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Measurement of the B_s Lifetime in Hadronic Decays Using Partially and Fully Reconstructed Modes

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This note describes the use of both fully and partially reconstructed modes in a B_s lifetime measurement using $\sim$1.3 fb^{-1} of data reconstructed as $B_s \to D_s^-(\phi\pi^-)\pi^+$. The $B_s \to D_s^-(\phi\pi^-)\pi X$ events analyzed were produced in $\overline{p}p$ collisions at $\sqrt{s} = 1.96$ TeV by the Fermilab Tevatron and collected with the CDF II detector. The fit procedure was tested extensively on three control samples: $B^0 \to D^-(K^+\pi^-\pi^-)\pi^+$, $B^0 \to D^{*-}[\overline{D}^0(K^+\pi^-)\pi^-]\pi^+$, and $B^+ \to \overline{D}^0(K^+\pi^-)\pi^+$ before performing the B_s fit. We measure

$$c\tau(B_s \to D_s^-(\phi\pi^-)\pi^+) = 455.0 \pm 12.2(\text{stat.}) \pm 8.2(\text{syst.})\ \mu\text{m}$$

Preliminary Results for Winter 2008 Conferences

I. CHANGES BETWEEN VERSIONS 1.0 AND 2.0

The following changes were made between Feb 15, 2008 and Oct 23, 2008 .

- The systematic uncertainty for the SVX alignment has been changed to 1.0μm from 2.4μm to avoid the double counting of systematic effects.
- We now include a 4.1μm systematic uncertainty for SVT modeling. The estimation of the size of this uncertainty is described in Sec. VIII.
- The total systematic uncertainty for $\tau(B_s)$ has been changed from 7.4μm to 8.2μm.
- Sec. IX has been both corrected and updated.

II. INTRODUCTION

This note describes a measurement of the B_s lifetime in $\overline{p}p$ collisions at $\sqrt{s} = 1.96$ TeV with the CDF II detector at the Fermilab Tevatron. After trigger and analysis cuts, this sample yields more than 1100 fully reconstructed $B_s \to D_s^-(\phi\pi^-)\pi^+$ candidates. In addition, the sample reconstructed as $B_s \to D_s^-\pi^+$ includes a similar number of partially reconstructed B_s candidates, e.g. $B_s \to D_s^-\rho^+(\pi^+\pi^0)$ where the π^0 is not reconstructed. Since these candidates come from actual B_s mesons, they can contribute to the lifetime measurement and double the statistics available for analysis. The increased statistics will come with an uncertainty due to missing tracks or misassigned masses, but it can be properly accounted for and folded into the likelihood formulation.

The lifetime of the B_s meson is determined from two fits done sequentially. The first fit is of the reconstructed mass of the B_s candidates. This fit determines the relative fractions of the data that can be found in various decay modes and backgrounds and is described in Section V. The second fit is of the proper decay time of the B_s candidates. Using the fractions determined in the mass fit as inputs, a maximum likelihood fit for the B_s meson lifetime is performed as described in Section VI.

A data sample rich in hadronic B decays and low in background was selected with a displaced-vertex trigger, the SVT [1]. This means that the proper time distribution is no longer an exponential extending down to $ct = 0$. Instead there is a visible “trigger turn-on” in the distribution that we use Monte Carlo simulation to model correctly, and our lifetime PDFs include a multiplicative “trigger efficiency curve”. The PDFs will be discussed in detail in Section VI. We also consider the possibility that the trigger turn-on selects $B_{s,H}$ over $B_{s,L}$ and discuss the resulting imbalance and bias on the measurement of $c\tau_s$ if a 50/50 mixture was assumed in Section IX.

The fit procedure was tested extensively on three control samples: $B^0 \to D^-(K^+\pi^-\pi^-)\pi^+$, $B^0 \to D^{*-}[\overline{D}^0(K^+\pi^-)\pi^-]\pi^+$, and $B^+ \to \overline{D}^0(K^+\pi^-)\pi^+$ before performing blinded B_s fits. Good agreement with the PDG values of the B^0 and B^+ lifetimes was found.

The CDF detector is described in detail in [2].

III. DATA SAMPLE & EVENT SELECTION

This analysis is based on an integrated luminosity of $\sim$1.3 fb^{-1} collected with the CDF II detector between February 2002 and November 2006. The data are collected with three displaced-vertex triggers specifically designed for B physics [1]. The triggers require two tracks to separately have $p_T \geq 2$ GeV/c and impact parameters d_0 with respect to the B meson production point to satisfy 120μm$< d_0 <$1mm. The opening angle between the tracks ($\Delta\phi$ in the $r - \phi$ plane transverse to the beam direction) must be between 2° and 90°, and the intersection must be 200μm from the B meson production point. The three triggers differ in requirements for the charges of the tracks and their Σp_T, and they also have different prescales.

From this dataset we select events offline with reconstructed B candidates that pass the cuts listed in Table II. To further separate B_s mesons from background, we cut on the following variables: the quality of the reconstructed B and D candidate vertices, $\chi^2_{xy}(B)$ and $\chi^2_{xy}(D)$; the transverse distance traveled by the B before decay $L_{xy}(B)$ and its significance $L_{xy}(B)/\sigma_{L_{xy}}(B)$; the transverse distance between the B and D decay points $L_{xy}(B \leftarrow D)$; the transverse and longitudinal impact parameters of the B $d_0(B)$ and $z_0(B)$; the transverse momentum of the B $p_T(B)$; the transverse momentum of the π from the B $p_T(\pi_B)$; the angle between the D and the π from the B $\Delta R(D, \pi_B)$ where $\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$; and the *isolation* of the B, defined as $p_T(B)$ divided by the scalar sum of the transverse momenta of all the tracks in a cone of $\Delta R < 1$ around the B. We also cut tightly on the mass of the ϕ and D_s and veto D_s candidates consistent with $D^{*+} \to D^0(K^-\pi^+\pi^0)\pi^+$ decays. The D^* veto is accomplished by taking the D_s daughter tracks ($KK\pi$), assigning the pion mass to the second kaon, and cutting on $\Delta m = m(K\pi\pi) - m(K\pi)$.

IV. MONTE CARLO SAMPLES

The Monte Carlo samples for this analysis consist of single b hadrons generated with p_T spectra consistent with NLO QCD and decayed with `EvtGen` [3]. Full detector and trigger simulations are performed for the Monte Carlo events as well. The Monte Carlo B candidates are reconstructed with the same procedure and the same cuts as the data candidates.

The Monte Carlo differs from the data in both its momentum spectrum and its trigger composition. To correct for this effect, the Monte Carlo (after analysis cuts have been applied) is reweighted to match the data. Due to the low statistics of the B_s data, the reweighting is performed in two steps. In the first, we reweight in the variable $p_T(B)$. In the second, we reweight in three trigger categories.

