Activity #1: Understanding Sinusoids Worksheet (Teacher version) Math

Note to students: Lab teams of three or four students are required for this activity.

National Standards addressed:

Content Standards:
Algebra Expectations: Students will understand and perform transformations such as combining, composing, and inverting commonly used functions, using technology to perform such operations on more complicated symbolic expressions; students will use symbolic algebra to represent and explain mathematical relationships; students will judge the meaning, utility and reasonableness of the results of symbolic manipulations, including those carried out by technology; students will identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships; students will draw reasonable conclusions about the situation being modeled.

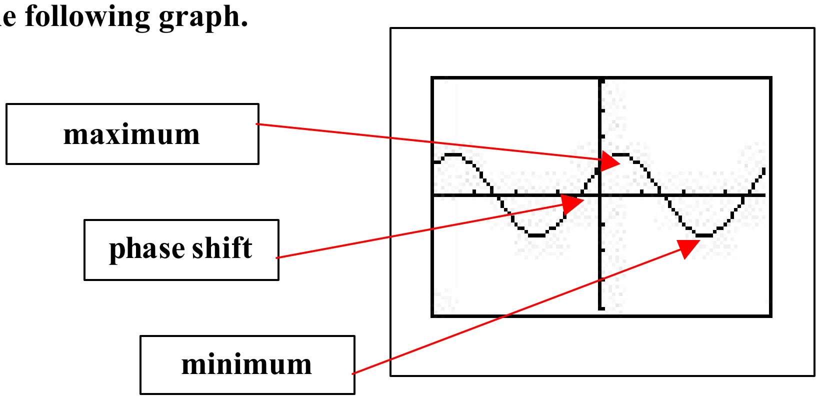
Process Standards:
Measurement Expectation: Students will make decisions about units and scales that are appropriate for problem situations involving measurement.
Problem Solving Expectation: Students will reflect on the process of mathematical problem solving.
Communication Expectation: Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
Connection Expectation: Students will recognize and apply mathematics in contexts outside of mathematics.
Representation Expectation: Students will use representations to model and interpret physical, social, and mathematical phenomena.

Purpose: **To understand the definition of sinusoidal curve**
To recognize a possible sinusoidal curve
To express a curve in sinusoidal form
To generate, using technology, a sine curve and find its mathematical representation
To recognize phenomena that might generate these curves and discuss advantages of the sine representation

Understanding Sinusoids → y = a sin (bx - c) + d

Definition: A sinusoid is the name given to any curve that can be written in the form, y = a sin (bx - c) + d.

To investigate, you will look at several curves, determine whether or not these curves are representative of the sine curve, and, if so, rewite in sine form. Consider the curve with equation y = sin x + cos x. Using radian mode, the curve has the following graph.



This graph does look like a sine curve. You will need to identify some points and do some calculations before you can make your sinusoidal representation (if possible)
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1. Identify the coordinates of the maximum point, (x_1, y_1). Give coordinates correct to three decimal places.
(x_1 = ___________, y_1 = ___________) For best results, store this x-value in A and this y-value in B. **(x_1 = .785, y_1 = 1.414) Hopefully, students will recognize these two values as $\frac{p}{4}$ and $\frac{p}{2}\sqrt{2}$, respectively.**

2. Identify the coordinates of the minimum point, (x_2, y_2). Give coordinates correct to three decimal places.
(x_2 = ___________, y_2 = ___________)
For best results, store this x-value in C and this y-value in D.

(x_2 = 3.927, y_2 = -1.414) Hopefully, students will recognize these two values as $\frac{5p}{4}$ and $-\sqrt{2}$, respectively.

3. Identify the coordinates of the phase shift point, (x_3, y_3). Give coordinates correct to three decimal places.
(x_3 = ___________, y_3 = ___________)
For best results, store this x-value in E and this y-value in F.

(x_3 = -.785, y_3 = 0) Students should easily recognize x_3 as - $\frac{p}{4}$.

You are now ready to decide on values for a, b, c, and d in the sinusoidal representation. Be sure to be as accurate as possible, i.e., use the values you have stored in your calculator.

4. The value of a represents the amplitude and opening direction of the curve. Write your value for a, correct to three decimal places. ______________
For best results, store this value in your calculator. What variable did you select to store a? ____
a = 1.414, which is the y-value stored in B.

