Internal Field Measurements and Magnetic Reconstruction in SSPX

C.T. Holcomb, R.H. Bulmer, D.N. Hill, E.B. Hooper, T.R. Jarboe[a], A.T. Mattick[a], H. McLean, R.D. Wood, S.Woodruff

Lawrence Livermore National Laboratory
[a] University of Washington

Outline

I. Introduction: Spheromak MHD Equilibrium
   A. Bessel Function model for the minimum energy state
   B. The Grad-Shafranov equation for finding the equilibrium in general
      1. Need field boundary condition data
      2. Need internal field data: FF'
      3. If β is large enough, include pressure data

II. Tools available to do this on SSPX:
   A. Edge polodial and toroidal field probes
   B. Transient Internal Probe
   C. Density and temperature measurements

III. Reconstruction using CORSICA
   A. CORSICA description
   B. Present reconstruction information based on edge probes
   C. CORSICA reconstruction with internal field data

Introduction: Our objective is to determine the equilibrium of a sustained spheromak

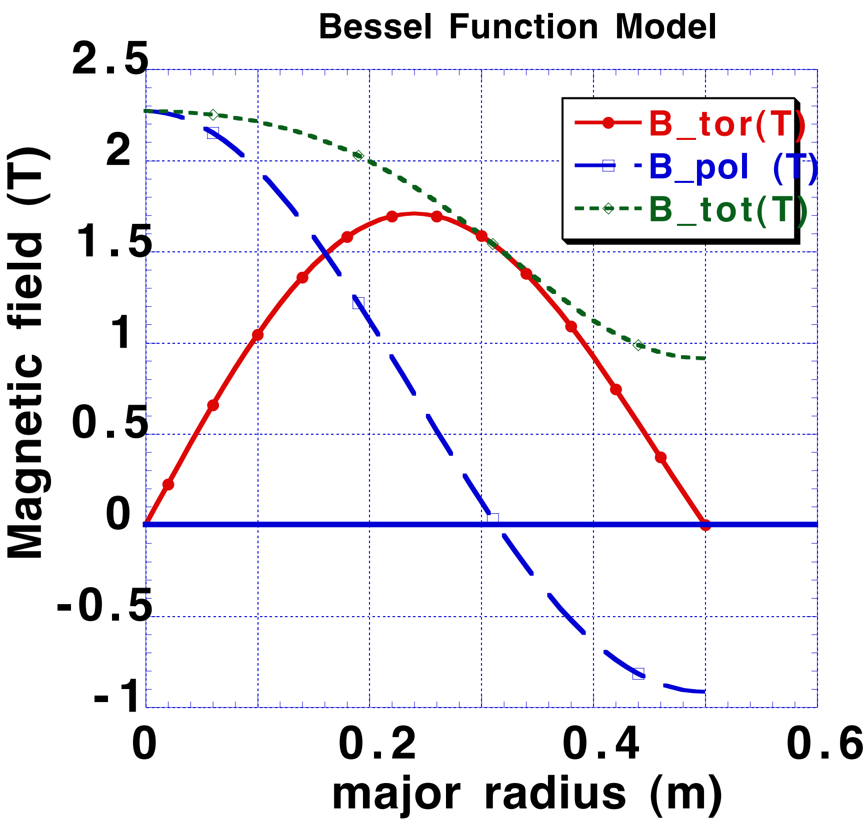
- Like all spheromaks to date, SSPX is near the Taylor Minimum Energy State, which has a well-known MHD equilibrium characterized by a flat $\lambda=\mathbf{j}\cdot\mathbf{B}/B^2$ profile.

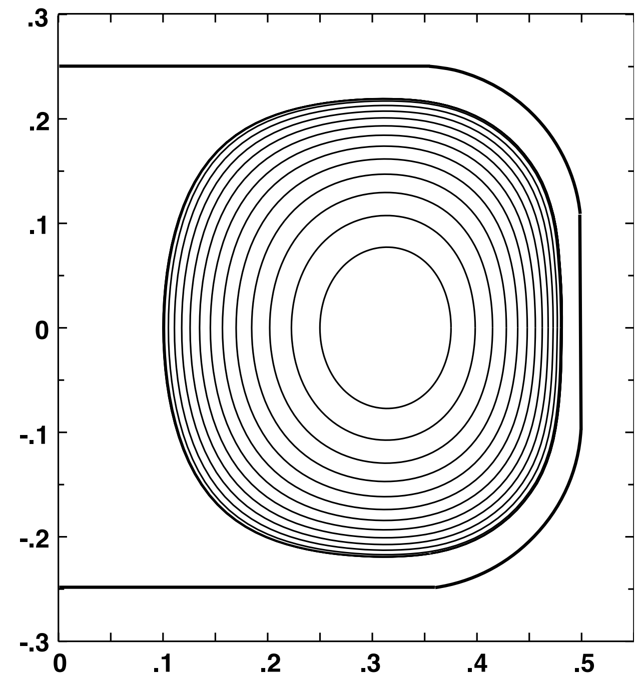
- However, the true equilibrium of a coupled spheromak/coaxial gun system with open and closed flux regions must be determined by solving the Grad-Shafranov equation for the flux surfaces

- To do this correctly, the solver code must have both boundary and internal field measurements to fit to

- SSPX has the advantage of a new diagnostic called the Transient Internal Probe which is designed to measure the toroidal field of a hot spheromak in an unobtrusive manner

