Electro-Optic Kerr Effects in Spun High-Birefringent Fiber Current Sensors

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Abstract

We have measured the influence of the electro-optic (EO) Kerr effect on the response of a spun high-birefringence (hi-bi) fiber current sensor in a simulated Gas Insulated System (GIS) environment. We show that the EO Kerr effect distorts the response of the sensor, and that the second-harmonic signal has a small dependence on the input polarizer angle. We also, have theoretically modeled a polarimetric current sensor using spun hi-bi fiber and compared the models to our experimental results. With the models we predict the response of a fiber current sensor in a 550 kV GIS application.

1. Introduction

In earlier publications we discussed the affects of the EO Kerr effect on annealed fiber current sensors in high electric field environments, such as gas-insulated systems GIS [1, 2]. In this paper we briefly discuss the first measurements of EO Kerr-effect signals in a fiber current sensor using spun hi-bi fiber. The EO Kerr effect is an electric-field-induced linear birefringence that arises when an electric field changes the polarizability of the glass molecules [3]. Linear birefringence, either from stress, bending, waveguide form, or the EO Kerr effect, alters the response of Faraday-effect current sensors [4]. In spun hi-bi fiber current sensors this added EO Kerr birefringence, like bend birefringence, alters the elliptical eigen-modes of the fiber, changing the response of the spun hi-bi sensor [5].

Current sensors made with spun hi-bi fiber are relatively immune to vibrational effects. The birefringence induced by vibration is only a small perturbation on the large intrinsic birefringence in the fiber. Polarimetric sensors made with this fiber suffer from a temperature dependent rotation of the output polarization-state. In recent years the use of Sagnac fiber sensors have made this fiber practical for current sensing [6-8]. In this paper we model the polarimetric spun hi-bi sensor using several approaches and experimentally confirmed some of the theoretical predictions with a polarimetric current sensor placed in a high electric field.

2. Theory

Equations governing evolution of polarization state along the bent spun hi-bi fiber can be derived using the differential Jones-matrix method. Accounting for the current-induced Faraday rotation and the Kerr-effect linear birefringence; the linear polarization equations are

$$
\begin{aligned}
\frac{dE_x}{dz} &= -j\left[\frac{\pi}{L_u}\cos(4\pi\tau z)+\frac{\delta}{2Z}\right]\cdot E_x-\left[j\frac{\pi}{L_u}\sin(4\pi\tau z)+\frac{\theta_F}{Z}\right]\cdot E_y \\
\frac{dE_y}{dz} &= -\left[j\frac{\pi}{L_u}\sin(4\pi\tau z)-\frac{\theta_F}{Z}\right]\cdot E_x+j\left[\frac{\pi}{L_u}\cos(4\pi\tau z)+\frac{\delta}{2Z}\right]\cdot E_y
\end{aligned}
\tag{1}
$$

where z is the coordinate along the fiber and Z is the total fiber length, L_u is the un-spun beat length, τ is the spin rate in revolution/m, θF is the total Faraday rotation in the fiber, and δ is the total linear retardance in radians. We assume $\delta = \delta_B + \delta_{EOK}$, where δ_B is the bend retardance, δ_{EOK} is the EO Kerr retardance, and the retardation axes are aligned to the X-axis. The linear birefringence beat length, due to bending and the EO Kerr effect, is $L_l = 2\pi Z / \delta$. The bend retardance is $\delta_B = (0.78\ r_f^2 Z) / (\lambda R^2)$, where r, is the fiber cladding radius, R is the bend radius, and λ is the operating wavelength [9]. The retardance due to the EO Kerr effect has the functional form $\delta_{EOK} = 2\pi K E^2 Z$, where K is the EO Kerr constant, equal to $(5.3 \pm 0.2) \times 10^{-16}\ m/V^2$ at 23°C in silica, and E is the electric field strength [10]. δ_{EOK} becomes large enough to affect the response of an optical-fiber current sensor for fiber lengths approaching 10 m and when the sensor is placed in uniform electric fields greater than I MV/m rms, such as the

GIS environment.

