Simulation Group's First Look at Filters

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The specific filter choice enters the supernova analysis through the K-correction. Neglecting zeropoint calibration, the ingredients of the K-correction are the filter passband, supernova spectrum, and supernova redshift. In practice spectra are not necessarily taken at each epoch of photometry and are thus the least constrained. This is a real problem since supernovae have P-Cygni features that vary in strength and wavelength position. So it would be nice to choose a filter set that gives K-corrections that are insensitive to exactly what supernova you are looking at.

We consider five filters, a top hat, triangle, symmetric and asymmetric trapezoids, and the Johnson-B. Chuck Bower has simulated realizations of these filters that are available on the SNAP BSCW page. These filters are distinguished by significant fringing in the throughput. We need to know if these fringes have a significant effect on our supernova measurements.

There are several kinds of error we are interested in:

1. The error we get if we mistakenly think we have the template when we have the fringed filters.
2. How sensitive the filter magnitudes are to the slight shifting of spectral features in and out of the passband. Are the fringed filters much worse than the template filters?
3. The error we get if we use the filter transmission function for the wrong angle of incidence. This amounts to a shift in the effective wavelength of the filter.

1. Constructing effective filters

The interference filters that we are considering using for SNAP have a couple of properties that make them "challenging". First, their throughputs wiggle as a function of wavelength due to fringing. Second, their throughputs depend on their angle of incidence. I want to know the effective passband anywhere on the focal plane. Maybe the fringes smooth out and we really should know how the effective wavelengths of the filters change across the Gigacam annulus.

I determine the incidence angle θ for an imaged point as a function of position on the exit pupil. The distance between the exit pupil and focal plane is given by l. The coordinates on the exit pupil is $(\rho\cos\phi, \rho\sin\phi, 0)$ and the image is at $(r, 0, l)$. The unit vector between them is $\frac{1}{\sqrt{r^2+\rho^2-2r\rho\cos\phi+l^2}}(r-\rho\cos\phi, -\rho\sin\phi, l)$ and the incidence angle is given by

$$\cos\theta = \frac{l}{\sqrt{r^2+\rho^2-2r\rho\cos\phi+l^2}}.$$

Mike Lampton has given some useful numbers describing his optics:

$$\begin{aligned} \text{Exit Pupil Outer Diameter} &= 95mm & (1)\\ \text{Exit Pupil Inner Diameter} &= 95/4mm & (2)\\ l &= 910mm. & (3) \end{aligned}$$

We can determine the incidence-angle distribution as a function of the distance from the optical center. Shown in Figure 1 are the distributions for the inner (0.06 mrad) and outer (0.13 mrad) parts of the focal plane. The annular shape of the pupil gives a double-peaked distribution. The average incidence angle for the distributions is given in Figure 2.

We construct the effective filters at given points on the focal plane. I use Chuck's prescription for the wavelength shift of the filters as a function of incident light of $\lambda = \lambda_0(1 + 1/3(\cos\theta - \cos\theta_0))$. The effective filters at the inner and outer annulus are shown in Figure 3. The wiggles of the interference filters are noticeably damped but not nearly close to being eliminated at the outer annulus. Hardly any difference is seen at the inner annulus. Unless specified, what follows we use Chuck's transmissions curves rather than the effective filter functions.

Given that the effective filter throughput is similar to the real throughput at the mean incidence angle, we can conclude that there will be a $\pm$ 0.7% shift in effective wavelength over the entire annulus.

2. Fringed vs. Template

We first consider the unrealistic case where we think that we have a target filter T but really have a fringed filter F. After doing the analysis with this bad assumption the resulting magnitude error would be

$$dm = 2.5 \log\left(\frac{\int SN * T d\lambda}{\int Vega * T d\lambda} \frac{\int SN * F d\lambda}{\int Vega * F d\lambda}\right).$$

These errors are shown in Figure 4. The trapezoids give the smallest magnitude errors, with a range 0.01 mag, whereas the the other range 0.03 magnitudes.