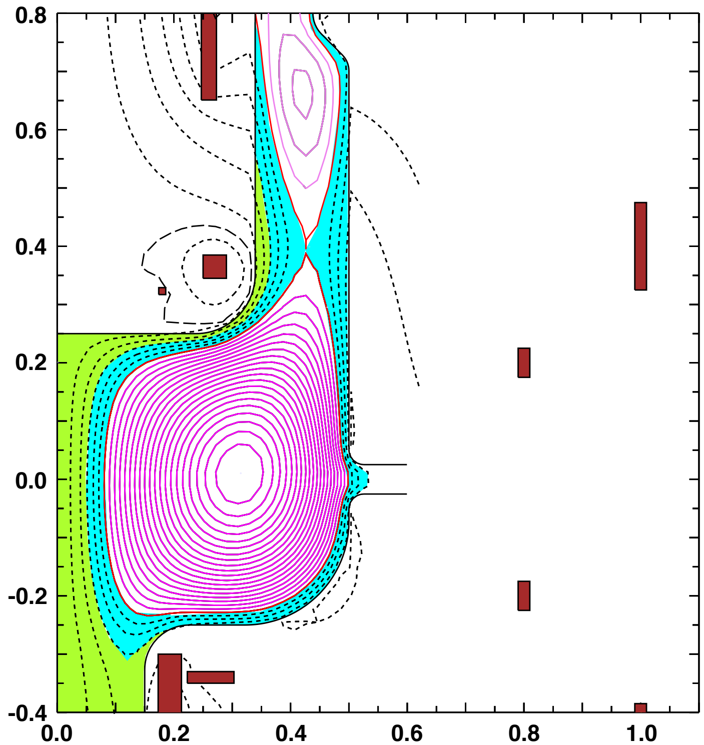
A lone Taylor-State spheromak sitting in a flux conserving shell has a known equilibrium





The Grad-Shafranov equation must be solved to find the equilibrium of the coupled system

A possible equilibrium in SSPX:



The Grad - Shafranov Equation :

$$-\Delta^* = \mu_o R^2 \frac{\partial p}{\partial \psi} + F(\psi)\frac{\partial F}{\partial \psi}$$

where the operator is

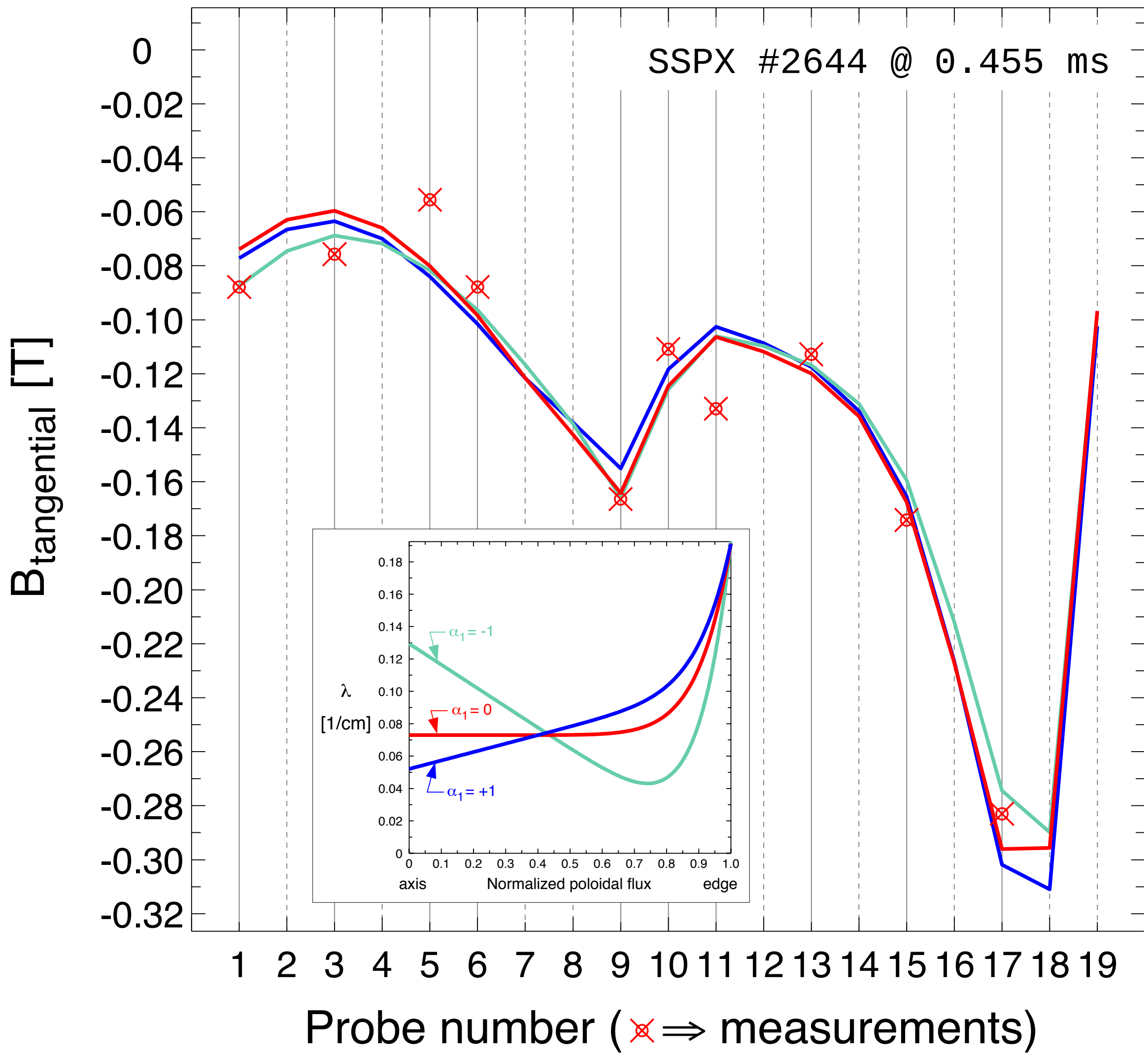
$$\Delta^* \equiv R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial}{\partial R}\right) + \frac{\partial^2}{\partial z^2}$$

and the magnetic field is

$$R\mathbf{B}(\psi) = \mathbf{e}_\phi F(\psi) + \mathbf{e}_R\left(-\frac{\partial \psi}{\partial z}\right) + \mathbf{e}_z \frac{\partial \psi}{\partial R}$$

