Cumulative Beam Breakup Study of SNS SC Linac

DRAFT for internal discussion only
not a final version

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1. Introduction

SNS H$^-$ linac is different from highly relativistic electron linac in that the beam particle velocity is significantly less than the velocity of light c. The wake field and bunch energy loss effect of beam with $b < 1$ is relatively unknown. Recently Kurennoy did some calculation on bunch energy loss for particle beams with $b < 1$ [1]. It is shown that

$$\frac{k_s(b,s)}{k_s(1,s)} = \exp\left[-\left(\frac{w_s s}{c}\right)^2 \frac{1}{b^2 g^2}\right] \frac{(R/Q)(b)}{(R/Q)(1)},$$

where $k_s(b,s)$ is the loss factor of mode s and s is half of rms bunch length of a Gaussian beam. Here b and g are relativistic factors, and R/Q is the shunt impedance. For very short bunches, exp[…]≈1. And it is a reasonable approximation to use

$$(R/Q)(b) = \frac{2c^2}{e_o w^3} \frac{\left| \int_0^l e^{-iwz/bc} \frac{\partial E_z(0,0,z)}{\partial x} dz \right|^2}{\int_V E^2 dV}$$

for H$^-$ (or proton) replacing $(R/Q)(1)$ of highly relativistic electron linacs.

A study is performed to investigate the effects of HOM (Higher Order Mode) of SNS superconducting (SC) linac on cumulative beam breakup. This is to provide tolerable Q value of HOM and to provide a systematic view of the dependence on parameters involved such as frequency, Q value, R/Q, frequency spread of HOM. For simulation, TDBBU code [2] developed at JLab is used with the relevant input of SNS SC linac.

SNS SC linac consists of 11 medium beta cryomodules and 17 high beta cryomodules. A medium beta cryomodule consists of three β_g=0.61 6-cell cavities and a high beta cryomodule consists of four β_g=0.81 6-cell cavities. The warm section is 1.6 m long where doublet quadrupole is placed to provide transverse focusing. The injection energy to the SC linac is 184 MeV.

Fundamental mode frequency of SNS SC linac is 805 MHz while bunch frequency is 402.5 MHz because beam bunch is in every other bucket. The linac bunch train is 645 ns long and the gap is 300 ns. And the macro bunch train is 1 ms long. In the simulation, the 302.4 ns gap is ignored for the sake of simplicity. For example, when Q=10^5, ω=2400 MHz and g=302.4 ns, only 2.3% of HOM field is damped during this 302.4 ns gap because the remaining field is proportional to exp(-ωg/2Q). This is a pessimistic condition from the beam breakup viewpoint.

2. Benchmarking

Benchmarking of TDBBU code is done by comparison with the analytical theory. For a few simple cases, analytical expression for the beam break-up threshold exists. One of them is when there is no transverse focusing, nor acceleration. For the sake of simplicity, the example presented in [3] is used.

Deflecting factor is defined as the ratio of the maximum transverse displacement over the transverse displacement without HOM excitation at one place of a linac. The Q for a given transverse deflecting factor x_∞ / x_0 at the end of linac is given by

$$Q = \frac{2p}{<I>q} \frac{w}{\Gamma} \frac{l}{L^2} \ln^2 (2 \frac{x_\infty}{x_0}),$$

where p is momentum of particle, $<I>$ the current, q charge of particle, w angular frequency of HOM, Γ the geometry factor of the mode, l the length of a cavity, L the total length of an accelerator. 10-A deuteron beam with 7.5 MeV energy is used. When the worst possible beam-cavity resonance is assumed, the angular frequency of HOM is $w = 2w_b(1+1/2Q)$. Here Q is the quality factor of HOM and w_b is the angular frequency of beam bunch (352MHz * 2π is used in this particular calculation). $l = 0.152$ m and $L = 0.608$ m are used. $\Gamma / w = 5.97\text{x}10^{-7}$ Ω/m^2 is used. When the deflecting factor x_∞ / x_0 is kept less than 2, the corresponding Q is 1.49x10^5. When the same problem is repeated using the TDBBU code, deflecting factor x_∞ / x_0 =1.83 is obtained as is shown in Fig. 1. In his paper geometry factor Γ (Ohm/m^2) is defined and the relationship between shunt impedance R/Q (Ohm) to be used in TDBBU code and Γ is R/Q=Γ/k^2=Γc^2 / w^2 where k is the wavenumber of HOM.

