

Also, the path of equilibrium asset holdings in moving from period τ_S to period $\tau_S + 1$ is given by:

$$Q(\tau_S+1) = G_0(\tau_S) + G_1(\tau_S)Q(\tau_S),$$

where $G_0(\tau_S) = H_0(\tau_S)$ and $G_1(\tau_S) = H_1(\tau_S) + I$.

In the last two expressions, the difference with the expressions in equations (A26) and (A28) is that $H_0(\tau_S)$ contains an additional term which reflects the fact that the distressed sales increase the amount of assets that are collectively held by all other market participants. This implies that for $t \leq \tau_S$, when there are distressed sales, then $H_0(t)$ does not equal 0.

Given the solutions for the equilibrium price schedule, reaction functions, and asset holding transitions, the solution for the value functions in period τ_S proceeds as in any period t and has the same functional form. For all periods t such that $\tau_R \leq t \leq \tau_S$, the value functions are solved by backwards induction using the same approach as used earlier. Finally, before period τ_R investors are not aware of the distressed sales, so the value function has the same form that was solved for in the undistressed sales case. Put differently, after trade in period

$\tau_R - 1$ is over, when investors learn about the future distressed sales, their value functions jump, and the future distressed sales create a basis for trade among the investors.

An important question is whether the distressed sales affect market liquidity after market participants learn of the sales, but before the sales occur. If liquidity is measured by the slope of the price function faced by large investors, then the answer is no. An examination of equations (A13) and (A14), and equation (A31) shows that the "slope" measure of market liquidity is determined by parameters that are invariant to the distressed sales when the quantity of distressed sales is known. Therefore, knowledge of the impending sales do not alter liquidity. Intuition for this result is that liquidity is based on market strucuture, and on the riskiness of each share of stock. Because the quantity of impending sales is known, it does not alter the riskiness of holding a share, and hence it has no effect on liquidity.[29] However, if the quantity of distressed sales at time τ_S is random instead, then this creates price risk at the time of the sales (prices are no longer deterministic) that will have liquidity effects. The effects of random sales on liquidity is examined further in what follows.

B.6 Random Distressed Sales

For simplicity, assume that distressed sales are normally distributed:

$$\Delta Q_D \sim \mathcal{N}(\mu_D, \Sigma_D) \tag{A114}$$

where $\mu > 0$ is interpreted as distressed purchases.

To solve for the effect that distressed sales have on investors value functions, I substitute the expressions for $H_0(\tau_S)$ and $\alpha(\tau_S)$ into large and small investors value functions and then take expectations with respect to the distribution of distressed sales. When doing so, I use the results from proposition 8 and 9 to simplify the analysis.

Large Investors Value Function of Entering Period τ_S

After substituting the new expressions for $H_0(\tau_S)$ and $\alpha(\tau_s)$ into large investors value functions, the value function conditional on ΔQ_D is an exponential linear quadratic function of ΔQ_D with the following form:

$$V_m(q_m, Q, \tau_s|\Delta Q_D) = \left[\frac{r^*_m(t)+1}{r^*_m(t)}\right] [\delta k_m(t) r^*_m(t)]^{\frac{1}{1+r^*_m(t)}} \times e^{-.5\Delta Q'_D \Phi^*_m \Delta Q_D - A_m(\tau_s) Q'(\bar{v}_m(\tau_s+1) + L^*_m \Delta Q_D)} \times e^{.5 A_m(\tau_s)^2 Q' \theta^*_m(\tau_s) Q} \tag{A115}$$

[29] In Kyle and Xiong (2002) additional share sales do alter liquidity because the sales alter investors wealth, and thus increase investors absolute risk aversion.

where, Φ^*_m is given by[30]:

$$\Phi^*_m = \text{Symm}\left(2A_m(\tau_s)H_0^{*'}S'_m\alpha^*/r - A_m(\tau_s)^2H_0^{*'}\theta_m(\tau_s+1)H_0^*/r\right),$$

$$\theta^*_m(\tau_s) = \frac{-2H_1(\tau_S)'S'_m\Gamma(\tau_S)}{rA_m(\tau_S)} + G_1(\tau_S)'\theta_m(\tau_S+1)G_1(\tau_S)/r + S'_m\Omega(\tau_S)S_m$$

and

$$L^*_m = H_1(\tau_s)'S'_m\alpha^*/r + \Gamma(\tau_s)'S_mH_0^*/r - A_m(\tau_s)G_1(\tau_s)'\left(\frac{\theta_m(\tau_s+1)+\theta_m(\tau_s+1)'}{2}\right)G_0^*/r$$

A sufficient condition for $E_{\Delta Q_D}\{V_m(q_m, Q, \tau_s|\Delta Q_D)\}$ to be bounded is that Φ^*_m is positive semidefinite. There is no guarantee that this condition will be satisfied. An alternative sufficient condition for the expectation to exist is that $\Sigma_D^{-1} + \Phi^*_m$ is positive definite. If Σ_D is a scalar multiple of some nonsingular matrix M then, it is clear that when the scalar multiple is close enough to zero, positive definiteness is guaranteed. I will assume that Σ_D is small enough to so that positive definiteness is guaranteed.

Taking expectations with respect to ΔQ_D then shows that:

$$V_m(q_m, Q, \tau_s) = k_m(\tau_S) \times e^{-A_m(\tau_S)q_m - A_m(\tau_s)Q'\bar{v}_m(\tau_S) + .5A_m(\tau_S)^2Q'\theta_m(\tau_S)Q} \tag{A116}$$

where,

$$\begin{aligned} k_m(\tau_S) &= |I+\Sigma_D\Phi^*_m|^{-.5} \times \left(\tfrac{r}{r-1}\right) \times (\delta r)^{\frac{1}{r-1}} e^{.5\mu'_D\Sigma_D^{-1}\left[(\Sigma_D^{-1}+\Phi^*_m)^{-1}-\Sigma_D\right]\Sigma_D^{-1}\mu_D}, \\ \bar{v}_m(\tau_S) &= \bar{v}_m(\tau_S+1) + L^*_m(\Sigma_D^{-1}+\Phi^*_m)^{-1}\Sigma_D^{-1}\mu_D, \\ \theta_m(\tau_S) &= \theta^*_m(\tau_S) + L^*_m(\Sigma_D^{-1}+\Phi^*_m)^{-1}L^{*'}_m \end{aligned} \tag{A117}$$

The random asset sales significantly change the parameters of the large investors value functions at time τ_S. $\bar{v}_m(\tau_s)$ is equal to its value without distressed sales $((\bar{v}_m(\tau_s+1))$ plus an additional term that reflects the random distressed sales. Similarly, $\theta_m(\tau_s)$ is equal to its value without distressed sales $(\theta^*_m(\tau_s))$ plus an additional term that reflects the distressed sales. The most significant change is that tedious calculations show that the $NM \times NM$ matrix $\theta_m(\tau_s)$ cannot be written as the kronecker product of an $M \times M$ matrix with Ω. This means that the asset pricing relationships between time periods τ_R (the time of the rumor) and τ_S (the time of the random distressed sales)are different than earlier periods. Even without the risk, pricing relationships are still affected by distressed sales as shown by taking limits of the value function as $\Sigma_d \to 0$. In this limit, $\theta_m(\tau_S)$ approaches its original value, but the asset sales affect $\bar{v}_m(\tau_s)$ by the amount $-L^*_m\mu_D$.This is sufficient to cause the asset pricing relationships to break down in the period between τ_R and τ_S.

Although the asset sales during time τ_S are random, large investors value function for time τ_S remains an exponential linear quadratic function of large investors risky asset holdings Q. This property will be convenient for solving the model backwards.

[30]The operator Symm() operates on squares matrices and returns the symmetric version of the matrix. For example, $\text{Symm}(X) = (X + X')/2$.