The boundary field is a necessary, but not a sufficient condition for a well posed equilibrium problem

- The edge probes are insensitive to internal variations in the equilibrium



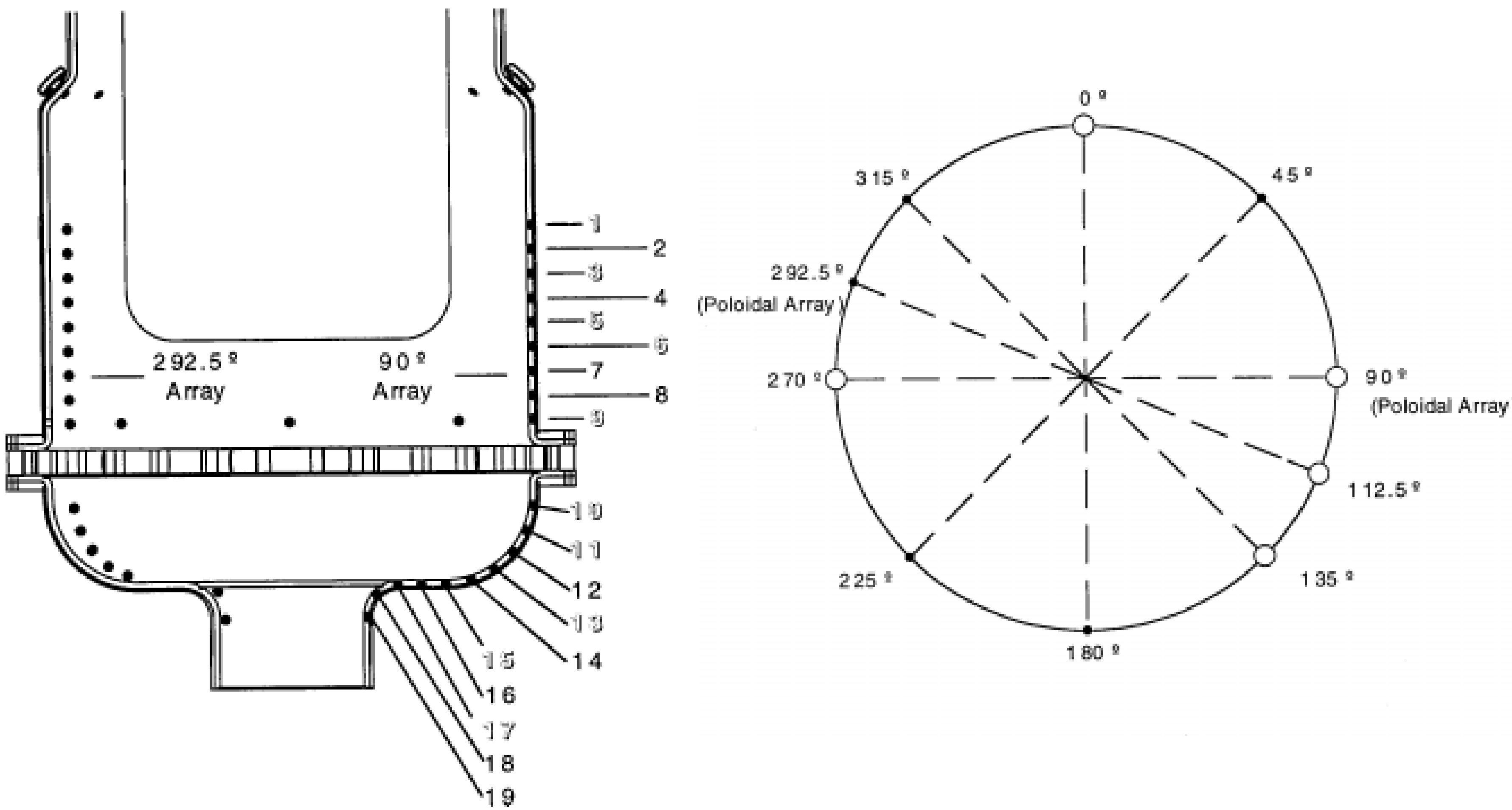
There are two approaches to using internal field profiles to solve the G.S. equation:

- One is to use the measured F(r) directly to estimate F(ψ). The G.S. equation is solved using this function as input.

