Basel II: A Case for Recalibration

by

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ABSTRACT

Primary objectives for the U.S. implementation of Basel II are the enhancement of financial system stability and the promulgation of sound standards for risk measurement and minimum capital regulations. U.S. implementation of Basel II is focused on the AIRB approach which will be mandatory for large internationally active banks. Given the potential for large reductions in minimum bank capital that may materialize under the AIRB approach, it is important to assess whether or not these reductions are justified by improvements in risk measurement. There is a strong presumption in most Basel II related documents and policy discussions that the AIRB approach represents a sound scientific standard for measuring bank minimum capital needs. Unfortunately, this confidence is misplaced. Given the state of knowledge concerning credit risk measurement and capital allocation, there is a large body of evidence that shows that the AIRB framework will undercapitalize credit risks.

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1. INTRODUCTION

Under the June 2004 Basel II agreements, national supervisory authorities may choose among three alternative frameworks to set minimum regulatory capital for their internationally active banks. The standardized approach sets minimum capital standards using a modified version of the 1988 Basel Capital Accord that links capital requirements to external credit ratings. The remaining two approaches, the so-called Foundation and Advanced (AIRB) Internal Ratings Based approaches use a regulatory model to assign minimum capital requirements using bank estimates of an individual credit's probability of default (*PD*), loss given default (*LGD*), and expected exposure at default (*EAD*).

Under the March 2006 draft Notice of Proposed Rulemaking, the U.S. implementation of the Basel II will include a modified version of the AIRB framework that will be mandatory for the largest internationally active banks.[1] A revised version of the Basel Accord, so-called Basel 1A, has been proposed as an alternative regulatory standard for non-AIRB banks. Basel 1A, which may include the current Basel Accord as an optional approach, has yet to be finalized as a proposal. Preliminary indications suggest that it may share common features with the Basel II Standardized Approach.

In the June 2006 discussion of the Basel II framework, the Basel Committee on Banking Supervision (BCBS) outlines its objectives for the revised Capital Accord. These include [BCBS 2006b, pages 2-4]:

- Strengthen the soundness and stability of the international banking system
- Promote the adoption of stronger risk management practices

[1] In the U.S., banking supervisors have determined that Basel II implementation will require only the largest banks, the so-called core banks, to adopt the AIRB approach, while other banks may petition supervisors for AIRB capital treatment (so-called opt-in banks). Core banks are defined as institutions with total consolidated assets (excluding insurance subsidiary assets) in excess of $250 billion or total on-balance-sheet foreign exposure of $10 billion or more.

- Institute more risk-sensitive capital requirements that are conceptually sound
- Provide a detailed set of minimum requirements designed to ensure the integrity of bank internal risk assessments
- Broadly maintain the aggregate level of capital requirements
- Prevent capital adequacy regulation from becoming a significant source of competitive inequality among internationally active banks
- Create incentives for the adoption of the more advanced framework approaches.

This paper will review the available evidence and assess the degree to which the U.S. implementation of the Basel II framework, the AIRB approach for setting capital, promises to meet the ambitious goals articulated by the international bank supervisory community. The assessment will focus on the Basel II goals of improving financial stability and promoting sound risk measurement practices.

The paper begins with a review of the AIRB approach including the logic used to set minimum capital requirements, the mathematical foundations of the rule itself, and the calibrations that have been selected in the U.S. implementation. Following this discussion, we review the existing evidence on the likely capital implications of Basel II and contrast these results with the goal of financial stability. Section 3 analyzes the AIRB as a risk measurement standard. We consider the benefits it may engender as it functions as the minimum risk measurement standards for bank internal risk measurement systems. A final section concludes the paper.

2. A Review of the AIRB Capital Framework

The introductory section of the draft US Basel II NPR [March 2006] explains the logic that the Basel II AIRB rules use for calculating minimum capital requirements. To set minimum capital needs, the AIRB focuses on the probability distribution of *potential credit losses*. The Basel II "soundness standard" for participating institutions is defined as the percentage of potential losses that must be covered by bank capital. The soundness standard determines the minimum probability that a bank will remain solvent over the coming year (e.g., 99.9 percent) [US NPR, p. 63].

To restate the logic of the Basel II AIRB rule in statistical terms, let $\widetilde{PL}$ represent a credit portfolio's random potential loss, and $\widetilde{PL} \sim \Psi(PL)$ the cumulative distribution function for potential credit losses. The AIRB capital rule sets minimum capital equal to $\Psi^{-1}(.999)$, or the inverse of the cumulative portfolio credit loss distribution evaluated at the 99.9 percentile.

The AIRB framework uses a regulatory model to approximate a bank's credit loss distribution and arrive at an estimate of $\Psi^{-1}(.999)$. The framework is a modified version of the single factor Gaussian credit loss model first proposed by Vasieck (1991). Using a very restrictive set of assumptions, this model creates a synthetic probability distribution for the default rate on a perfectly diversified portfolio of identical credits. AIRB capital requirements are set using a tail value of this synthetic distribution for alternative portfolios.

The single common factor Gaussian model of portfolio credit losses uses a latent random factor to determine whether an individual credit defaults. There is a unique latent factor for each credit with the properties,

$$\begin{aligned} &\tilde{V}_i = \sqrt{\rho}\,\tilde{e}_M + \sqrt{1-\rho}\,\tilde{e}_i \\ &\tilde{e}_M \sim \phi(e_M) \\ &e_i \sim \phi(e_i), \\ &E(\tilde{e}_i\,\tilde{e}_j) = E(\tilde{e}_M\,\tilde{e}_j) = 0\ \forall\, i, j. \end{aligned} \tag{1}$$

$\tilde{V}_i$ is normally distributed with $E(\tilde{V}_i) = 0$, and $E(\tilde{V}_i^{\,2}) = 1$. $\tilde{e}_M$ is a factor common to all credits' individual latent factors, and the correlation between individual latent factors is ρ.

Firm *i* is assumed to default when $\tilde{V}_i < D_i$ and so the unconditional probability that firm *i* will default is, $PD = \Phi(D_i)$. The loss incurred should the firm default, *LGD*, is exogenous to the model. Time does not play an independent role in this model but is implicitly recognized through the calibration of input values; *PD,* for example*,* will differ according to the capital allocation horizon.

The model calculates the portfolio default rate distribution for a portfolio of *N* credits, where *N* is a very large number, and each credit is identical regarding its default threshold, $D_i = D$, and its latent factor correlation, ρ. For such a portfolio, credit losses depend only on the default rate experienced by the portfolio. The capitalization rate required for a single credit added to this so-called "asymptotic" portfolio is identical to the capitalization rate for the entire portfolio because idiosyncratic risks have been fully diversified. The model calculates capital for a perfectly diversified portfolio and ignores capital needs generated by risk concentrations.

The probability distribution for the portfolio default rate is defined using an indicator function,

$$\tilde{I}_i = \begin{cases} 1 & if \quad \tilde{V}_i < D \\ 0 & otherwise \end{cases} \tag{2}$$

$\tilde{I}_i$ has a binomial distribution with an expected value of $\Phi(D)$. Conditional on a specific value for e_M, these default indicators are independent and identically distributed binomial random variables. The default rate on a portfolio of N credits is $\tilde{X} = \frac{\sum_{j=1}^{N} \tilde{I}_j}{N}$.

If $\tilde{I}_j \mid e_M$ is used to represent the distribution of $\tilde{I}_i$ conditioned on a realized value $\tilde{e}_M = e_M$, then as $N \to \infty$, the Strong Law of Large Number requires,

$$\lim_{n\to\infty}\left(\tilde{X} \mid e_M\right) = \lim_{n\to\infty}\left(\frac{\sum_{i=1}^{n}\left(\tilde{I}_i \mid e_M\right)}{n}\right) \xrightarrow[a.s.]{} E\left(\tilde{I}_i \mid e_M\right) = \Phi\left(\frac{D-\sqrt{\rho}\, e_M}{\sqrt{1-\rho}}\right) \qquad (3)$$

Minimum capital requirements are set using the inverse of this unconditional distribution function, $\Psi^{-1}(\alpha), \quad \alpha \in [0,1]$. Substituting for the default barrier, $D = \Phi^{-1}(PD)$, and the identity, $\Phi^{-1}(\alpha) = -\Phi^{-1}(1-\alpha)$, the inverse of the unconditional cumulative distribution function for the portfolio default rate, $\Psi^{-1}(\alpha), \alpha \in [0,1]$, is given by,

$$\Psi^{-1}(\alpha) = \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right), \quad \alpha \in [0,1] \qquad (4)$$

Assuming a constant exposure (*EAD*) for each credit in the portfolio and an exogenous loss given default (*LGD*) per \$1 of *EAD* that is also identical for all portfolio credits, the inverse of the portfolio unconditional credit loss rate distribution is,

$$LGD \cdot EAD \cdot \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\,\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right), \quad \alpha \in [0,1] \qquad (5)$$

Basel II targets a soundness standard of 99.9 percent and sets minimum capital equal to the 99.9 percentilelevel of the portfolio loss distribution. Adding the requirement that bank loan loss reserves (which count as regulatory capital) must be equal to (or greater than) expected portfolio loss, the bank minimum capital requirement in excess of loan loss reserves is,