Small Investors Value Function of Entering Period τ_S

Following the same basic approach as for large investors, small investors value function conditional on ΔQ_D is exponential linear quadratic in ΔQ_D and takes the form:

$$V_s(q_s, Q_s, Q, \tau_S|\Delta Q_D) = \left(\frac{r}{r-1}\right) \times (\delta r)^{\frac{1}{r-1}} \times e^{-.5\Delta Q_D' \Phi_s^* \Delta Q_D - Q' \bar{v}_s^* \Delta Q_D - .5 Q' \theta_s^* Q}$$
$$\times e^{-A_s(\tau_s) q_s(\tau_s) - A_s(\tau_s) Q_s' \left[\frac{\bar{D}}{1-(1/r)} - \frac{\alpha^*}{r}\Delta Q_D - \frac{\Gamma(\tau_s)}{r} Q\right] + .5 A_s(\tau_S)^2 Q_s' \Omega(\tau_S) Q_s}, \tag{A118}$$

where,

$$\begin{aligned} \Phi_s^* &= \text{Symm}\left(\frac{a_0^{*\prime}\Omega(\tau_S+1)^{-1}a_0^* + G_0^{*\prime}\theta_s(\tau_s+1)G_0^*}{r}\right) \\ a_0^* &= \alpha^* - \frac{\Gamma(\tau_s+1)G_0^*}{r} \\ \bar{v}_s^* &= \frac{a_1(\tau_s)'\Omega(\tau_s+1)^{-1}a_0^* + G_1(\tau_s)'[(\theta_s(\tau_s+1)+\theta_s(\tau_s+1)')/2]G_0^*}{r} \\ \theta_s^* &= \frac{a_1(\tau_s)'\Omega(\tau_s+1)^{-1}a_1(\tau_s) + G_1(\tau_s)'\theta_s(\tau_s+1)G_1(\tau_s)}{r} \end{aligned} \tag{A119}$$

Then, taking expectations with respect to ΔQ_D shows that:

$$V_s(q_s, Q_s, Q, \tau_S) = -K_s(\tau_S) F(Q, \tau_S) e^{-A_s(\tau_s) q_s - A_s(\tau_s) Q_s' [\bar{D} + \hat{P}(Q,\tau_s)] + .5 A_s(\tau_s)^2 Q_s' \Omega^*(\tau_s) Q_s}, \tag{A120}$$

where,

$$K_s(\tau_s) = |I + \Sigma_D \Phi_s^*|^{-.5} \times \left(\frac{r}{r-1}\right) \times (\delta r)^{\frac{1}{r-1}} e^{.5\mu_D' \Sigma_D^{-1}\left[(\Sigma_D^{-1} + \Phi_s^*)^{-1} - \Sigma_D\right]\Sigma_D^{-1}\mu_D}, \tag{A121}$$

$$\begin{aligned} F(Q, \tau_s) &= e^{-Q'\bar{v}_s(\tau_s) - \frac{1}{2}Q'\theta_s(\tau_s)Q}, \\ \bar{v}_s(\tau_s) &= v_s^*(\Sigma_D^{-1} + \Phi_S^*)^{-1}\Sigma_D^{-1}\mu_D, \\ \theta_s(\tau_s) &= \theta_s^* - \bar{v}_s^*(\Phi_s^* + \Sigma_D^{-1})^{-1}\bar{v}_s^{*\prime}; \end{aligned} \tag{A122}$$

$$\begin{aligned} \hat{P}(Q, \tau_s) &= \tfrac{1}{r}\left(\alpha(\tau_s+1) - \Gamma(\tau_s)Q\right) - \tfrac{\alpha^*}{r}(\Sigma_D^{-1} + \Phi_s^*)^{-1}\Sigma_D^{-1}\mu_D + \tfrac{\alpha^*}{r}(\Sigma_D^{-1} + \Phi_s^*)^{-1}\bar{v}_s^{*\prime}Q \\ &= \tfrac{1}{r}\left(\left[\alpha(\tau_s+1) - \alpha^*(\Sigma_D^{-1} + \Phi_s^*)^{-1}\Sigma_D^{-1}\mu_D\right] - \left[\Gamma(\tau_s) - \tfrac{\alpha^*}{r}(\Sigma_D^{-1} + \Phi_s^*)^{-1}\bar{v}_s^{*\prime}\right]Q\right); \end{aligned} \tag{A123}$$

and,

$$\Omega^*(\tau_s) = \Omega(\tau_S) + \frac{\alpha^{*\prime}}{r}(\Phi_s^* + \Sigma_D^{-1})^{-1}\frac{\alpha^*}{r}. \tag{A124}$$

It is clear that the random distressed sales significantly affect the value function of the small investors. The variance of asset prices increases the variance of excess returns $[\Omega^*(\tau_s)]$

beyond the amount that they would be in the absence of distressed sales $[\Omega[\tau_s)]$. Additionally, because asset prices in period τ_S are random, and correlated with changes in the state variable Q, the correlation of asset prices with the state variable affects small investors demand for the risky assets. The correlations show up when deriving the pseudo-price function $\hat{P}(Q, \tau_S)$, which appears in small investors value functions. It should be noted that $\hat{P}(Q, \tau_S)$ is not the equilibrium price function in period τ_S and it is not the expected value of the equilibrium price function; instead it is a grouping of terms in the value function so that the small investors value function has the same form as in periods $t > \tau_S$. Because large investors value functions also have the same form as in periods $t > \tau_s$, the model can be solved backwards from this point using the same sets of Riccatti equations that were used in the earlier analysis.

More specifically, to analyze how random distressed sales affect the liquidity received by large investors, I use the pseudo price function in period τ_S to solve for $\beta_{Q_B}(\tau_s - 1)$, which is the slope of the price schedule faced by large investors in the period before the distressed sales take place. Applying the analysis in equations (A11) through (A14) shows that

$$\begin{aligned}\beta_{Q_B}(\tau_s - 1) = &\left\{(1/r)\Gamma(\tau_s)\begin{pmatrix} -S \\ I \end{pmatrix} - A_1(\tau_s)\Omega(\tau_s)S\right\} \\ &- \frac{\alpha^*}{r}(\Sigma_d^{-1} + \Phi_s^*)^{-1}\bar{v}_s^{*\prime}\begin{pmatrix} -S \\ I \end{pmatrix} - A_1(\tau_s)\frac{\alpha^{*\prime}}{r}(\Sigma_d^{-1} + \Phi_s^*)^{-1}\frac{\alpha^*}{r}S\end{aligned} \tag{A125}$$

Equation (A125) shows that in time period $\tau_s - 1$, the liquidity received by large investors is equal to the liquidity without the rumor (the term in braces) plus a term which reflects the covariance of prices with large investors asset holdings (the second term) plus a term which reflects an increased variance of prices (the third term). Examination of the third term shows that the distressed sales affect the liquidity received by all large investors by the same amount. But, the distressed sales affect the liquidity received by large investors by potentially different amounts because the covariance between prices and the state variables depends on each large investors willingess to absorb the distressed asset sales, and this varies across large investors who differ in their risk aversion.

To for how distressed sales affect liquidity and trades in earlier periods, it is necessary to use the analysis in this section to solve the model numerically.

B.7 Optimal Liquidations

Suppose there is a large investor who learns at time τ_R that he must liquidate his portfolio of risky assets by time τ_S. What is the optimal liquidation strategy that the investor should follow? To solve for the optimal liquidation strategy, I simply solve for the value functions of the liquidator, and the other investors in the period that the remainder of the position is liquidated. I then backwards induct to solve for investors value functions in earlier periods. Since the parameters of the value functions determine liquidity conditions, liquidity is endogenous: i.e. the liquidating investors price impact function and all other large investors price impact functions depend on the fact that the liquidating investor plans to liquidate by a certain date. Given the liquidity conditions, the trading path is also endogenous. The value functions of the investors are derived below. With these value functions in hand, it is straightforward to solve for the path of optimal liquidations.