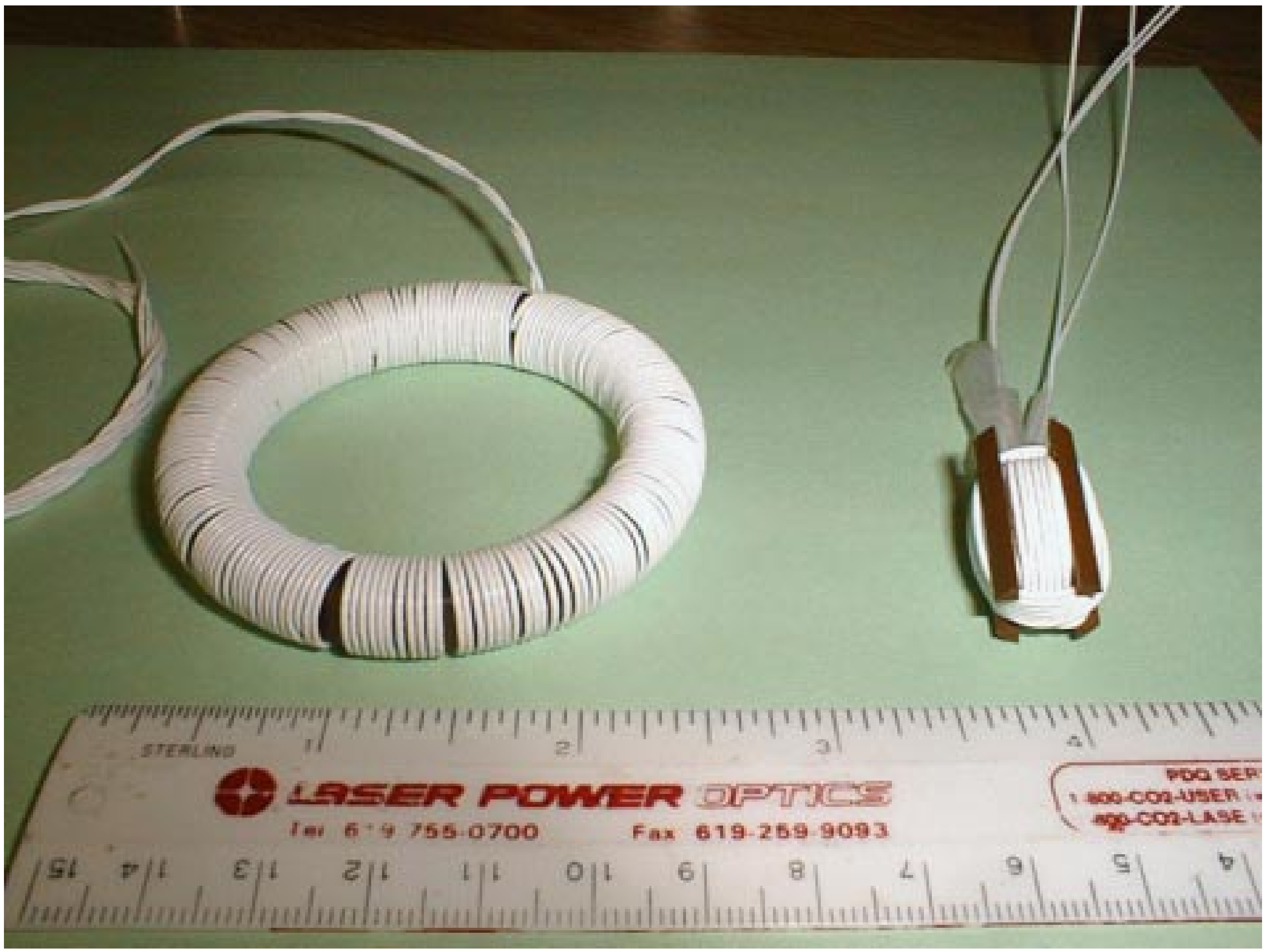
- The other is to supply trial functions for F(ψ) and fit equilibrium solutions to the measured B_{tor}(r) and edge fields. This is currently being done for SSPX with CORSICA using only the edge fields.

- If Thomson Scattering data (n_e, T_e) reveal that there is a significant pressure gradient, then p(r) can be used in the same manner as F(r)

