Forward-looking bidding in online auctions

Robert Zeithammer

Assistant Professor of Marketing
The University of Chicago
Graduate School of Business
5807 South Woodlawn Avenue
Chicago, IL 60637

e-mail: rzeitham@gsb.uchicago.edu
phone: (773)-834-3769
fax: (773)-702-045

March 1, 2005

I would like to thank John Little, Birger Wernerfelt, Elie Ofek, Dan Ariely, Sergei Izmalkov, Jeroen Swinkels, Jean-Pierre Dube, Uri Simonsohn and seminar participants at several marketing departments around the country, where I presented an early version of this paper in 2002.

Forward-looking bidding in online auctions

Brief abstract

At Internet auction sites like eBay, nearly identical goods are often sold in a sequence of auctions, separated by small amounts of time. Upcoming auctions are announced several days in advance, so buyers can benefit from forward-looking strategies that take that information into account. This paper develops a model of such bidding, provides empirical evidence of the model's relevance to actual behavior on eBay, and discusses the general implications of forward-looking bidding for sequential auction-driven marketplaces.

I. Introduction

Internet auction sites like eBay are increasingly being used to sell mass-produced consumer durables: the largest eBay categories in dollar terms are: cars, consumer electronics, computers, clothing/accessories, and books/movies/music (2004 company report). Since the ending times of the individual auctions are not synchronized, each of these markets evolves as a sequence, allowing bidders to focus on the auction that will end first, while accounting for the fact that there will be other auctions later. The eBay webpage design reinforces the sequential conceptualization by listing auctions in a sequence, with the default ordering by ending time (see Appendix 1 for a snapshot of the eBay auction-listing webpage), and by allowing bidders to place future auctions on their private watch-lists. Because online auctions are usually listed for several days before concluding, detailed information about what and when will be sold in the near future is available to bidders. Two important questions thus arise, namely how should rational bidders use such information, and how do eBay bidders seem to use the information available to them. To answer these questions, this paper develops a new model of equilibrium bidding in a very long sequence of auctions, and provides empirical evidence of the model's relevance to actual behavior of eBay bidders.

The model assumes that each product-category is horizontally differentiated into several types of goods, with each bidder having a unit demand for only one type of the good. For example, a consumer may be shopping for one DVD of her favorite movie, or for one unit of a specific brand and model of an MP3-player. The model departs from previous models of sequential auctions by assuming that bidders know not only the type of the current product they are bidding on, but also what type will be sold next and when. In other words, the bidders are not only forward-looking in that they anticipate a future auction, but also *forward-seeing* in that they know detailed information about several future auctions.

When bidding in any particular auction within a sequence, each bidder faces a tradeoff between winning now and winning later. The tradeoff arises from the fact that the individual desired units are perfect substitutes: the winner of each auction exits the marketplace and hence foregoes the expected surplus from participating in future auctions that also sell her desired good, possibly for a lower price. Therefore, the bidder faces a positive opportunity cost of winning now, and the opportunity cost is exactly the expected surplus from participating in future auctions conditional on losing the current auction. For example, a bidder who values a DVD of "Gladiator" $20 and wins it for $12 receives a surplus of $8, but incurs an additional opportunity cost from foregoing future auctions of Gladiator, and this opportunity cost is exactly the future surplus this bidder would expect if she lost the current auction instead, and bid on another Gladiator DVD in the future. Because they face a positive opportunity cost of winning, rational bidders should reduce their bids relative to the myopic bidding strategy that would be optimal in the absence of future auctions selling the desired type of good. Bidders do not need to be forward-seeing in order to be forward-looking and acknowledge the chance that future auctions may sell the desired type of good. Both Engelbrecht-Wiggans (1994) and Jofre-Bonet and Pesendorfer (2003) model forward-looking bidding in a sequence of heterogeneous substitutes, and conclude that bidders reduce their bids by the opportunity cost arising from the fact that the objects sold are substitutes.

When forward-looking bidders also see forward, the expected future surplus is a function of the available information about what and when will be sold in the near future – both the timing of the upcoming auctions as well as the types of products sold in those auctions. The equilibrium analysis of the game with forward-seeing bidders is complicated by several dependencies that prevent a closed-form solution. In particular, the expected surplus function depends on the bidding strategy while the bidding strategy in turn depends on the expected surplus function. While the equilibrium bidding strategy is thus intractable in closed form, this paper shows that there exists a well-behaved symmetric pure-strategy Markov Perfect equilibrium bidding function whose comparative statics can be characterized without relying on specific assumptions about the distribution of personal valuations in the bidder population.