The liquidator

The liquidating investor is assumed to be investor $M+1$ in the model. He maximizes the discounted expected utility of future consumption. Like other investors in the model, he has discount rate δ and CARA utility of per-period consumption with coefficient of absolute risk aversion A_{M+1}. At the time that he liquidates his portfolio of risky assets, he is assumed to choose his future consumption path subject to the constraint that he only holds the riskless asset in his portfolio. Under this assumption, straightforward dynamic programming show that after the risky asset portion of his portfolio has been totally liquidated, the value of the liquidators remaining wealth has form:

$$V_{M+1}(W) = \left(\frac{r}{r-1}\right)(r\delta)^{\frac{1}{r-1}} \times e^{-A(1-(1/r))W} \tag{A126}$$

If the liquidating investor enters period τ_S with risky asset holdings Q_{M+1}, and the other investors enter the period with the $NM \times 1$ vector of risky asset holdings Q, then from equation (A113) and proposition 2 we know that when the liquidating investor sells off his remaining assets in period τ_S, then equilibrium prices have the form:

$$P(Q, Q_{M+1}, \tau_S) = \frac{1}{r}\left(\frac{\bar{D}}{1-(1/r)} - \alpha(\tau_S)^* Q_{M+1} - \Gamma(\tau_s)Q\right). \tag{A127}$$

Therefore, the liquidating investors after liquidation wealth, W, is equal to

$$W = q_{M+1} + Q'_{M+1}[D(\tau_s) + P(Q, Q_{M+1}, \tau_S)],$$

where q_{M+1} is the cash carried into the period, and the following two terms are revenues from dividends within the period and revenues from liquidating the risky asset position. Substituting the expression for equilibrium price into wealth, and then substituting the result into the liquidating investors value function and taking expectations over the distribution of dividends, shows that to the liquidating investor, the value of entering period τ_S when the vector of risky asset holdings in the economy is given by $Q^* = (Q', Q'_{M+1})'$, is exponential linear quadratic in the state variables and has form:

$$V_{M+1}(q_m, Q^*, \tau_S) = -K_{M+1}(\tau_s)e^{-A_{M+1}(\tau_S)Q^{*'}\bar{v}_{M+1}(\tau_s)+.5A_{M+1}(\tau_s)Q^{*'}\theta_{M+1}(\tau_S)Q^* - A_{M+1}(\tau_S)q_{m+1}}, \tag{A128}$$

where,

$$
\begin{aligned}
A_{M+1}(\tau_s) &= A_{M+1}(1-(1/r)) && \text{(A129)}\\
K_{M+1}(\tau_s) &= \left(\frac{r}{r-1}\right) \times (\delta r)^{\frac{1}{r-1}} && \text{(A130)}\\
\bar{v}_{M+1}(\tau_S) &= S'_{M+1}\bar{D}/[1-(1/r)], && \text{(A131)}
\end{aligned}
$$

and,

$$
\theta_{M+1}(\tau_S) = \begin{bmatrix} 0_{[NM \times NM]} & \Gamma(\tau_S)'/A_{M+1}(\tau_S) \\ \Gamma(\tau_S)/A_{M+1}(\tau_S) & [\alpha^*(\tau_S)/A_{M+1}(\tau_S)] + \Omega \end{bmatrix}. \quad \text{(A132)}
$$

The value functions for the large and small investors can be derived similarly.

Large Investors Value Function at Time τ_S

The expression for large investors value function conditional on distressed sales of amount ΔQ_D at time τ_S is provided in equation (A115). Making the subsitution $\Delta Q_D = Q_{M+1}$, and simplifying then shows that large investors value function (investors 2 through M) in time period τ_S has form:

$$
V_m(q_m, Q^*, \tau_S) = -K_m(\tau_s)e^{-A_m(\tau_S)Q^{*'}\bar{v}_m(\tau_s)+.5A_m(\tau_s)Q^{*'}\theta_m(\tau_S)Q^* - A_m(\tau_S)q_m}, \quad \text{(A133)}
$$

where,

$$
\begin{aligned}
A_m(\tau_s) &= A_m(1-(1/r)) && \text{(A134)}\\
k_m(\tau_s) &= \left(\frac{r}{r-1}\right) \times (\delta r)^{\frac{1}{r-1}} && \text{(A135)}\\
\bar{v}_m(\tau_S) &= S'_m\bar{D}/[1-(1/r)], && \text{(A136)}
\end{aligned}
$$

and,

$$
\theta_m(\tau_S) = \begin{bmatrix} \theta_m(\tau_s)^* & -L^{*'}_m/A_m(\tau_S) \\ -L^*_m/A_m(\tau_S) & -\frac{\Phi^*_m(\tau_S)}{A_m(\tau_S)^2} \end{bmatrix}. \quad \text{(A137)}
$$

Small Investors Value Function at Time τ_S

Small investors value function conditional on distressed sales ΔQ_D at time τ_S is provided in equation (A118). Simplifying this expression using the same steps as for large investors shows that small investors value function is given by:

$$
V_s(q_s, Q_s, Q^*, \tau_S) = K_s(\tau_s)F(Q^*, \tau_s)e^{-A_s(\tau_s)q_s - A_s(\tau_s)Q'_s[\bar{D}+P(Q^*,\tau_s)]+.5A_s(\tau_s)^2Q'_s\Omega^*(\tau_s)Q_s}, \quad \text{(A138)}
$$

where,

$$A_s(\tau_s) = A_s[1-(1/r)] \tag{A139}$$

$$K_s(\tau_s) = \left(\frac{r}{r-1}\right) \times (\delta r)^{\frac{1}{r-1}} \tag{A140}$$

$$F(Q^*, \tau_S) = e^{-Q^{*'}\bar{v}_s(\tau_S) - \frac{1}{2}Q^{*'}\theta_s(\tau_s)Q^*} \tag{A141}$$

$$P(Q^*, \tau_S) = \frac{1}{r}\left(\frac{\bar{D}}{1-(1/r)} - \Gamma^*(\tau_S)Q^*\right) \tag{A142}$$

and,

$$\begin{aligned} \Gamma^*(\tau_S) &= [\Gamma(\tau_s), \alpha^*(\tau_S)] \\ \bar{v}_s(\tau_S) &= 0_{[N(M+1)\times 1]} \\ \theta_s(\tau_S) &= \begin{pmatrix} \theta_s^* & \bar{v}_s^* \\ \bar{v}_s^{*'} & \Phi_s^* \end{pmatrix}. \end{aligned} \tag{A143}$$

Because all three types of investors at time τ_S have value functions that have the same form as in proposition 2, it is possible to solve for their value functions and trades using the same approach as in section B of the appendix.

B.8 Liquidity Received by the Large Investors

The paper uses two measures of the liquidity received by large investors, the first is the slope of the price schedule with respect to large investors trades, or β_{Q_B} from equation (A11). The second is the price response when a large investor has to sell a share of stock for exogenous reasons to the other large investors. I evaluate this price effect when all investors asset holdings are initially Q_W. To model this effect, I assume that the reaction function for the large investor with the exogenous shock takes the form $\Delta Q_m = -\iota_N$, and N-vector of ones. I substitute this reaction function for that of investor m in equation (A23) to solve for ΔQ_B. The resulting price effect comes from equation (A11) and is equal to $-\beta_{Q_B}\Delta Q_B$.

B.9 Competitive Benchmark Model

It is useful to contrast the behavior in the multi-market model with large investors with the behavior of asset prices and trades in the same model when all investors are price takers. The derivation of prices and trades in this case is a special case of the derivation with large investors. It is also a special case of the derivation in Stapleton and Subrahmanyam (1978). Therefore, I will not provide a detailed derivation, but will instead just provide results.

There are two cases to consider. The first is that investors are infinitely lived and trade risky assets forever. The second is that investors live forever, but that after period $T-1$ trade in risky assets ceases, and investors consume their dividends and invest in the risk-free asset.

Case 1: Infinitely lived investors / Risky Asset Trade in All Periods.