V. MASS FIT DESCRIPTION

The mass fit is a maximum likelihood fit to the invariant mass of the candidate reconstructed as $D_s^-(\phi\pi^-)\pi^+$. The mass fit components can be characterized as coming from one of three possible sources:

- **Single B:** These D plus track combinations come from a single-b hadron. The mass PDFs for these components come from Monte Carlo samples. Since these samples have several times the statistics of data, we choose to use histograms instead of functional forms for these PDFs.
- **Real D + track:** This background consists of a real D, perhaps promptly produced or from a b hadron decay, plus a track from the underlying event or other b hadron. The mass template for these events comes from an auxiliary fit to the wrong-sign (WS) sample, data events reconstructed as $D_s^-\pi^-$. Sideband subtraction in the D mass is performed to isolate the real D's. The invariant mass of the $D_s\pi$ is then fit with an exponential plus a flat background in the restricted region [5.00,6.45] GeV. Below 5.00 GeV partially reconstructed single-B modes contaminate the WS distribution. The normalization of the real-D component is allowed to float in the final mass fit, but the shape parameters are fixed.
- **Fake D + track:** This background could still consist of tracks from b hadron decays, but the distribution of the masses of the tracks reconstructed as D's in these candidates does not peak. The mass template for these events comes from the D sidebands. However, the D sidebands are close enough to the D peak that there is some leakage from the single-B modes that must be subtracted out in an auxiliary fit. The D-sideband fit employs an exponential plus flat background for the fake-D component and a histogram taken from the D sidebands of single-B Monte Carlo. When the fake-D PDF is used in the final mass fit, we fix its normalization by extrapolating the D sidebands into the D signal region.

VI. LIFETIME FIT DESCRIPTION

The lifetime of the B_s meson is determined from an unbinned likelihood fit with separate probability distribution functions for the fully reconstructed (FR) modes, partially reconstructed (PR) modes, and the backgrounds. The FR and PR PDFs depend on τ_B, the B_s lifetime, while the background PDFs have fixed shapes. The likelihood can be written as

$$\mathcal{L} = \prod_{i=1}^{N_{\mathrm{events}}} \left[f_{\mathrm{FR}} P_{\mathrm{FR}}(ct_i) + \sum_{j=1}^{m} f_{\mathrm{PR},j} P_{\mathrm{PR},j}(ct_i) + \sum_{k=1}^{n} f_{\mathrm{Bkgd},k} P_{\mathrm{Bkgd},k}(ct_i) \right] \tag{1}$$

where m is the number of PR modes and n the number of background modes. See Section VI B 1 for definitions of the variables. The functional forms for P_{FR}, P_{PR}, and P_{Bkgd} are discussed in Section VI B.

The fractions f_{FR}, $f_{\mathrm{PR},j}$, and $f_{\mathrm{Bkgd},k}$ are determined from an initial mass fit and are fixed in the lifetime fit. The systematic uncertainty associated with f_{Bkgd} will be further discussed in Section VIII. The mass fit procedure is summarized in Section V. The fractions satisfy the condition

$$f_{\mathrm{FR}} + \sum_{j=1}^{m} f_{\mathrm{PR},j} + \sum_{k=1}^{n} f_{\mathrm{Bkgd},k} = 1$$

The lifetime fit is performed in three B mass regions, the third region being the union of the first two. The fully reconstructed (FR) region $m_B \in [5.35, 5.45]$ GeV contains the bulk of the D_sK and $D_s\pi$ peaks. The partially reconstructed (PR) region $m_B \in [5.00, 5.35]$ GeV consists mostly of $D_s\rho$ and $D_s^*\pi$ and has a larger background fraction. The FR and PR regions are fit separately to check that the fitting tools perform correctly in both cases. The third fit is to the union of the FR and PR regions $m_B \in [5.00, 5.45]$ GeV. This last fit properly propagates the errors from the FR and PR regions, and its result is the final value we quote.

A. Lifetime Fit Components

As the likelihood formulation above suggests, there are three main types of lifetime fit components. How each component is treated depends on its decay structure and whether it can provide information about the B_s lifetime. There are separate PDFs for the three categories of proper time distribution.

- **Fully Reconstructed:** The only mode fully reconstructed is the $D_s\pi$. The fully reconstructed mode was the main contributor of statistics to the previous lifetime analyses, as there are no uncertainties associated with tracks missing from the reconstructions. The shape of this PDF comes from a fit to the Monte Carlo sample where the lifetime used for generation is known. All the parameters for the PDF except the lifetime of the B_s meson are then fixed for the final fit to data. The partially reconstructed, `PHOTOS`-modeled $D_s\pi(n\gamma)$ decays are grouped with the FR $D_s\pi$ for this analysis. This simplification is considered as a possible source of systematic uncertainty.

- **Partially Reconstructed:** As they also come from B_s mesons, the D_sK, $D_s\rho$, $D_s^*\pi$, and other partially reconstructed modes can contribute to the B_s lifetime measurement. Since the partially reconstructed candidates are missing tracks after reconstruction, a multiplicative correction factor K to the decay length is needed. See Section VI B 1 for a complete definition of the K factor. The distributions of the K factors for these modes are taken from Monte Carlo and folded into the PR PDFs. Strictly speaking, the D_sK mode is not a partially reconstructed mode, but a mode with a single mis-identified track. However, we choose to group it with the PR modes, since it also requires a K factor. Equivalent to the FR PDF, the shapes of the PR PDFs come from fits to Monte Carlo samples where the lifetime used for generation is known. All the parameters for the PDFs except the lifetime of the B_s meson are then fixed for the final fit to data.

- **Background:** The backgrounds can either come from other single-B modes such as $B^0/B^- \to D^-X$, $B^0 \to D_sX$, and $\Lambda_b \to \Lambda_cX$ reconstructed as $D_s(\phi\pi)\pi$ for which we have Monte Carlo, or they be combinatorial in nature. In the mass fit, the combinatorial backgrounds are grouped into real D and fake D. For the lifetime fit, there are two combinatorial background proxies available.

  - The B upper sideband, taken from the m_B interval [5.7,6.4] GeV, contains a mixture of real and fake D's. The fractions of each can be determined by counting the number of events in the D-peak and D-sideband regions and extrapolating.
  - The D sidebands in the fully and partially reconstructed regions contain fake D's. There is some leakage from the D signal as well, and the fraction can be determined in an auxiliary fit as described in V

  Since neither background proxy is 100% real or fake D, we must use the mass fit fractions as inputs to calculate the fractions of B upper sideband and D sideband needed for the lifetime fit.

  The background PDFs come from fits to Monte Carlo samples(for single-B background) or from the background proxies discussed above (for fake D and real D not coming from a single-B meson). All the background shape parameters are fixed in the final lifetime fit. Uncertainties in the background fractions from the mass fit are treated as systematic uncertainties.

B. Functional Forms of Probability Distribution Functions

1. *Definition of Variables*

With perfect knowledge of the decay of a B meson, the true proper decay length ct' could be easily calculated as

$$ct' = \frac{L'_{xy} \cdot m'_B}{p'_{T_B}} \tag{2}$$

where p'_{T_B} is the transverse momentum of the B, L'_{xy} the decay length projected along the direction of p'_{T_B}, and m'_B the B mass. The prime indicates that the symbol represents the true value of the quantity, not the measured value.