The properties of the equilibrium bidding strategy depend on how much detail of the available information about near-future auctions do the bidders actually use, i.e. how sophisticated are the bidders in taking the information into account. Three nested levels of such information-usage sophistication are considered: First, when bidders ignore the information completely, the model reduces to a special case of the model of Engelbrecht-Wiggans (1994), in which bids do not depend on short-term variation in the near-future frequency of auctions or on variation in the near-future incidence of specific product-types. Second, auctions ending within the next hour are highlighted in red on eBay, so frequency of near-future auctions is easier to discern than the specific attributes of the objects sold. Therefore, it is important to consider an intermediate level of sophistication, in which bidders see only the general frequency of auctions in the near-future, but not the types of the individual future objects. Such bidders should reduce their bids more whenever there are more auctions ending soon, because that decreases the expected waiting time until another unit of their desired types comes up for sale, and hence increases the expected future surplus.

Finally, each bidder can actually examine the near-future auctions closely and base her bidding strategy not only on timing, but also on the types of objects actually coming up for sale.

When rational bidders consider the specific objects coming up for sale in the future, the opportunity cost of winning today becomes a function of personal preferences for the future items - the more desirable products coming up and the sooner they are coming up, the higher the future expected surplus.

How much detail of the available information about near-future auctions do actual eBay bidders use is an empirical question, and this paper proposes an empirical strategy to answer this question using standard eBay data. The empirical strategy relies on measuring the relationship between bids and both object-types and ending-times of near-future auctions, a relationship for which the three levels of information-use sophistication generate the above-described nested restrictions of the "most sophisticated" bidding function. Consumer preferences in the data are identified using the assumption that each consumer desires only a subset of product-types and considers all desired units to be identical in terms of utility. Then, the theory predicts that observing a positive bid on a given type reveals the bidder's preference for that type, and it is thus possible to classify the types of products immediately following the current auction as either desired or not desired by that particular bidder. Assuming that the variation in short-term frequency of different types among the already-listed auctions is exogenous to current bids, it is possible to determine the best-fitting model by examining the empirical correlations between bids and both object-types and ending-times of near-future auctions. To test the model's predictions, two different datasets are used, one from the MP3-player category with each player brand-model combination considered to be a different type, and one from the DVD movie category with different product-types assigned to different movie-titles. The empirical test on both product categories rejects the two nested simpler models in favor of the "most sophisticated" model, in which bidders take their personal preferences for specific future products into account. Therefore, the model of forward-looking behavior proposed here is relevant to understanding the demand-side of auction-driven marketplaces like eBay.

The paper is structured as follows: Section II provides a brief review of the relevant literature. Section III then presents the model that constitutes the main theoretical contribution of this paper, and Section IV discusses the robustness of the model predictions to perturbations in the assumptions. Section V presents the results of an empirical test that demonstrates the relevance of the proposed model to actual bidding behavior observed on eBay. Section VI then concludes by discussing the implications of the present findings for researchers studying the growing auction-driven online marketplaces, as well as for both buyers and sellers in those marketplaces.

II. Literature review

This work draws on literatures studying online auctions specifically and multi-object auctions in general. Online auctions have only existed for about eight years, so academic research focusing on them is relatively scarce. The issue of multi-auction online bidding has not been addressed except for work by Bajari and Hortacsu (2003), who study bidder entry in common-value auctions, and Dholakia and Soltysinski (2001), who find a “herding bias” – consumers flocking to popular auctions despite the existence of other auctions for substitute items. The “herding bias” is especially relevant to this work because it provides another layer of behavioral complexity on top of the rational behavior described here.

The theoretical model proposed here is not confined to online auctions, and contributes to the general auction theory literature. The vast auction theory literature focuses predominantly on single isolated auctions, and investigations of the issues arising from multiple related auctions are comparatively rare as pointed out by Klemperer (1999) in a recent review of the literature. Previous work on sequences of auctions has mostly focused on price-trends in finite sequences of auctions, motivated by empirical discovery of the "price-decline anomaly" by Ashenfelter (1989) and others.