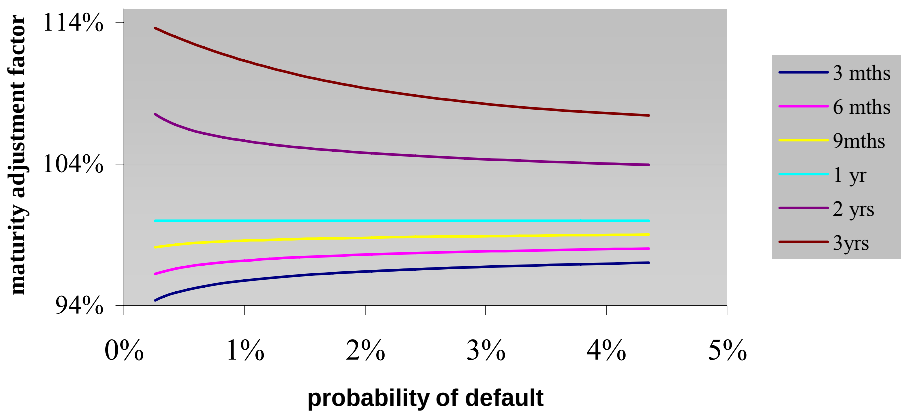
$$LGD \cdot EAD \cdot \Phi\left(\frac{\Phi^{-1}(PD)+\sqrt{\rho}\,\Phi^{-1}(.999)}{\sqrt{1-\rho}}\right)-PD \cdot LGD \qquad (6)$$

The Basel II AIRB capital rule is expression (6) modified by an ad hoc regulatory correlation assignment function that is different for each class of exposures (wholesale, revolving retail, mortgages, and other retail) and an ad hoc multiplicative maturity adjustment factor that applies for wholesale exposures.

The maturity factor for wholesale exposures (corporate, bank and sovereign credits) is plotted in Figure 1. There is no mathematical basis for this maturity correction factor. The correction term was calibrated to make the AIRB rule mimic the capital allocation behavior of capital estimates calculated using KMV Portfolio Manager for different maturity and credit risk profiles [BCBS 2005, p.9]. This maturity adjustment factor lowers capital for shorter-term credit and raises capital for longer term credits; it has a value of 1 for one-year credits.

The AIRB uses a regulatory assignment function to specify the correlation ρ used in the capital rule. The correlation assignment depends on the type of credit (wholesale, residential mortgage, other retail, or qualifying revolving retail). The regulatory correlation may be a constant or a declining function of *PD*, depending on the credit category. AIRB correlation assumptions are plotted in Figure 2.

Figure 1: Maturity Adjustment Factors for Corporate, Bank and Sovereign Credits

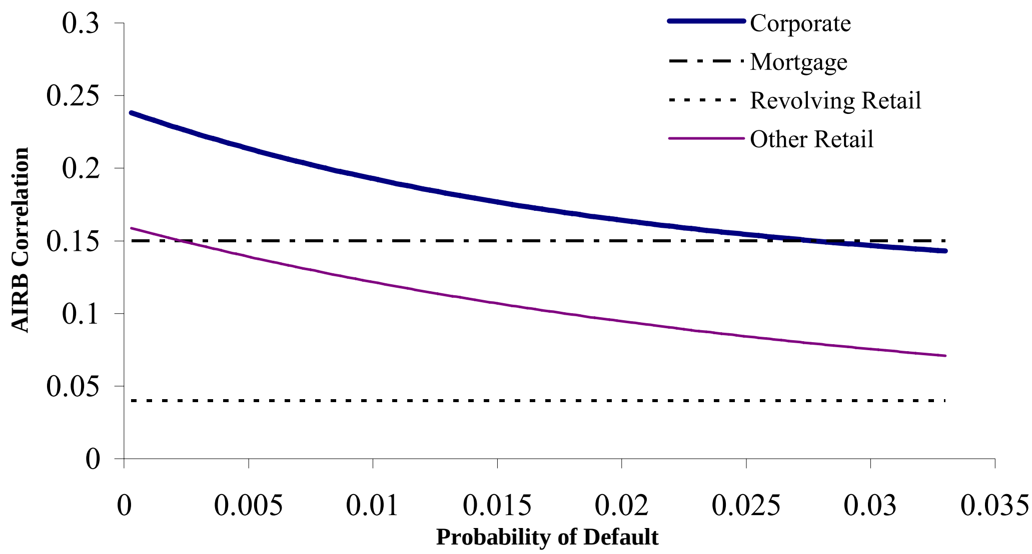


Source: Author's calculations using June 2006 AIRB maturity adjustments

The AIRB correlation functions were calibrated by the BCBS [see BCBS July 2005] using data sets made available by G10 bank supervisors. The BCBS interpretation of the data suggested "stylized facts" that reportedly guided the calibration of the wholesale correlation curve. The regularities noted were: (1) default correlation increases with firm size; and, (2) default correlations decrease as *PD* increases. The wholesale correlation assignment function was built to mimic these observed regularities while bounding correlations to be below 24 percent for the lowest *PD*s, and above 12 percent for the highest *PD* wholesale exposures.

The retail correlation assignments [See BCBS 2005, p. 14] were "reverse engineered" by choosing a correlation parameter that, when used in conjunction with expression (6), produced an AIRB capital requirement that was approximately equivalent to the capital requirement that was assigned by the internal capital allocation models of a group of large internationally active banks.

Figure 2: Basel II US AIRB Correlation Assumptions



Source: Author's calculations using June 2006 AIRB correlation assignment rules

Discussion

The AIRB is based on a very simple model of portfolio credit risk in which potential credit losses are driven by the proportion of portfolio credits that may default in a large and perfectly diversified portfolio. The model focuses entirely on a portfolio's default rate and does not include other factors that may generate capital needs. The model, moreover does not measure of the diversification benefits that may arise from income that is generated when credits fully perform.

Among the more important risk factors that are omitted from the AIRB framework are systematic risks driven by random *LGD*s or random *EAD*s on portfolios of undrawn credit commitments. Depending on the characteristics of the *LGD* and *EAD* distributions, uncertainty in these factors may generate sources of risk that require additional capital.

Appropriately measured, required capitalization rates may far exceed those calculated using the Vasicek approximation for the portfolio loss distribution.

Empirical evidence concerning *LGD*s finds significant time variability in realized *LGD*s. Default losses clearly increase in periods when default rates are elevated. Studies by Frye (2000), Schuermann (2004), Araten, Jacobs, and Varshney (2004), Altman, Brady, Resti and Sironi (2004), Hamilton, Varma, Ou and Cantor (2004), Carey and Gordy (2004), Emery, Cantor and Arnet (2004) and others show pronounced decreases in the recovery rates during recessions and periods of heightened defaults.

There is relatively little published evidence regarding the empirical characteristics of EADs for revolving exposures. The evidence that is available, including studies by Allen and Saunders (2003), Asarnow and Marker (1995), Araten and Jacobs (2001), and Jiménez, Lopez, and Saurina (2006) suggests that obligors draw on their lines of credit as their credit quality deteriorates. The evidence suggests that *EAD*s and *PD*s are positively correlated, suggesting that there is at least one common factor that simultaneously determines *EAD* and default realizations.

The BCBS is clearly aware that the random nature of *LGD* and *EAD* may affect minimum capital needs; nonetheless the committee decided to avoid any formal mathematical generalizations of the Vasicek model, and instead focused on providing written guidance regarding the measurement of the *EAD* and *LGD* inputs into the capital rule. For revolving credits, Basel II requires that *EAD* estimates include recognition that obligors may draw on their lines, but there is no formal method, process, or standard for modeling *EAD*.

Basel II is similarly vague on the methods that must be used to measure *LGD*. The Basel II discussion defines ELGD as the simple average of historical *LGD* observations.

Basel II requires that *LGD* equal *ELGD* plus some adjustment for the potential that losses might be elevated from *ELGD* should they occur during a recession. Again, no formal method of adjustment or standard is provided to guide the estimation of the so-called "downturn" *LGD* input in the capital rule.

The BCBS chose to calibrate the Vasicek model using a regulatory correlation function. For wholesale credits [corporate, bank and foreign sovereign exposures] and other retail credits [auto loans, boat loans, personal loans, etc], the BCBS choose to specify a correlation that declined as a credit's *PD* increased. In their correlation assignments, low *PD* credits may have up to twice the default correlation of high *PD* exposures. However, independent empirical evidence does not support this calibration. In contrast to the BCBS characterization of the stylized facts [BCBS 2005, p. 12], independent studies that have analyzed default correlation, including Allen, DeLong and Saunders (2004), Cowan and Cowan (2004), Dietsch and Petey (2004), and Das, Duffie, Kapadia and Saita (2004), find that default correlation increases as the credit quality of a portfolio declines. The choice of the shape of the Basel II correlation curve is not consistent with empirical evidence, but was likely made to attenuate fears that the AIRB might create strongly procyclical capital requirements.