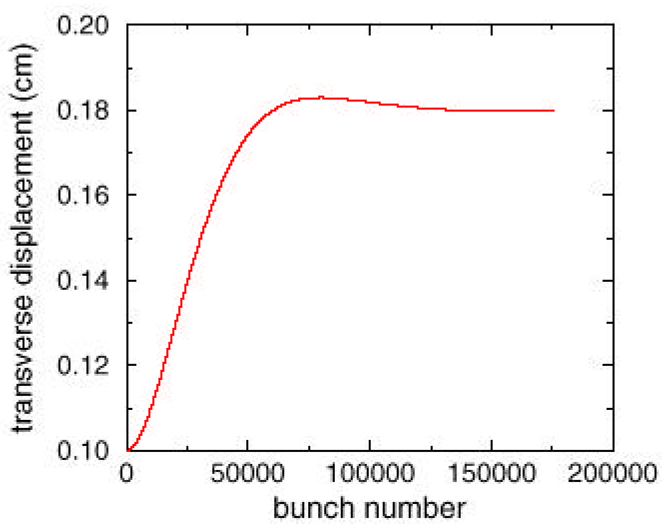

FIG.1 Plot of the transverse displacement at the end of the linac vs. bunch number.

This benchmarking study shows that TDBBU code produces consistent results compared with analytic theory. TDBBU code has been extensively used at JLab for benchmarking with experiment and simulations [4]. The TDBBU simulation has also reproduced the Gluckstern simulation results exactly [5], which indicates that the simulation is correct.

3. Dependency on HOM frequency

Beam breakup simulation is done only for the superconducting part of SNS linac starting from 184 MeV. It is assumed that all the medium beta and high beta cavities have identical HOM frequency, shunt impedance R/Q, and Q value for this simulation. The effect of single HOM is simulated, not the combined effect of several HOMs. All the parameters are fixed except for the HOM frequency, which is varied from 800 MHz to 2200 MHz. A safety factor of more than ten is applied to beam current, which is set to 500 mA instead of 36 mA of actual average (macropulse) current. R/Q is set to 2 Ω, and Q is set to $1.0\text{x}10^7$. Initial beam bunch is assumed to be displaced 1 mm transversely.