SSPX has poloidal and toroidal field probes imbedded in the flux conserver

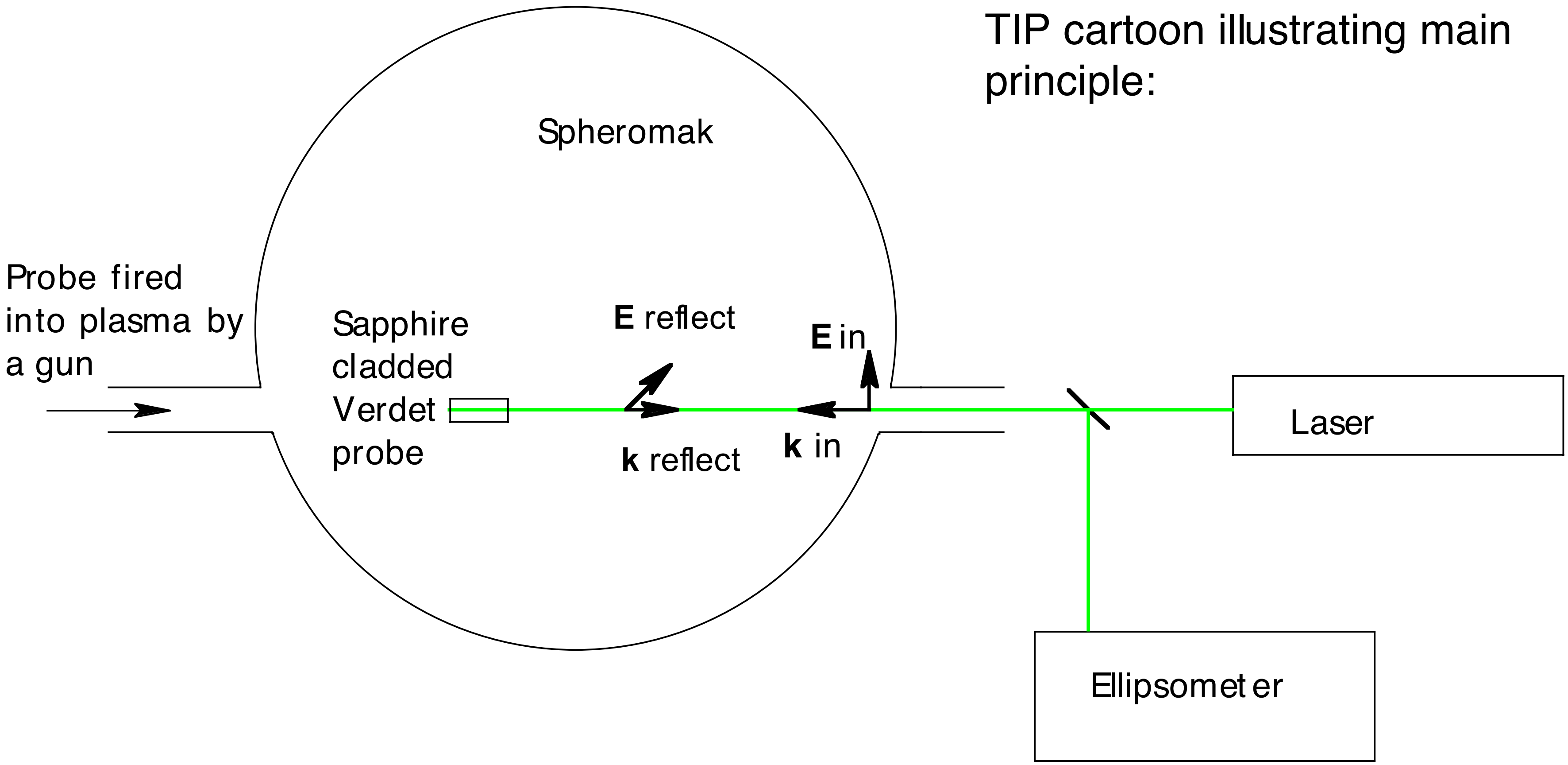


A probe alongside a Rogowski loop that measures poloidal wall current through a bridge in the diagnostic gap



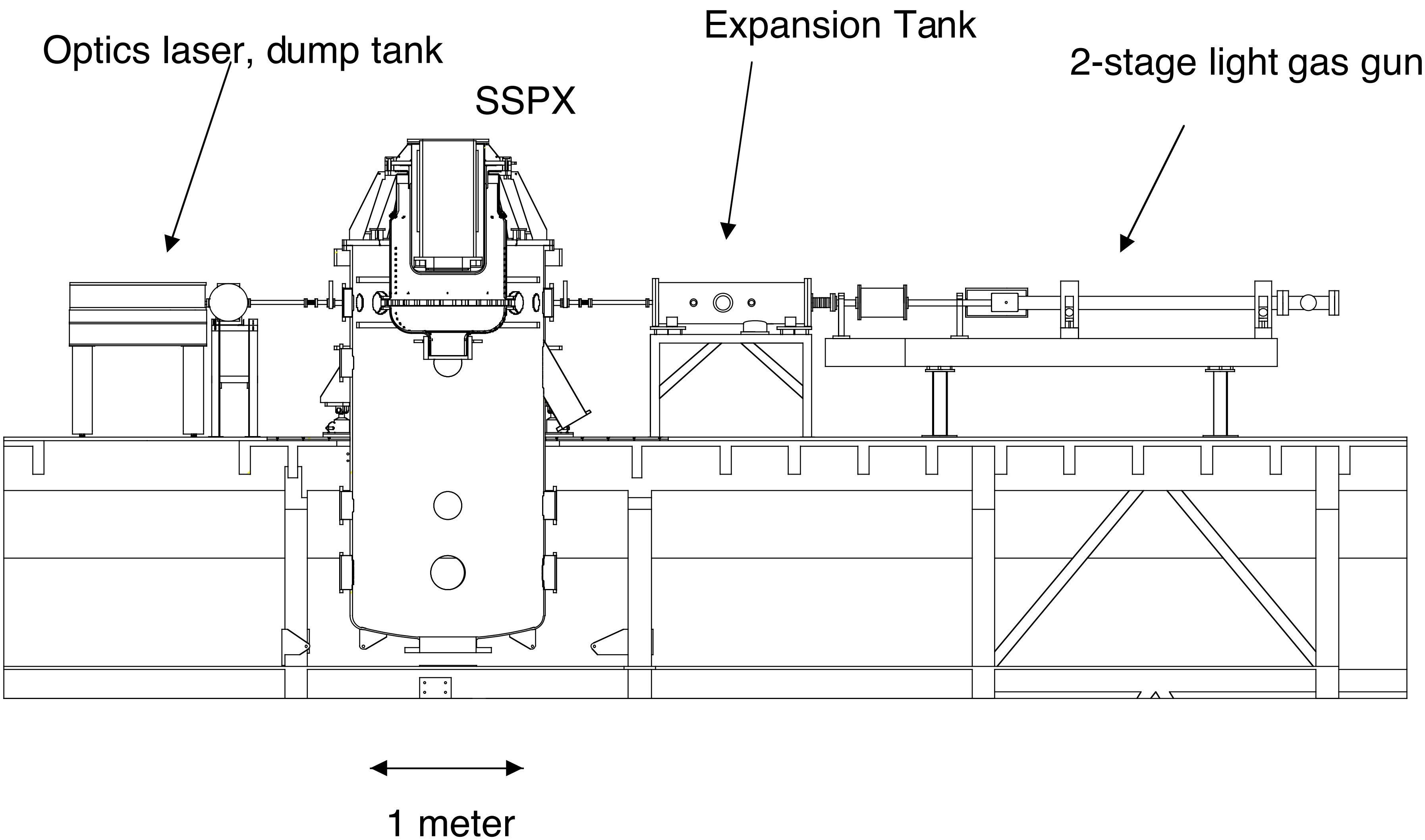
The Transient Internal Probe diagnostic uses Faraday rotation to spatially resolve the field

TIP cartoon illustrating main principle:



Measure $\Delta\theta = 2\mathbf{lvB}_{//}$

Layout of the system



TIP measurement details:

- The Faraday rotator probe is fired through the plasma at about 2 km/s along a chord tangential to the magnetic axis: time of flight from flux conserver to M.A. is about 250 μs.
- The probe length sets the spatial resolution to 1 cm
- Magnetic Resolution= (°uncertainty in ellipsometer)/2lv= ±7 Gauss
- The bandwidth of the measurement is presently limited to 1 MHz by the digitizers (field soak in time ~ 1 ns)
- The probe is encased in sapphire, which along with its high speed and small size (4 mm diameter), guards against ablation, sputtering, and plasma contamination.