3. Supernova Uncertainty

For a quick and easy analysis, I model supernova heterogeneity with a effective shift in wavelength. This captures the effect of heterogeneous ejection velocities. First-order color effects will be removed with standard calibration techniques. We also get a supernova redshift determination requirement for free.

For each of target filter and corresponding Chuck-realized filter, I calculate the K-correction as a function of z. I then calculate $dK(z)/dz$, shown in Figure 5. Given an uncertainty in the supernova spectrum, parameterized by a δz, the K-correction error is then $\frac{dK(z)}{dz}\delta z$. A small value for $\frac{dK(z)}{dz}$ is desired to reduce the propagated contribution of K-correction error.

The dominant behavior is that of the template filter; the fringed filters have very similar derivatives. The fringed filters are slightly more volatile since supernova features could move in and out of phase with the filter wiggles but the effect is small as the wiggles are integrated over the entire filter. The fringes in fact help temper the template top-hat filter which is sensitive to the positions of spectral lines.

Comparison between different template filters goes beyond the scope of this note. We note that filters with sharp edges, such as the tophat function, will have larger propagated K-correction errors. The choice of optimal template filter will require further study of supernova variability as a function of wavelength. This will be saved for another day.

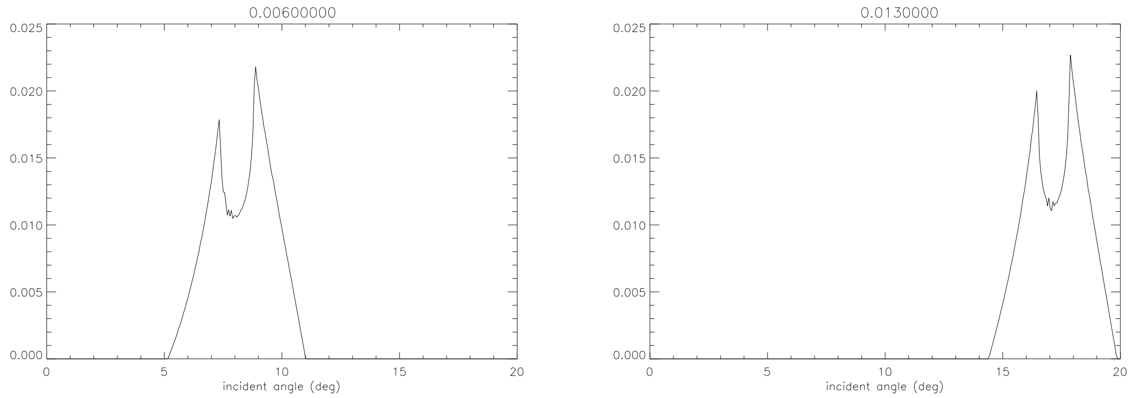

Fig. 1.— The distribution of incidence angles for points at the inner (0.006 mrad) and outer (0.013 mrad) parts of the annular focal plane.

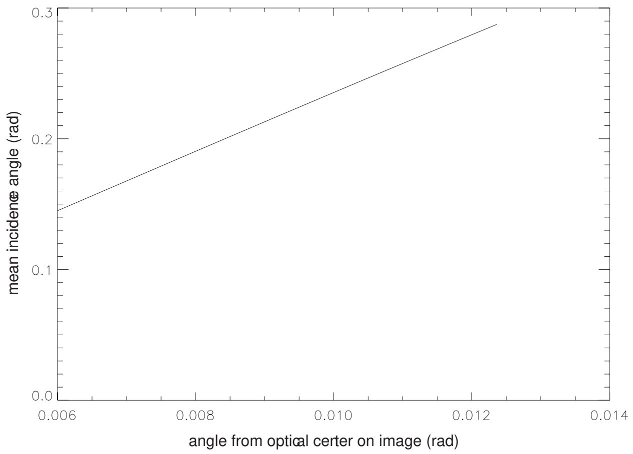

Fig. 2.— The effective incidence angle as a function of radians from the optical center.

