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Closed Orbit Effects Due to Longitudinal Bend Center Displacement

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Contents

Abstract

A longitudinal displacement of a dipole bend center will create a closed orbit distortion in a synchrotron. The bend center of a dipole or gradient magnet may be displaced from the mechanical center by non-uniform distribution of the dipole field. This will be illustrated using data from a prototype Main Injector 8 GeV Line dipole prototype. Orbit distortions due to magnet placement (survey) and magnet fabrication (longitudinal gradient) errors are calculated for single magnets. Effects of systematic errors on orbit dynamics will not be covered. Magnet measurement options for characterizing these errors will be discussed.

1 Introduction

For typical iron-dominated accelerator electromagnets, the longitudinal center of the magnet is very near the mechanical center since the same excitation is applied along the length by the coil, and the yoke geometry is designed to provide a uniform cross section. For hybrid permanent magnet assemblies, the same geometric arguments apply to the iron structures, but the excitation is provided by magnetized material which may not be of uniform strength. Variations of a few percent are possible due to material differences. In addition, asymmetries which are ascribed to assembly order effects which may produce non-uniform excitation of the remanent fields in the iron pieces have been reported in the prototype efforts for the permanent magnet 8 GeV transfer line to the Fermilab Main Injector. Example data will be shown.[1]

In order to provide specifications for the hybrid permanent dipole and gradient magnet design efforts which are underway at this time, calculations which define the relation between longitudinal non-uniformities, placement error, and orbit distortions will be reported here for single magnet errors. Analytical results for the combined effects of random errors in multiple magnets will be provided but systematic effects are also possible, including several sets with different magnitudes and signs of systematic center offset. Details of such calculations will be left for separate consideration. Effects due to longitudinal displacement of the focusing of the gradient magnet are also not covered here.

[1] One might assume that the first order asymmetries could be eliminate by using a symmetric assembly order: begin assembly at the center rather than at one end.