This paper does not study price-trends in finite sequences, but instead builds an infinite-horizon model and investigates equilibrium bidding as a function of bidder information about near-future auctions. The model extends the finite-horizon identical-goods model of Milgrom and Weber (1999) to an infinite horizon and horizontally differentiated goods. A very simple differentiation into a finite number of mutually exclusive types is assumed, so the extension amounts to assuming several randomly-interlaced sequences of identical-goods sequences. The proposed model is thus the simplest model that involves unit-demand bidders and non-trivial information about the near-future auctions. The extension to infinite-horizon is designed to match eBay reality, and it is non-trivial because it requires bidder-pool turnover and replenishment whereas the original finite-horizon model of Milgrom and Weber depends heavily on the depletion of the bidder pool together with the certainty that each losing bidder advances to the next auction. Because the model considers a sequence of auctions for non-identical objects with knowledge of future objects, it also extends the model of Gale and Hausch (1994), who examine the case of continuously heterogeneous objects by focusing on the special case of two bidders and two auctions. The extension beyond two auctions is accomplished here thanks to the simplifying assumption on the product-heterogeneity being captured by a finite number of types.

The most important simpler benchmark is provided by the model of similar but unseen future objects (Engelbrecht-Wiggans 1994), a special case of which is nested in the proposed model: when only some fixed common distribution of future products is known, bidders can still engage in forward-looking strategies, but they are unable to use the forward-seeing strategies captured by the model proposed here.

On the empirical front, the most related work is a recent paper by Jofre-Bonet and Pesendorfer (2003). They investigate sequential auctions for highway construction procurement contracts in California, and they find evidence of forward-looking behavior using a structural model. Forward-seeing behavior is not part of their model because bidders are assumed to not take public information about upcoming auctions into account. Another important difference is that this paper conducts an empirical test non-parametrically without functional-form assumptions that would be necessary for a structural econometric model.

Outside of the game-theoretic equilibrium paradigm, several papers have tackled the problem of optimal bidding in sequential auctions by applying optimal control methodology after noting that a game-theoretic treatment would be much more difficult (see Oren and Rothkopf 1975 for a seminal contribution and Arora et. al., 2002 for a recent example and a good review of this stream). This line of research assumes a single strategic bidder facing non-strategic competition, limiting the applicability of the conclusions and the kind of competitive phenomena that can be investigated.

III. Theory of forward-looking bidding with public information about near-future auctions

Assumptions

To obtain a tractable model of strategic bidding in multiple auctions, it is necessary to assume that the individual auctions are relatively simple. The key simplifying assumption of this paper is that valuations for the objects are private and independent across bidders (IPV). This assumption would be inappropriate for some economically small eBay product-categories like antiques and collectibles, in which valuations are most likely “affiliated” across bidders (Milgrom & Weber 1982). Nevertheless, the economically largest eBay categories involve mass-produced goods like consumer electronics, which are usually purchased for private use, and with the private valuations driven by personal preferences. Therefore, the IPV assumption is reasonable as a model of consumer preferences in a large class of online auctions, in particular the auctions for MP3 players and DVDs considered in the empirical section of this paper.

Online auctions usually remain open for several days, potentially leading to strategically rich within-auction behavioral dynamics (see Ariely and Simonson 2001 for a discussion). All models in this paper abstract from these within-auction dynamics, and model each individual auction as an instantaneous sealed-bid auction occurring at the time of the actual auction’s end. Validity of this abstraction is supported by the fact that bidding on eBay both should and does tend to happen at the very end of each auction, not giving the competitive bidders time to react to each other’s bids (Roth & Ockenfels 2002). Moreover, the simplification is not overly restrictive in the context of independent valuations since bidders do not try to learn about their own preferences from other people’s bids. To approximate the price-determination in the eBay’s ascending English auction, the models examine second-price auctions, in which the highest bidder wins the object, but only pays the second highest bid as the price.