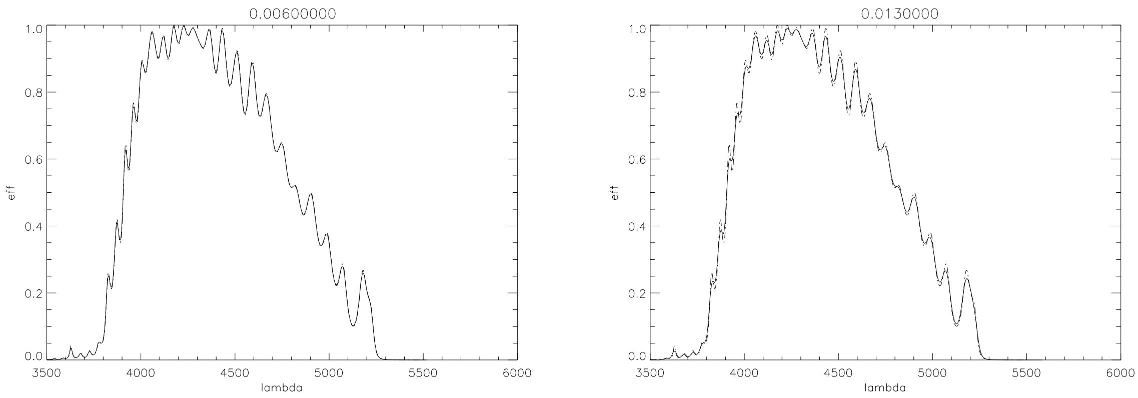

Fig. 3.— The effective filters for points at the inner (0.006 mrad) and outer (0.013 mrad) parts of the annular focal plane (solid). The transmission of the interference filter at the average incidence angle is shown in dashed lines.