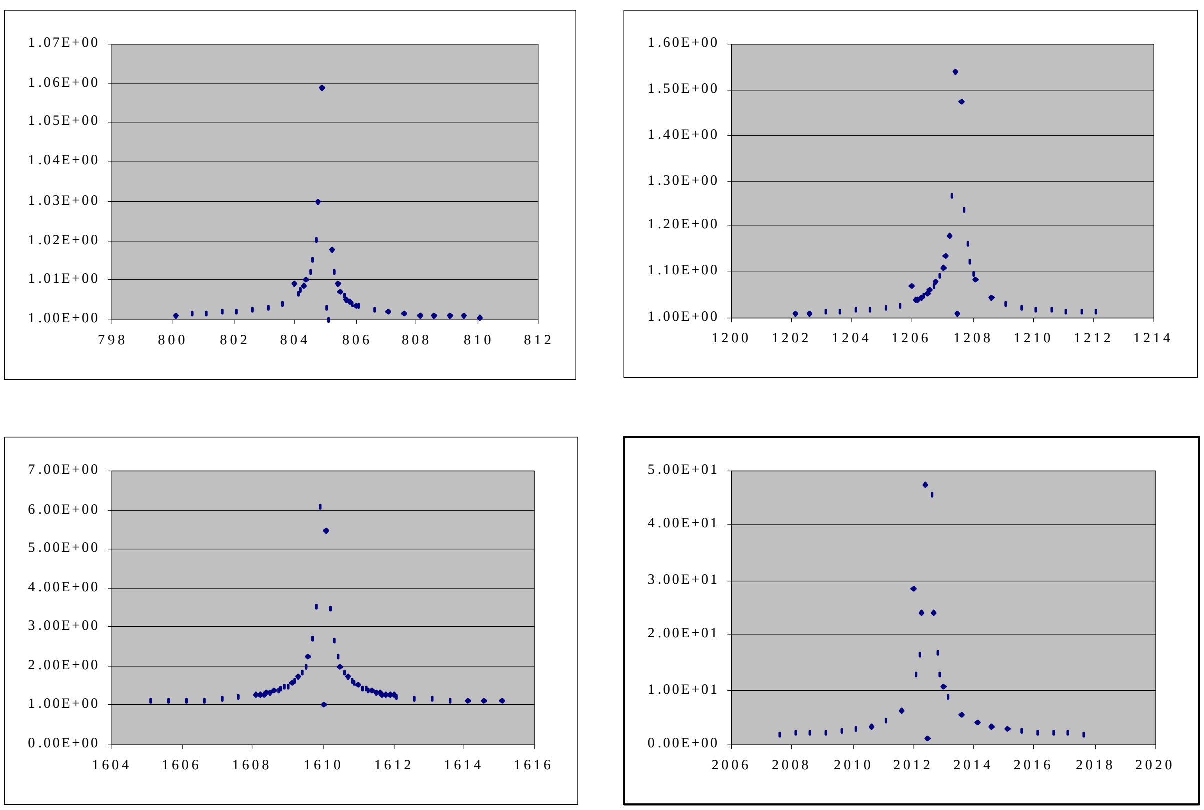

FIG.2 Plots of transverse deflecting factor at the end of linac vs. HOM frequency in MHz. Clear resonance structure is displayed at every multiples of bunch frequency.

Considering the relation between bunch frequency 402.5 MHz and HOM frequency, resonance condition holds when HOM frequency is quite near a multiple of bunch frequency of 402.5 MHz as is shown in Fig. 2. When HOM frequency is away from resonance frequency by 2 MHz, deflecting factor decreases down to 3.5 at the bottom right plot.

One should note that the peak value gets greater and the peak gets wider for higher multiples of 402.5 MHz bunch frequency as shown in Fig. 2. R/Q does tend to diminish at higher HOM frequencies. In this sense, R/Q is maybe not the best parameter. The reason is as follows. Because constant values of R/Q is used throughout the whole frequency range, the value of $\Gamma=R/Q*k^2$ increases as squares of k wavenumber of HOM.

Figure 3 shows plot of transverse displacement of beam bunches vs. bunch number when HOM frequency is 2013 MHz, $Q=1.0\times10^7$ and $R/Q=2\ \Omega$.

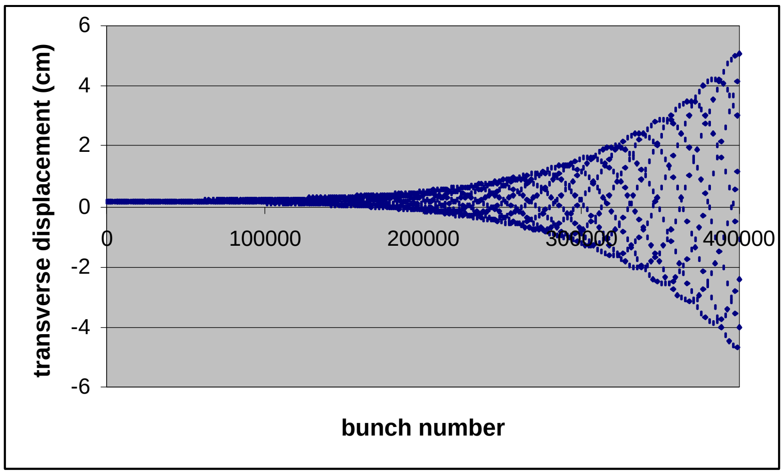

FIG.3 Plot of transverse displacement of beam bunches vs. bunch number. The HOM frequency is 2013 MHz, $Q=1.0\times10^7$ and $R/Q=2\ \Omega$.

4. Threshold current versus Q value of HOM

The dependency of threshold beam current versus HOM Q value is displayed at Fig. 4. In the simulation, all the cavities both in the medium beta section and high beta section are assumed to have the same HOM frequency 2012.6 MHz. This frequency is only 0.1 MHz away from the resonance frequency 2012.5 MHz. Threshold beam current is defined as the beam current that produces deflecting factor of 2.0 at the end of SC linac during 1ms beam bunch train. Five different values of shunt impedance R/Q are used. As is shown in Fig. 2, this corresponds to the strongest resonance among the multiples of 402.5 MHz up to 2012.5 MHz.

