Direct Measurement of the CKM Matrix at High Energy e^+e^- Colliders

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Abstract. We evaluate the potential precision to which the hadronic decay branching fractions of W bosons and top quarks (and hence the CKM matrix elements) can be measured at proposed high energy linear e^+e^- colliders. Identification of the individual light quark flavours is achieved in a model-independent way by calibration in Z^0 decays. The method is competitive with and complementary to the traditional way of extracting the CKM matrix elements from hadron decay measurements.

I INTRODUCTION

Up to now the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] elements have been derived from measurements of hadron decays. The direct observation of W bosons and top quarks offers a complementary way to determine the CKM matrix elements. In case of W bosons one may determine the decay widths into identified pairs of quarks, (q, q'), which are proportional to the square of the CKM matrix element, $|V_{qq'}|^2$. Similarly one can relate the branching fractions $B(\mathrm{t} \to q\mathrm{W})$ to the CKM elements $|V_{\mathrm{t}q}|^2$. In practice this requires firstly that all decay modes of the W bosons and top quarks can be reconstructed with a good signal to background ratio, which is the case at e^+e^- colliders. Second is the requirement that all (especially light) quark species can be identified.

A method was suggested in [2] and used by the OPAL collaboration [3] to determine individual up, down and strange quark tagging efficiencies directly from Z^0 data, thus avoiding dependence on hadronisation models. The main idea for determining the fraction of light flavours is that the particles with the highest momentum in a jet carry information about the primary flavour. The resulting tagging efficiencies then can be applied at other centre-of-mass energies. The high luminosity at proposed linear e^+e^- colliders both for running at the Z^0 and at high energies above the W pair threshold offers unique possibilities to pursue this program with substantially higher precision than currently available with the LEP data only.

For definiteness we assume the parameters of the TESLA [4] option for a future e^+e^- linear collider, although our proposal is in no way restricted to this choice. For running at the Z^0 peak we assume an instantaneous luminosity of $\mathcal{L}_{Z^0} = 7 \times$

10^{33} sec^{-1}cm^{-2} [5]. For a nominal year of running this implies 70 fb^{-1} of data. We will therefore assume for this study a sample of 2×10^9 hadronic Z^0 decays. At centre-of-mass energies of 500 GeV a luminosity of $\mathcal{L}_{HE} = 3 \times 10^{34}$ sec^{-1}cm^{-2} is expected [6]. We will base the following discussion on 5 million usable hadronic W decays, which could be collected in a few years of high energy data taking, and one million well-tagged and reconstructed top decays.

The detector capabilities which are relevant for our method to determine the CKM matrix elements are in particular hadron and, to a lesser extent, lepton identification. For definiteness we assume the same particle identification capabilities as obtained with the OPAL detector at LEP [7], and do not attempt to calculate the small differences in particle identification efficiencies at proposed detectors.

II MEASUREMENTS WITH W AND Z BOSONS

As detailed in [2,3] the light flavour tagging probabilities are determined by using the highest momentum identified particles ($\pi^\pm$, $K^\pm$, K^0_S, protons and Λ baryons) with $x_p > 0.2$ in event hemispheres of a hadronic Z^0 decay. The event counts, N_i for single event hemispheres tagged with particle type i, and double-tagged events N_{ij}, are related to the probabilities, η^i_q, that a given primary quark flavour q gives rise to a particle i by the system of equations:

$$N_i/N_{Z^0} = 2 \sum_q \eta^i_q \, R_q \tag{1}$$

$$N_{ij}/N_{Z^0} = (2 - \delta_{ij}) \sum_q \eta^i_q \, \eta^j_q \, R_q \ , \tag{2}$$

where $\delta_{ij} = 1$ if $i = j$. We also have neglected correlations between the tagging probabilities in opposite hemispheres, due to kinematic or geometrical effects, for example. Note that the η^i_q include possible distortions due to detector effects. The R_q are the Standard Model hadronic branching fractions of the Z^0.

Neglecting experimental effects, the tagging efficiencies in the rest system of the W and Z^0 bosons are almost identical, since the masses are so similar. Therefore the η^i_q determined at the Z^0 peak can be used to measure flavour production in the W rest system. Considering both single and double tagged W decays leads to a second system of equations:

$$N_k/N_W = \sum_{qq'} [\eta^k_q (1 - \sum_l \eta^l_{q'}) + (1 - \sum_l \eta^l_q) \eta^k_{q'}] \cdot B_{qq'} \tag{3}$$

$$N_{ij}/N_W = (1 - 0.5\delta_{ij}) \sum_{qq'} [\eta^i_q \eta^j_{q'} + \eta^j_q \eta^i_{q'}] \cdot B_{qq'} \ , \tag{4}$$