The toroidal field from the flux conserver to the magnetic axis will be determined

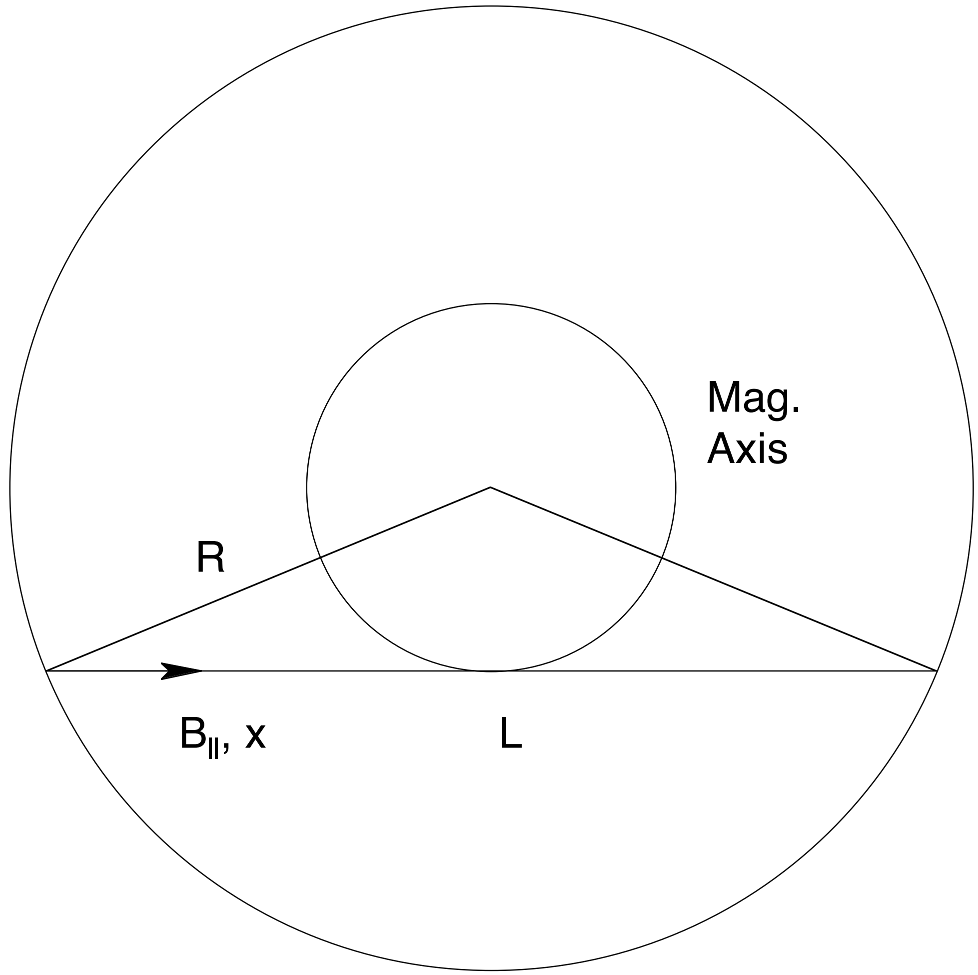
- We measure $\boxed{B_{//}(x) = \frac{\Delta\theta}{2lv}}$

From x=0 to L/2:

$$B_{tor} = \frac{2B_{//}\sqrt{x^2 - Lx + R^2}}{\sqrt{4R^2 - L^2}}$$

Form x=L/2 to L:

$$B_{tor} = \frac{2B_{//}\sqrt{(L-x)^2 - L(L-x) + R^2}}{\sqrt{4R^2 - L^2}}$$



Flux conserver

A multi-point Thomson Scattering system will soon provide T_e and n_e profiles

- If the pressure gradient is high enough, we will use it along with F(r) in the reconstruction
- T.S. measures electron densities down to 5 X 10^{12} cm^{-3}
- Temperatures from 2 eV - 2 KeV, < 10% error
- 10 Channel capability using GA Polyboxes
- Density calibration w/ Rayleigh scattering of argon
- Single pulse Nd:YAG laser, 0.8 J, 8 ns

CORSICA is a general-purpose simulator of toroidal plasma configurations

- It includes the ideal MHD equilibrium model which solves the Grad-Shafranov equation in cylindrical coordinates on an RZ grid.
- Near the Taylor state, the flux function dF/dψ=$\lambda(\psi)$
- In SSPX, we specify the λ profile with the model

$$\lambda(\psi) = \lambda_0\left(1 + \alpha_1\psi^1 + \alpha_2\psi^2 + \alpha_3\psi^3 + \alpha_4\psi^n\right)$$

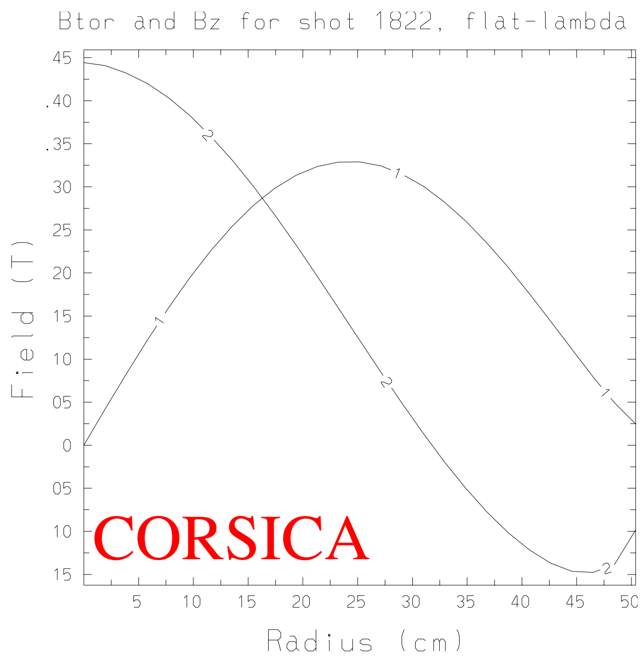
- ψ is the normalized poloidal flux (0 at the magnetic axis, 1 at the separatrix), and n typically has a large value (e.g., 10)
- The value of λ at the edge is therefore $\lambda_0(1+\Sigma\alpha)$
- Presently, we take the value of λ_{edge} to also be the constant value of λ on the open field lines and the injector private flux: this may change when we get internal data
- λ_0 is the value on axis; the eigenvalue λ of the spheromak is determined by the geometry, and is usually the average of the whole λ-profile