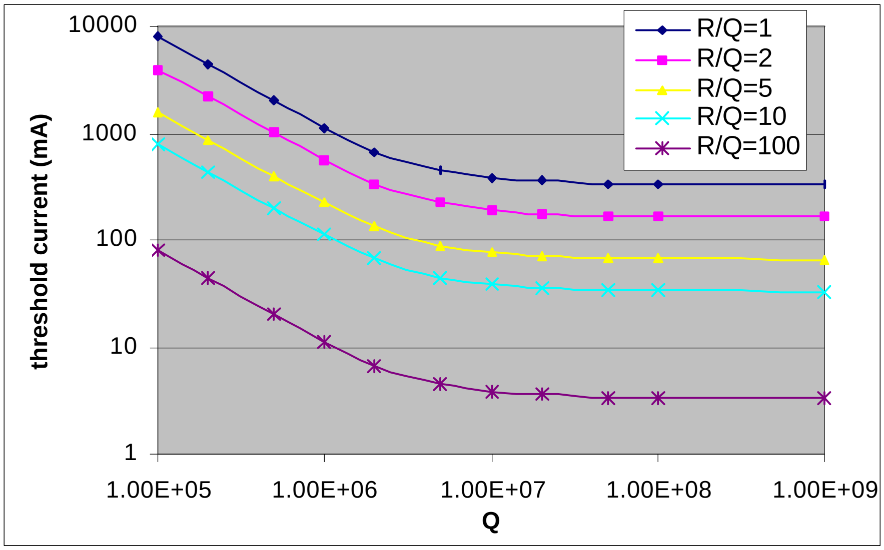

FIG. 4 Plots of threshold beam current (mA) versus Q for different values of shunt impedance R/Q in Ohm. Threshold beam current is defined as the beam current that doubles transverse displacement. Threshold beam current starts to saturate around $Q=1.0\times10^{7}$. The HOM frequency used in this simulation is 2012.6 MHz, which is 0.1 MHz away from 5x402.5 MHz.

Table I lists all the threshold current values versus Q for five different values of shunt impedance R/Q displayed in Fig. 4. One interesting fact is that threshold beam current starts to saturate around $Q=1.0\times10^{7}$ for all five values of shunt impedance. Because at Q around 1.0×10^{7} for a 2012.6 MHz mode, the "rise time" of the HOM is $1.0\times10^{7}/(\pi*2.0126\times10^{9})\sim1.58$ ms, which is the beam pulse length. In other words, the instability cannot grow without a long enough pulse for the HOMs to be appreciably excited. When R/Q=100 Ω, threshold current of 36mA corresponds to $Q=2.5\times10^{7}$. It should be noted that threshold current is inversely proportional to shunt impedance R/Q.

Table I. Threshold beam current vs. Q

Q	R/Q=1 Ω	R/Q=2 Ω	R/Q=5 Ω	R/Q=10 Ω	R/Q=100 Ω
1.0E+05	7913.2 mA	3956.4 mA	1583.4 mA	792.0 mA	79.07 mA
2.0E+05	4365.4 mA	2183.3 mA	874.7 mA	437.0 mA	43.70 mA
5.0E+05	2017.0 mA	1009.3 mA	403.6 mA	202.1 mA	20.21 mA
1.0E+06	1128.5 mA	564.0 mA	225.7 mA	112.9 mA	11.29 mA
2.0E+06	666.4 mA	332.9 mA	133.3 mA	66.6 mA	6.66 mA
5.0E+06	448.2 mA	223.7 mA	89.7 mA	44.8 mA	4.48 mA
1.0E+07	384.3 mA	192.5 mA	76.9 mA	38.4 mA	3.84 mA
2.0E+07	356.3 mA	177.8 mA	71.3 mA	35.6 mA	3.56 mA
5.0E+07	338.4 mA	169.5 mA	67.7 mA	33.8 mA	3.38 mA
1.0E+08	334.2 mA	167.3 mA	66.8 mA	33.4 mA	3.34 mA
1.0E+09	328.0 mA	164.1 mA	65.6 mA	33.0 mA	3.28 mA

It is good to know the Q values versus R/Q that allows threshold current of 360 mA under this simulation condition. Taking into account safety factor of 10 to 36 mA of SNS linac, 360 mA is chosen. Figure 5 shows the curve of Q value of HOM that allows threshold current of 360mA versus shunt impedance R/Q when the frequency of HOM is 2012.6 MHz. For example, when R/Q=10 Ω, Q value should be less than 2.51×10^5 as is listed in Table II. The curve between R/Q=1 Ω and 5 Ω is due to the saturation of threshold current mentioned above. As is shown in Fig. 4, threshold beam remains almost constant for Q value above 10^7.