where $B_{qq'}$ are the hadronic branching fractions of the W. Here we have neglected experimental effects, hemisphere correlations, background from non-W events, and assumed the right assignment of particles and jets to W bosons. In a real experiment

	current uncertainty [11]	projected other	LC (TESLA)
$\lvert V_{\mathrm{ud}}\rvert$	$\pm$ 0.0008		$\pm$0.0028
$\lvert V_{\mathrm{us}}\rvert$	$\pm$0.0023		$\pm$0.0124
$\lvert V_{\mathrm{ub}}\rvert$	$\sim\pm$0.008	$\pm$0.0004 [12]	$\pm$0.011
$\lvert V_{\mathrm{cd}}\rvert$	$\pm$0.0016		$\pm$0.0072
$\lvert V_{\mathrm{cs}}\rvert$	$\pm$0.01		$\pm$0.0017
$\lvert V_{\mathrm{cb}}\rvert$	$\pm$0.0019	$\pm$0.0012 [12]	$\pm$0.0011
$\lvert V_{\mathrm{td}}\rvert$	$\lvert V_{\mathrm{td}}\rvert/\lvert V_{\mathrm{ts}}\rvert$ <0.24	<0.016	$\pm$0.026 ($\pm$0.00003)
$\lvert V_{\mathrm{ts}}\rvert$			$\pm$0.006 ($\pm$0.0002)
$\lvert V_{\mathrm{tb}}\rvert$	(+0.29,-0.12) [13]	$\sim\pm$0.05 [14]	$\pm$0.000008 ($\pm$0.005)

TABLE 1. Current and expected precision of future measurements of the CKM matrix elements. For the TESLA precision on $|V_{\mathrm{tq}}|$ the uncertainty due to Γ_{top} is shown in parentheses.

none of the conditions holds exactly, but the experience of the LEP experiments indicates that such sources of uncertainty should be well under control.

We use a χ^2 fit to solve the two systems of equations simultaneously, which avoids having to introduce further constraints, as in [3]. Here we have assumed that there are no W decays to other particles, implying that $\sum B_{qq'} = 1$. We note that by omitting such a constraint we can determine $(1 - \sum B_{qq'})$ with a precision of $\sim 5 \cdot 10^{-4}$ and thus place limits on any exotic W decays.

Without assuming unitarity, the CKM matrix elements can be related to the branching factions $B_{qq'}$ via:

$$|V_{qq'}|^2 = \frac{1}{C_{\mathrm{QCD}}} \frac{B_h}{B_{(\mathrm{e},\mu)}} B_{qq'}. \tag{5}$$

where C_{QCD} expresses the QCD radiative corrections and $B_h/B_{(\mathrm{e},\mu)}$ is the inclusive hadronic over leptonic decay fractions of the W. Their contributions to the overall uncertainty should be smaller than the projected uncertainties in $B_{qq'}$. We obtain the expected precisions given in the right column of Table 1. At this stage we have completely neglected any possible theoretical corrections and uncertainties due to mass effects, electroweak contributions etc.

III TOP QUARK MEASUREMENTS

In general the determination of the branching fractions of $\mathrm{t} \to q\mathrm{W}$ follows the procedure for the W bosons. The CKM matrix elements involving top decays can be related to the branching fractions $B(\mathrm{t} \to \mathrm{qW})$ by:

$$|V_{\mathrm{t}q}|^2 = B(\mathrm{t} \to q\mathrm{W}) \cdot \Gamma_{\mathrm{top}} \cdot \frac{8\pi\sqrt{2}}{G_{\mathrm{F}} {\mathrm{m_t}}^3 \mathcal{F}(\mathrm{m_t}, \mathrm{m}_q, \mathrm{m_W})} \tag{6}$$

The greatest uncertainty comes from the knowledge of the total width of the top quark, Γ_{top}. Scaling the precision estimated in [9] to the luminosities foreseen at

TESLA, Γ_{top} should be measurable to $\sim 1\%$. The results are shown in Table 1. Sources of systematic uncertainty include QCD scaling violations and the fact that the top quark is a coloured object. As already known from the discussions on colour reconnection for W pairs at LEP, this affects in particular low momentum particles, whereas leading particles are rather undistorted [10].

IV CONCLUSIONS

The large samples of W bosons and top quarks at a future e^+e^- collider of high energy and luminosity provide a direct means to determine the fundamental CKM matrix elements in a way that is complementary to currently used methods. Significant improvements over present [11] and future [12–14] measurements can be made in the charm sector. Although the expected results for $|V_{\mathrm{ud}}|$ and $|V_{\mathrm{cd}}|$ are only some factor three to four worse than those obtained from nuclear beta and deep inelastic scattering, their direct measurement in W boson decays may be an interesting complement. The direct determination of CKM matrix elements involving top quarks may be pioneered at a future linear collider.

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