Since ending times of online auctions are not synchronized, the sealed-bid abstraction results in a model of sequential auctions, in which one auction ends before the next one starts. As mentioned in the introduction, the eBay webpage design reinforces this sequential conceptualization. Anectodal evidence from an eBay community newsgroup also suggests that the abstraction to a sequence of auctions with known future items rings true with at least some eBay bidders. When I posted a question to an eBay newsgroup asking what to do with auctions for similar digital cameras, one user replied: *“Place bids on only one item at a time and put all the rest on your watch-list. If you are outbid on the first item, move to the next ending time on your watching page”.*

One more simplifying assumption is necessary for a tractable model, namely that bidders do not consider past prices. Suppose they did. Then, since past prices are upper bounds on the past bids of competing bidders who did not win the past auction and hence may have survived until today, the outcomes of past auctions could be informative about the level of current and future competition. In the current formulation of the model, current competition does not enter equilibrium bidding, but future competition does. To make matters even more complicated, the past price-determining bidder would have slightly different information about the remaining competition than the bidders whose past bids were less than the past price. This asymmetry would escalate over time as pointed out by Milgrom and Weber (1999). Therefore, considering a model of a very long sequence of auctions with memory of past outcomes would be highly problematic and most likely not tractable. Assuming the effects of past prices away does not have a big impact on the model because the effects are likely to be second-order, especially in an eBay-like environment characterized by a fluctuating and unobservable bidder pool.

Basic Model: 1-period look-ahead

There is an infinite sequence of instantaneous second-price sealed-bid auctions occurring at distinct and countable points of continuous time. The waiting time ω between auctions varies stochastically and independently, according to a known distribution. Bidders discount future utility exponentially by factor δ per unit of time, so outcome of a future auction ω units of time away is discounted by a factor δ^{ω}.

Each auction sells one object. The objects offered for sale are horizontally differentiated into $K+1$ types, with probability of type k captured by rate ρ_k. For example, the category of MP3 players is differentiated by brand-model combinations like "Rio 500", and the category of movies on DVD is differentiated by title of the movie. Each bidder desires one of the first K product-types in the sense that all non-desired types give the bidder no utility while the desired type gives the bidder a positive utility. For example, each bidder is interested in buying only one particular movie-title or only one particular model of MP3 player. No bidders desire the last $K+1^{st}$ product type, which captures various suspect "free" offers as well as poorly described and misplaced goods that are bound to clutter any marketplace.[1]

Select an arbitrary auction and let the rest of the auctions be indexed relatively to the focal "current" auction, so the current auction has index 0, the immediately following auction index 1, the auction after that index 2, etc. To capture the type information, let $\varphi_{j,k} \in \{0,1\}$ be the indicator function equal to one when auction j is of type k, and equal to zero otherwise. To capture the waiting-time information, let ω_j be the waiting time between auction $j-1$ and j. The key innovation of this model is that bidders of type k know not only the desirability of the current

[1] The existence of the K+1[st] type is not essential to the model with K>1, but with K=1, there would be no value to information about future product-type because all products would be identical.

product $\varphi_{0,k}$, but also the information $\left(\varphi_{1,k}, \omega_1\right)$ that arises from seeing forward, namely whether they desire the next product and how far in the future will the next sale occur. The fundamental difference between the two kinds of forward-seen information is that $\varphi_{1,k}$ is inherently type-specific whereas ω_1 is the same for all types.

N_k bidders participate in each auction of type *k*. Bidder *i* of type *k* considers all her desired objects to be identical, and can derive a private value of $v_{i,k}$ dollars from any one of them. The individual private values $v_{i,k}$ are drawn independently across bidders from a known probability distribution F_k with full support on [0,1] and a continuous density *f*. Therefore, the private valuation to bidder *i* of type *k* of the current object is $\varphi_{0,k} v_{i,k}$, and the private valuation to the same bidder of the next object is $\varphi_{1,k} v_{i,k}$, where $v_{i,k} \sim F_k$. All bidders can only derive utility from a single unit of their desired good, so once a bidder owns one unit of her desired type, all subsequent units are worth zero to her. Assume resale is too costly for such a private buyer to warrant speculative purchases of multiple objects for future resale, so each auction's winner exits the game and is replaced by another randomly-drawn bidder.

Bidders also exit at random with an exogenously given probability $(1-\lambda)$ per time-period. Some bidder-replacement beyond the replacement of the winner is an essential feature of a realistic steady-state model, because when only the winner is replaced and bidders stay until they win an auction, the distribution of the steady-state survivors degenerates to a group of bidders with zero valuations. The bidders have no memory, so they cannot base their actions on outcomes of past auctions.