Concerns about "procyclicality" are based on the idea that, during recessions, any given set of bank credits is more likely to be reclassified into lower-rated buckets.[2] In boom periods, the reverse will likely occur. If a portfolio of given credits migrates through various

[2] See for example, Turner (2000), Lowe (2002), Allen and Saunders (2003), Kashyup and Stein (2004), or Gordy and Howells (2004).

PD grades in response to changing economic conditions, AIRB minimum capital will rise during recessions and decline during booms. Such a cycle in minimum capital has the potential to retard the extension of new bank credit during recessions and overly stimulate bank lending during boom periods and so unintentionally reinforce the bank lending cycle. It is probable that the BCBS hoped to dampen the procyclicality effect by specifying a correlation function that declines as *PD* increases. This calibration will reduce the minimum capital fluctuations that a credit may generate as it moves through an up-grade/down-grade cycle.

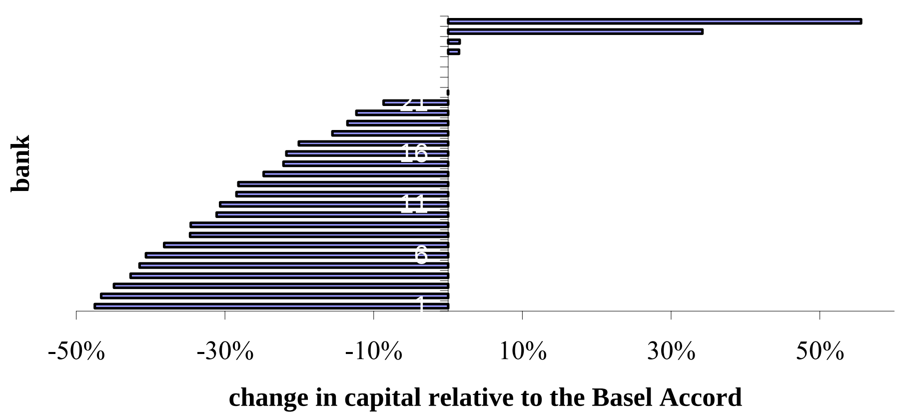
3. THE AIRB AND FINANCIAL STABILITY

Basel II will enhance financial stability if it improves upon the 1988 Basel Accord's ability to ensure that systemically important institutions retain adequate minimum capital. In a variety of published papers and public addresses, members of the BCBS have explained that the complexity of the AIRB is to ensure risk and minimum capital are properly aligned given the complexity of large international banking organizations and the need to foreclose opportunities for regulatory arbitrage that exist under the 1988 Basel Accord.[3] Capital savings accorded under the AIRB are intended to offset costs associated with developing and operating AIRB systems. Reductions in capital also reflect a presumption that AIRB will improve the accuracy of bank credit risk measures and thereby improve the assignment of minimum capital allocations within banks.

[3] See for example, Greenspan (1998), BCBS (1998, June 1999), Mingo (2000), Jones (2000), or Meyer (2001) or more recently, Bies (2005).

The BCBS has conducted two Quantitative Impact Studies (QIS) following the June 2004 publication of the Basel II framework. QIS 4 included banks in the United States, Germany and South Africa. QIS 5 included banks in adopting countries in the remainder of the world. Both studies reported substantial declines in minimum capital requirements for AIRB banks relative to required capital under the 1988 Basel Accord. Figure 3 plots a histogram of estimates of the effective change in the levels of minimum capital that would be required under the AIRB approach for banks participating in the QIS 4 exercise, relative to capital levels required under the U.S. implementation of the 1988 Basel Accord.

Figure 3: Estimates of Effective AIRB Changes in Minimum Required Capital of QIS4 Banks



Source: QIS 4 Interagency Analysis

The Spring 2005 QIS 4 study included 26 U.S. institutions, all of which reported using the AIRB approach.[4] The results show that, in aggregate, minimum regulatory capital for these institutions fell by 15.5 percent under the AIRB. Among these banks, the median

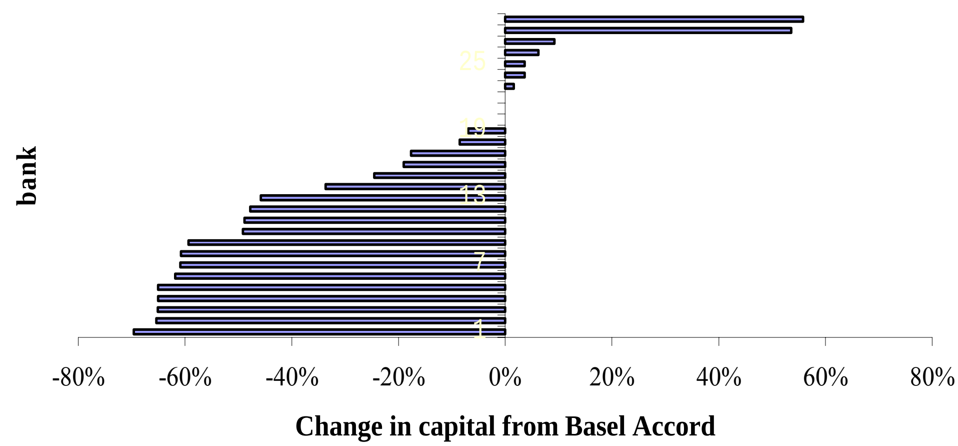
[4] See the Federal Reserve Board Press release, "Summary Findings of the Fourth Quantitative Impact Study," available at www.federalreserve.gov

reduction in capital was 26 percent and the median reduction in Tier I capital requirements was 31 percent. Of the few banks that experienced increases in minimum capital requirements under the AIRB, the increases were driven primarily by increases in capital for consumer retail portfolios and to a lesser extent by equity exposures.

In addition to large declines in capital, QIS 4 results show a high degree of dispersion in reported estimates of minimum capital requirements. Banks reported widely divergent capital estimates for their constituent portfolios (corporate, mortgages, etc.). Although these differences could be due to true difference in bank risk profiles as a result of differentiation among customer bases and business strategies, additional analysis using shared national credit data and a hypothetical mortgage portfolio indicated that banks reported widely divergent capital estimates for positions with substantially similar risk characteristics. Further analysis suggests that a significant share of the variation in QIS 4 results may be attributed to differences in bank estimates of *PD*s and *LGD*s among credits with approximately equivalent risk characteristics. For the wholesale portfolio, for example, QIS 4 *LGD* estimates on non-defaulted credits varied from about 15 to 55 percent across banking institutions.

The minimum regulatory capital treatment of securitization exposures provides one indicator of the degree to which the AIRB approach meets Basel II objectives. Bank securitization activities have been specifically identified as the means through which Basel Accord minimum capital standards have been eroded [e.g., Jones (2000), Mingo (2000)]. The Basel AIRB approach includes a complex set of capital rules for measuring capital requirements on exposures related to securitized positions. Figure 4 plots the histogram of the changes in effective minimum capital required by the AIRB for QIS 4 participating banks. Changes are calculated relative to the capital that is required under the U.S. implementation of the Basel Accord. In these estimates, AIRB rules that require deductions from capital are treated as a capital requirement of 100 percent. Figure 4 shows, for a large majority of banks, the AIRB will result in substantial reductions in the capital that will be required for exposures related to securitizations. Although a full analysis is not possible using QIS 4 data, a large part of the indicated reductions likely stems from reductions in the AIRB capital requirements for the assets that are included in these securitization structures.[5]

Figure 4: QIS 4 Estimates of AIRB Change in Capital for Securitization Exposures



Source: Author's calculations using QIS 4 Interagency data

The QIS 5 study, completed Spring 2006, included 382 banks in 32 countries outside of the U.S.[6] Of the banks that participated, the largest internationally active banks, so-called Group 1 banks, posted capital declines of 7.1 percent on average under the AIRB approach. Smaller banks, so-called Group 2 banks, primarily nationally focused institutions, posted

[5] The Basel II capital rules for securitization exposures have a "look through" property, meaning that the minimum capital requirements that apply to the collateral in these structures in part determines the capital requirements for a bank's securitization position.

[6] See, BCBS (2006a). QIS 5 AIRB capital rules include a 1.06 scaling factor that was not included in the June 2004 calibration or the instructions that guided QIS 4. The inclusion of this scaling factor means the reported capital declines will appear less severe than those reported in the U.S.

much larger declines in minimum regulatory capital.[7] Within Europe,[8] Group 1 banks posted average capital declines of 8.3 percent under the AIRB. For European Group 2 banks, capital declines averaged 26.6 percent under the AIRB. The QIS 5 attributed large declines in minimum regulatory capital requirements to bank concentrations in retail lending, especially residential mortgages.

The BCBS summary of QIS 5 results does not provide detailed analysis of the dispersion of bank minimum capital estimates. The study does however report significant variation in AIRB input values. *LGD* estimates for wholesale credits, for example, range from 10.8 to 67.6 percent across reporting banks.

The results of the QIS 4 and QIS 5 studies show that, under the AIRB, most banks will face large reductions in their minimum required capital levels on their current portfolio positions. In practice, the AIRB will lead to further capital reductions as banks optimize and adjust their positions to maximize the benefits available through new (unanticipated) regulatory arbitrage opportunities that are available under the AIRB.