Generally, Eq. (I) doesn't have an analytical closed-form solution for bent fiber. However, under moderate bending (R ≥ 3 cm) approximate solutions can be obtained using a perturbation theory with the parameter δ. Alternatively, the propagation equations can be solved by numerical integration.

Perturbation approach
If the spin period is much smaller than the un-spun beat-length ($1/\tau \ll L_u$) then the spun hi-bi fiber can be viewed as a fiber with continuously distributed circular birefringence. The corresponding circular beat-length can be shown to be $L_c = 4\tau\, L_u^2$. The bending and EO Kerr effect both introduce a continuously distributed linear birefringence to the fiber. Considering this linear birefringence a small perturbation, one can derive the following formula for the response of a polarimetric sensor,

$$R_v \approx \frac{A}{2} + \frac{A}{2}\sin\left(\frac{L_C \delta_{EOK}}{2L_l}\right) \tag{2}$$

where A is the transmittance of the sensor, and L_l is the linear beat-length corresponding to the bending birefringence only ($L_l = 2\pi Z / \delta_B$). As follows from Eq. (2), the sensor's response becomes independent of the EO Kerr-induced birefringence in the absence of bending ($L_l = \infty$). Conceptually the $1 / L_l$ dependence in Eq. (2) can be understood in terms of the EO Kerr retardance being averaged to zero as the linear input polarization state rotates along the length of the spun hi-bi fiber.

Equation (2) does not include contributions from the current. If E = 0, the current-induced polarization rotation measured by the sensor will be $\theta_F = 2\mu VNI$ where μ is the permeability of free space, μ V is the Verdet constant in units of rad/A, N is the number of fiber turns around the conductor, and I is the current. If a coherent light source is used, the sensor response is $R_I = AS\,\theta_F$, where [5]

$$S = \frac{(L_u \tau / \pi)^2}{1 + (L_u \tau / \pi)^2} \tag{3}$$

Equations (2) and (3), or R_V and R_I form what we call a simple perturbation theory from which we can predict the response of a current sensor in high E fields.

When the current and electric field are synchronous ac functions of time, such as $I = I_0 \sin(\omega t)$ and $E = E_0 \sin(\omega t + \phi)$, the sensor signal due to current will beat ω and sensor signal due to the electric field will beat 2ω. (ω is the angular frequency, t is time, and ϕ is the phase between the current and electric field.)

To find higher harmonics of the output signal with ac current and voltage, one has to solve Eq.(1) to second order in linear birefringence. When this approach is taken, the output intensity of the sensor is

$$\begin{aligned} R \approx{}& \frac{A}{2} + \frac{A\theta_{F0}}{2}\sin(\omega t) + A\left(\frac{L_C \delta_{EOK0}}{4L_l} + \frac{\delta_{EOK0}}{2\sqrt{2}\tau L_l}\sin(4\pi\tau Z)\sin(4\alpha - 8\pi\tau Z - \frac{\pi}{4})\right)\cos(2\omega t + 2\phi) \\ & - A\left(\frac{L_C \delta_{EOK0}\theta_{F0}}{16\pi\tau L_L Z} + \frac{\delta_{EOK0}\theta_{F0}}{8\tau L_L}\sin(4\alpha - 8\pi\tau Z)\right)\cos(3\omega t + \phi) + \frac{A L_C \delta_{EOK0}^2}{64\pi Z}\cos(4\omega t + 4\phi) \end{aligned} \tag{3}$$

where a is the input polarizer angle with respect to the bending birefringence axes, $\theta_{F0} = 2\mu VNI_0$, and $\delta_{EOK0} = 2\pi K E_0^2 Z$. From Eq.(4) we see that the spun hi-bi fiber sensor will have high harmonic content due to the EO Kerr and Faraday effects. From this approach we also see that the EO Kerr effect signal at 2ω has a small angular-dependent part, which in turn depends on the fiber length Z and the spin rate τ. Again, as the bend radius is increased the EO Kerr signal is reduced by $1 / L_l$. Eq. (4) forms what we call the perturbation theory for a polarimetric current sensor using spun hi-bi.