In an experimental situation, the reconstructed proper decay length ct of a fully reconstructed B meson is defined as

$$ct \equiv \frac{L_{xy} \cdot m_B^{rec}}{p_{T_B}} \tag{3}$$

Note that the reconstructed mass of the B meson in the event m_B^{rec} is used instead of the PDG B mass.

A salient feature of this analysis is the treatment of partially reconstructed B mesons as signal events that contribute to the lifetime measurement. Since in the partially reconstructed cases L_{xy}, m_B^{rec}, and $p_T(B)$ are extracted from candidates that are missing tracks after reconstruction or have the wrong mass assignment for a single leg, a multiplicative correction factor K to the decay length is needed. The distribution of the K factor is taken from Monte Carlo. The K factor should only correct for the track dropped during reconstruction and not include any of the uncertainties introduced by detector simulation. For each Monte Carlo event, one can construct a "partially reconstructed MC candidate."

By constructing the ratio of the true decay length of the B meson to the decay length that would be calculated from the partially reconstructed MC candidate, we can quantify the effect of missing tracks and misassigned masses. K is then given as

$$\begin{aligned} K &= \frac{ct(\mathrm{MC})}{ct(\mathrm{PR})} \\ &= \frac{L_{xy}(\mathrm{MC})}{L_{xy}(\mathrm{PR})} \cdot \frac{p_T(\mathrm{PR})}{p_T(\mathrm{MC})} \cdot \frac{m_{rec}(\mathrm{MC})}{m_{rec}(\mathrm{PR})} \\ &= \frac{1}{\cos\theta_{\mathrm{PR}}} \cdot \frac{p_T(\mathrm{PR})}{p_T(\mathrm{MC})} \cdot \frac{m_B(\mathrm{PDG})}{m_{rec}(\mathrm{PR})}, \end{aligned} \tag{4}$$

where θ_{PR} is the angle in the $x - y$ plane between the transverse momenta of the fully and partially reconstructed B's. Note that although the position of the B-decay vertex is the same in both candidates, the decay length projected along the transverse momentum changes by a factor of $\cos\theta_{PR}$. "MC" denotes Monte Carlo truth. "PR" denotes that a quantity is calculated from the partially reconstructed MC candidate. The K factor distributions from Monte Carlo are shown in Figure 1

2. *PDF for Fully Reconstructed Decays*

With perfect knowledge of decays of particles with lifetime τ, the distribution of the true decay lengths ct' would be described by an exponential decay model. The PDF is given by the following equation:

$$P_{\mathrm{FR}}(ct') = \frac{1}{c\tau} e^{\frac{-ct'}{c\tau}} \tag{5}$$

Experimentally, we find a measured decay length ct that has been smeared by the detector resolution σ. This gives

$$P_{\mathrm{FR}}(ct) = \frac{1}{c\tau} e^{\frac{-ct'}{c\tau}} \otimes_{t'} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(ct-ct')^2}{2\sigma^2}} \tag{6}$$

where $\otimes_{t'}$ indicates the convolution over the true decay time variable t'.

In this analysis the decay length distribution is also sculpted by the trigger selection and analysis cuts. These enter the PDF as a function of the measured decay length, commonly called the efficiency curve eff(ct). The parameterization of the efficiency curve will be discussed in Section VI B 5.

$$P_{\mathrm{FR}}(ct) = \left[\frac{1}{c\tau} e^{\frac{-ct'}{c\tau}} \otimes_{t'} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(ct-ct')^2}{2\sigma^2}} \right] \cdot \mathrm{eff}(ct) \tag{7}$$

A more detailed discussion of the fit for the parameters of this function follows in Section VI C. In brief, we determine a σ and the efficiency curve parameters for each mode separately by fitting Monte Carlo where the τ used

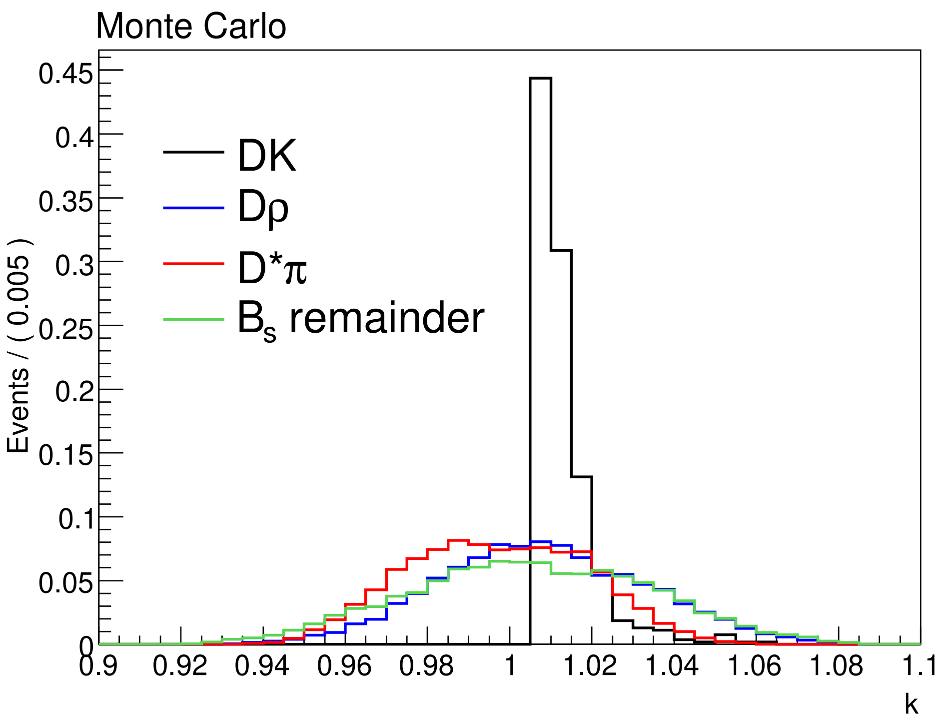

FIG. 1: K factor distributions for the modes partially reconstructed as $B_s \to D_s^-(\phi\pi^-)\pi^+$. The remainder piece encompasses all $B_s \to D_s^-(\phi\pi^-)\pi^+X$ modes that are not $D_s^-\pi^+$, $D_s^-K^+$, $D_s^{*-}\pi^+$, or $D_s^-\rho^+$

for generation is known and fixed in the auxiliary fit. The efficiency curve parameters and σ are then fixed in the fit to the data. The only free parameter in the unbinned likelihood fit to the data is the lifetime of the B_s meson. Note that although σ is intended to be a detector resolution, it floats along with the efficiency curve parameters during the fits to the Monte Carlo. During this process it becomes correlated with the other parameters describing the overall PDF shape and loses some of its physical meaning.