In this infinite period set-up with competitive markets, the equilibrium risk-premium should be time invariant. Denote this risk premium by ρ, where,

$$\rho = P(t+1) + \bar{D} - rP(t) \tag{A144}$$

Solving this equation forward while imposing the transversality condition $\lim_{t\to\infty} r^{-t}P(t) = 0$, shows that

$$P(t) = \frac{\bar{D} - \rho}{r - 1}$$

for all time periods t.

Given the hypothesized behavior of prices, it remains to solve for ρ and then to show that the hypothesized behavior of prices is consistent with equilibrium.

The function,

$$V_m(W,t) = -\frac{r}{r-1}(A_m\ r\ \delta)^{\frac{-1}{r-1}} \exp^{-A_m(1-(1/r))W - \frac{.5\rho'\Omega^{-1}\rho}{r-1}}$$

where $\rho = \frac{(1-(1/r))\Omega X}{\sum_{m=1}^{M}(1/A_m)}$

satisfies the Bellman equation,

$$V_m(W,t) = \max_{\substack{C_m(t),\\ Q_m(t),\\ B_m(t+1)}} \quad -e^{-A_m C_m(t)} + E_t\{\delta V_m(W(t+1), t+1)\},$$

such that,

$$W(t+1) = Q_m(t)'(P(t+1) + D(t+1)) + B_m(t+1),$$

and

$$B_m(t+1) = r(W(t) - Q_m(t)'P(t) - C_m(t)).$$

In addition, agents optimal choices of $Q_m(t)$ at each time t satisfy the market clearing condition for the hypothesized ρ. Substituting the hypothesized ρ into the expression for equilibrium price, it follows that in a competitive equilibrium, the equilibrium price is given by

$$P(t) = \frac{\bar{D}}{r-1} - \frac{\Omega X}{r\sum_{m=1}^{M}\frac{1}{A_m}} \tag{A145}$$

Case 2: Infinite Lived Agents, but no Risky Asset Trade After Period T-1

In this case, the value of carrying risky assets into period T is given in equation (A57). At time $T-1$, when investors have this value function at time T, it turns out that the equilibrium price of risky assets at time T_1 is

$$P(T-1) = \frac{\bar{D}}{r-1} - \frac{\Omega X}{r \sum_{m=1}^{M} \frac{1}{A_m}}, \tag{A146}$$

which is the same as the price at time $T-1$ in case 2. Moreover, the price at all time periods before $T-1$ is the same as at period $T-1$.

C Details on The Simulations

For the simulations reported in section 3.2, and for the analysis on distressed sales, the instantaneous risk aversions of investors 1 through 6 are 10, 1.8328889, 4.5822222, 11.455556, 28.638889, and 71.597222, respectively. This corresponds to the risk tolerances of large investor $m+1$ having a risk tolerance equal to 0.4 times the risk tolerance of large investor m for $m=2$ to $m=5$. The annualized risk free rate is $r=1.02$; the annualized discount rate is $\delta=0.9$; annualized dividends are normally distributed with mean$\bar{D}=1$ and variance $\Omega=1$. These parameters have to be scaled based on the trading frequency. I assume trades occur once a day and that there are 250 trading days per year, which corresponds to a period length of $h=1/250$. The appropriate scaling of interest rates and discount rates are r^h and δ^h. For daily dividends, the mean and variance are $\bar{D}h$ and Ωh. Finally, each participants daily risk aversion is scaled to be A/h. The number of outstanding shares of assets only affects the level of prices, but not price dynamics or risk premia. For simplicity, the supply of assets is normalized to 1. When there are distressed sales, the total amount of distressed sales is 5.5 shares.

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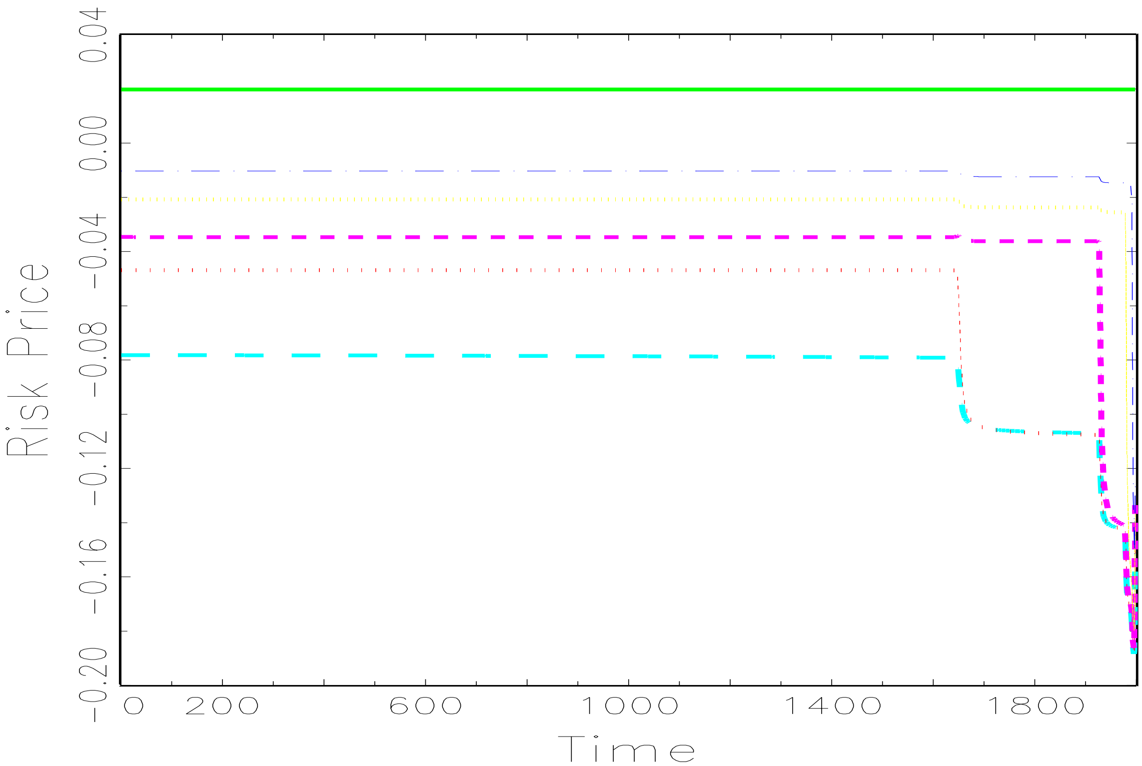
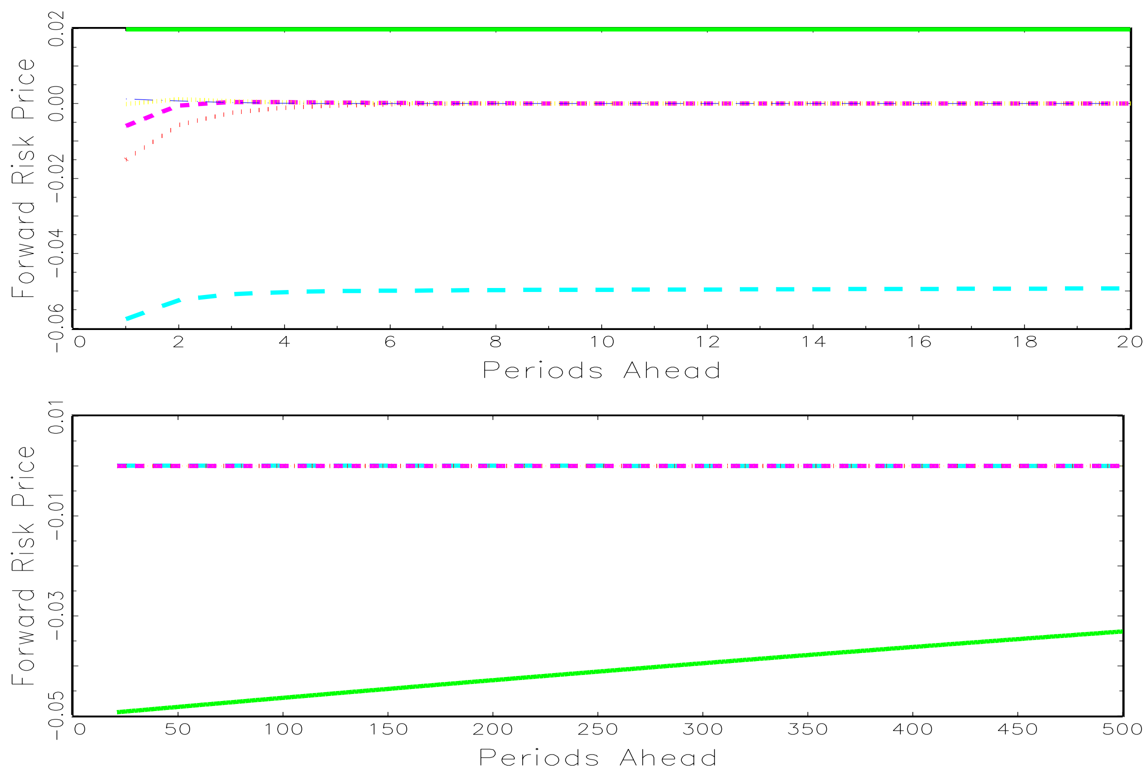
Figure 1: **Prices of Risk**