2 Measured Data from Hybrid Permanent Dipole

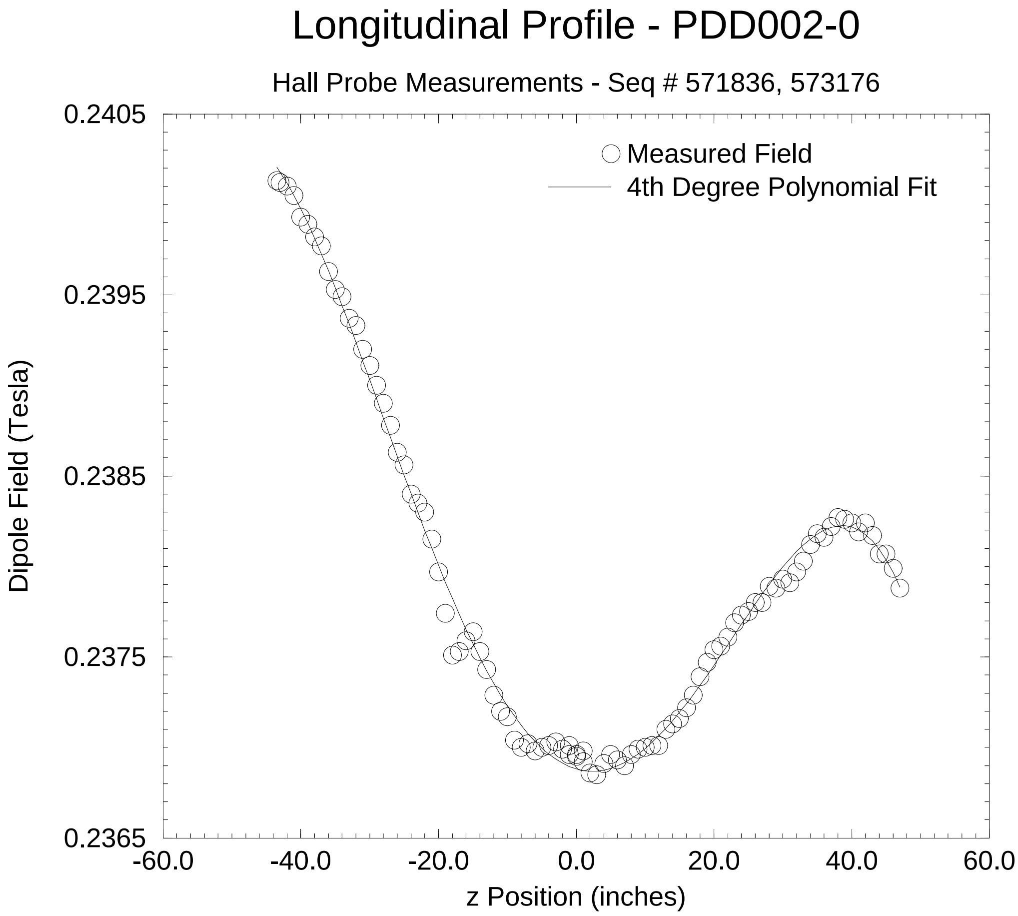

Figure 1: Magnetic field $B_y(z)$ on axis of PDD002-0 as measured with Hall Probe. Polynomial fit is to form and parameters shown in the table.

Let us begin by examining data from a hybrid permanent magnet dipole constructed as a prototype for the Booster to Main Injector 8 GeV Transfer Line project. Data for this magnet is shown in Figure 1 and we show fit parameters for it in Table 1. We see that for this magnet the longitudinal profile has significant nonuniformity. Among effects of this on beam dynamics, the longitudinal asymmetry, which displaces the bend center from the

n	C_n	c_n	$c_n \times (L/2)^n$
0	0.236881306	1	1
1	-1.1088e-05	-4.6809e-05	-2.4340e-03
2	2.36611e-06	9.9886e-06	2.7009e-02
3	-6.8111e-09	-2.8753e-08	-4.0429e-03
4	-6.1348e-10	-2.5898e-09	-1.8936e-02

Table 1: Fit Parameters for Longitudinal Shape of PDD002-0 as measured with Hall Probe scan along the transverse centerline. Fit is to the form $B_y(z) = \sum_n C_n = C_0 \sum_n c_n$. z is expressed in inches from the center of the magnet. 4th Column shows the fractional contribution of each term to the field at the magnet end. Data is shown in the figure above.

geometric center is, perhaps, most important.

3 Closed Orbit Analysis

Deviations from the design orbit of a synchrotron are created by bending fields which create angle changes, θ_i distributed around the ring. In terms of the standard accelerator formalism, one calculates the deviation $d(s)$ at the longitudinal location s from

$$d(s) = \sum_i \frac{\sqrt{\beta_i \beta(s)}\theta_i}{2\sin(\pi\nu)} \cos(|\Psi_i - \Psi(s)| - \pi\nu) \tag{1}$$

where ν, β and Ψ are the usual tune, amplitude function and phase function for a strong focusing lattice, and θ_i is the bend angle error at location i. We examine the effect of dipole asymmetry and/or longitudinal displacement by considering two bends, with angles θ and $-\theta$ separated by a distance L. Think of this as modeling a shift in magnet position by adding a bend portion at the end toward which it is shifted and cancelling the other end with the negative bend. Plugging this into the formula gives

$$d(s) = \frac{\sqrt{\beta_{im}\beta(s)}(-\theta_i)}{2\sin(\pi\nu)} \cos(|\Psi_{im} - \Psi(s)| - \pi\nu) + \frac{\sqrt{\beta_{ip}\beta(s)}\theta_i}{2\sin(\pi\nu)} \cos(|\Psi_{ip} - \Psi(s)| - \pi\nu) \tag{2}$$

where im and ip mark the locations of the minus and plus bends. If we take the phase advance between these locations as $\Psi_o = \Psi_{ip} - \Psi_{im}$ and we assume that it is small, we can simplify our result by substituting for Ψ_{im} in Equation 2. We utilize the usual expansion for $\cos(A - \Psi_0)$, apply the small angle approximation for the terms in Ψ_0, and demand that $\beta_{im} = \beta_{ip} = \beta_i$, which yields

$$d(s) = -\frac{\sqrt{\beta_i \beta(s)}(\theta_i)}{2\sin(\pi\nu)} \sin(|\Psi_i - \Psi(s)| - \pi\nu)\Psi_o. \tag{3}$$