Given the potential for large reductions in minimum bank capital that may materialize under the AIRB approach, it is important to assess whether or not these reductions are justified by improvements in risk measurement. There is a strong presumption in most Basel II-related documents and policy discussions that the AIRB approach represents a sound scientifically supported standard for measuring bank minimum capital needs. Unfortunately, this confidence is misplaced. Given the collective state of knowledge concerning credit risk

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[7] BCBS (2006a).

[8] So-called CEBS (Committee of European Bank Supervisors) banks.

measurement and capital allocation, there is a large body of evidence that shows that the AIRB framework will undercapitalize credit risks.

The bias in the AIRB capital rule that causes credit risks to be undercapitalized owes to two separate sources. One source is underestimation of the 99.9 percent tail loss values for bank portfolio credit losses. The AIRB synthesizes an estimate of a bank's credit loss distribution using a model that ignores systematic risks in *LGD*s, the draw rates on revolving lines of credit, as well as exposure concentrations. A second source of bias is a flaw in the logic used to set AIRB minimum capital requirements. The AIRB capital rule ignores the need for a bank to pay interest on its own liabilities. Claims that market discipline or national supervisory discretion exercised under pillar 2 will attenuate the flaws in the AIRB rule are untested and should not be a basis for codifying into regulation a flawed risk measurement standard. The following sections discuss these issues in more detail.

4. Establishing a Sound Benchmark for Risk Measurement Practices

The Need for Capital for Bank Interest Expenses

Although the U.S. Basel II NPR discussion mirrors a textbook description of a credit VaR calculation, the procedure described will not set minimum capital requirements to ensure the 99.9 percent targeted soundness standard. An important flaw in credit VaR capital allocation methods is that they fail to recognize a bank's need to pay interest on its own liabilities. Ignoring this need to pay interest causes little harm when VaR measures are used to set capital over short horizons as they are for example, in the 1-day and 10-day horizons used in the market risk rule. Over longer horizons like the 1-year horizon used for Basel II, ignoring the need to pay interest will cause a substantial divergence between the intended and actual AIRB soundness standard. The magnitude of the deterioration in the intended safety margin will, moreover, depend on the level of interest rates and may magnify the procyclical nature of the AIRB capital rules.

Consider the problem of setting capital for a single credit. To avoid any questions about the magnitudes of the capital variations involved, we frame the example in terms of an exact pricing model for credit risk. While we will use the Black and Scholes (1973) and Merton (1974) model (hereafter BSM) to frame the analysis, the qualitative result is true for any equilibrium asset pricing model.

Under some simplifying assumptions, the BSM model establishes equilibrium pricing relationships that must hold for risky discount debt instruments. When the default-free term structure is not stochastic and flat at a rate, r_f , and a firms' assets have an initial value of A_0 and evolve in value following geometric Brownian motion with and instantaneous volatility of σ, the BSM model has shown that the equilibrium price, B_0 , of a one-year discount bond with a promised maturity value of Par and default risk is,

$$B_0 = e^{-r_f} \Phi\left(\frac{\ln(A_0) - \ln(Par) + \left(r_f - \frac{\sigma^2}{2} \right)}{\sigma} \right) - A_0 \, \Phi\left(\frac{\ln(Par) - \ln(A_0) - \left(r_f + \frac{\sigma^2}{2} \right)}{\sigma} \right) \quad (7)$$

The value-at-risk measure for this bond is calculated using the probability distribution for the value of this bond at the end of one year, $\widetilde{B}_1$. Under the BSM model assumptions, $\widetilde{B}_1$ the physical probability distribution for the bond's value after one year is,

$$\tilde{B}_1 = Min\left[A_0\, e^{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\,\tilde{z}}, Par\right] \tag{8}$$

where $\tilde{z}$ is a standard normal variable, $\mu = r_f + \lambda\sigma$, where λ is the market price of risk. The critical value of this distribution used to set a $VaR(\alpha)$ measure is,

$Min\left[A_0\, e^{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\,\Phi^{-1}(1-\alpha)}, Par\right]$ which simplifies to $A_0\, e^{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\,\Phi^{-1}(1-\alpha)}$ when the probability of default on the bond exceeds $(1-\alpha)$.

To determine the capital needed to fund this bond, note that any debt issue with a par value greater than $A_0\, e^{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\,\Phi^{-1}(1-\alpha)}$ will default with a probability greater than $(1-\alpha)$ if $\tilde{B}_1$ is the only source of funds available to repay the funding debt. Thus

$Par_F(\alpha) = A_0\, e^{\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}\,\Phi^{-1}(1-\alpha)}$ is the maximum permissible par value for the funding debt. The cash flows from $\tilde{B}_1$ "pass through" the firm to payoff the funding debt issue, and so the BSM model can be used to price the bond issued by the bank. The difference between B_0 and the market value of the funding debt issue is the minimum equity capital needed to fund the risky bond. This minimum amount of capital needed to achieve a soundness standard of α is,

$$B_0 - Par_F(\alpha)\, e^{-r_f}\, \Phi\left(\frac{\ln(A_0) - \ln(Par_F(\alpha)) + \left(r_f - \frac{\sigma^2}{2} \right)}{\sigma} \right) - A_0\, \Phi\left(\frac{\ln(Par_F(\alpha)) - \ln(A_0) - \left(r_f + \frac{\sigma^2}{2} \right)}{\sigma} \right) \qquad (9)$$

The potential importance of the omission of bank funding costs from the Basel II AIRB capital calculations is illustrated in Figure 5. Figure 5 illustrates an AIRB-style VaR calculation for a bank whose sole asset is a risky 1-year BSM discount bond. The bond has a par value of 70 and for the rights this claim, the bank lends \$66.14. The underlying assets of the borrower have an initial value of 100, and these assets evolve in value following geometric Brownian motion with an instantaneous drift rate of $\mu = .10,$ and an instantaneous volatility $\sigma = .25.$ The one-year Treasury rate is 5 percent. The probability distribution of $\widetilde{B}_1$ is plotted in the top panel of Figure 1. In this example we consider a soundness standard of 99 percent which dictates that the bank's equity must be large enough to absorb 99 percent of all potential losses. The 99 percent critical value of the loss distribution is equivalent to the 1 percent critical value of the bond's future value distribution, $\widetilde{B}_1$. This critical value is \$59.82. Under the AIRB approach for setting capital, this bond requires \$7.32 in capital (\$66.14-\$59.82) to cover both expected and unexpected losses. To fund the bond, the bank must sell debt that has an initial market value of \$59.82.

The bottom panel of Figure 5 illustrates the potential outcome one year after the bond is purchased and funded according to an AIRB approach for setting minimum capital. If the bank raises \$59.82 in debt finance to fund the bond, it owes bank debt holders \$63.04 at the end of the year.[9] After accounting for the interest payments that are due on the bank debt, the true probability that the bank defaults on its debt is 1.7 percent.[10] The actual default rate is 70 percent higher than the default rate consistent with the minimum regulatory soundness standard.

There is nothing “staged” about this example. The AIRB approach for setting minimum regulatory capital requirements excludes any consideration of the need to compensate bank debt holders for the time value of money and credit risk. As a consequence the credit VaR based AIRB rule will always understate capital requirements. Kupiec [2006b] provides additional discussion including the portfolio generalization of this result.