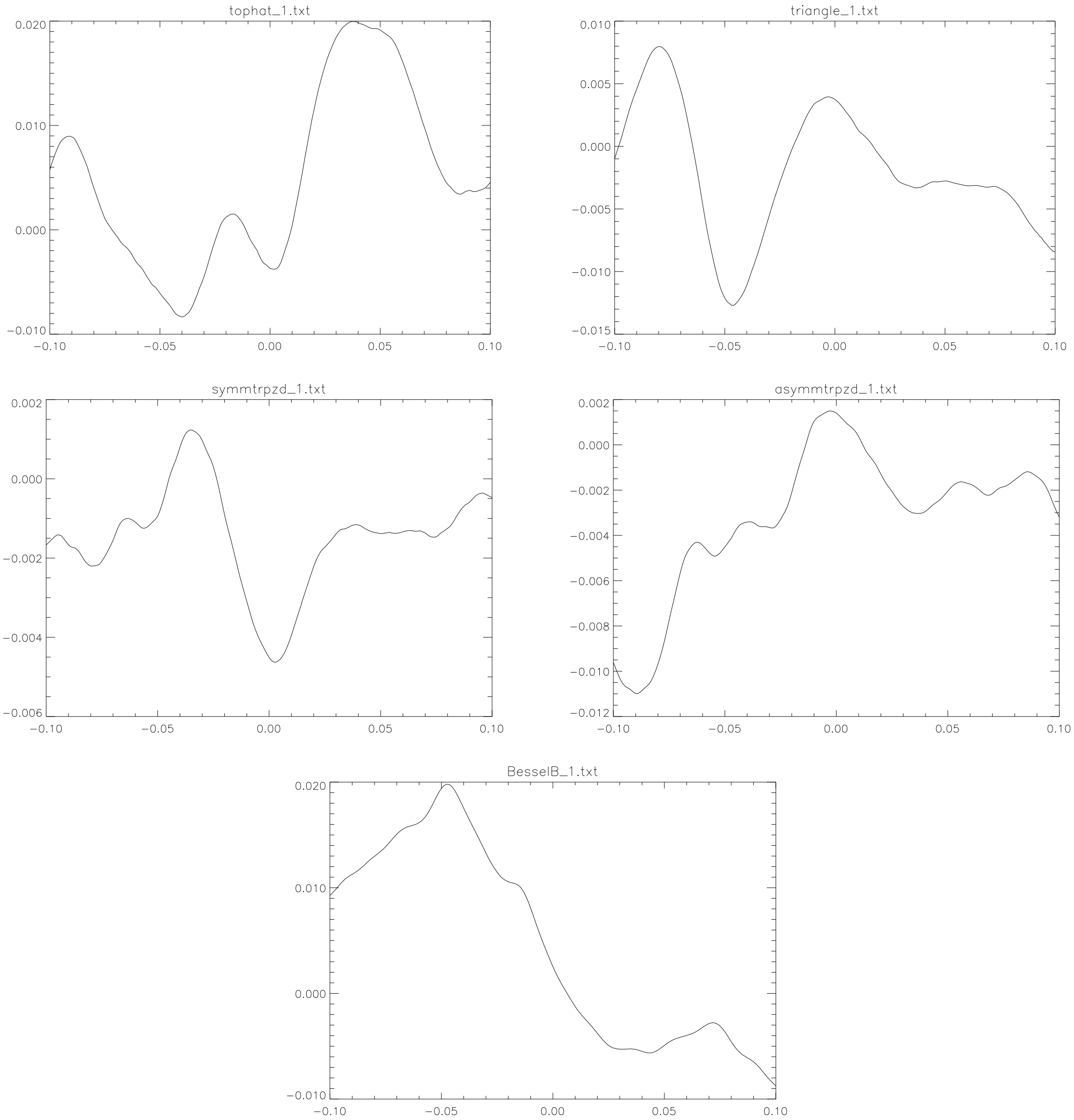

Fig. 4.— Magnitude errors as a function of K-correction redshift offset if a fringed filter is assumed to be the template.