Since we are considering only a short arc length (the length of one bending magnet), we evaluate Ψ_0 from the elementary definition, assuming only that β is constant over the magnet length.

$$\Psi_0 \equiv \int_S^{S+L} ds \frac{1}{\beta(s)} = \frac{L}{\beta_i}. \tag{4}$$

$$d(s) = -\frac{\sqrt{\beta(s)}\theta_i L}{\sqrt{\beta_i} 2\sin(\pi\nu)} \sin(|\Psi_i - \Psi(s)| - \pi\nu). \tag{5}$$

We observe that the magnet parameter which determines this closed orbit error is $\theta_i L$.

4 Magnet Analysis

Having identified a parameter with dimensions of length which characterizes the unwanted orbit distortion, let us evaluate it for the cases of installation error and magnet asymmetry. If we define the body field, $\overline{B}$, of a dipole as some average of $B_y(z)$ in the magnet body, we can define an effective length by the relation

$$\overline{B} L_{eff} = \int_{-L/2}^{L/2} B_y(z) dz, \tag{6}$$

We note that the bend angle created by a bending magnet is given by

$$\theta_M = \frac{1}{(B\rho)} \int_{-L/2}^{L/2} B_y(z) dz = \frac{\overline{B} L_{eff}}{(B\rho)} \tag{7}$$

where $(B\rho)$ is the magnetic rigidity of the particle (related to its momentum). The angle error θ_i, used in Equation 1 or 2, is given by the difference between the field and the average field, $B(z) - \overline{B}$. The distance over which it acts is simply the distance from the center (mechanical) of the magnet, which we take as z. However, we wish to consider only the anti-symmetric portion of this, $B(z) - B(-z)$, which removes the term in $\overline{B}$. Thus we identify the parameter $\theta_i L$ as follows:

$$\theta_i L = \frac{1}{(B\rho)} \int_0^{L/2} (B_y(z) - B_y(-z)) z \, dz \tag{8}$$

By re-ordering the limits of integration and substituting for$(B\rho)$ we have

$$\theta_i L = \frac{\theta_M}{\overline{B} L_{eff}} \int_{-L/2}^{L/2} B_y(z) z \, dz \tag{9}$$

4.1 Magnet Displacement

We wish to evaluate $\theta_i L$ for the case of a uniform dipole of length L displaced by a distance δz. As described above, this is modeled by a pair of dipoles of length δz and strength $\mp \overline{B}$ separated by length L, which gives

$$\theta_i L = \frac{\theta_M}{\overline{B} L_{eff}} \overline{B} L \delta z = \theta_M \, \delta z \tag{10}$$

where we have taken $L = L_{eff}$ which is quite precise for permanent magnets. Note that this is the intuitive result for the displacement of the beam. It is reduced by $\times 2$ in the closed orbit effect.

4.2 Magnet Longitudinal Uniformity

To relate this to the magnet uniformity requirements, let us express the bend field as a polynomial in the distance from the magnet (mechanical) center.

$$B_y(z) = \overline{B}(1 + c_1 z + c_2 z^2 + c_3 z^3 + \cdots). \tag{11}$$

$$\delta z = \frac{\theta_i L}{\theta_M} = \frac{1}{\overline{B} L_{eff}} \int_{-L/2}^{L/2} B_y(z) z \, dz. \tag{12}$$

$$\delta z = \frac{1}{L_{eff}} \int_{-L/2}^{L/2} z \, dz (1 + c_1 z + c_2 z^2 + c_3 z^3 + \cdots). \tag{13}$$

$$\delta z = \frac{1}{L_{eff}} (\frac{c_1 L^3}{12} + \frac{c_3 L^5}{80} + \cdots) = \frac{c_1 L^2}{12} + \frac{c_3 L^4}{80} + \cdots. \tag{14}$$

If we consider only the c_1 term and we define ΔB as the end to end difference in the magnet strength, we find $\Delta B/\overline{B} = c_1 L$. This then gives

$$\delta z = \frac{\Delta B}{\overline{B}} \frac{L}{12}. \tag{15}$$

Note that we choose to leave this result in terms of a longitudinal displacement of the magnet center, since that provides a useful description of the important error possibilities.