Procyclicality of the AIRB Soundness Standard

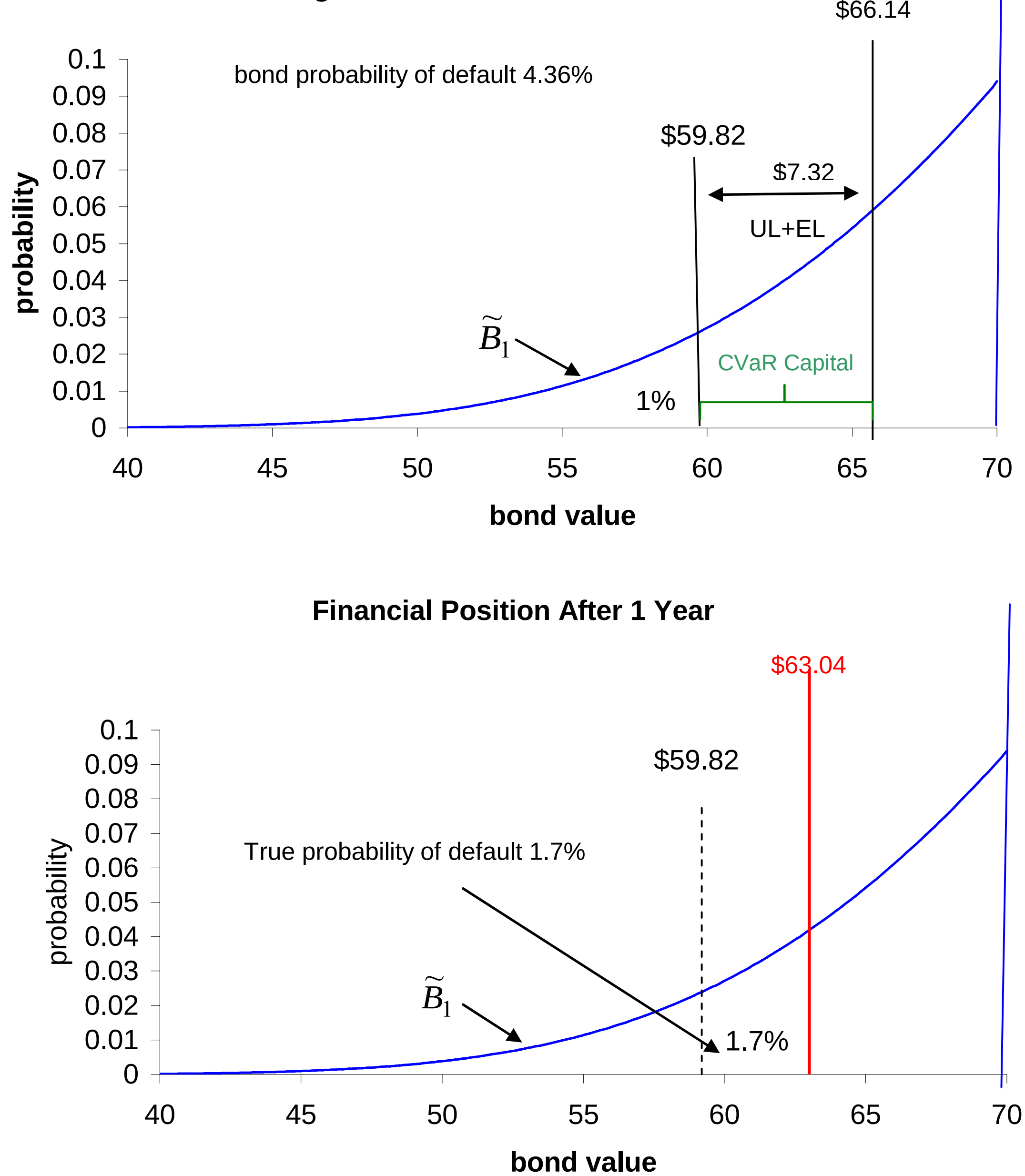
The omission of bank interest expense in the AIRB capital rule engenders a soundness standard that varies over the business cycle. The soundness standard set by AIRB minimum capital requirements will decline (i.e., the probability of default will increase) when interest rates are high and the central bank is attempting to dampen economic activity and bank lending. Conversely, AIRB capital standards engender the strictest solvency standard when interest rates are low and the central bank is attempting to stimulate bank lending and economic activity. As a consequence, the potential safety net benefits to the banking system are increased during the boom phase of the economic cycle when banks

[9] This value is calculated by inverting the BSM pricing model to find the par value of debt that would raise $59.82 when it is sold to investors. The bank’s debt is risk so it must pay a rate higher than the one-year risk free rate.

[10] The probability distribution for $\tilde{B}_1$ includes the interest that is paid to the bank on the purchased risky bond.

compete on underwriting standards and stock up on the "bad loans" that default when a subsequent downturn materializes.

Figure 5: Credit VaR Calculation



Source: Author's calculations

The procyclicality of the soundness standard is illustrated in Figure 6. The top panel of the figure illustrates the credit VaR capital calculation for a bond identical to that analyzed in Figure 1. The only change in Figure 6 is the one-year Treasury rate is 10 percent instead of 5 percent. Since this new bond must satisfy equilibrium conditions, the higher default-free rate requires an increase in the instantaneous drift rate (μ = 15 percent) on the value of the underlying assets. Under these new equilibrium conditions, the credit VaR approach requires only $.18 for its minimum capital requirement, so the bond can be purchased for $63.07 and funded with $62.89 in debt.

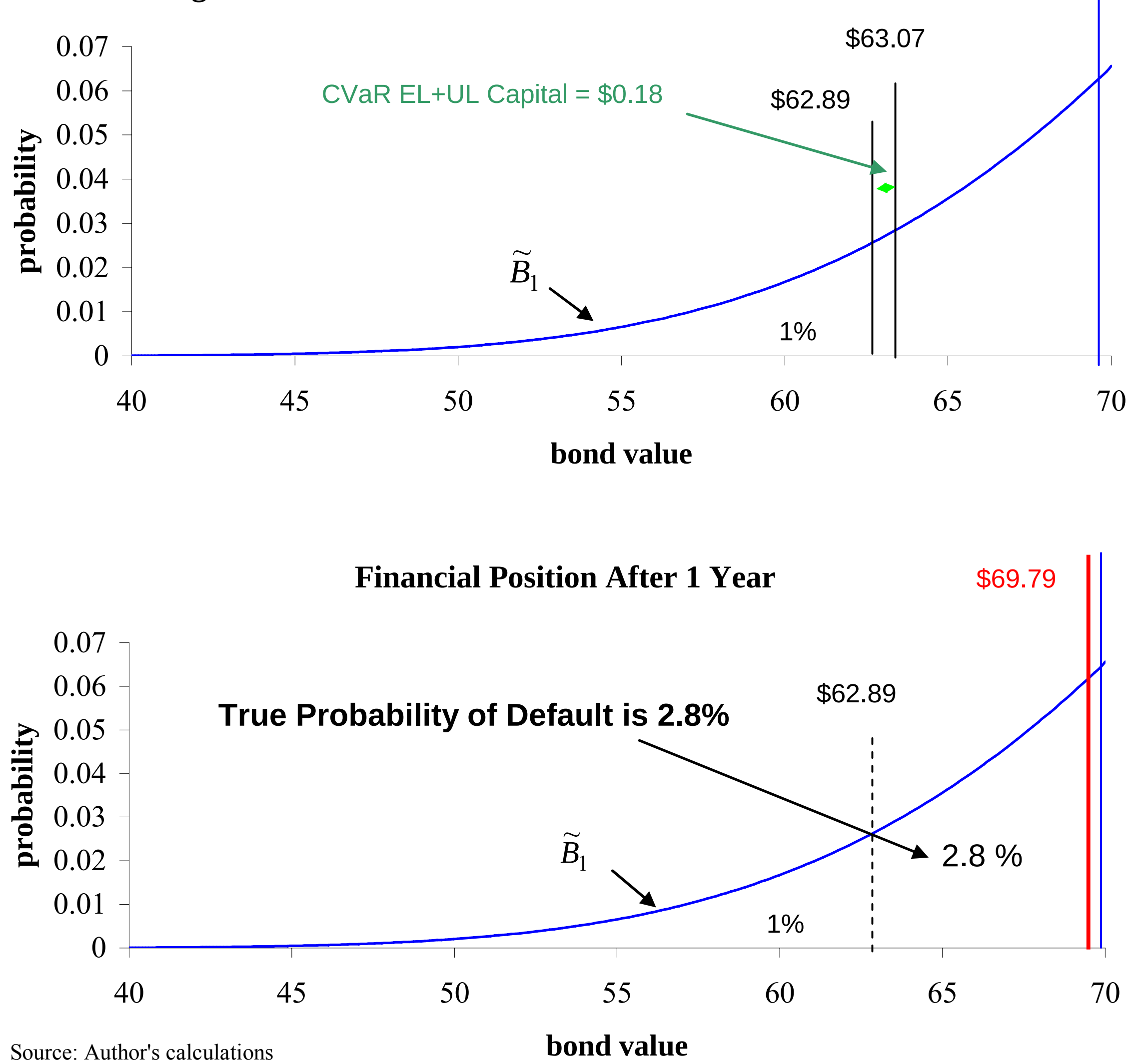
The bottom panel of Figure 6 shows the possible outcomes one year later. After one year, the bank must pay its debt holders $69.79 to avoid default and retire its debt with accrued interest. The probability that the value $\widetilde{B}_1$ is less than $69.79 is 2.8 percent. Thus the actual soundness standard set by the AIRB minimum capital rule is 97.20 percent and not the targeted 99.9 percent. The actual soundness standard set by the AIRB rule declined from 1.7 percent to 2.8 percent as risk free interest rates rose by 5 percentage points.[11]

The omission of bank interest costs will induce procylicality in the AIRB regulatory soundness standard. To the extent that minimum regulatory capital requirements impose binding constraints on bank capital positions, this procyclicality may work to magnify the bank lending cycle. During the initial upturn phase of the business cycle, the demand for

[11] Notice that this increase in capital is for credit risk and not for interest rate risk as the one-year default free rate was changed *ceteris peribis* and not converted into a random variable.

credit is strong and banks may expand lending and grow without relaxing their underwriting standards or offering concessionary spreads.

Figure 6: Credit VaR Calculation with 10% Risk Free Rate



Source: Author's calculations

As the recovery phase matures toward the peak of the business cycle, growth opportunities wane, and banks compete aggressively to continue to grow. In this portion of the cycle, banks' risk of booking marginal quality credits increases. Concurrently, at this stage of the cycle, the central bank typically begins to increase interest rates in order to attenuate aggregate demand imbalances. Under the AIRB approach to setting capital, the increase in risk free interest rates will automatically reduce banks' minimum regulatory solvency standard.