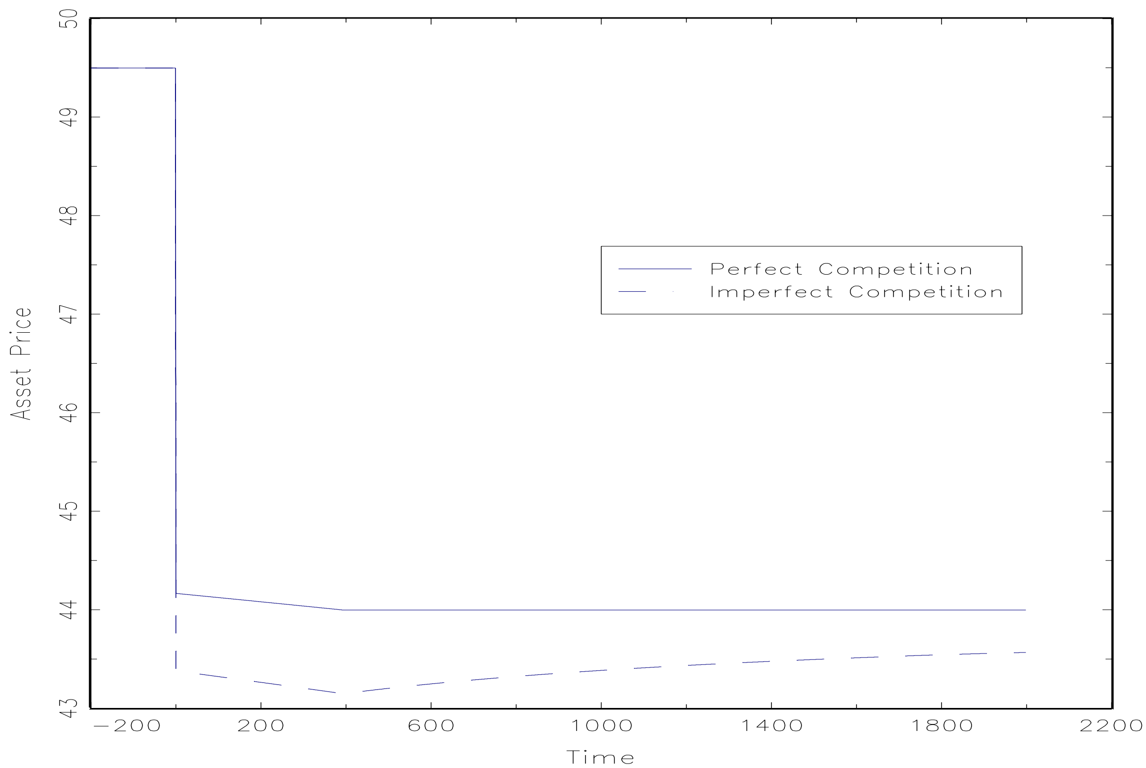
Notes: The figure presents the time-series behavior of equilibrium prices for factor risk, where the factors are the market portfolio, and deviations of each large investors risky asset holdings from those associated with perfect risksharing. The price for the market factor is denoted by the solid line. The large investors vary in their risk aversion, investor 2 is the least risk averse and investor 6 is the most risk averse. The risk prices also vary by risk aversion. The lowest risk prices (most negative) correspond to investor 2 while the highest nonpositive risk price corresponds to investor 6.

Figure 2: **Forward Prices of Risk**



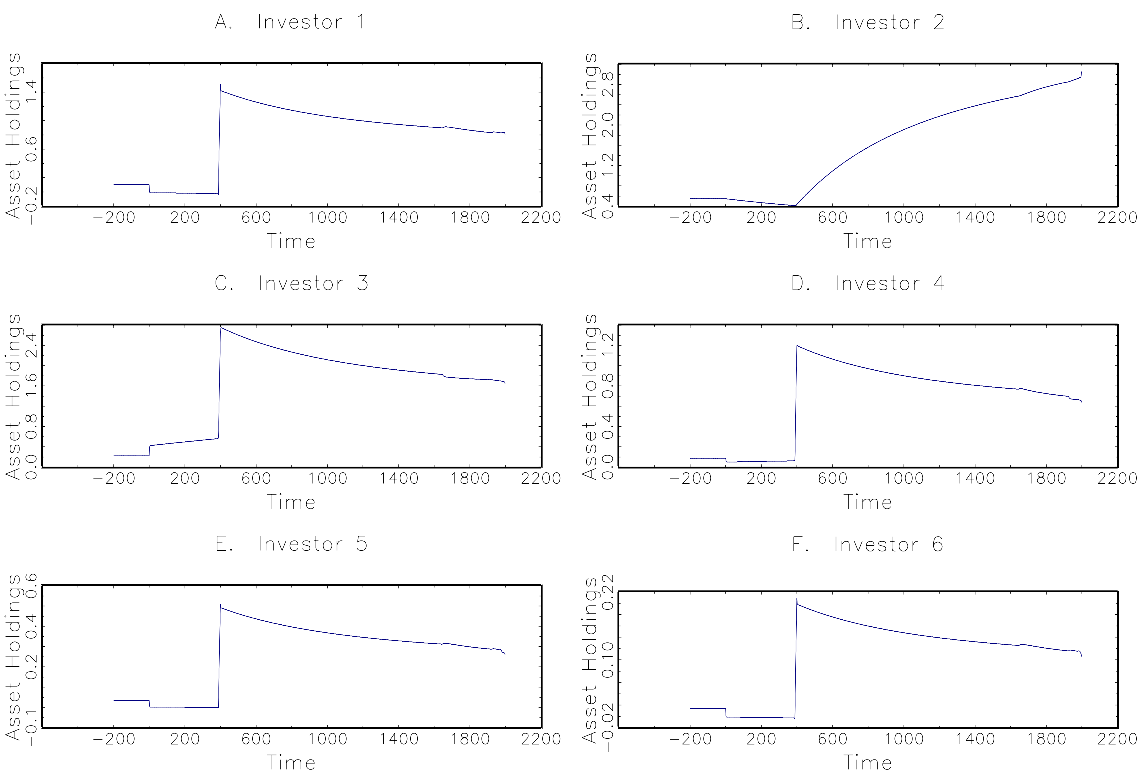
Notes: The figure presents the time-series behavior of equilibrium forward prices for factor risk, where the forward price measures the effect that deviations from optimal risk sharing at time t have on asset returns at forward time $t + \tau$. In the figure $t = 1000$. The deviations from optimal risk sharing correspond to risk factors as discussed in figure 1. The forward price for the market factor is denoted by the solid line. The large investors vary in their risk aversion, investor 2 is the least risk averse and investor 6 is the most risk averse. The forward risk prices also vary by risk aversion. The lowest forward risk price in each panel (most negative) correspond to investor 2 while the highest nonpositive forward risk price corresponds to investor 6.