NOTE ABOUT NORMALIZATION: Although we have been cavalier about writing the above expression without explicit normalization, it should be done at least once. The following normalization condition applies:

$$\int P(ct)\,\mathrm{d}ct \;=\; 1 \tag{8}$$

Equation (7) should then be written as

$$P_{\mathrm{FR}}(ct) = \frac{\left[\frac{1}{c\tau}e^{\frac{-ct'}{c\tau}} \otimes_{t'} \frac{1}{\sqrt{2\pi}\sigma}e^{\frac{-(ct-ct')^2}{2(\sigma)^2}}\right]\cdot \mathrm{eff}(ct)}{\int\left[\frac{1}{c\tau}e^{\frac{-ct'}{c\tau}} \otimes_{t'} \frac{1}{\sqrt{2\pi}\sigma}e^{\frac{-(ct-ct')^2}{2(\sigma)^2}}\right]\cdot \mathrm{eff}(ct)\,\mathrm{d}ct} \tag{9}$$

but we will not explicitly write the normalization again; it is implied.

3. PDF for Partially Reconstructed Decays

For the partially reconstructed case the distribution of K factors must be folded in as well.

$$P_{\mathrm{PR}}(ct) = \left[\frac{1}{c\tau}e^{\frac{-ct'}{c\tau}} \otimes_{t'} \frac{1}{\sqrt{2\pi}K\sigma}e^{\frac{-(K\cdot ct-ct')^2}{2K^2\sigma^2}} \otimes_K p(K)\right]\cdot \mathrm{eff}(ct) \tag{10}$$

For this analysis the D_sK, $D_s^*\pi$, and $D_s\rho$ modes are also treated as signal, using separate PR PDFs with separate K factor distributions, σ's, and efficiency curves for each mode. The remaining B_s decay modes also contribute to the $c\tau$ measurement, but they are grouped together with a collective K factor distribution and efficiency curve into a single partially reconstructed PDF.

4. PDF for Background Events

The functional form of the background PDF is essentially the same as the FR PDF, however the background PDFs do not depend on $\tau(B_s)$. The background ct shape parameters are determined in fits to background samples (either single-B Monte Carlo background samples or combinatorial backgrounds from data) and are fixed in the final fit to data. One should note, however, that the decay constant of the exponential, τ_{Bkgd}, has little physical meaning for the combinatorial backgrounds, since the background may not come from a single physics source (such as a decaying B meson). Because τ_{Bkgd} is degenerate with other lifetime-like parameters in the efficiency curve, we fix it at 500 μm with no detrimental effect on the fit quality.

$$P_{\mathrm{Bkgd}}(ct) = \left[\frac{1}{c\tau_{\mathrm{Bkgd}}} e^{\frac{-ct'}{c\tau_{\mathrm{Bkgd}}}} \otimes_{t'} \frac{1}{\sqrt{2\pi}\sigma_{\mathrm{Bkgd}}} e^{\frac{-(ct-ct')^2}{2\sigma_{\mathrm{Bkgd}}^2}} \right] \cdot \mathrm{eff}(ct) \tag{11}$$

Although its name suggests otherwise, eff(ct) for the real D and fake D backgrounds is not actually an efficiency curve. Efficiency curves parameterize the effect of the trigger and analysis cuts on the shape of the measured ct distribution for signal events. Since we don't have background Monte Carlo and can't tell how the trigger selections and analysis cuts affect the background shape, the best we can do is to choose a sample to serve as a proxy for the background and use its ct distribution as the background PDF. We choose to parameterize the shape of the background instead of using histograms, and a natural choice for the parameterization is a functional form similar to those used for the fully and partially reconstructed modes. However, eff(ct) for the background PDF doesn't have the same physical significance.

5. Efficiency curve parameterization

The efficiency curves describing the sculpting of the decay length distribution due to trigger and analysis cuts are parameterized the same way as in the previous hadronic B lifetime analysis [4].

$$\mathrm{eff}(ct) = \begin{cases} 0 & \text{if } ct \leq \beta_i \\ \sum_{i=1}^{3} N_i \cdot (ct-\beta_i)^2 \cdot e^{\frac{-ct}{\tau_i}} & \text{if } ct > \beta_i \end{cases} \tag{12}$$

Since the total PDF must be normalized to one, the efficiency curve actually only contributes 8 additional parameters to the PDF: β_1, β_2, β_3, τ_1, τ_2, τ_3, N_2/N_1, and N_3/N_1. Further discussion of the fits for the efficiency curve parameters follows in Section VI C.

C. Measurement of Efficiency Curves

The parameters of the efficiency curves for the FR and PR modes are derived from unbinned likelihood fits to the reweighted Monte Carlo with τ_B set to the lifetime used for its the generation. For some fits to low statistics Monte Carlo, fewer than eight parameters are needed for a good fit (high χ^2 probability), and N_3 and possibly N_2 may be set to zero.

After the efficiency curve parameters are found in fits to the reweighted Monte Carlo (with τ_B set to the generator lifetime), they are fixed for the fit to the data. The only free parameter in the unbinned likelihood fit to the data is the lifetime of the B_s meson.

VII. RESULTS

A. Mass Fit Results

When performing the mass fit, the the $D_s^- K^+/D_s^- \pi^+$ fraction is constrained to the recent CDF measurement [5], and the size of the $D_s^- \pi^+$ radiative tail is fixed to the default PHOTOS value.

The auxiliary fit to the D sidebands to determine the fake-D shape is found in Figure 2. The auxiliary fit to the B_s sideband-subtracted WS sample to determine the real-D shape is found in Figure 3. The normalization of the fake-D component is fixed by extrapolating the D sidebands into the D signal region.

A plot of the mass fit of the full data sample is found in Figure 4. The results of the fit in the full mass range [4.85,6.45] GeV are shown in Table III.

B. Lifetime Fit Results

The lifetime of $c\tau_{B_s} = 455.0 \pm 12.2(\text{stat.})$ μm is obtained from the full fit. The results of the lifetime fit are separated by mass region in Table IV. The fit results are plotted in Figure 5. The results for the FR region (436.5 $\pm$ 20.0 μm) and PR region (463.0 $\pm$ 15.2 μm) disagree with each other by 1.0σ.

VIII. SYSTEMATIC UNCERTAINTIES

We use toy Monte Carlo to assess the systematic uncertainties. For each systematic uncertainty, 1000 pseudo-experiments are generated with the number of events Poisson-distributed around the number of events in the combined fully and partially reconstructed regions. With one exception, each set of experiments is generated with a non-standard PDF or component fraction, and the model used to fit the the pseudo-experiments is the same as was used for the $B_s \to D_s^- \pi^+$ full data sample fit. The mean biases ($c\tau_{\text{returned}} - c\tau_{\text{gen}}$) returned from the toy Monte Carlo are used to set the size of the systematic uncertainties. We list the sources of systematic uncertainty below, and Table I contains the final list of systematic uncertainties for this measurement.

- **Background Model Choice and Fractions:** In the default model, the B upper sideband ($m_B \in$ [5.7,6.4] GeV) and the D sideband in the FR + PR regions ($m_B \in$ [5.00,5.45] GeV) are combined to form the mixture of real D's and fake D's measured by the mass fit. There are several inputs that go into determining the fractions of B upper sideband and D sideband, and the uncertainties on these inputs translates into a systematic uncertainty on the final lifetime measurement. We also test a $\sim 2\sigma$ variation where we use an alternate mass shape for the real-D background, and generate with the new fraction.