5 Examples

The above formulas permit one to estimate closed orbit error created by a single magnet error of a specified value. Let us observe these for generous errors.

5.1 Magnet Displacement

Consider a 6 m Main Injector Dipole. It provides a bend of about 0.02 radians. Imagine that it has a longitudinal displacement of 25 mm. It occurs with a $\beta = 50$ m in a lattice with fractional tune $\nu = 0.4 (\sin \pi\nu = .95)$. For this case we calculate a beam displacement at the magnet using Equation 5.

$$d = \frac{\delta z \theta_M}{2} = 2.5 \times 10^{-4} m. \tag{16}$$

If such an error were the sigma of the installation error for N_{mag} magnets, and we multiply by $\sqrt{N_{mag}}$ we find an RMS orbit distortion of about 4 mm before we properly average over the β values. Such a crude installation is marginal. We will refuse to consider the effects of a systematic longitudinal placement error.

5.2 Longitudinal Gradient

Let us consider a similarly gross error in the bend field of the Recycler gradient magnets[1][2]. They are about 4 m long and produce the same bend as the 6 m Main Injector dipoles. To have a magnetic center displaced in z by 25 mm we would need a gradient which produces $\Delta B/\overline{B}$ of 0.075. This will create the same .25 mm single magnet error and is an unlikely possibility.

However, we can imagine creating systematic effects by either brick selection or by assembly procedures which affect a whole design series in the same way and perhaps related series with opposite signs. With 344 gradient magnets contributing, if even 1/3 add coherently, we have a 25 mm distortion. The possible combinations in which similarly built magnets can be combined are large enough that reviewing all combinations would be onerous. In lieu of that, let us set a sufficiently tight longitudinal uniformity requirement and consider if it is a problem for the magnet builders. Consider limiting the closed orbit error from this effect to 1 mm. Imagine that all 344 dipoles conspire to each contribute coherently (which is not possible). If each dipole contributes 3×10^{-6} m of error, we will still meet our goal.

$$d = 3 \times 10^{-6} = \frac{\delta z \theta_M}{2}. \tag{17}$$

This requires $\delta z < 3 \times 10^{-4}$ m (.3 mm). When interpreted as a longitudinal gradient which is then expressed as a limit on the fractional end-to-end difference it becomes

$$\frac{\Delta B}{\overline{B}} < .87 \times 10^{-3}. \tag{18}$$

Although it is possible that this could be easily achieved, if it is not inexpensive, then one will need to refine the estimate. It is clear, however, that asymmetries of 2% are unlikely to be acceptable.

6 Determination of Longitudinal Centers

Several straightforward techniques can be used to determine the bend center for dipole magnets. For gradient magnets, there are some complications for each technique so we will review several possibilities.

6.1 POINTSCAN

The POINTSCAN measurement technique[3] has been developed at MTF to provide a detailed field map on some coordinate grid. For production measurements, the emphasis has traditionally been on producing a longitudinal scan of bend magnets. Hall Probe/NMR Probe combinations have been used for dipoles with good success. Recording the dipole field over a grid in z permits direct evaluation of the bend center displacement in accordance with Equation 12.

For gradient magnets, a couple of additional complications are evident. The transverse gradient (about 6% at 25 mm in the arc gradient magnets) demands that a sufficiently careful transverse probe positioning mechanism be provided. In addition, the use of NMR is precluded by the effects of the transverse gradient on the NMR line width. It may be possible to use and ESR (Electron Spin Resonance) or FMR (Ferrimagnetic Resonance) probe but those are not available at this time. The precision required for this center determination should be available from a Hall Probe. A disadvantage of POINTSCAN is that setup and measurement times are not small.

6.2 Stretched Wire

The FLATCOIL measurement systems at MTF[4][5] have been adapted to acquire data from stretched wire probes[6]. In particular, extensive work has been done using a single stretched wire in Main Injector quadrupoles. This system can be readily adapted to measure the longitudinal center of dipoles. Again, additional complications apply to gradient magnets. We will examine those issues here.