5. The value for b affects the period of the graph. Determine the period from the points found in steps 1 and 2. Remember that b is a positive real number. What period did you find, correct to three decimal places? ______________ Remember, the period is the length of one complete cycle and is given by, period = $\frac{2p}{b}$.
For best results, store this value in your calculator. What variable did you select to store b? ____
period = 6.283, which is 2p. 1 is the value for b and does not need to be stored.

6. The value of c affects the phase shift. Remember, the phase shift is given by $\frac{c}{b}$.
If c is negative, the phase shift is left and if c is positive, the phase shift is right. Determine the value for c. What was your value for c, correct to three deciamal places? __________ For best results, store this value in your calculator. What variable did you select to store c? ____

c = -.785, which is - $\frac{p}{4}$. This value is already stored in E.

7. Lastly, you need to calculate the value for d. Remember that d describes the vertical shift in the graph. What value did you calculate for d, correct to three decimal places? __________
Since the maximum is $\sqrt{2}$ and the minimum is - $\sqrt{2}$, the graph has no vertical shift. The central axis remains the horizontal line described as y = ($\sqrt{2}$ + $^{-}\sqrt{2}$) / 2 = 0.

8. You are now ready to compare the curve given by y = sin x + cos x to the sinusoidal representation you have just found. Substitute the exact values for a, b, c, and d in the sinusoidal model, y = a sin (bx - c) + d, and compare the two graphs. Write your sinusoidal equation. ____
Do you think you have rewritten y = sin x + cos x in sinusoidal form? __________
Why or why not?

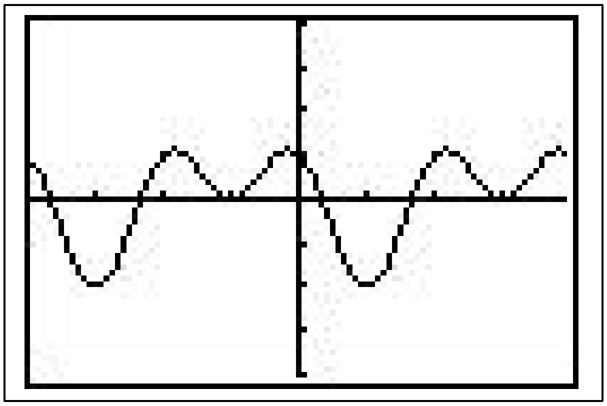
____
____
____

y = $\sqrt{2}$ sin (1x - $^{-}\frac{p}{4}$) + 0

Determine if any of the following curves might be represented in sinusoidal form. Rewrite in sine form, those you have selected.

1. $y = \cos 2x - \sin x$

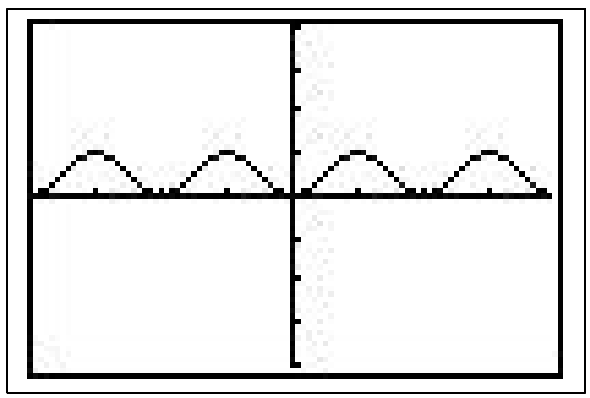
This curve is the graph of a periodic function. However, it is not a sine curve.



2. $y = \sin^2 x$

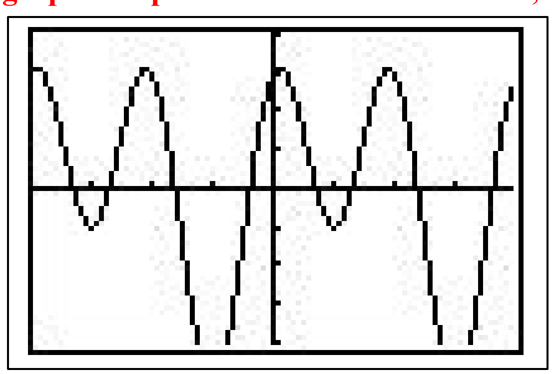
This curve appears to be the sine curve given by the equation,

$$y = .5 \sin\left(2x - \frac{p}{2}\right) + .5.$$



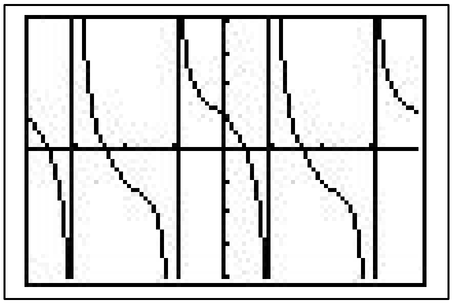
3. $y = 3\cos 2x + 2\sin x$

This curve is the graph of a periodic function. However, it is not a sine curve.