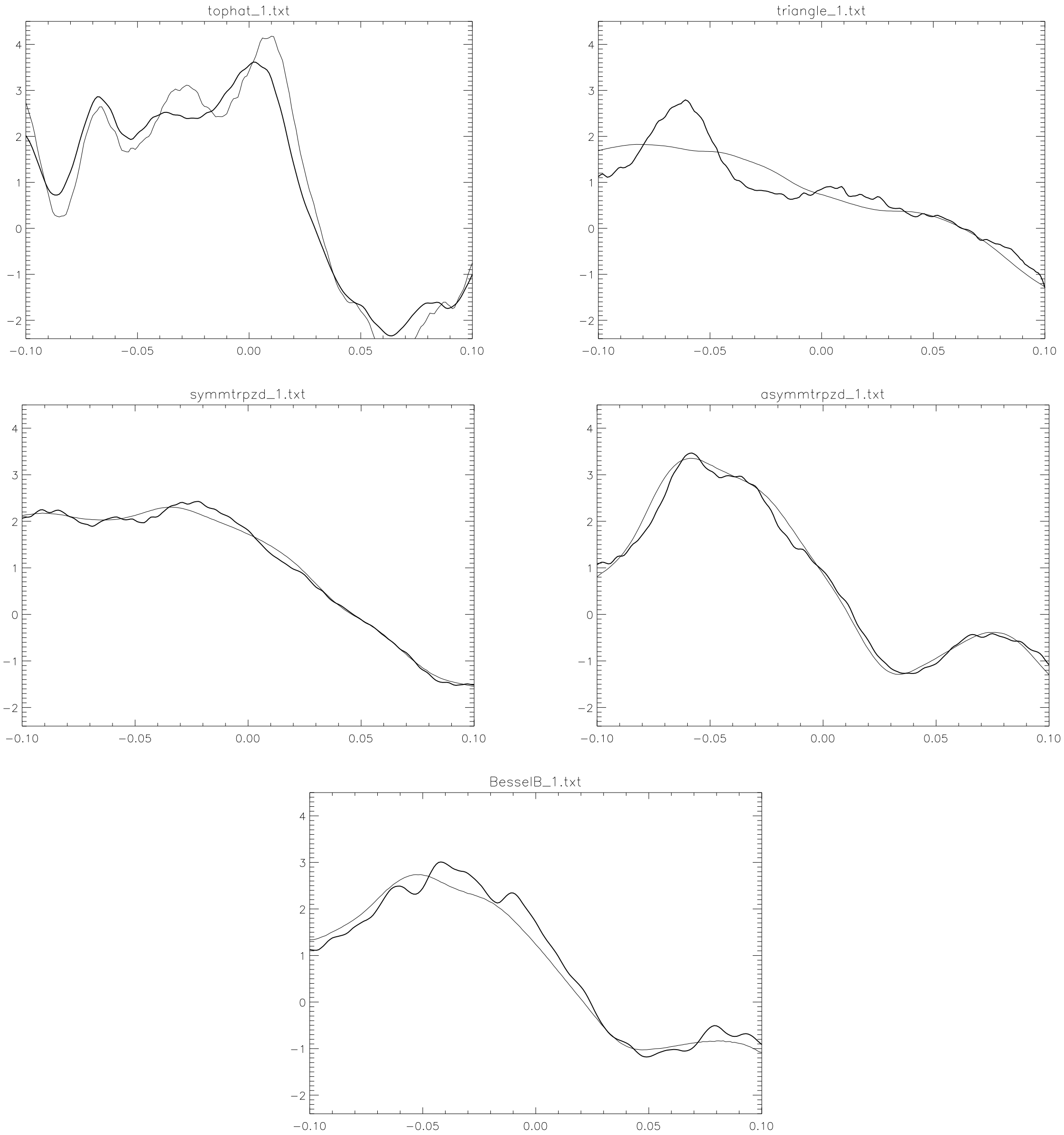

Fig. 5.— $dK(z)/dz$ for the different filters. The thick lines are for Chuck's filters and the thin lines are the target filters.

4. Incidence Angle

We can do the same analysis as in § 2 to determine the magnitude errors induced by assuming the inner annulus passband at the outer annulus. This assumes the use of a single interference filter, untuned for its radial placement on the focal plane. Figure 6 shows the effective transmissions at the extremes of the annulus for Chuck's Johnson filter. The error for the Johnson filter is given in Figure 7. The magnitude errors for this example on the order of 0.01 mag. The errors for the other filters are expected to be similar.

5. Filters and Detectors

It makes sense to choose effective bassbands that incorperate the transmission functions of the LBL CCD and HgCdTe detectors. The throughout for the CCD and HgCdTe drop off at 1 and 1.7 μm respectively. A proposed SNAP B filter is shown in Figure 8. It has a lower effective area and wavelength than the Johnson-B filter. With a wavelength spacing between filters of 1.16 (as opposed to the canonical 1.15 we have been assuming) we get the filter set as shown in Figure 9.

6. Conclusion

1. Certain template filters are better reproduced than others, at least for the purposes of K-correction. The trapezoidal filters give the least amount of error if the shape of the filter is unknown and is assumed to be the template. The errors are ~ 0.002 mag which should be acceptable for SNAP.

2. Fringed filters are not much worse than their corresponding smooth filters if as long as the shape of the filter is well known. Constructive and destructive interference of the fringes and supernova spectra does not appear to be a serious problem.

3. The wavelength shift of the effective filters can be significant depending on how the data is taken. If the points in the supernova light curves are observed at random radial positions, the effective induced dispersion on each point is negligible. Supernova always observed at similar radial positions will have systematically biased magnitudes. However, these errors can be corrected for by fitting out any observed radial dependency.