Equilibrium bidding strategy

Since the game has an infinite horizon and bidders have no memory, the only state-variables that distinguish one period from another are the desirability indicators $\left(\varphi_{0,k},\varphi_{1,k}\right)$ and the waiting-time information ω_1. I describe a symmetric pure-strategy Markov Perfect Equilibrium, in which the strategy can depend only on the publicly-known state $\left(\varphi_{0,k},\varphi_{1,k},\omega_1\right)$ and on each bidder's private information $v_{i,k}$. Since the product-types have their own bidder populations that evolve without interacting across type boundaries, the optimal bidding problem is symmetric across types: for each type k, the remaining types $\{1,2,...,k-1,k+1,...K,K+1\}$ can all be lumped into "other" undesired type. Without loss of generality, I can therefore solve for the equilibrium bidding strategy in the case K=1, suppressing all k subscripts for clarity.

Let $K=1$, and let $S\left(\varphi_0,\varphi_1,\omega_1,v_i \mid c_0\right)$ be the bidder i's continuation value of the game in case of a loss today, i.e. steady-state expected future surplus of bidder i conditional on bidder i losing today's auction. Within each product-type, bidders face symmetric optimization problems because they are all drawn independently from the same population, and they differ only by their valuation of a unit of their desired good v_i. Therefore, all bidders use the same S in a symmetric equilibrium. Finally, it will become clear that the continuation value relevant at the margin depends on the current competition, so let c_0 be the highest competing bid assuming $\varphi_0=1$ (when $\varphi_0=0$, let c_0 be the amount the highest competitor would bid should $\varphi_0=1$ instead). Let G be the distribution of c_0. Then, the bidder with valuation $v_i=v$ solves the following utility-maximization problem to find the optimal steady-state bid $b\left(\varphi_0,\varphi_1,\omega_1,v\right)$:

$$b\left(\varphi_0,\varphi_1,\omega_1,v\right)=\arg\max_b \int^{b}\left(\varphi_0 v-c_0\right)dG\left(c_0\right)+\left(\delta\lambda\right)^{\omega_1}\int_{b} S\left(\varphi_0,\varphi_1,\omega_1,v\mid c_0\right)dG\left(c_0\right) \qquad (1)$$

In a symmetric equilibrium, the expected surplus function must account for the fact that other bidders are also reducing their current bids, so the current competition is weaker than if the competitors were not strategically forward-looking, and the future competition depends on the current competition. Furthermore, in an infinite-horizon setting employed here to capture a mature ongoing market, future bidders will again be reducing their bids as a function of the future's future and at least some of those future bidders will be current competitors who lost the present auction. Therefore, the expected surplus function depends on the bidding strategy while the bidding strategy in turn depends on the expected surplus function. These dependencies make a closed-form solution of the model unavailable, but a well-behaved equilibrium exists as shown in Proposition 1:

Proposition 1 (proof in Appendix 2): There is a symmetric pure-strategy Markov Perfect equilibrium characterized by a bidding function $b\left(\varphi_0,\varphi_1,\omega_1,v\right)$ that satisfies:

$$\begin{aligned}&b\left(1,\varphi_1,\omega_1,v\right)=v-\left(\delta\lambda\right)^{\omega_1}S\left(1,\varphi_1,\omega_1,v\mid c_0=b\left(1,\varphi_1,\omega_1,v\right)\right)\\&b\left(0,\varphi_1,\omega_1,v\right)=0\end{aligned} \qquad (2)$$

Where the expected steady-state function satisfies a set of Bellman equations (3):

$$S\left(\varphi_{0,1},\omega_1,v\mid c_0\right)=$$

$$=E_{\varphi_2,\omega_2}\left[\int^{b\left(\varphi_{1,2},\omega_2,v\right)}\left(v-c_1\right)dG\left(c_1\mid c_0,\varphi_{0,1,2},\omega_{1,2}\right)+\left(\delta\lambda\right)^{\omega_2}\int_{b\left(\varphi_{1,2},\omega_2,v\right)}S\left(\varphi_{1,2},\omega_2,v\mid c_1\right)dG\left(c_1\mid c_0,\varphi_{0,1,2},\omega_{1,2}\right)\right]$$

The bidding strategy has several striking properties. First, it does not directly depend on G – a consequence of the general truth-revealing property of the second-price sealed-bid auction. However, $b\left(\varphi_0,\varphi_1,\omega_1,v\right)$ does depend on the current competition inasmuch as the current competition is informative about the future competition: When evaluating the option value of the future, the bidder assumes that she will lose the current period to a competitive bid that exactly matches her current bid. This "tie" is the only situation in which raising the current bid slightly changes the outcome of the game, and *S* given $c_0 = b\left(1, \varphi_1, \omega_1, v\right)$ is therefore the opportunity cost relevant at the margin. In other words, the bidder assumes she is pivotal to the outcome of the game, and the pivotal nature of a second-price auction thus comes through even in a sequential context. Finally, it is notable that bidders only submit positive bids on products of their desired type – a result that will lead to identification of personal preferences in the empirical test.