4. y = cos x – tan x

This curve is the graph of a periodic function. However, it is not a sine curve.

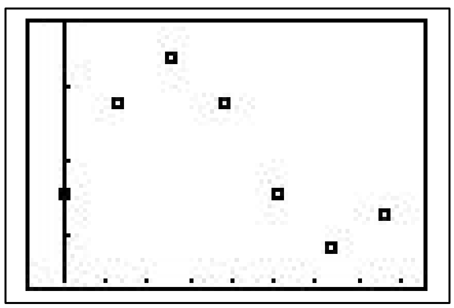


5. Throughout the day, the depth of water at the end of a dock varies with the tides. The table shows the depths (in meters) at various times during the morning.

t (time)	Midnight	2 A.M.	4 A.M.	6 A.M.	8 A.M.	10 A.M.	Noon
d (depth)	2.55	3.80	4.40	3.80	2.55	1.80	2.27

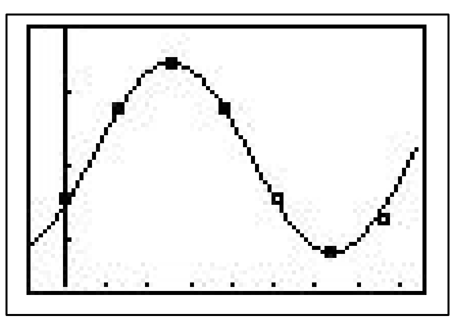
Use a sine curve to model this data. During what times in the afternoon can a boat safely dock if it needs at least 3 meters of water to moor at the dock? (Problem from page 359 of Precalculus with Limits, by Larson, Hostetler, Edwards, and Heyd from Houghton Mifflin Ó1997)

Create stat plot using t-values (the x's) in list 1 and depths (the y's) in list 2.

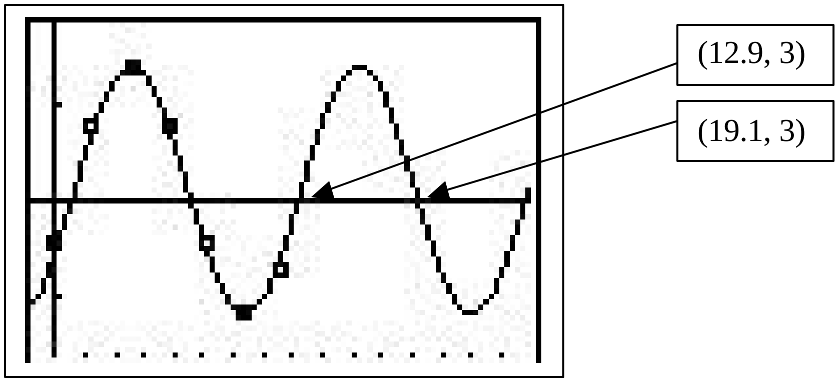


The maximum point occurs at (4, 4.4) and the minimum point occurs at (10, 1.8). From these two points, amplitude is (4.4 – 1.8)/2 = 1.3; the vertical shift is .5 (4.4 + 1.8) = 3.1. The period is 2 (10 – 4) = 12, making the value for b = $\frac{p}{6}$ and c = $\frac{p}{6}$.

This gives the equation y = $1.3\sin(\frac{p}{6}x - \frac{p}{6}) + 3.1$ $1.3\sin(\frac{p}{6}x - \frac{p}{6}) + 3.1$. Now, the two graphs appear as follows.



Graphing the line y = 3 and changing the window so that the afternoon hours are represented by the sinusoidal curve, the depth is at least 3 meters in the afternoon between 12.9 P.M. and 19.1 P.M. or 12:51 P.M. and 7:09 P.M.



Link to Activity #1, Title: An Investigation into Transverse Waves, from the science component.