Figure 3: **Equilibrium Price Paths**



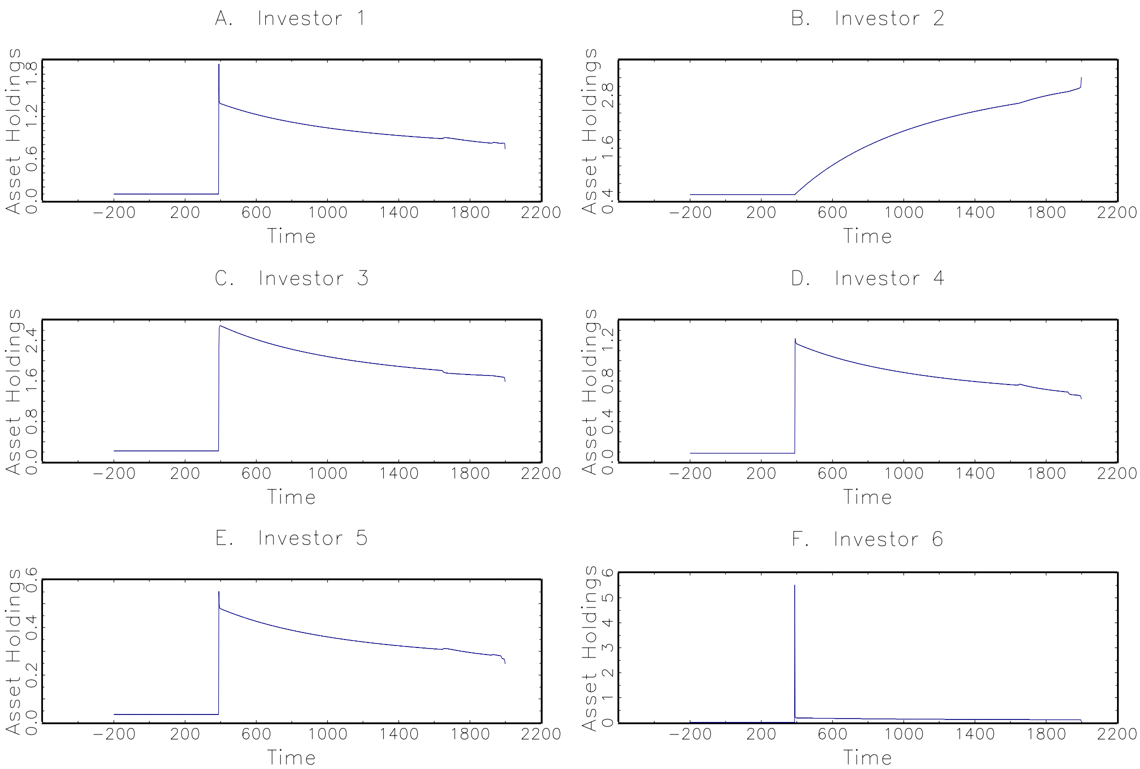
Notes: The figure illustrates equilibrium price paths when investors hear a rumor at time 0 (which they know to be true) that future distressed sales will occur during time periods $390 - 400$. Price dynamics are presented when asset markets are competitive and when they are imperfectly competitive. The imperfectly competitive price path assumes that large investors vary sharply in their risk aversion. The associated trades are provided in figure 4. Further details are in section 4 in the text.

Figure 4: **Equilibrium Trades when there is a Distressed Investor**



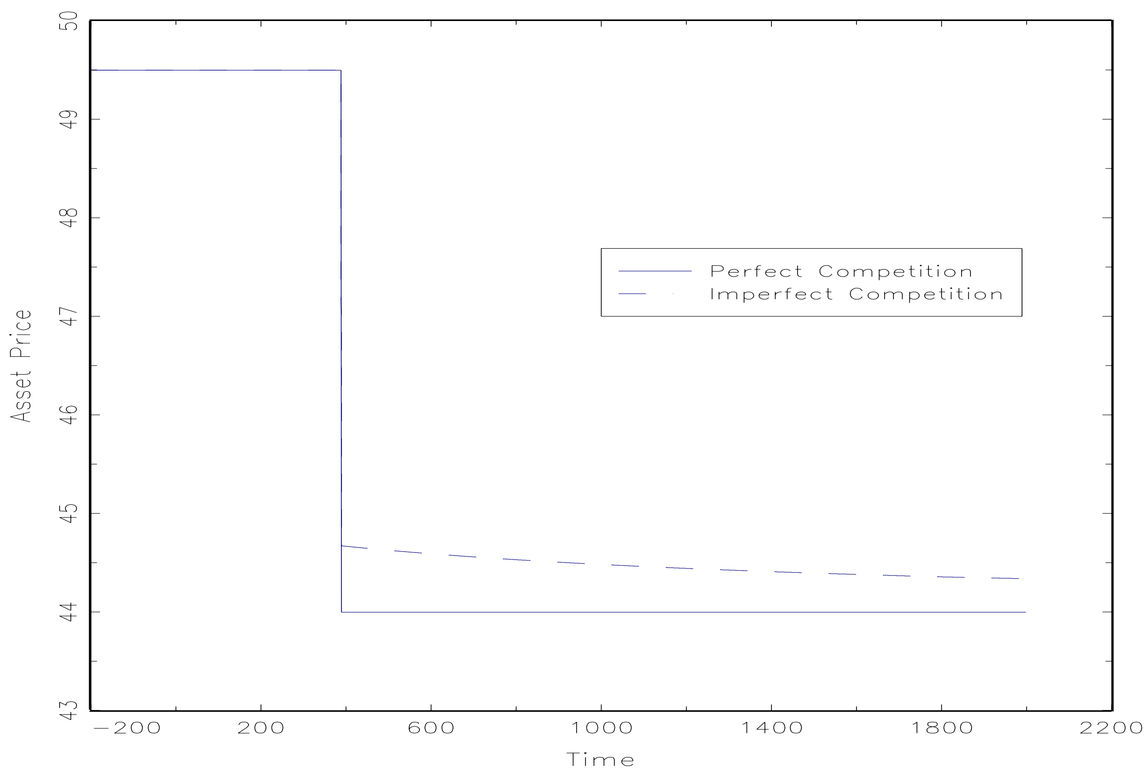
Notes: When markets are imperfectly competitive, the figure presents the equilibrium trade patterns that emerge in response to the rumor of distressed sales at time 0 followed by the distressed sales from time periods 390-400. Investor 1 takes prices as given; investors 2-6 are non-price taking investors who differ in their risk aversion. Large investors risk aversion is increasing in investor number. Details on the trade patterns are provided in section 4 of the text.