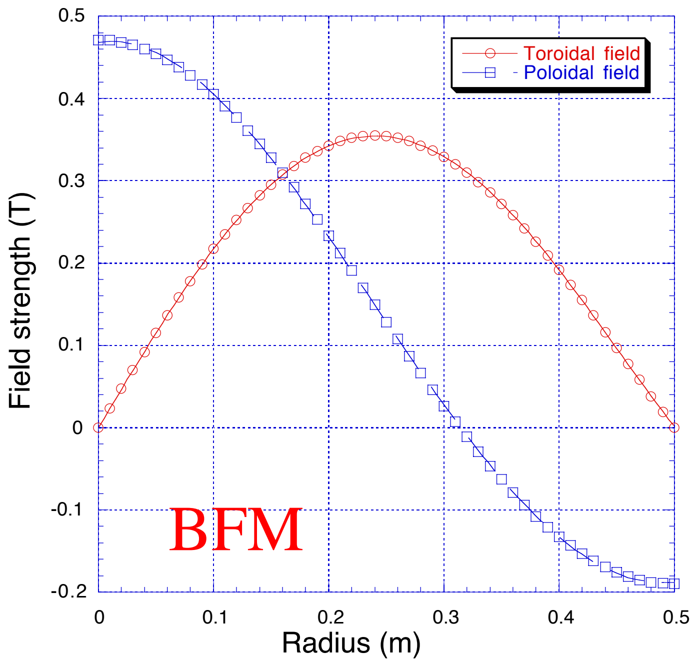
The spheromak equilibrium is determined by:

- The supplied λ-profile
- The supplied coil currents for the injector flux
- The total (open and closed flux) toroidal current inside the flux conserver, which, along with the coil currents, specifies the boundary conditions at the flux conserver
- The pressure profile
- The geometry of the flux conserver

Presently, equilibira are fit only to edge field data - but a few results are already apparent

During the decay phase of a SSPX shot, the edge poloidal field measurements fit well to a CORSICA equilibrium characterized by a flat λ-profile.