- **Fixed Single-B Background $c\tau$:** The PDFs for the single-B backgrounds come from fits to Monte Carlo samples. If a lifetime used by the Monte Carlo generator is different from the true hadron lifetime by a few percent, this would then affect the B_s lifetime measurement. To estimate the size of this effect, we generate toy MC with $c\tau_{B^0}$, $c\tau_{B^+}$,and $c\tau_{\Lambda_b}$ set to 98% of the default values, and fit with the default model. We repeat the toy Monte Carlo study with the lifetimes set to 102% of the default values.

- **Effect of Reweighting for p_T and Trigger Category:** Due to the finite statistics of data and Monte Carlo, there is a possibility that a bias is introduced when the Monte Carlo samples are reweighted to match the data p_T and trigger category distributions. As an estimate of the systematic uncertainty associated with reweighting to a binned histogram, we use a functional form fit to the p_T ratio histogram instead. We choose to fit the histogram with a first degree polynomial. The efficiency curves derived from Monte Carlo reweighted to the function are then used for generation of toy Monte Carlo.

- **Lifetime Contribution of the PHOTOS Tail:** In the default lifetime fit configuration, we do not currently separate the radiative tail (modeled by PHOTOS) from the $D_s^- \pi^+$ Monte Carlo before fitting for the P_{FR} efficiency curve parameters. For the mass fit, the PHOTOS tail is treated separately, although its value is fixed to 4.2% of the non-PHOTOS $D_s^- \pi^+$ size. Here we estimate the systematic uncertainty associated with (1) treating the $D_s \pi$ as a single, fully reconstructed mode and (2) fixing the size of the PHOTOS tail.

- **Efficiency Curve Parameterization:** To test whether a PDF adequately describes a Monte Carlo ct distribution, one would ideally fit for the efficiency curve parameters on one EvtGen Monte Carlo sample and then freeze the parameters and fit for the lifetime on 1000 other EvtGen samples. Rather than generate extra

	Uncertainty (μm)
Background modeling and fractions	5.7
Fixed single-B background ct	1.0
Reweighting for p_T and trigger	3.7
Lifetime contribution of PHOTOS tail	0.6
Efficiency curve parameterization	0.6
SVT Modeling	4.1
Impact parameter correlation	1.0
Alignment + others	1.0
TOTAL	8.2

TABLE I: Final systematics for $B_s \rightarrow D_s^-(\phi\pi^-)X$ fit. The total is calculated assuming the individual contributions are uncorrelated.

`EvtGen` samples, we "make" 1000 samples from the one we have by bootstrapping from the Monte Carlo and background data samples. As with the other toy Monte Carlo, the samples generated have the number of events Poisson-distributed around the number of events in the combined data regions. The same event can be chosen twice; not all events have to be chosen. Each bootstrapped sample is then fit with the default PDFs.

- **SVT Modeling** Because the efficiency curve parameters are derived in fits to Monte Carlo and fixed in the final fit to data, the lifetime measurement relies heavily on the agreement between Monte Carlo and data. The concern addressed by this systematic uncertainty is how well the trigger turn-on is modeled in the Monte Carlo. Since the SVT simulation code that is run in the trigger simulation is identical to the SVT code run on data, we expect excellent agreement. However, because the trajectories and hit patterns of real and simulated particles can be different even if they have identical initial conditions, the SVT response can also be different.

  To determine whether there is an overall disagreement between the trigger turn-ons in data and Monte Carlo, we study a J/ψ sample collected by the unbiased dimuon trigger. We determine the efficiency for Scenario A events as a function of $L_{xy}(J/\psi)$ for J/ψ data and Monte Carlo. Fitting a line to the ratio of data and MC efficiencies, we take the uncertainty on the slope as an estimate of the uncertainty on the level of data-MC agreement. We use a line with zero slope $\pm 1\sigma$ variations to reweight the B_s Monte Carlo (after the p_T and trigger reweightings have already been performed) and calculate new efficiency curves. These new efficiency curves are used to generate toy MC which are then fit with the default curves. We take the larger mean bias from the $+1\sigma$ or -1σ variation as the systematic uncertainty for SVT modeling.

- **Impact Parameter Correlation** We use the 1.0 μm value of the previous hadronic lifetime analysis [4].

- **Alignment + others** We use the 1.0 μm value of a previous $B_s \rightarrow J/\psi\phi$ lifetime analysis [6].

IX. EFFECT OF TRIGGER BIAS ON 50/50 MIXTURE OF $B_{s,L}$ AND $B_{s,H}$

The displaced-vertex trigger alters the expected mixture $B_{s,L}$ and $B_{s,H}$ in flavor-specific decays by preferentially selecting the longer lived $B_{s,H}$. The size of the imbalance depends on the values of $\Delta\Gamma$ and Γ.

At production, a $|\bar{b}s\rangle$ state is an equal mixture of $B_{s,H}$ and $B_{s,L}$. The decay length distribution as a function of time (assuming no trigger bias) is $F(t) \propto e^{-\Gamma_H t} + e^{-\Gamma_L t}$ where $\Gamma_H = 1/\tau_H$, $\Gamma_L = 1/\tau_L$, and $\Gamma = (\Gamma_H + \Gamma_L)/2$. If the PDF is integrated over time, one does not expect equal parts $B_{s,H}$ and $B_{s,L}$. The fraction of $B_{s,H}$ will be

$$f_H = \frac{1/\Gamma_H}{1/\Gamma_H + 1/\Gamma_L} = \frac{\Gamma_L}{\Gamma_H + \Gamma_L} = \frac{\Gamma_L}{2\Gamma}. \tag{13}$$

The fraction of $B_{s,L}$ will be

$$f_L = \frac{1/\Gamma_L}{1/\Gamma_H + 1/\Gamma_L} = \frac{\Gamma_H}{\Gamma_H + \Gamma_L} = \frac{\Gamma_H}{2\Gamma}. \tag{14}$$

The expected fraction of $B_{s,H}$ in a flavor-specific sample is plotted as a function of $\Delta\Gamma/\Gamma$ as a dashed line in the upper panel of Figure 6 assuming the world average $1/\Gamma = 1.47$ ps^{-1} [7]. The mean B_s lifetime measured in such a flavor-specific sample will be

$$\begin{aligned} \tau_m(B_s) &= f_L \cdot \tau_L + f_H \cdot \tau_H \\ &= \frac{1}{\Gamma} \cdot \frac{1 + \frac{1}{4}\left(\frac{\Delta\Gamma}{\Gamma}\right)^2}{1 - \frac{1}{4}\left(\frac{\Delta\Gamma}{\Gamma}\right)^2} \end{aligned} \tag{15}$$