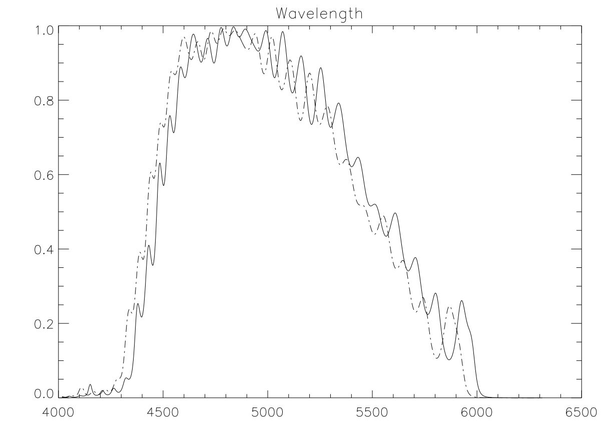

Fig. 6.— The effective filters at the inner (solid) and outer (dashed) edges of the annulus assuming Chuck's Johnson interference filter.

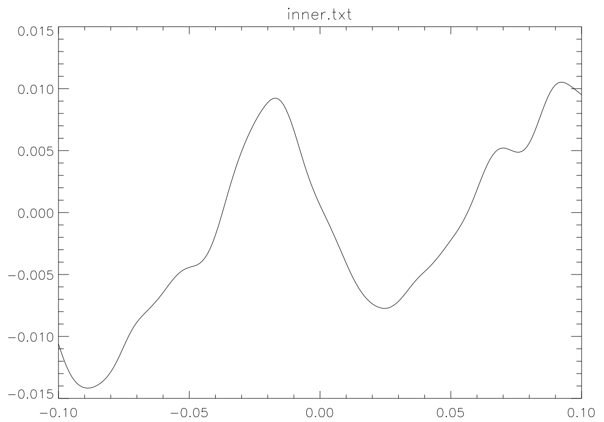

Fig. 7.— Magnitude errors as a function of K-correction redshift offset if the inner effective filter is assumed to be the outer effective filter.

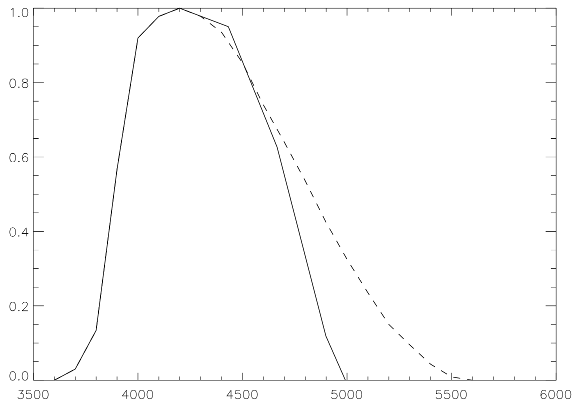

Fig. 8.— A B-band filter constructed so that the right side matches the LBL CCD throughput for the appropriate logarithmically spaced filter. The dashed curve shows the Johnson B-band.

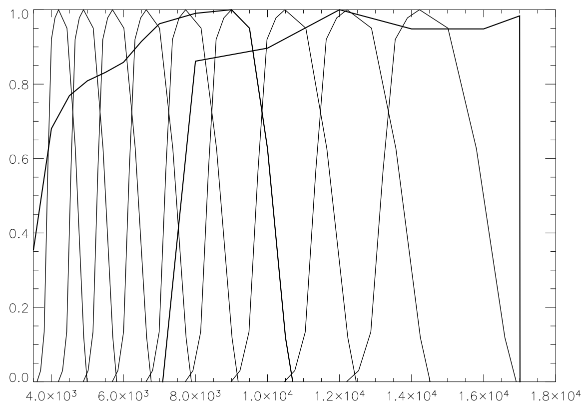

Fig. 9.— A proposed filter set superimposed on the LBL CCD and HgCdTe throughputs. Note how filter 6 fits snugly on the red CCD falloff. This filter can also be mated with the HgCdTe detector.