Figure 5: **Trade Response: Endowment Shock to Investor 6**



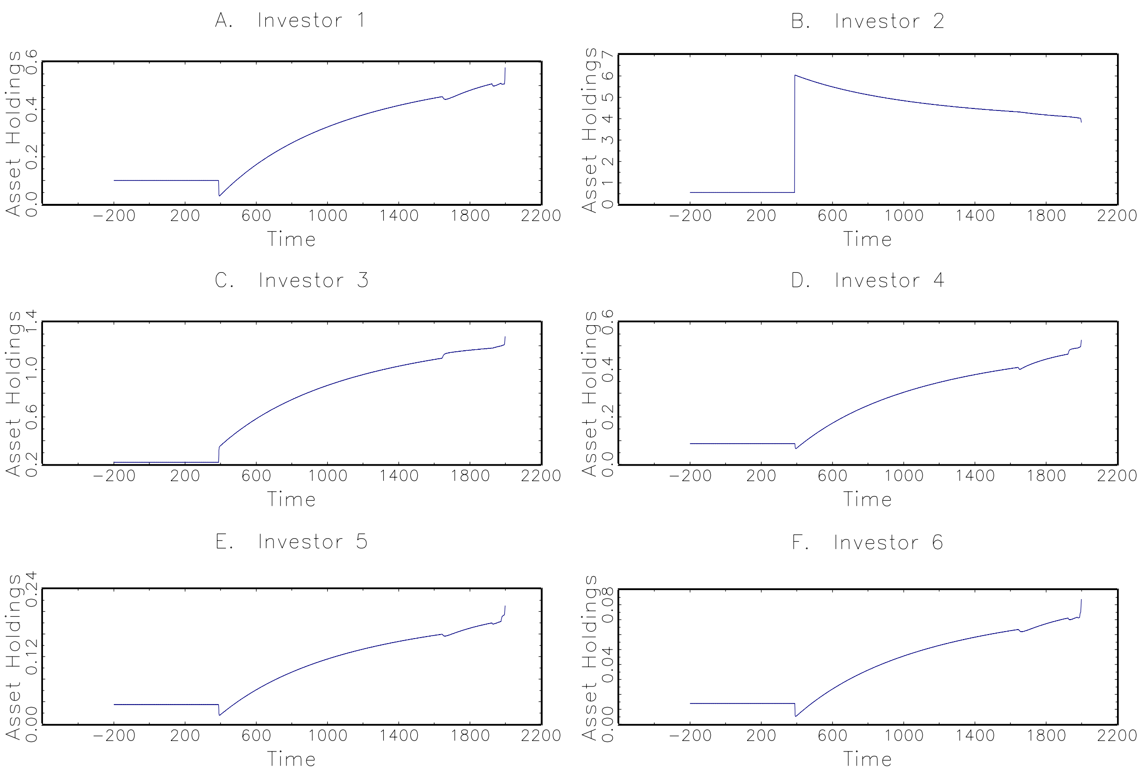
Notes: The figure presents the equilibrium trade response when large investor 6 receives a positive endowment shock at time 400, and is then free to sell part of her endowment through time to other investors.

Figure 6: **Price Response: Endowment Shock to Investor 2**



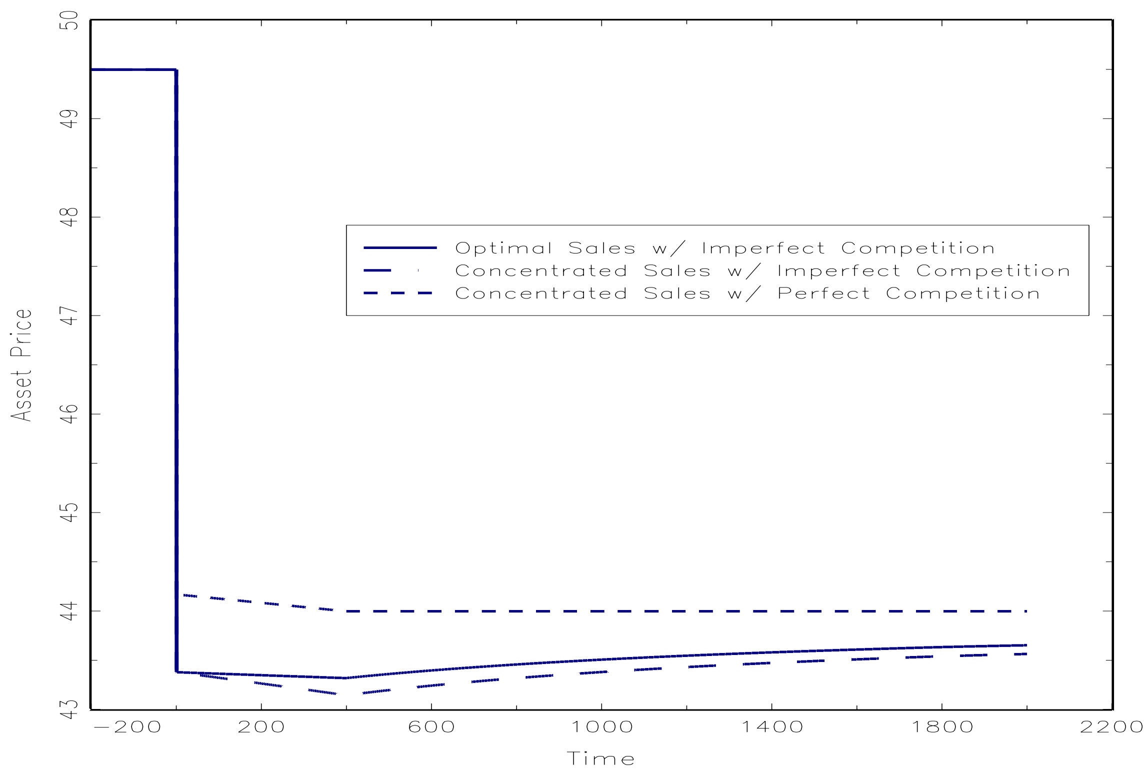
Notes: The figure presents the equilibrium price response when large investor 2 receives a positive endowment shock at time 400, and is then free to sell part of her endowment through time to other investors. Equilibrium trades are presented in figure 7.

Figure 7: **Trade Response: Endowment Shock to Investor 2**



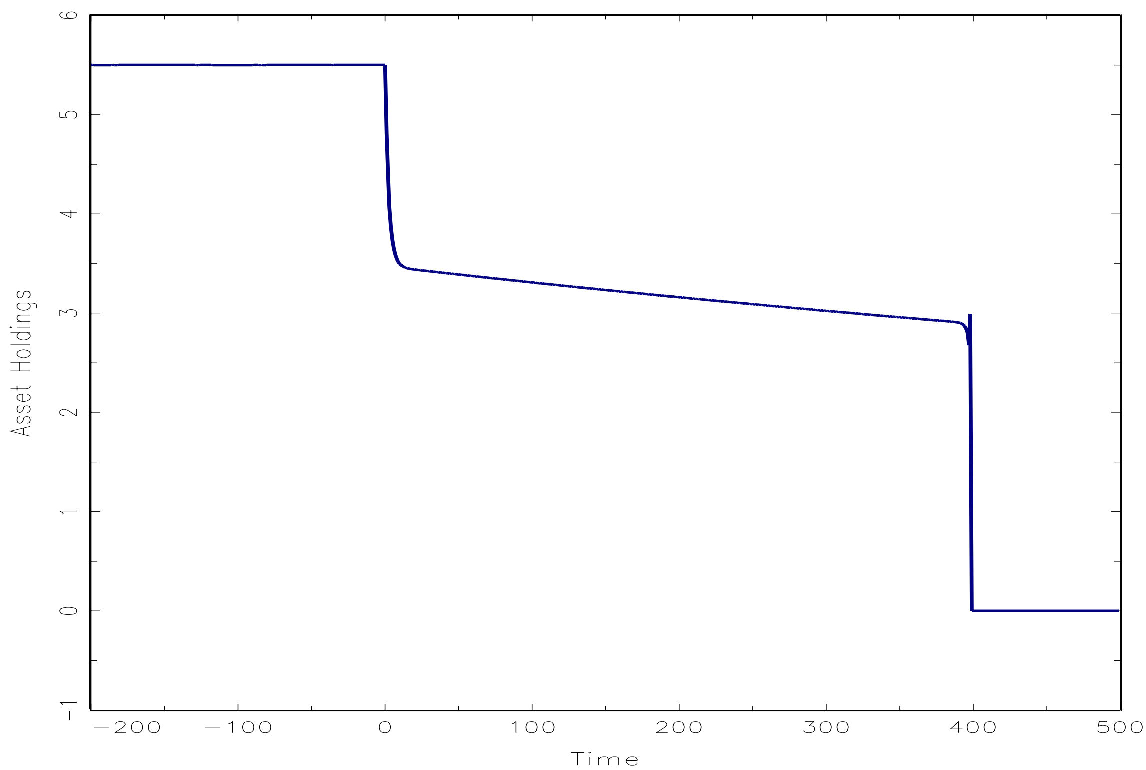
Notes: The figure presents the equilibrium trade response when large investor 2 receives a positive endowment shock at time 400, and is then free to sell part of her endowment through time to other investors. Equilibrium prices are presented in figure 6.