Note that $\tau_m(B_s) \neq 1/\Gamma$ and also $\tau_m(B_s) \neq (\tau_H + \tau_L)/2$, since

$$\begin{aligned} \frac{1}{2}(\tau_H + \tau_L) &= \frac{1}{2}\left(\frac{1}{\Gamma_H} + \frac{1}{\Gamma_H}\right) \\ &= \frac{1}{2}\left(\frac{1}{\Gamma - \frac{1}{2}\Delta\Gamma} + \frac{1}{\Gamma + \frac{1}{2}\Delta\Gamma}\right) \\ &= \frac{1}{\Gamma} \cdot \frac{1}{1 - \frac{1}{4}\left(\frac{\Delta\Gamma}{\Gamma}\right)^2} \end{aligned} \tag{16}$$

As was mentioned earlier, trigger and analysis cuts change the expected decay length distributions from simple exponential decays to the forms found in Equations 7 and 10. Once the efficiency curve parameters are known, these equations (with an appropriate factor of τ out front) can be integrated with $\tau = \tau_L$ and $\tau = \tau_H$ to determine the f_L^{SVT} and f_H^{SVT} for our SVT-triggered sample. We plot f_H^{SVT} as a function of $\Delta\Gamma/\Gamma$ as a solid line in the upper panel of Figure 6 assuming the world average $1/\Gamma = 1.47$ ps$^-1$ [7]. The mean B_s lifetime measured in the SVT-triggered sample will be $\tau_m^{SVT}(B_s) = f_L^{SVT} \cdot \tau_L + f_H^{SVT} \cdot \tau_H$

The lower panel of Fig. 6 shows the resulting bias $\tau_m^{SVT}(B_s) - \tau_m(B_s)$ in the flavor-specific lifetime measurement. The yellow band represents the most likely value from the 2008 world average $\Delta\Gamma/\Gamma = 0.069 + 0.058 - 0.062$ [7]. The graph indicates the most likely bias is 0-0.5 μm. Because this is much smaller than our current statistical and systematic uncertainties, we do not correct the central value or assess an additional systematic uncertainty. We provide a correction factor of the form $a + b\,(\Delta\Gamma/\Gamma) + c\,(\Delta\Gamma/\Gamma)^2$ from a fit to the lower panel in Fig. 6 where we fix $a = b = 0$ for physical reasons. Our result can be corrected back to a flavor-specific lifetime measurement with $\delta\tau(B_s) = 33.09\,(\Delta\Gamma/\Gamma)^2\ \mu m$.

X. CONCLUSIONS

A fit for the lifetime of the B_s of the $\sim$1.3 fb^{-1} of data reconstructed as $B_s \to D_s^-(\phi\pi^-)\pi^+$ is performed. The fit utilizes both fully and partially reconstructed modes. Toy Monte Carlo studies are used to set the size of the systematic uncertainty. We measure

$$c\tau(B_s \to D_s^-(\phi\pi^-)\pi^+) = 455.0 \pm 12.2(\text{stat.}) \pm 8.2(\text{syst.})\ \mu\text{m}$$

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	$B_s \to D_s^-(\phi\pi^-)\pi^+$
$\chi^2_{xy}(B)$	< 10
$\chi^2_{xy}(D)$	< 15
$L_{xy}(B)$ (μm)	> 450
$L_{xy}(B)/\sigma_{L_{xy}}(B)$	> 5
$L_{xy}(B \leftarrow D)$	> 0
$\lvert d_0(B)\rvert$ (μm)	< 60
$p_T(B)$ (GeV)	> 5.5
$p_T(\pi_B)$ (GeV)	> 1.0
$\Delta R(D, \pi_B)$	< 1.5
$\lvert z_0(B)/\sigma_{z_0}(B)\rvert$	< 3
iso(B) ($\Delta R < 1$)	> 0.5
$m(D_s)$ (MeV)	$\lvert m - 1968.3\rvert < 12$
$m(\phi)$ (MeV)	$1013 < m < 1028$
$m(K\pi\pi) - m(K\pi)$ (MeV)	$\Delta m > 180$
Good run list	B_s mixing version 17
SVT trigger confirmation	$\surd$

TABLE II: Final cuts used to select candidates for this analysis.

Decay modes	Number of events	Fraction of total
$B_s \to D_s\pi$	1127.84	0.202630 $\pm$ 0.007557
$B_s \to D_s\pi(n\gamma)$	47.32	0.008501
$B_s \to D_s K$	110.03	0.019769 $\pm$ 0.004348
$B_s \to D_s\rho$	1406.90	0.252767 $\pm$ 0.035888
$B_s \to D_s^*\pi$	648.21	0.116458 $\pm$ 0.030848
$B^0/B^+ \to D^+X$	307.44	0.055236 $\pm$ 0.019468
$B_s \to D_s^{(*)}K^{(*)}$	53.25	0.009566
$B_s \to D_s^{(*)}eX$	187.65	0.033713
$B_s \to D_s^{(*)}X$	872.90	0.156826 $\pm$ 0.022567
$B^0 \to D_s^{(*)}K$	26.79	0.004813
$B^0 \to D_s^{(*)}\pi$	24.10	0.004330 $\pm$ 0.022199
$\Lambda_b \to \Lambda_c X$	175.71	0.031569 $\pm$ 0.015183
Fake D_s +track	293.44	0.052720
Real D_s +track	284.44	0.051103 $\pm$ 0.005377

TABLE III: Results of the mass fit for $B_s \to D_s^-(\phi\pi^-)\pi^+$ in terms of the number of events in the region [4.85,6.45]MeV. Since the fit is non-extended, the errors on the fractions are quoted. The normalization of the fake D component was fixed in the fit. The other modes without quoted errors have their errors incorporated into modes they are fixed to.