When governments provide implicit or under-priced explicit guarantees on bank liabilities, banks debt is priced to reflect this guarantee. Because bank shareholders do not pay (or pay the fair price) for this guarantee, they profit from a government safety net subsidy. A reduction in a bank's soundness standard is equivalent to expanding the safety net subsidy enjoyed by banks. Banks may utilize the increased subsidy and continue to grow by adding marginal loans that otherwise might have been rejected under a stricter solvency standard. Reverse incentives will be promulgated at the depths of a recession, as decreases in interest rates strengthen the regulatory solvency standard and discourage bank lending. Over the business cycle, the procyclic nature of the AIRB solvency standard, a feature created in part by omitting capital to cover bank interest expenses, has the potential to magnify the bank credit cycle.

Incorporating Portfolio Interest Income

Quite apart from the need to recognize that bank capital requirements must be set to ensure that a bank can meet its interest expenses, well-formulated capital allocation estimates should also recognize the interest income received by a bank on fully performing credits. The AIRB framework calculates capital requirements using an approximation for the distribution of the default rate on a well-diversified portfolio. The model does not include any recognition of the loss diversification benefits that arise from the interest payments that are received on fully performing credits. Portfolio interest income can be recognized by formulating the model using an asymptotic approximation for the portfolio return distribution instead of the portfolio loss distribution.[12]

Consider the portfolio of identical credits analyzed in Section II. Let *YTM* represent the yield to maturity calculated using the initial market value of an individual credit and let *LGD* represent the loss from initial loan value should a loan default. All loans in a portfolio are assumed to have identical values for *YTM, PD,* and *LGD*.

Let $\widetilde{R}_p$ represents the return on the portfolio of credits. The end-of-horizon conditional portfolio return is given by,

$$\widetilde{R}_p = YTM - (YTM + LGD)\,\widetilde{X} \tag{10}$$

Following the same logic used in Section 2 to derive the Vasicek approximation for the portfolio's loss distribution, the unconditional cumulative return distribution for the portfolio, $\widetilde{R}_p$ can be derived from the inverse of the unconditional distribution for the portfolio default rate [expression (3)][13]. The critical value of the portfolio return distribution that is consistent with a regulatory soundness standard of 99.9 percent is,

$$\left(1 + YTM - \left(YTM + LGD\right)\Phi\left(\frac{\Phi^{-1}\left(PD\right) + \sqrt{\rho}\,\Phi^{-1}\left(.999\right)}{\sqrt{1-\rho}}\right)\right) \tag{11}$$

[12] This discussion draws on Kupiec [2006a].

[13] Kupiec (2006a) provides a full derivation.

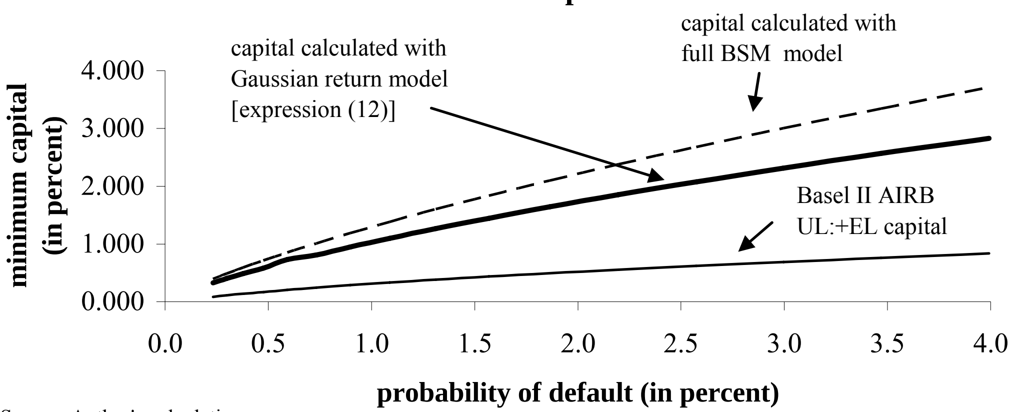
Using *YTM* as a conservative estimate of the equilibrium required rate of return on the bank's funding debt when it is issued, the minimum required portfolio (and individual credit) capitalization rate to ensure a 99.9 percent solvency standard is,

$$K(\alpha) \approx \frac{YTM + LGD}{1 + YTM} \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\, \Phi^{-1}(.999)}{\sqrt{1-\rho}} \right) \tag{12}$$

Expression (12) is an approximation for the capital needed in a single common factor framework. It includes capital for both expected and unexpected loss as well as capital to cover bank interest expenses. Unlike the Basel II AIRB capital rule, it fully recognizes the capital reducing benefits of interest income that is earned by the fully performing credits in a portfolio. Capital requirements set according to expression (12) are uniformly larger than the capital requirements set by the Basel AIRB formula even when including capital for expected loss [expression (5)]. The relationship between the capital recommended by expression (12) and (5) is illustrated in Figure 7.

Figure 7 compares minimum capital requirements for a 99.9 percent soundness standard as set by the Basel AIRB rule for expected and unexpected loss [expression (5)] and expression (12). The minimum capital estimates are for hypothetical credit portfolios that are composed of credits that are priced to satisfy BSM equilibrium conditions [see Kupiec (2006b) for additional details].

Figure 3: Capital Requirements Recognizing Bank Interest Income and Expense



Source: Author's calculations

It is important to remember that the Basel AIRB rule and expression (12) are approximations for the true capital needed to satisfy a regulatory soundness standard. Both of these models are developed under a set of restrictive assumptions that allow the models to be parameterized in terms of *PD* and *LGD* and admit a closed form expression for capital. For reference, Figure 7 also includes the exact capital that is required to ensure the 99.9 percent soundness standard. These exact capital requirements are calculated using a full BSM capital allocation model developed in Kupiec [2004, 2006a, 2006b]. The full BSM model expression for capital is significantly more complex than expression (5) or expression (12), and it is not directly parameterized using common high-level measures of credit risk (*PD*, *LGD*, default correlation) but instead is calibrated using a deeper set of model parameters (volatilities, drift rates, initial asset values, etc).

***Accounting for Correlation between* PD *and* LGD**

Many studies have recognized that credit loss rate realizations may be tied to the business cycle. Recovery values tend to be depressed for defaults that occur when default rates are elevated. Alternatively, there is a positive correlation between the *PD* and *LGD*.

The Basel II AIRB model framework takes *LGD* as an exogenous parameter. Correlation between *PD* and *LGD* is not modeled, but must be accounted for through some ad hoc adjustment to expression (5). In the Basel II framework, this adjustment is made through requirements on how the *LGD* parameter must be estimated.

The U.S. Draft NPR makes a distinction between two loss-given-default parameters. One parameter, expected loss given default, or *ELGD*, is the default-frequency weighted average default experience for an *LGD* grade. The second measure of loss given default, *LGD*, is the parameter that is to be used as the AIRB input. *LGD* is the greater of a bank's *ELGD* estimate for the exposure, or the loss per dollar of *EAD* that the bank would likely incur should the exposure default within a one-year horizon during an economic downturn [U.S. NPR, p. 365]. This regulatory definition of downturn *LGD* is not very prescriptive as to how *LGD* should be estimated. It is possible to formally incorporate random *LGD* into the AIRB model and to derive a rigorous statistical characterization of *LGD*.

Random Loss Given Default and "Stress" LGD

Assume that a generic credit has a potential loss given default, $LG\tilde{D}_i$, that is random. *LGD* uncertainty is driven by a latent Gaussian factor, $\tilde{Y}_i$ with the following properties,

$$\begin{aligned}
&\tilde{Y}_i = \sqrt{\rho_Y}\,\tilde{e}_M + \sqrt{1-\rho_Y}\,\tilde{e}_{iY} \\
&\tilde{e}_M \sim \phi(e_M) \\
&e_{iY} \sim \phi(e_{iY}), \\
&E(\tilde{e}_{iY}\,\tilde{e}_{jY}) = E(\tilde{e}_M\,\tilde{e}_{jY}) = E(\tilde{e}_{iY}\,\tilde{e}_j) = 0\ \forall\, i, j.
\end{aligned} \tag{13}$$

The common Gaussian factor, $\tilde{e}_M$, is identical to the common Gaussian factor in expression (1), and so the latent default factor $\tilde{V}_i$ and loss given default factor, $\tilde{Y}_i$, are positively correlated provided $\sqrt{\rho_Y} > 0$.

The unconditional distribution for $LG\tilde{D}_i$ can be approximated to any desired level of precision using a step function that is driven using the realized value of $\tilde{Y}_i$. For expositional simplicity, consider the following simple approximation,

$$LG\tilde{D}_i = \begin{cases} LGD_0 & for\ \tilde{Y}_i > B_{i1} \\ LGD_0 + \Delta LGD & for\ B_{i2} < \tilde{Y}_i < B_{i1} \\ LGD_0 + 2\,\Delta LGD & for\ B_{i3} < \tilde{Y}_i < B_{i2} \\ LGD_0 + 3\,\Delta LGD & for\ \tilde{Y}_i \le B_{i3} \end{cases} \tag{14}$$

For each threshold level of *LGD*, there is an associated cumulative probability defined by the cumulative probability of the latent variable crossing the threshold. This association is described in Table 1.