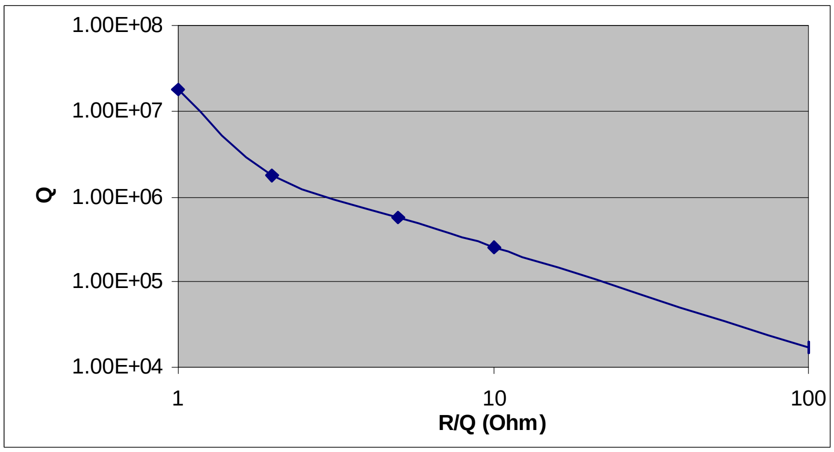

FIG. 5 Plot of Q values versus shunt impedance R/Q that allows threshold beam current of 360 mA when HOM frequency is 2012.6 MHz close to 5x402.5 MHz.

Table II. Q values that allows 360 mA threshold beam current

R/Q (Ohm)	1	2	5	10	100
Q	1.76E+07	1.78E+06	5.72E+05	2.51E+05	1.70E+04

Table I and II list data when the difference between HOM frequency and resonance frequency of 2012.5 MHz is 0.1 MHz.

5. Effect of HOM frequency spread

Due to tolerances on manufacturing process of cavities, the frequency of a HOM varies from cavity to cavity. This frequency spread effectively reduces the Q of the mode [6] and increases threshold beam current. In the simulation, $Q=1.0 \times 10^7$ is assumed and shunt impedance R/Q is set to 100 Ω for all the cavities. $Q=1.0 \times 10^7$ is chosen because threshold current starts to saturate from this value. 100 Ω shunt impedance is chosen because MAFIA study [7] indicates that biggest shunt impedance is less than or equal to about 100 Ω. Beam current is set to 500 mA to allow enough safety factor. One of resonant HOM frequencies is chosen. Median frequency of HOM is set to 2012.5 MHz.

Two types of frequency spread are considered. One is frequency spread of uniform distribution and the other of Lorentzian distribution. Lorentzian distribution is picked for the sake of simplicity in the theory.

5.1 Uniform HOM frequency spread

The HOM frequency spread effect is simulated with random numbers with uniform distribution within specified boundaries. The HOM frequency spread is taken to be ±0.2 MHz, ±0.5 MHz, ±1.0 MHz, ±3.0 MHz and ±5.0 MHz respectively.