Consider a single wire stretched along the transverse centerline of a magnet with motors which control movement symmetrically placed near the magnet ends. A flux measurement loop is created by attaching this wire to a return wire which is stationary, either within or outside the magnet field. One records flux changes at constant field level beginning with the wire centered at X=0 (horizonal center) at both ends of a dipole. Prescribe a motion of the wire such that when the the x position of at one end ($z = L/2$) is +X, the x position of the wire at the other end ($z = -L/2$) is -X. This will cause the wire to fall along a line $x = 2Xz/L$ as a function of z. The change in magnetic flux, $\phi(X)$, as X is changed is given by

$$\phi(X) = \int_{-L/2}^{L/2} dz \int_{0}^{2Xz/L} dx B(z,x) \tag{19}$$

If $B(z,x)$ is independent of x (good dipole field), then

$$\phi(X) = \frac{2X}{L} \int_{-L/2}^{L/2} dz\, zB(z) \tag{20}$$

We observe that this is closely related to the definition of the longitudinal magnetic center in Equation 12. We conclude that

$$\phi(X) = 2\overline{B}\,\delta z\, X \tag{21}$$

and measurements at a series of X values will result in a straight line for $\phi(X)$ *vs.* X. $\overline{B}L$ is readily measured with a symmetric wire motion so $\delta z/L$ is determined directly.

The assumption that $B(z,x)$ was independent of x is useful for a dipole with good field quality. For the gradient magnets of the Recycler Ring, the assumption ignores the design quadrupole and sextupole terms. To understand the possibilities for measuring the longitudinal bend center of these magnets with a stretched wire, let us assume that the field shape dependance on z and x are separable. This is reasonable because the transverse field shape is dominated by the iron geometry, which is expected to be quite uniform in z while the longitudinal variation at the center line will be unaffected by the transverse field shape. Let us define a field shape expansion by

$$B(z,x) = B(z)(1 + b_2(\frac{x}{a}) + b_3(\frac{x}{a})^2) \tag{22}$$

where a is the reference radius (usually 25 mm). Beginning again with Equation 19 for the flux change measured by the stretched wire, we now have

$$\phi(X) = \int_{-L/2}^{L/2} dz \int_0^{2Xz/L} dx B(z)(1 + b_2(\frac{x}{a}) + b_3(\frac{x}{a})^2) \tag{23}$$

How is this related to the bend center displacement? We first perform the x integration.

$$\phi(X) = \int_{-L/2}^{L/2} dz B(z)(\frac{2X}{L}z + \frac{b_2}{a}\frac{2X^2}{L^2}z^2 + \frac{b_3}{a^2}\frac{8X^3}{3L^3}z^3) \tag{24}$$

$$\phi(X) = \frac{2X}{L}\int_{-L/2}^{L/2} dz B(z) z + \frac{b_2}{a}\frac{2X^2}{L^2}\int_{-L/2}^{L/2} dz B(z) z^2 + \frac{b_3}{a^2}\frac{8X^3}{3L^3}\int_{-L/2}^{L/2} dz B(z) z^3 \tag{25}$$

We identify the first of these terms as the desired longitudinal center position. Separating it from the 2nd and 3rd terms will rely on fitting the form of $\phi(X)$ *vs.* X. How large are these effects for the design field shape of Recycler gradient magnets?

Using the longitudinal expansion in Equation 11, we find

$$\int_{-L/2}^{L/2} dz B(z) z^2 = \overline{B}\int_{-L/2}^{L/2} dz(z^2 + c_1 z^3 + c_2 z^4 + c_3 z^5) \tag{26}$$

$$\int_{-L/2}^{L/2} dz B(z) z^2 = \overline{B}L(\frac{L^2}{12} + c_2\frac{L^4}{80}) \tag{27}$$

$B_1 = .1523$ T	$B_2 = -.3774$ T/m	$B_3 = -.6087$ T/m^2
$b_1 = 1$	$b_2 = 0.06294$	$b_3 = -25.8 \times 10^{-4}$

Table 2: Design Specification for the Recycler Ring Defocusing Gradient magnets (RGF) magnetic field properties.