Table 1: Probability Distribution Approximation for *LGD*

Loss Step Function Increment	LGD Level	Probability Threshold for Latent Variable $\widetilde{Y}_i$
0	LGD_0	
ΔLGD	$LGD_0 + \Delta LGD$	$\Phi(B_{i1})$
$2\,\Delta LGD$	$LGD_0 + 2\,\Delta LGD$	$\Phi(B_{i2})$
$3\,\Delta LGD$	$LGD_0 + 3\,\Delta LGD$	$\Phi(B_{i3})$

In this example, the loss distribution for an individual account can be defined using four indicator functions,

$$\widetilde{I}_i = \begin{cases} 1 & if \quad \widetilde{V}_i < D_i \\ 0 & otherwise \end{cases}, \quad \widetilde{H}_{ij\,ij} = \begin{cases} 1 & if \quad \widetilde{Y}_i < B_{ij} \\ 0 & otherwise \end{cases}, \quad for \;\; j = 1,2,3. \qquad (15)$$

Each indicator variable has a binomial distribution with a mean equal to the cumulative standard normal distribution evaluated at the indicator functions threshold value. For example, $\widetilde{I}_i$ has a binomial distribution with an expected value of $\Phi(D_i)$; similarly, $\widetilde{H}_{i1}$ is binomial with an expected value of $\Phi(B_{i1})$, and so on for the remaining indictors.

The loss rate for account *i* measured relative to *EAD*, can be written

$$L\widetilde{R}_i = \widetilde{I}_i \left(LGD_{i0} + \Delta LGD \sum_{k=1}^{3} \widetilde{H}_{k1} \right) \qquad (16)$$

Define $\widetilde{I}_i \mid e_M$ and $\widetilde{H}_{ik} \mid e_M$ as the distributions of the default indicator functions conditional on a realized value for e_M for ($k = 1, 2, 3$). The conditional indicator functions are independent binomial random variables with the properties,

$$E\left(\widetilde{I}_i \mid e_M\right) = \Phi\left(\frac{D - \sqrt{\rho_d}\ e_M}{\sqrt{1-\rho_d}}\right), \quad E\left(\widetilde{H}_{il} \mid e_M\right) = \Phi\left(\frac{B_{il} - \sqrt{\rho_Y}\ e_M}{\sqrt{1-\rho_Y}}\right) \tag{17}$$

Using the conditional indicator function notation, the conditional loss rate for an individual credit can be written,

$$L\widetilde{R}_i \mid e_M = \left(\widetilde{I}_i \mid e_M\right)\left(LGD_{i0} + \Delta LGD \sum_{k=1}^{3}\left(\widetilde{H}_{k1} \mid e_M\right)\right) \tag{18}$$

Asymptotic Portfolio Loss Distribution

Consider a portfolio composed of *N* accounts with identical latent-factor correlations, $\{\rho, \rho_Y\}$, and default thresholds, $D_i = D.$ Assume that all credits' *LGD*s are drawn from a common distribution but each credit has an independent idiosyncratic risk factor. The common unconditional *LGD* distribution is defined by expression (14) with parameters: $B_{i1} = B_1, B_{i2} = B_{2,}$ and $B_{i3} = B_3$.

Under the asymptotic portfolio assumptions, $\widetilde{I}_i \mid e_M$ and $\widetilde{H}_{ik} \mid e_M$ are independent and identically distributed across credits in the portfolio and consequently *i* subscripts can be dropped. Define $L\widetilde{R}_P \mid e_M$ as the loss rate on the portfolio of accounts conditional on a realization of e_M, $L\widetilde{R}_P \mid e_M = \left(\frac{\sum_{i=1}^{N}\left(L\widetilde{R}_i \mid e_M\right)}{N}\right)$. Because $\left(L\widetilde{R}_i \mid e_M\right)$ is independent of

$\left(\widetilde{LR}_j \mid e_M\right)$ for all $i \neq j$, and these conditional losses are identically distributed, the Strong Law of Large Numbers requires, for all e_M,

$$\lim_{N\to\infty}\left(\widetilde{LR}_P \mid e_M\right)=\lim_{N\to\infty}\left(\frac{\sum_{i=1}^{N}\left(\widetilde{LR}_i \mid e_M\right)}{N}\right)\xrightarrow[a.s.]{} E\left(\widetilde{LR}_i \mid e_M\right) \tag{19}$$

Independence of the conditional indicator functions across credits implies,

$$E\left(\widetilde{I} \mid e_M \cdot \widetilde{H}_k \mid e_M\right)=E\left(\widetilde{I} \mid e_M\right)\cdot E\left(\widetilde{H}_k \mid e_M\right) \quad \forall k. \tag{20}$$

and the asymptotic portfolio return distribution converges almost surely to,

$$\lim_{N\to\infty}\left(\widetilde{LR}_P \mid e_M\right)=\lim_{N\to\infty}\left(\frac{\sum_{i=1}^{N}\left(\widetilde{LR}_i \mid e_M\right)}{N}\right)\xrightarrow[a.s.]{} E\left(\widetilde{I} \mid e_M\right)\cdot\left(LGD_0+\Delta LGD\sum_{k=1}^{3}E\left(H_k \mid e_M\right)\right) \tag{21}$$

The approach does not restrict the number of steps that may be included in the approximations for the LGD unconditional density functions. If the number of steps in the approximations is M, after substituting the binomial expressions for the conditional indicators' expected values, the conditional portfolio loss distribution converges almost surely to,

$$\lim_{N\to\infty}\left(\widetilde{LR}_P \mid e_M\right)\xrightarrow[a.s.]{}\Phi\left(\frac{D-\sqrt{\rho_d}\;e_M}{\sqrt{1-\rho_d}}\right)\cdot\left(LGD_0+\Delta LGD\sum_{k=1}^{M}\Phi\left(\frac{B_k-\sqrt{\rho_Y}\;e_M}{\sqrt{1-\rho_Y}}\right)\right) \tag{22}$$

The inverse of the unconditional distribution function for the portfolio loss rate can be derived using expression (22) and the density function for $\widetilde{e}_M$.

For soundness standard (α), the critical value of $\tilde{e}_M$ is $\Phi^{-1}(1-\alpha)$. Latent factor threshold values can be defined using the characteristics of the individual account's unconditional *PD*, *Ex* and *LGD* probability distributions. These threshold values are defined in Table 2.

Table 2: Latent Factor Model Parameters

Default process	LGD Process
$D = \Phi^{-1}(PD)$	$B_1 = \Phi^{-1}\left(\sum_{i=1}^{M} PLGD_i\right)$
	$\vdots$
	$B_{M-1} = \Phi^{-1}\left(PLGD_{M-1} + PLGD_M\right)$
	$B_M = \Phi^{-1}\left(PLGD_M\right)$

Making use of the identity $\Phi^{-1}(1-\alpha) = -\Phi^{-1}(\alpha)$, the inverse of the unconditional cumulative distribution function for the asymptotic portfolio loss rate can be written,

$$LR_p(\alpha) = \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho_d}\;\Phi^{-1}(\alpha)}{\sqrt{1-\rho_d}}\right) \cdot \left(LGD_0 + \Delta LGD\, B(\alpha)\right), \quad \text{for } \alpha \in [0,1], \qquad (23)$$

where, $$B(\alpha) = \sum_{l=1}^{M} \Phi\left(\frac{\Phi^{-1}\left(\sum_{i=0}^{l} PLGD_{M-i}\right) + \sqrt{\rho_Y}\;\Phi^{-1}(\alpha)}{\sqrt{1-\rho_Y}}\right) \qquad (24)$$

The first term in expression (23) is the inverse of the cumulative distribution function of the Vasicek portfolio loss rate model, the standard Gaussian model in which *LGD* is an exogenous constant. The second term in the expression adjusts the distribution to account for random *LGD*. When $\rho_Y \to 0$, it is straight forward to show $\left(LGD_0 + \Delta LGD\, B(\alpha)\right) \to E\left(L\widetilde{GD}\right)$. So when *LGD* is random, but uncertainty is completely idiosyncratic, expression (23) becomes,

$$LR_p(\alpha) = \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho_d}\;\Phi^{-1}(\alpha)}{\sqrt{1-\rho_d}}\right) \cdot E\left(L\widetilde{GD}\right). \qquad (25)$$

When $\rho_Y \neq 0$, *LGD*s are correlated across credit in the portfolio, and expression (25) no longer holds.