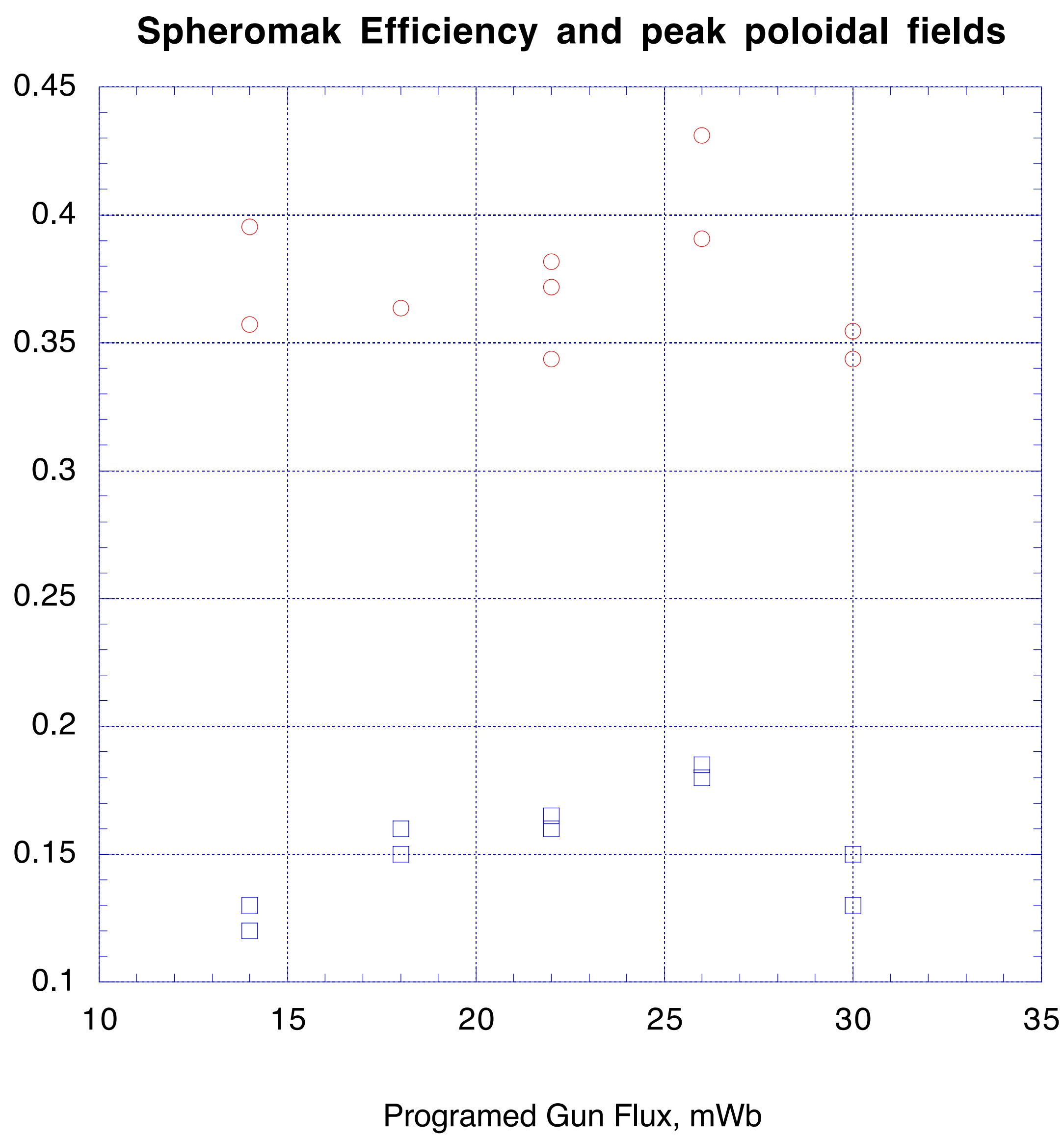




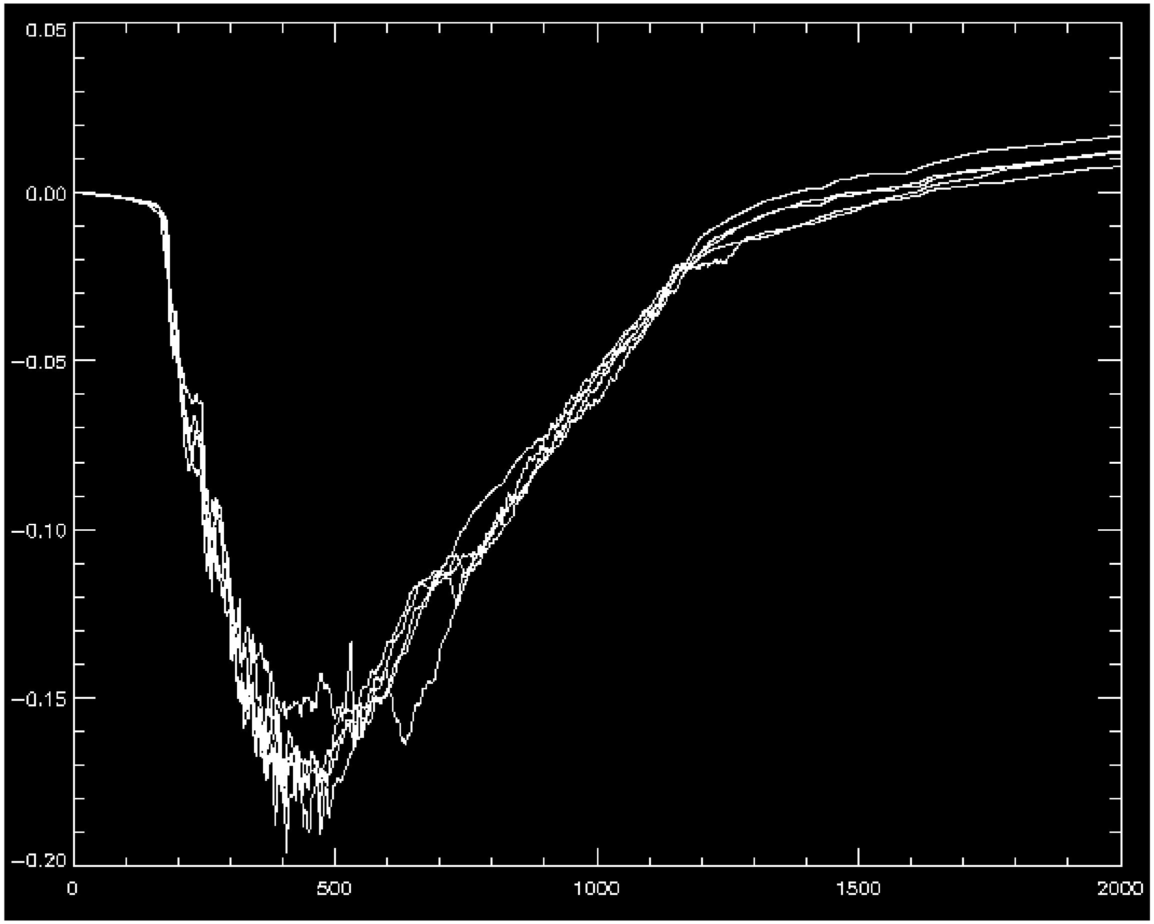
This is to be expected: theory predicts that a decaying spheromak will relax towards a minimum energy state described by the Bessel Function Model.

Fitting to a series of 9 kV shots that scanned injector flux gives efficiency information

Although more data analysis is needed to be conclusive, the efficiency, defined here as $\lambda_0/\lambda_{edge}=1/(1+\alpha_4)$, seems to be the highest (red) for shots with the greatest measured edge poloidal field (blue, Tesla). This point corresponds to the most gun flux being pulled out, and the shallowest λ-gradient between the open (edge) and closed (spheromak) flux.



Five different poloidal field probes at different toroidal locations show axisymmetry



Poloidal Field (T)

Time (μs)

TIP data will be available mid-summer

- Presently, we use a code that calculates the λ-profile and I_{tor} such that the equilibrium best fits the edge poloidal field data. An upgraded and expanded version of this routine will allow us to fit to the measured $B_{tor}(r)$ as well.

- This will gives us an equilibrium that we can be sure of

- An added set of information TIP will be able to give is toroidal mode amplitudes and frequencies up to a few hundred kHz. Bridge Rogowski's routinely give such information on the edge by measuring induced wall currents. However, TIP will determine to what extent these modes are driven within the core of the plasma.

- This will allow us to study possible current drive mechanisms that have been proposed in which an n=1 mode on the open flux couples to the electrons within the separatrix to drive current (Jarboe et al.)

Conclusions:

- Edge field data and CORSICA are being used presently to determine some basic equilibrium conditions, such as possible injector and spheromak λ's, proximity to the Taylor State during decay, and toroidal symmetry.

- This data is not enough to determine the equilibrium for certain: internal field data will soon be provided by the Transient Internal Probe (without perturbing even a hot plasma).

- This measurement relies on Faraday rotation within a high speed probe to spatially determine the toroidal field.

- CORSICA will fit an equilibrium to the combined edge fields, internal toroidal field, and possibly the pressure.