Gauge Theories & Particle Physics

Physics 539 | Spring Semester 1997 | Chris Quigg

March 4, 1997

Recommended Reading:

Gauge Theories, §6.1 – 6.4.
Aitchison and Hey, §14-1 – 14-5.
On the σ Model: J. F. Donoghue, E. Golowich, and B. R. Holstein, *Dynamics of the Standard Model,* (Cambridge University Press, Cambridge, 1992), pp. 10–12, 21–22.
K. Gottfried and V. F. Weisskopf, *Concepts of Particle Physics* vol. II (Oxford University Press, New York, 1986): on the choice of the electroweak gauge group, §VI.B.2 (pp. 491-496); on the analogy with superconductivity, §VI.C.1 (pp. 509-516).

Cultural Reading:

To celebrate the hundredth anniversary of the discovery of the electron (and the beginning of particle physics), read the articles by Abraham Pais, Steven Weinberg, and Chris Quigg in the Spring 1997 issue of the SLAC *Beam Line,* available in PDF format at
http://www.slac.stanford.edu/pubs/beamline/97i.pdf.
Robert N. Cahn and Gerson Goldhaber, *The Experimental Foundations of Particle Physics* (Cambridge University Press, Cambridge and New York, 1989), chapter 12.
On spontaneous symmetry breaking: Mary M. Crone and Marc Sher, "The environmental impact of vacuum decay," *Am. J. Phys.* **59**, 25 (1991).

Problems (due March 13, 1997)

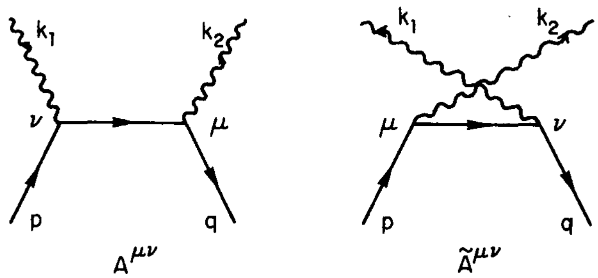
21. Derive the equation of motion for the photon field A_ν in the Abelian Higgs model and show that it amounts to a relativistic generalization of the Ginzburg–Landau description of a superconductor.

22. (a) Compute the decay rate for the disintegration of the tau lepton, $\tau^- \to e^- \bar{\nu}_e \nu_\tau$, neglecting the electron mass, and compare with the familiar rate for muon decay.
(b) How would the rate change if, by virtue of an unconventional lepton number assignment, $\nu_\tau \equiv \bar{\nu}_e$?

23. (a) Compute the differential and total cross sections for $\nu_e e$ and $\bar{\nu}_e e$ elastic scattering in the Weinberg–Salam model. Work in the limit of large M_W and M_Z, and neglect the electron mass with respect to large energies. The computation is done most gracefully by Fierz reordering one of the graphs, as indicated in §6.4 of *Gauge Theories.* [Reference: G. 't Hooft, *Phys. Lett.* **37B**, 195 (1971).]
(b) Without calculating the cross section, compute the $\bar{\nu}_e$ energy for which W^- is formed as an s-channel resonance.

24. (a) Compute the cross section for the reaction
$$e^+e^- \to H \to f\bar{f} \ ,$$
where f is a massive fermion.
(b) How does the result differ for leptons and quarks?
(c) Write the cross section for the reaction $e^+e^- \to \text{all}$ in the form
$$\sigma(e^+e^- \to H) = \frac{4\pi\Gamma(H \to e^+e^-)\Gamma(H \to \text{ all})}{(s - M_H^2)^2 + M_H^2\Gamma(H \to \text{ all})^2} \ .$$
How does the result change if the colliding particles are muons, rather than electrons?
(d) What integrated luminosity would be required to discover a 120-GeV Higgs boson in the reaction $\mu^+\mu^- \to H$, if the only decay products are $e\bar{e}, \mu\bar{\mu}, \tau\bar{\tau}, u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c},$ and $b\bar{b}$ pairs?

25. Consider the process $e^+e^- \to \gamma\gamma$, which is described, in QED, by the two Feynman diagrams



and for which the amplitude may be written $\mathcal{M} = \epsilon^*_{1\nu}\epsilon^*_{2\mu}(A^{\mu\nu} + \tilde{A}^{\mu\nu})$, where ϵ_i is the polarization vector of a photon and $A^{\mu\nu}(\tilde{A}^{\mu\nu})$ corresponds to the first (second) diagram.
(a) Calculate the tensors $A^{\mu\nu}$ and $\tilde{A}^{\mu\nu}$.
(b) Show that gauge invariance requires that

$$k_{1\nu}(A^{\mu\nu} + \tilde{A}^{\mu\nu}) = 0 = k_{2\mu}(A^{\mu\nu} + \tilde{A}^{\mu\nu}).$$

(c) Verify that these conditions are met, although the quantities $k_{1\nu}A^{\mu\nu}$, $k_{1\nu}\tilde{A}^{\mu\nu}$, $k_{2\mu}A^{\mu\nu}$, and $k_{2\mu}\tilde{A}^{\mu\nu}$ are all different from zero.

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I strongly recommend that you work through the calculations in §6.1 of *Gauge Theories*. So that we may cover many topics during the semester, I will not do all of these calculations in class. The calculations are done very explicitly in the book, so you should have no trouble in reproducing them. It might also be a useful experience to try doing the calculations at the blackboard, explaining steps to classmates who are free to object, offer advice, or peek at the answer.

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Comment on Problem 12: Because the interaction rate must be very high in a multi-TeV hadron collider, the continuous fluxes of neutrinos may be high enough to allow very high-energy neutrino experiments, including a three-neutrino experiment. The possibilities are worked out in some detail by A. De Rújula, E. Fernández, and J. J. Gómez-Cadenas, *Nucl. Phys.* **B405**, 80 (1993).

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Typographical errors in the fifth paperbound printing of *Gauge Theories* (identified at the bottom of p. vi by `EFGHIJ-MA-89`):

p. 140, Problem 6-3, below (A.4.29):
. . . where $\Gamma_i = (1, \gamma_\mu, \sigma_{\mu\nu}, i\gamma_\mu\gamma_5, \gamma_5)$ . . .

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Course information, including class schedules and assignments, is available on the World Wide Web. Point your browser to http://lutece.fnal.gov/Princeton/.