Figure 8: **Price Path with Optimal Liquidation**



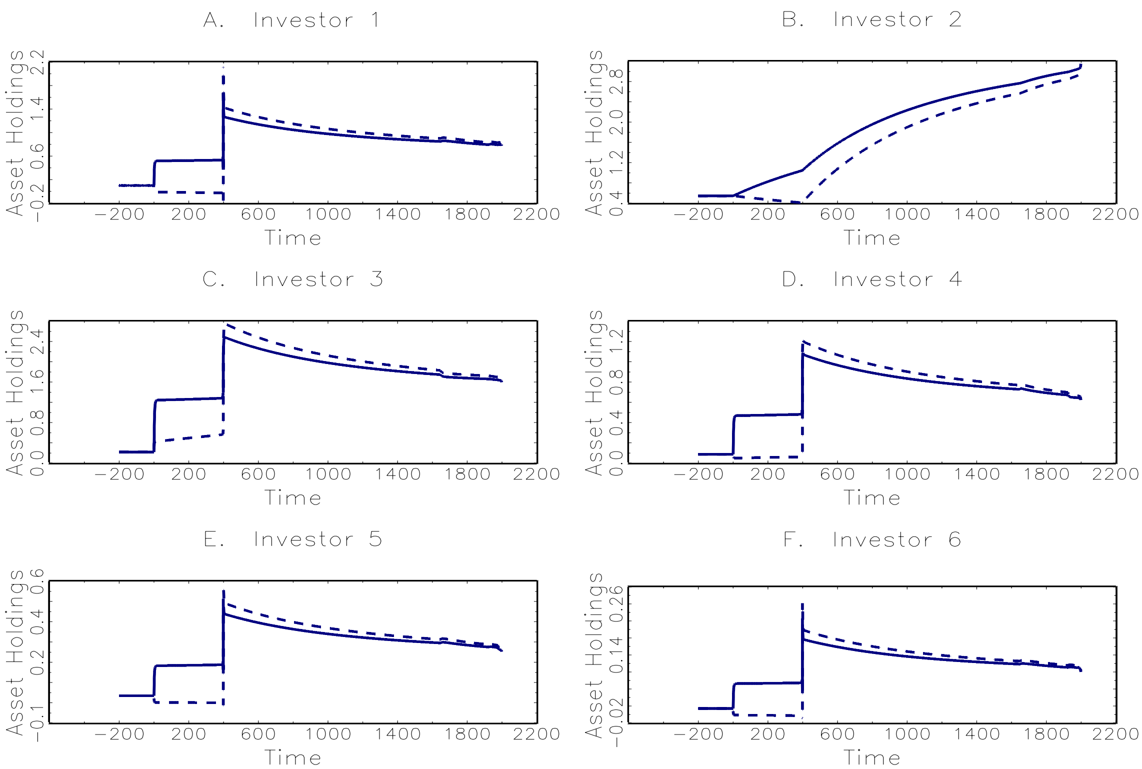
Notes: The figure presents price paths when large investors learn at time 0 that one large investor must liquidate his risky asset position by time 400 and then exit the market. The liquidation scenarios are optimal sales into illiquid, imperfectly competitive, markets (solid line); sales into illiquid markets that are concentrated at time 400 (dashed line); and sales into liquid, perfectly competitive, markets that are concentrated at time 400 (short dashed line). The asset holdings for the optimally liquidating large investor are presented in figure 9, and the asset holdings for the other investors are presented in figure 10.

Figure 9: **Trades by Optimally Liquidating Distressed Investor**



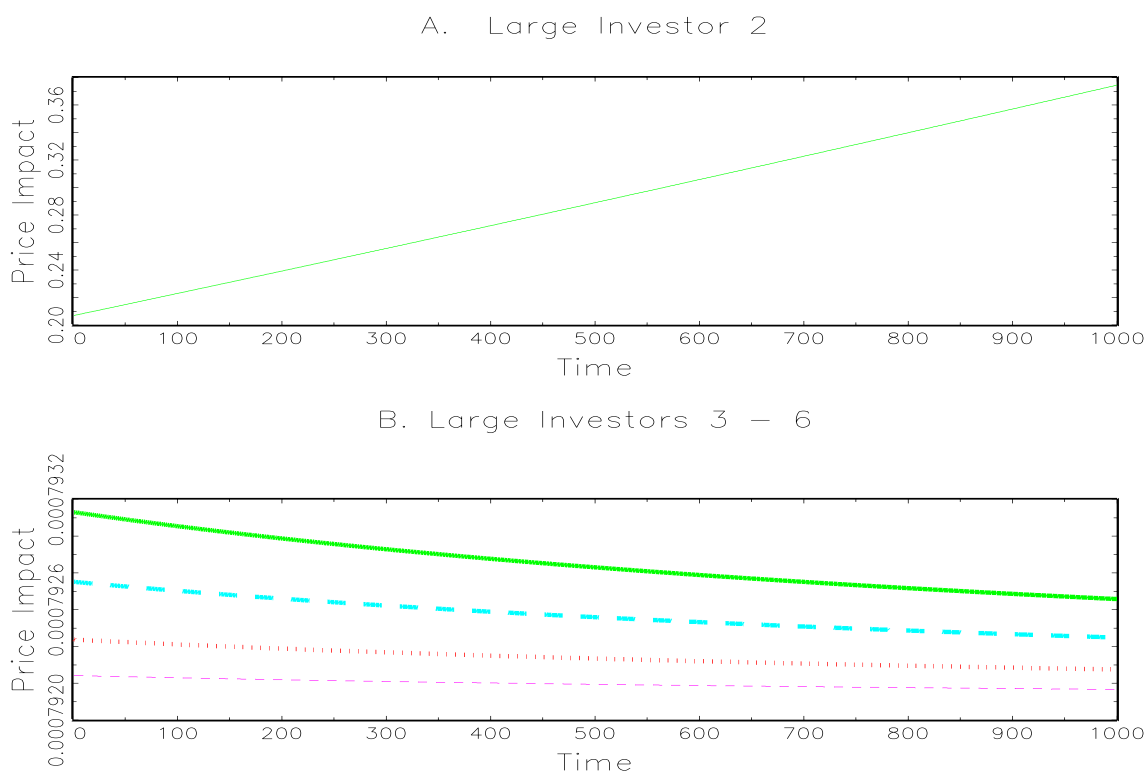
Notes: The figure presents the path of optimal asset holdings for a large investor when all investors learn at time 0 that he must liquidate his risky asset position by time 400 and then exit the market. The optimal asset holdings for the other investors are presented in figure 10. The equilibrium price path is presented in figure 8.

Figure 10: **Investors Asset Holdings During an Optimal Liquidation**



Notes: The figure presents other investors paths of equilibrium asset holdings when one large investor learns at time 0 that he must liquidate his risky asset holdings by time 400 and then exit the market. Asset holdings are presented for a scenario in which the liquidating investor can follow an optimal liquidation strategy (solid lines), and for a scenario in which his asset sales are concentrated at time 400 (dashed lines). The optimal trades for the liquidating investor are presented in figure 9. The price paths for the optimal and concentrated sales scenarios are presented in figure 8.

Figure 11: **Slope of Price Impact Functions by Large Investor**



Notes: The figure presents the price impact of large investors trades through time. Price impact is the slope of the demand curve that large investors face when deciding to purchase risky assets. The slope measures the per unit change in asset prices if the large investor buys 1 additional share when the positions of the other large investors are held fixed. When there is more than one large investor, the price impact per share sold varies by large investors risk aversion. More risk tolerant investors have a larger slope, i.e. their trades have a larger price impact.