The bidding function is fully characterized by the expected surplus function *S*, which is in turn characterized by the steady-state distribution of the future competition c_1 conditional on the current competition c_0 and all the state variables involved in the relationship between them $G\left(c_1 \mid c_0, \varphi_0, \varphi_1, \varphi_2, \omega_1, \omega_2\right)$. In equilibrium, the surplus function must reflect the actual expected surplus given that everyone uses the optimal bidding strategy (2). Therefore, the equilibrium expected surplus function must satisfy the Bellman equation shown in Proposition 1. Such an *S* function exists because of the continuity of *f*, compactness of its support, and the fact (shown in the proof of Proposition 1) that the slope of *S* in any of its arguments cannot diverge to infinity. However, equilibrium *S* cannot be expressed in closed form even for a simple distribution *F* and small *N*. Despite the lack of a closed-form solution, some general comparative statics of the bidding function can be derived from an analysis of the Bellman equation:

Proposition 2: For all F, the equilibrium bidding function $b(\varphi_0, \varphi_1, \omega_1, v)$ has the following properties:

1) $b(1, \varphi_1, \omega_1, v)$ increases in ω_1
2) $b(0, \varphi_1, \omega_1, v) = 0 < b(1, 1, \omega_1, v) < b(1, 0, \omega_1, v) < v$ for all v>0.
3) $b(1, \varphi_1, \omega_1, v)$ decreases in ρ

The first property shows that bids decrease when the future gets closer in the sense that ω_1 decreases. In the empirical section, an eBay-relevant generalization of this result will be tested, namely the prediction that bids decrease as the number of auctions in the next hour increases. The second property contains several important results. The first inequality shows that all bidders with positive valuations submit positive bids on objects they desire and trade is thus guaranteed. In the empirical section, this will be used to identify bidder preferences over types by interpreting a positive bid on a type as an indication of that type's desirability to the given bidder. The second inequality in 2) is the main result of this paper because it shows that all forward-seeing bidders of all types bid less when they see their desired object in the next period compared to when they see an object they don't desire. In the empirical section, a generalization of this result will be tested. Finally, the third inequality provides a comparison of forward-seeing behavior to the myopic benchmark: forward-seeing bidders always bid less than they would if they were myopic because myopic bidders have a dominant strategy to bid their valuation in a second-price sealed-bid auction. The fundamental reason for the third inequality is the positive opportunity cost of winning, so it will hold for any forward-looking bidding strategy, even without forward-seeing.

The third property shows that as the long-term rate ρ of desired products increases, the bids decrease. The reason is that forward-seeing bidders are also forward-looking beyond the near future they can see. The result would hold even if the bidders did not know their (φ_1, ω_1) and hence could not be forward-seeing. The resulting model would be analogous to the model of Engelbrecht-Wiggans (1994), so this result shows how that important benchmark model is nested within the model proposed here. The empirical section will not be able to separate this effect from a type-specific effect because each type is, by definition, only observed with one long-term rate. A generalization of the basic model that will inform empirical testing is discussed next.

IV. Robustness of the model predictions

Several assumptions of the basic model can be relaxed without altering the key predictions of Proposition 2, namely that more desirable and more proximate near-future auctions lead to a reduction in current bids as does more desirable long-term future. These relaxations include a stochastically varying number of bidders, bidders idiosyncratically desiring more than one type of product, risk-averse bidders, the addition of speculators into the bidder pool, and bidders seeing more than one period ahead. This section discusses these relaxations in turn along with the boundaries beyond which the model breaks down.