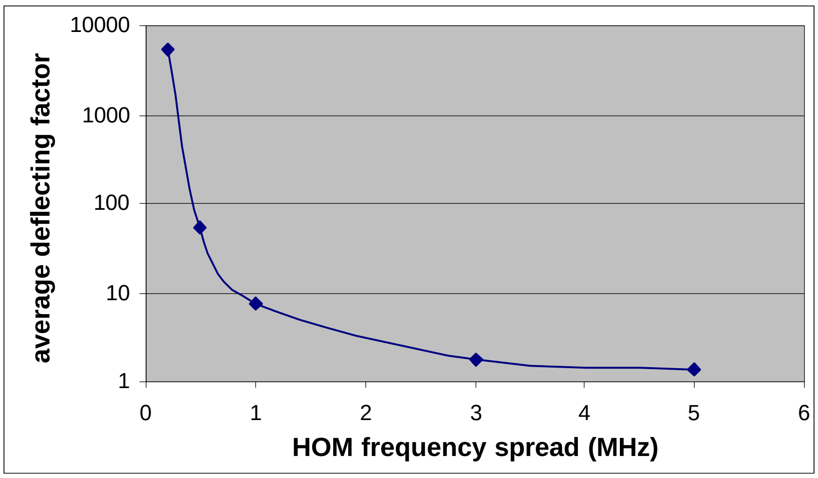

FIG. 6 Plot of average deflecting factor at the end of linac for 1000 Monte Carlo linacs with HOM frequency spread. The HOM frequency spread is taken as ±0.2 MHz, ±0.5 MHz, ±1.0 MHz, and ±5.0 MHz.

Table III. Average deflecting factor vs. HOM frequency spread

Frequency spread	0.2 MHz	0.5 MHz	1.0 MHz	3.0MHz	5.0MHz
Avg Deflecting factor	5396	53.42	7.66	1.79	1.39

HOM frequency spread dramatically reduces average deflecting factor as is illustrated in Fig. 6 and Table III. Average is taken over 1000 Monte Carlo linacs. HOM frequency spread of ±3.0 MHz reduces average deflecting factor to 1.79. So even when the median HOM frequency is 2012.5 MHz which is on resonance, uniform HOM frequency spread of ±3.0 MHz reduces average deflecting factor below 1.79 for 500-mA beam current.

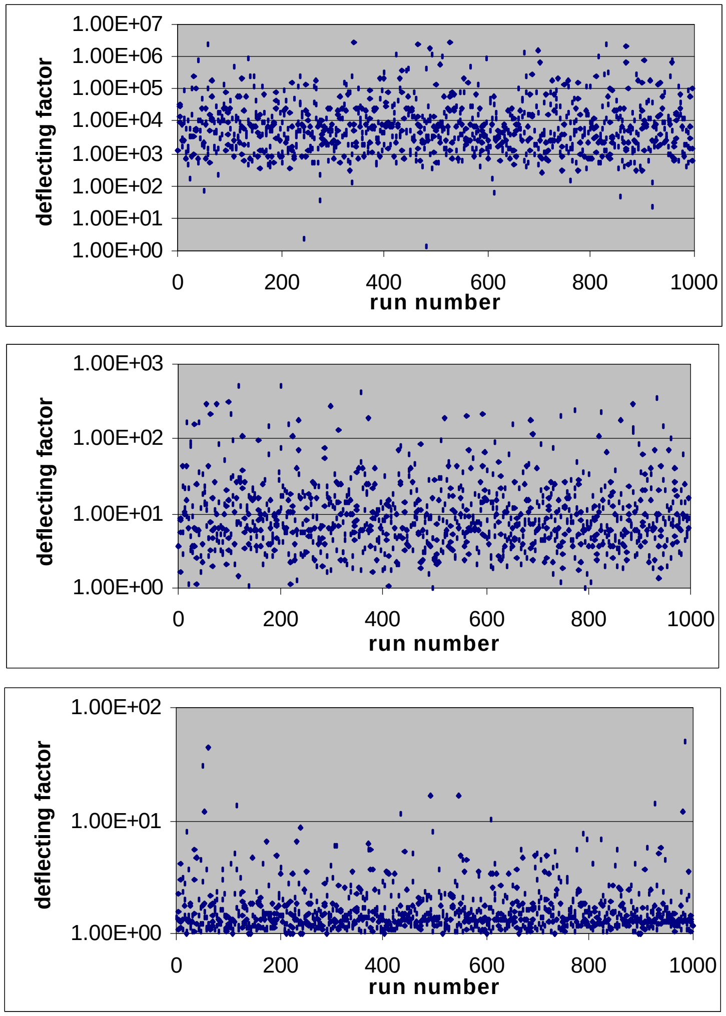

FIG. 7 Distributions of transverse deflecting factor at the end of SC linac for 1000 Monte Carlo linac runs with HOM frequency spread. The top plot is for HOM frequency spread of ±0.2 MHz, middle plot for ±1.0 MHz and bottom plot for ±5.0 MHz.