Numerical approach
We also solved Eq. (1) numerically for the output response of the sensor to compare with our approximate solutions and with experimental data taken on a laboratory sensor. The numerical results have less than a 5 dB difference between the signal amplitudes predicted by Eq. (4).

3. Experiment

To test these theoretical models we used the same apparatus as our previous papers [1, 2] except a spun hi-bi fiber was used as the sensing coil. The spun hi-bi fiber had a spin pitch of about $\tau \approx 250$ twist/m, an un-spun beat-length of about $L_u \approx 3$ cm, a diameter of about 80 µm, and was single mode at 830 nm. We placed about 38 turns into the ~15 cm diameter fixture filled with silicone gel. The length of the fiber in the coil was ~17.9 m. The fiber leads, about 0.5 m in length, came off of the fixture tangent to the coil and were carefully held so that no additional birefringence was induced. The bend and circular beat-lengths were $L_u = 23.5$ m and $L_u \approx 90$ cm, respectively. In our apparatus we use a polarizing beam-splitter at 45° to the input polarizer, and electronically take the difference-divided-by-the-sum (Δ/Σ) of the two intensities from the beam-splitter. This normalizes the response of the sensor to source intensity changes and fiber coupling losses.

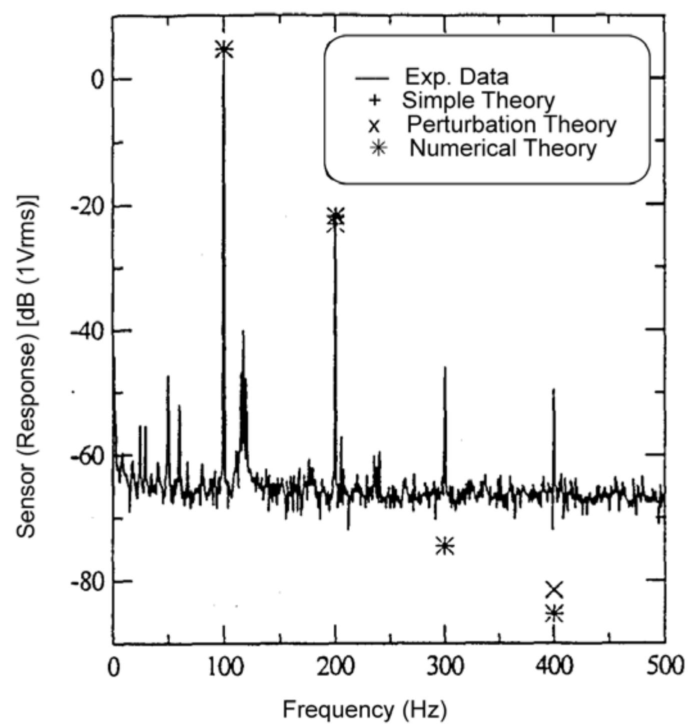

Figure 1 The spectral response of our sensor with ~812 kV/m rms and ~81 A rms at 100 Hz. The marks show the various theoretical predictions.

The response of our sensor taken with a spectrum analyzer is shown in Fig. 1. The current was set to about 81 A rms and the voltage at about 2.03 kV rms or an electric field on the fiber of about 812 kV/m rms. This produces a Faraday rotation $\theta_{F0} \approx 15.8$ mrad rms ($\mu V = 2.58$ µrad/A at 830 nm [11]) and an EO Kerr retardance $\delta_{EOK0} \approx 39.3$ mrad rms. Using these values and the theoretical models we estimated the value for each harmonic in Fig. 1. All the models (Eqs. (2), (3), (4) and numerically (1)) are able to predict the fundamental current signature and the second-harmonic signal due to the EO Kerr effect. The perturbation and numeric models do not correctly predict the amplitudes of the third- and fourth-harmonic signals. At this time we do not have an explanation for this discrepancy.