$$\int_{-L/2}^{L/2} dzB(z)z^3 = \overline{B}L \int_{-L/2}^{L/2} dz(z^3 + c_1 z^4 + c_2 z^5 + c_3 z^6) \tag{28}$$

$$\int_{-L/2}^{L/2} dzB(z)z^3 = \overline{B}L(c_1\frac{L^4}{80} + c_3\frac{L^6}{448}) \tag{29}$$

Combining these results we find

$$\phi(X) = \frac{2X}{L}\overline{B}L\,\delta z + \frac{b_2}{a}\frac{2X^2}{L^2}\overline{B}L(\frac{L^2}{12} + c_2\frac{L^4}{80}) + \frac{b_3}{a^2}\frac{8X^3}{3L^3}\overline{B}L(c_1\frac{L^4}{80} + c_3\frac{L^6}{448}) \tag{30}$$

Re-organizing terms we find

$$\frac{\phi(X)}{\overline{B}L} = \frac{2\delta z}{L}X + \frac{b_2}{6a}(1 + c_2\frac{L^2}{240})X^2 + \frac{b_3}{3a^2}(c_1\frac{L}{10} + c_3\frac{L^3}{56})X^3. \tag{31}$$

Using the result in Equation 14 and substituting in the X^3 term we find

$$\frac{\phi(X)}{\overline{B}L} = \frac{2\delta z}{L}X + \frac{b_2}{6a}(1 + c_2\frac{L^2}{240})X^2 + \frac{b_3}{3a^2}(\frac{6\delta z}{5L} + c_3\frac{4L^3}{175})X^3. \tag{32}$$

Evaluating the parameters in Equation 32 for an X excursion of 25 mm and with a 4 m gradient magnet such as the RGD whose parameters are shown in Table 2 we find that for a δz of .3 mm (see Section 5.2.) we would have a flux, 2.25×10^{-6} V-s for the first term in Equation 32. This is near the noise limit for the existing Single Stretched Wire system[6]. The term in X^2 gives 266×10^{-6} so some care will be required to extract the linear term. For this example, the term in X^3 is small and the term in c_2 contributes negligibly to the X^2 term. Linear and quadratic terms are comparable for $\delta z = 0.02$ m. Of course, determining δz to an accuracy of 0.0003 m will require survey or placement of the magnet to that precision in a reference frame established by the stretched wire apparatus.

6.3 Rogowski Coil

If the magnet manufacturing process is well controlled and there is no expectation of large asymmetries, it may be possible to monitor the longitudinal uniformity of the field by measuring the magnetic potential of the poles using a Rogowski coil[7] at several points along the length. Even measurements only at the two ends will provide some evidence of asymmetry if such asymmetry exists. For magnets which produce 0.15 T fields, the construction of a coil with sufficient sensitivity is not difficult.

7 Summary

We have obtained formulas for the closed orbit distortion due to longitudinal displacement of magnet bend centers. The magnetic bend center has been defined in terms of measured field and related to characteristics of the gross field shape such as the longitudinal asymmetry. Measurement techniques which can provide the required information have been identified.

7.1 Data from PDD002-0 POINTSCAN Measurement

The data shown in Figure 1 were fit to the parameters shown in Table 1. The longitudinal magnetic center was determined from this data. Points in the central 86 inches were used (only for analysis convenience). The direct integration produced a result as from Equation 12 of $\delta z = -.0486$ inches or -1.2345 mm. When evaluated for the same range of z, Equation 14 gives $\delta z = -.0485$ inches or -1.232 mm. We see that the 4th order polynomial fit is adequate for these purposes. This is not important for the 8 GeV Line application. A 4 m gradient magnet with similar longitudinal asymmetry would have a larger δz.

7.2 Conclusions

Since the effects which created the asymmetry in PDD002-0 are not understood at this time, both accelerator requirements and magnet construction effects will have to be studied. If the average δz were of this size or larger, it might impose re-alignment requirements during Recycler Ring commissioning. Alternatively, the longitudinal magnetic center can be determined by one of the measurement techniques discussed above and the required mechanical offset applied during installation.

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