For ±1.0 MHz of HOM frequency spread, the average deflecting factor is 7.66. When running TDBBU code with R/Q=100 Ω and with no frequency spread, the deflecting factor we get is 7.52 when Q is equal to 1.5×10^4. Thus ±1.0 MHz uniform HOM frequency spread is equivalent to Q value of 1.5×10^4 without HOM frequency spread.

Distribution of deflecting factor is displayed in Fig. 7. These are obtained for 1000 Monte Carlo linacs with ±0.2 MHz, ±1.0 MHz and ±5.0 MHz uniform HOM frequency spread.

5.2 Lorentzian HOM frequency spread

Effect of HOM frequency spread is studied also when the frequency spread is Lorentzian distribution

$$g(f) = \frac{1}{p} \frac{\Delta f}{(f - f_o)^2 + \Delta f^2},$$

where Δf is frequency spread (half-width-half-maximum) and f_o is the median HOM frequency. Table V lists the average deflecting factor versus frequency spread (half-width-half-maximum) in MHz. This is displayed in Fig. 8.

In reference [6], it is shown that for Lorentzian frequency spread, the effective Q is $Q_{eff}=f_o/2\Delta f$ where f_o is the median value of frequency spread and Δf the half width half maximum. With f_o=2012.5 MHz and Δf=0.2 MHz, the corresponding Q_{eff} is 5031. When there is not any HOM frequency spread, Q value of HOM determines deflecting factor. Running TDBBU code with R/Q=100 Ω, the Q is equal to $2.80\text{x}10^4$ that corresponds to the deflecting factor of 10.1. This is factor 5.6 larger than Q_{eff}=5031. It should be pointed out that Q_{eff} of theory is obtained for steady state not for the transient state, and that SNS SC linac is in transient state over 1 msec. Only 1000 Monte Carlo linacs are considered and increased number of Monte Carlo runs will improve the accuracy.

Table IV. Average deflecting factor vs. HOM frequency spread

Frequency spread (Δf)	0.2 MHz	0.5 MHz	1.0 MHz	3.0MHz
Avg Deflecting factor	181.03	10.14	3.10	1.42

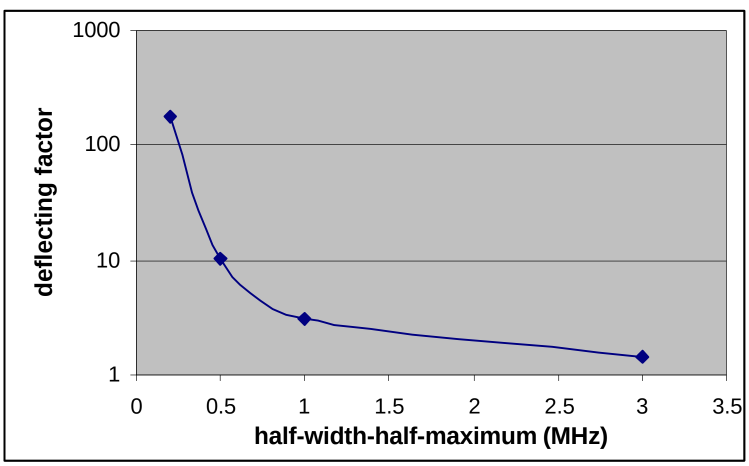

FIG. 8 Plot of deflecting factor vs. half-width-half-maximum frequency spread of Lorentzian HOM frequency distribution.

6. Discussions

Numerical simulation indicates that cumulative beam breakup instability is not a concern to SNS SC linac. Even when the median HOM frequency is exactly on resonance, HOM frequency spread with ±3.0 MHz uniform distribution can ensure operation of linac with average deflecting factor less than 2.0 for 500-mA beam current and Q value $1x10^7$. For HOM frequency spread of Lorentzian distribution, half-width-half-maximum spread of 2.5 MHz can ensure linac operation with average deflecting factor less than 2.0.

Without HOM frequency spread, Q value of $1.7x10^4$ allows threshold current of 360 mA for shunt impedance R/Q=100 Ω. Q value of $2.5x10^5$ allows threshold current of 360 mA for R/Q=10 Ω. It is assumed that all the cavities have the same HOM frequency of 2012.6 MHz which is 0.1 MHz away from resonance frequency.

The period of linac bunch train is 945 ns with 300 ns gap. This time structure generates harmonic frequency of multiples of 1.058 MHz. This effect is not studied yet.

References

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