Consider the interpretation of expression (25) when $\rho_Y \neq 0$. The function $B(\alpha)$ can be interpreted as a function that shifts the probability distribution for *LGD.*

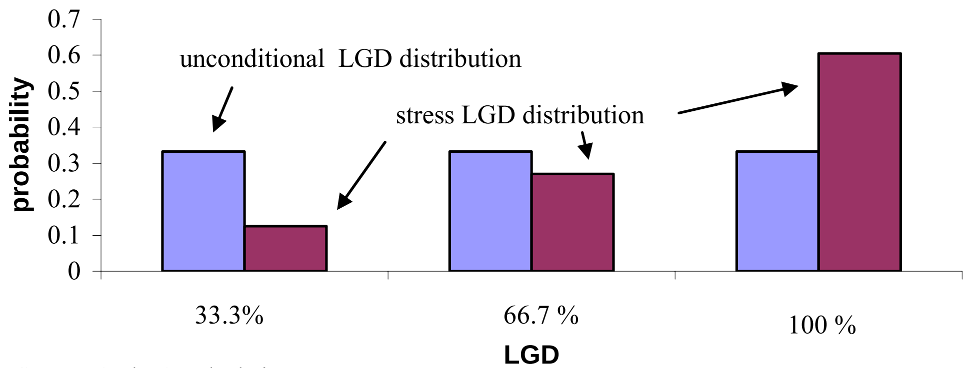
Table 3: Step Function Approximation for the Corporate LGD Distribution

LGD rate thresholds	cumulative probability of LGD level	cumulative probability of LGD increment	threshold for $\widetilde{Y}$
33%	33%	33.3%	0.432
67%	67%	33.3%	-0.432
100%	100%	33.3%	
mean	66.70%		

To keep terminology consistent with Basel II, define this new probability distribution after the function $B(\alpha)$ is applied as the "stress" (or "downturn") distribution. Define the expected value of *LGD* taken with respected to this new distribution as "stress *LGD*". Using "stress *LGD,"* and $\alpha = .999$, the portfolio loss rate in expression (23) is identical in form with the Basel AIRB rule using stress *LGD* in place of the Basel definition of LGD.

To see the stress *LGD* interpretation, consider the unconditional LGD distribution in Table 3 where one-third of all loss rates are 33.3 percent, one third are 66.7 percent, and the final one third are 100 percent. In step function form, $LGD_0 = .333$, and $\Delta LGD = .333$. The probability associated with the first threshold value is .667; the second threshold has a probability of .333.

Figure 8: Unconditional and Stress LGD Distributions



Source: Author's calculations

Assuming the correlation among *LGD*s is positive, when $\rho_Y > 0$, the $B(\alpha)$ function shifts probability mass into the right tail of the *LGD* distribution.[14] For example, when $\rho_Y = .05$, $LGD_0 + \Delta LGD\, B(\alpha) = .333 + .333(.8753 + .6049) = .827$. The calculation in this example is equivalent to taking the expected value of *LGD* using new probabilities, where weight has been shifted to higher *LGD* realizations. Figure 8 plots the unconditional and stress *LGD* distributions for $\alpha = 99.9$ percent and $\rho_Y = .05$. The amount of probability mass

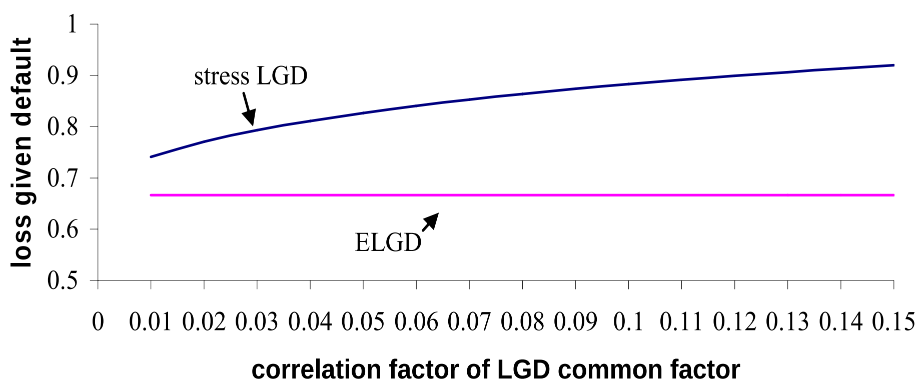
[14] Should $\rho_Y < 0$, $B(\alpha)$ would shift weight towards the left tail of the *LGD* distribution.

that is shifted under the stress measure depends on α, the cumulative probability at which the portfolio loss rate is being evaluated, and on the latent *LGD* factor correlation ρ_Y.

If we define $E\left(\widetilde{LGD}^S\right) = \left(LGD_0 + \Delta LGD\, B(\alpha)\right)$ as the stress *LGD*, the approximate minimum capital requirement necessary to ensure a soundness standard of 99.9 percent is,

$$K(\alpha) \approx \frac{YTM + E\left(\widetilde{LGD}^S\right)}{1 + YTM} \Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho}\,\Phi^{-1}(.999)}{\sqrt{1-\rho}}\right) \tag{26}$$

Figure 9: Correlation and Stress LGD



Source: Author's calculations

To provide a sense of the potential importance of a positive correlation between *PD* and *LGD*, consider an example in which credits' unconditional *LGD* is consistent with the unconditional *LGD* distribution in Table 3. Figure 9 plots the *ELGD* and stressed *LGD*s that are appropriate for use in setting capital for an asymptotic portfolio of these credits. Small increases in the correlations between exposures' potential *LGD*s can lead to large changes in minimum capital requirements. For example, an increase in ρ_Y from 0 to 10 percent will increase required capital by 28.2 percent when capital is calculated using expression (26) for the *LGD* distribution in Table 3.[15]

Random Exposures at Default (EAD)

The AIRB modeling framework treats *EAD* an exogenous parameter. For revolving exposures, banks using the AIRB are required to estimate *EAD*, but Basel II rules give very little guidance as to how *EAD* should be estimated. For example, the guidance suggests that banks must have methods for estimating *EAD* but the only quantitative standard imposed is that an *EAD* estimate must be at least as large as an obligor's current exposure. As discussed in Section 2, there is a growing body of evidence that suggests that credit facility draw rates are higher for low quality credits and credits nearing default implying a positive correlation between *PD* and *EAD*.

Similar to the case of random *LGD*, if the random exposure realizations of the credits in a portfolio are positively correlated, then the ability to reduce credit risk using portfolio diversification is limited. Kupiec [2006c] includes a random *EAD* into the Vasicek framework and shows that, similar to the case of correlated *LGD*s, an expression for minimum capital can be defined in terms of "stressed *EAD*." Correlation among *EAD*s will lead to the need for substantially higher minimum capital requirements. [See Kupiec (2006c) for further details).

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[15] If one uses the AIRB rule for setting capital [expression (5)], the increase in capital necessary to account for random *LGD* is nearly 33 percent.

The upshot is that there are good reasons to believe that the Basle II AIRB capital rule will underestimate capital needs for revolving credit portfolios unless banks somehow compensate and input *EAD* rates that are significantly overstated relative to their average facility *EAD*s. The Basel AIRB standard is underdeveloped relative to the treatment of revolving credit exposures and substantial improvement in the accuracy of capital estimates can be achieved even in the context of the simple single factor Gaussian approximation for measuring portfolio credit risks.

8. CONCLUSIONS

Basel II objectives include the enhancement of financial stability and the promotion of sound risk measurement standards. Unless Basel II fortifies the minimum bank capital requirements for any given set of exposures, it is unclear how it will lead to enhanced stability in the banking sector. Quantitative Impact Studies (QIS) show that large internationally active banks will benefit from large capital reductions under Basel II, especially under the AIRB approach. Once banks are allowed to optimize under the AIRB approach, capital levels will be further eroded.

The results of the QIS studies call into question whether the Basel AIRB approach in its current form should even be considered a minimum regulatory capital standard. The idea of a standard implies that positions with identical risks are subject to identical minimum capital requirements. QIS studies show that AIRB estimates of minimum capital requirements for positions with similar risks vary by wide margins across banks. These results suggest that the AIRB rule and its associated guidance for implementation standards have been vaguely formulated and allow substantial capital differences or subjective interpretations. It is difficult to envision that supervisors around the globe will use pillar 2 powers and impose national implementation standards that ensure equal capital for equal risk. With wide latitude to interpret the input values for the AIRB capital rule, the AIRB approach cannot be viewed as a well-formulated standard.

Concerns about reduction in required capital under the AIRB approach are amplified when the economic foundations of the AIRB rule are examined. The current AIRB capital rule cannot accurately measure the credit risks taken in large complex banking institutions. It does not formally model capital needs that arise because *EAD* and *LGD* are themselves random factors that create potentially large unexpected credit losses that are not modeled in the AIRB framework. The current framework, moreover, is without a sound economic foundation. It ignores the capital needed to satisfy bank interest expenses. This oversight leads to a large understatement in AIRB capital requirements. The AIRB also omits any measure of the capital benefits that are generated by bank interest earnings on its credit portfolio. The adequacy of banks' pricing of credit risk is a primary factor of importance in measuring portfolio credits risk and assigning minimum capital needs

This analysis suggests that it is improbable that the AIRB approach would either enhance financial stability or serve as a sound standard against which bank credit risk measurement processes are evaluated. Although the list of apparent weakness in the AIRB approach discussed here may seem long, there are others. This paper's analysis has not addressed issues attendant to the AIRB approach not setting capital surcharges for credit risk concentrations--undoubtedly an important source of risk in many banking institutions. This is left for pillar 2. Issues regarding the accuracy of AIRB operational risk measurement standards are undoubtedly numerous, but they will be left as topics for other authors and future papers.

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