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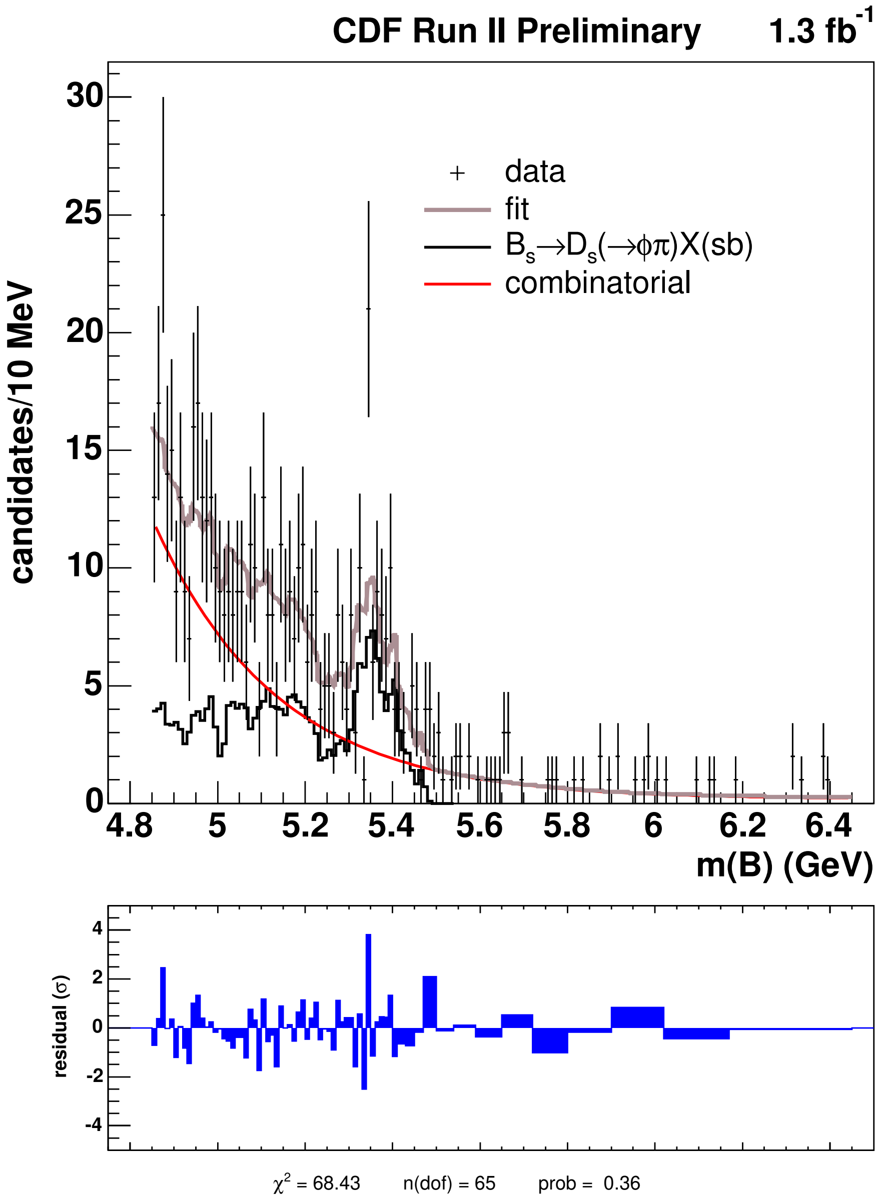


FIG. 2: This auxiliary fit to the D_s sidebands with D_s leakage subtracted is used to determine the shape of the fake-D component for the mass fit. These events are reconstructed as $B_s \to D_s^-(\phi\pi^-)\pi^+$ in full 1.3 fb^{-1}

FR	$\tau_{B_s}(\mu\text{m})$	436.5 ± 20.0
	χ^2/ndf	68.3/(67-1)
	prob	39.9%
PR	$\tau_{B_s}(\mu\text{m})$	463.0 ± 15.2
	χ^2/ndf	93.5/(89-1)
	prob	32.4%
FR + PR	$\tau_{B_s}(\mu\text{m})$	455.0 ± 12.2
	χ^2/ndf	99.6/(95-1)
	prob	32.8%

TABLE IV: Results of $B_s \to D_s^-(\phi\pi^-)\pi^+$ lifetime fit: The lifetime fit is performed in three mass regions. The fully reconstructed (FR) region includes all events with $m_B \in [5.35, 5.45]$. The partially reconstructed (PR) region includes all events with $m_B \in [5.00, 5.35]$. The combined (FR+PR) region includes all events with $m_B \in [5.00, 5.45]$

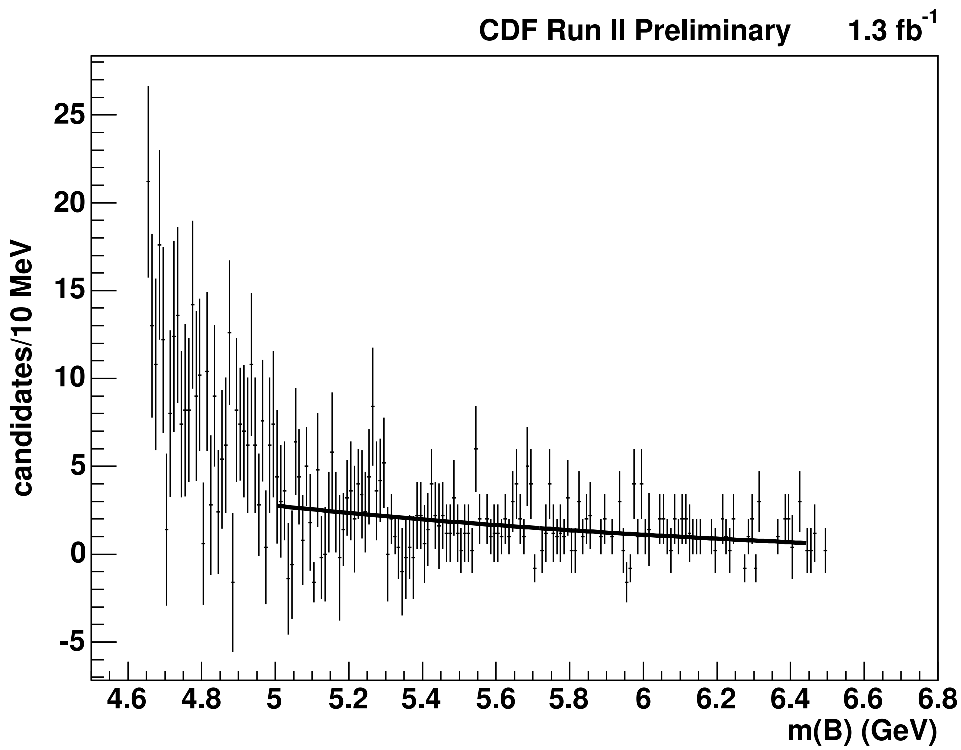


FIG. 3: This auxiliary fit to the sideband-subtracted WS sample from the full 1.3 fb^{-1} of data. The WS sample is made of events reconstructed as $D_s^-\pi^-$ that pass the analysis cuts. The functional form employed is an exponential plus flat background in the region $m_B \in [5.00, 6.45]$.