Fig. 2 shows the dependence of the second-harmonic signal on input polarizer angle with $E_0 \approx 828$ kV/m rms or $\delta_{EOK0} \approx 41$ mrad rms. The dashed line in the figure is a spline fit to the data. The solid line is Eq. (4) with the sensor in a Δ/Σ arrangement. As the input polarizer was rotated the output polarizing beam splitter was also rotated to maintain equal dc intensities on the two photodetectors. The measured angular response of the second-harmonic signal is approximately $\sin(4\alpha)$ with a peak-to-peak magnitude of about 150 µrad. Equation (4) predicts an amplitude of about 10 µrad ppk. The numerical model predicts about 16 µrad ppk for the amplitude of the angle dependence.

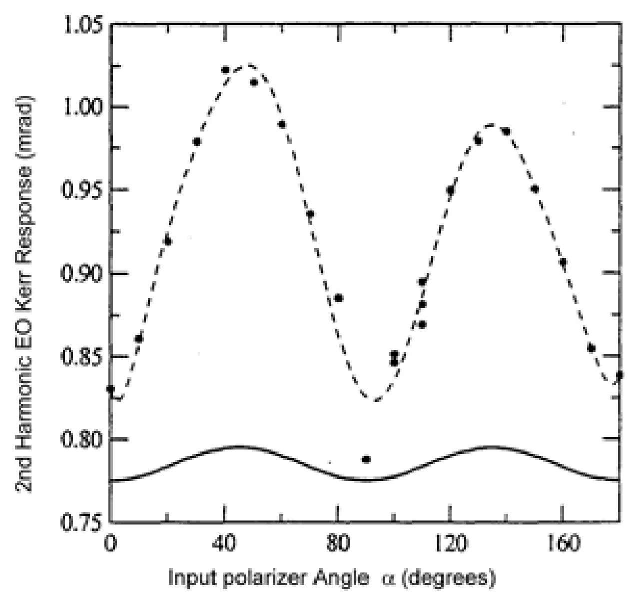

Figure 2 Second harmonic EO Kerr signal variation with input polarizer angle for $E_0 \approx 828$ kV/m rms. The dashed line is a spline fit to the data and the solid line is Eq. (4).

Equation (4) also predicts a length dependence Z for the 2ω signal with a period of $1/4\pi\tau \approx 0.3$ mm. We can not reliable cut the fiber in small enough increments to see this length dependence.

4. Conclusions

The first measurements of the influence of the EO Kerr effect on a current sensor using spun hi-bi fiber have been made and modeled. However, our models do not accurately predict the amplitudes of the 3ω and 4ω signals. Also, we have measured the angular dependence of the second-harmonic signal, and compare it to theory.

With the confirmation of Eq. (4)'s ability to predict the amplitudes of the ω and 2ω signals we can make a useful comparison between polarimetric current sensors using annealed and spun hi-bi fiber in a GIS application. For example, assume that a fiber sensor with N = 8 is placed in a GIS line of 550 kV rms and 4.5 kA rms, operating at 60 Hz with a 0.5 m diameter outer conductor, so that Z= 12.7 m [1]. At 830 nm and full current, the Faraday rotation would be 186 mrad rms. At full voltage, E_0 = 3.3 MV/rn rms, the EO Kerr retardance would be δ_{EOK} = 460 mrad rms.

A sensor using annealed fiber with a static retardance δ_S = 174 mrad (10°) placed inside the outer conductor would have a fundamental signal of 182 mrad rms, and a second-harmonic signal of 40 mrad rms, when the polarizers are oriented to optimize this 2ω signal [1]. A sensor using spun hi-bi fiber, with L_C and L_u the same as our fiber, placed in the GIS line would have L_l = 1044 m, a fundamental signal of 186 mrad rms, and a secondharmonic signal of 198 µrad rms.

Comparing the spun hi-bi and annealed fiber sensors, the current signals are nearly equal, but the secondharmonic signal for the spun hi-bi sensor is lower by -46 dB. This is a worst case (polarizers oriented for optimum EO Kerr signals) for the polarimetric sensor. However, with the spun hi-bi fiber sensor the 2ω EO Kerr signal can not be reduced significantly by polarizer angle as with the annealed fiber sensor. Equation (4) predicts that the amplitude of all of the EO Kerr-effect signal's will be reduced as the sensor radius and fiber length are increased.

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