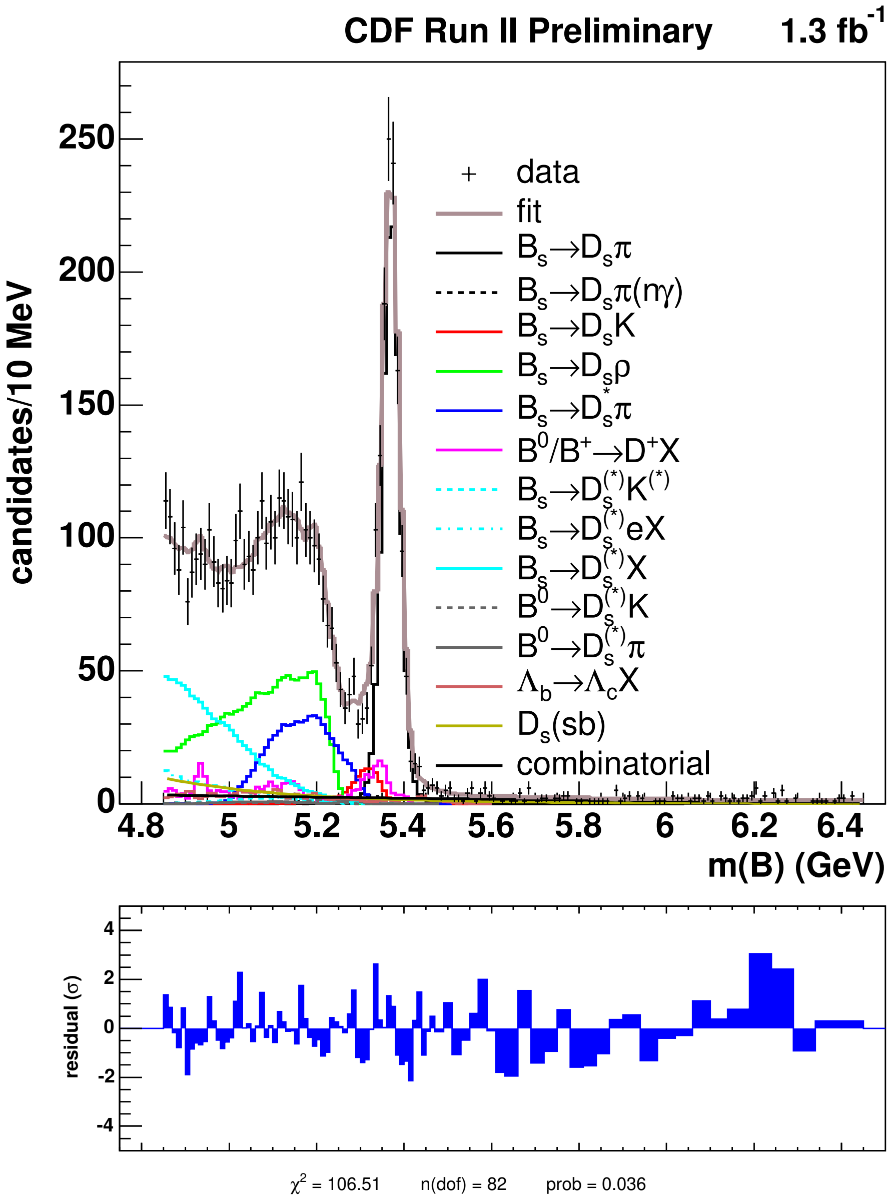
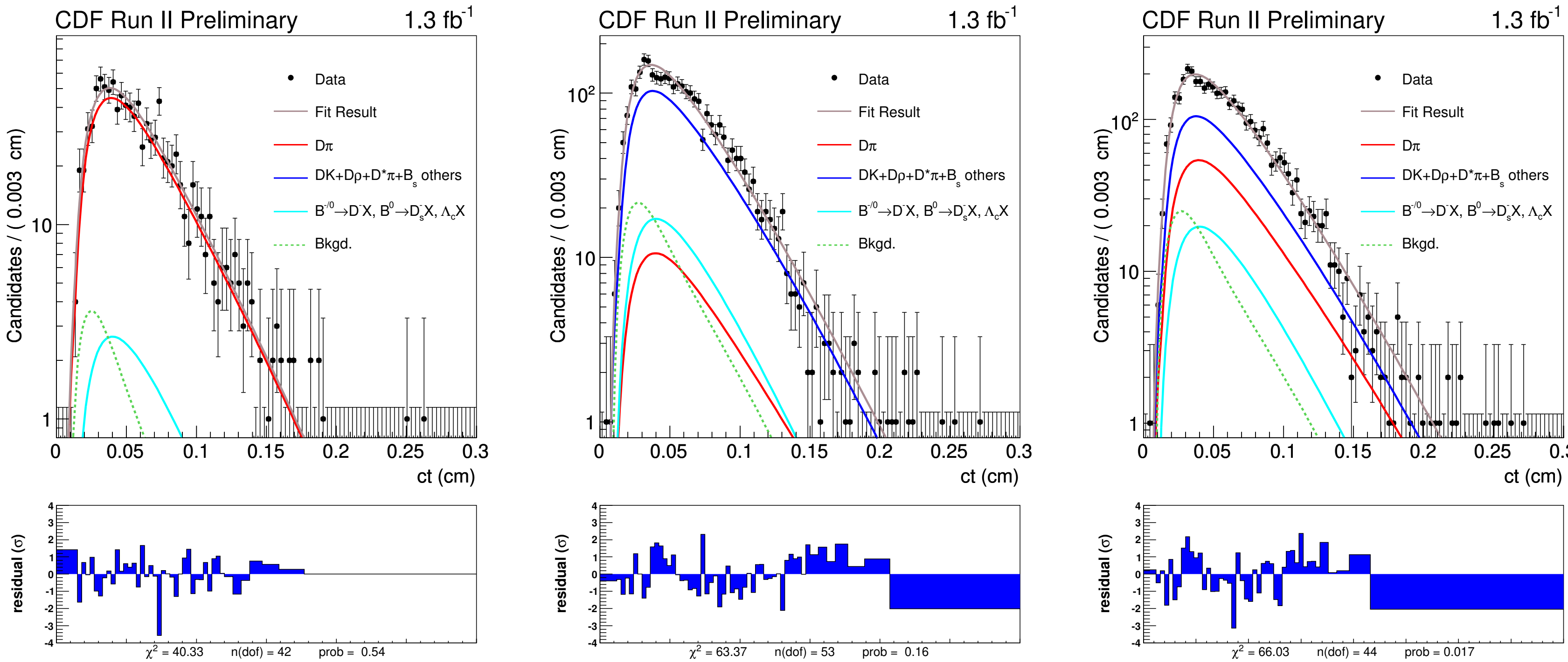


FIG. 4: Mass fit of events reconstructed as $B_s \to D_s^-(\phi\pi^-)\pi^+$ in full 1.3 fb^{-1}



(a) Fully Reconstructed Region (b) Partially Reconstructed Region (c) FR + PR

FIG. 5: ct projection of lifetime fit results for events reconstructed as $B_s \rightarrow D_s^- (\phi\pi^-)\pi^+$ in the three mass regions

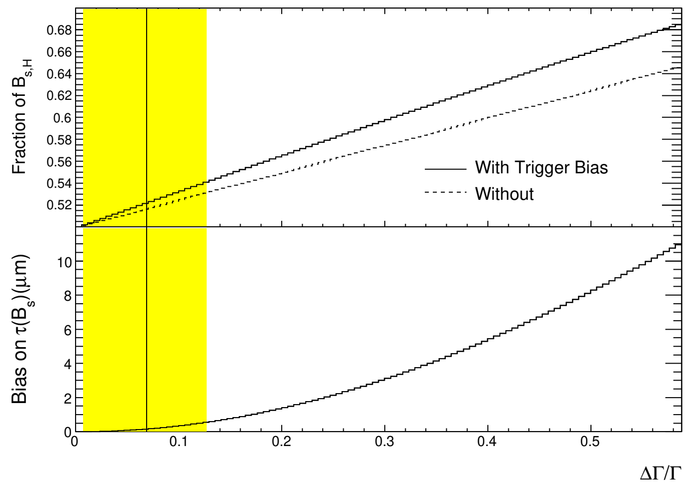


FIG. 6: Effect of trigger selection on the mixture of $B_{s,L}$ and $B_{s,H}$ in a flavor-specific sample as a function of $\Delta\Gamma/\Gamma$.