Stochastic and unknown number of competing bidders

Within each product-type, number of bidders N_k is assumed to be the same in each period. This assumption is reasonable if entry is, in fact, endogenous, and the bidders enter until their expected surplus is above some participation threshold. Moreover, specifying the model with a fixed N_k simplifies the exposition without sacrificing generality because the model's qualitative conclusions will not be sensitive to variations in the assumption about the bidder pool as long as some bidders stay for multiple periods and there is a well-defined steady-state distribution of the number of bidders present. Since current competition generically does not enter the bidding strategy, the only difference a stochastic N_k would make to the results is that the entire RHS of equation (3) would have to be integrated over the steady-state distribution of future N_k, adding another argument to the expectation. Since future N_k varies from one auction to the other on eBay and is ex-ante unknown, the empirical part of this paper will actually involve this extension of this model and the number of bidders will be taken into account.

Bidders desiring more than one type of product

Another variation of the model that can be accommodated is allowing each bidder to idiosyncratically desire more than one type, but still have only unit demand in the category and still consider all desired types identical in terms of utility. Thus, all private single-unit valuations v_i would be drawn from some common distribution F, and φ would have to be specified for every bidder as $\varphi_{j,i} = 1$ when bidder i desires the type of product sold in auction j. Then, the private value to bidder i of product sold j auctions from now would be $\varphi_{j,i} v_i$. This structure of preferences is thus relevant to the MP3 player category, where each bidder may only have use for one player but be indifferent among several models. In contrast, each movie-category bidder may have use for multiple DVDs as long as they all contain different movies. Allowing each bidder to have idiosyncratic preferences over multiple types would terminate the symmetry and independence across types that allowed the analysis of K=1 to be without loss of generality, and the model would have to be specified in terms of a set of type-specific equilibrium surplus functions S_k. Then, it could again be shown that there is a set of type-specific equilibrium bidding functions $b_k\left(\varphi_{0,i}, \varphi_{1,i}, \omega_1, v_i\right)$ that all satisfy an analogue of Proposition 2, with $\varphi_{1,i}$ assuming the role of φ_1. This model is investigated empirically in the empirical section by focusing on multi-type bidders and identifying from individual behavior which other types besides the current one does each bidder consider desirable. As predicted above, the data lends no support to this model in the movie data while finding at least some weak evidence for it in the MP3 player data.

Note that the model with idiosyncratic multi-type preferences reduces to a special case of the basic model, in which all F_k are identical. A full extension of the basic model to idiosyncratic preferences with each bidder having different valuations $v_{i,k}$ for different types is not possible because the pure-strategy equilibrium breaks down. The reason is that there is a chance of zero bids on a desired object. There is thus a chance of no trade, and no trade implies stiffer competition tomorrow since the bidders are not trading precisely because their valuations of tomorrow's object are relatively high. Hence, when there is such a chance of no trade associated with a more competitive future, some bidders will have an incentive to deviate from bidding zero to bidding a small amount, and the pure-strategy characterization in (2) breaks down. Besides these technical reasons, a unit-demand model with different private values of different desired units is not realistic because the winner may want to bid on a second unit if the unit won is lower-valued given and disposal is free. The winner not bidding again is a key simplification that makes the elegant solution of the proposed model feasible, and it would not be tenable under these generalized assumptions about preferences.

Risk-aversion

Risk aversion reduces the difference $b(1,0,\omega_1,v)-b(1,1,\omega_1,v)$ because the certainty equivalent of the future plays the role of the expected surplus in equation (2). To see why certainty equivalent replaces the expected surplus in the characterization of the optimal bidding, let the bidder have a utility function u such that $u(0)=0$, and hence replace S in equation (1) with an expected-utility function $E[u(.)]$ to compensate the bidder for the option-value of losing in terms of expected future utility in the case of a loss today (note that the $\lambda\delta$ discounting factors become part of the utility expectation). Then, the first-order conditions that characterize optimal bidding are (suppressing states φ for clarity): $u(v-b)=E\left[u(\text{future}, v \mid b)\right]$. Therefore, the optimal bid satisfies $b = v - CE(\text{future}, v \mid b)$, where CE is the certainty equivalent. Since the certainty equivalent is always lower than the expected surplus, risk-averse bidders will all bid more than risk-neutral bidders, but all qualitative conclusions of Proposition 2 remain exactly as stated, so risk-aversion does not affect Proposition 2 qualitatively.

Presence of speculators

The model assumes that the bidders buy for private use and not for resale. This assumption can be justified by the fact that effective selling on eBay requires a much greater set of capabilities than buying on eBay: while buyers can treat eBay as any other online store, sellers need to have at least minimal web-publishing skills along with ability to ship goods and accept payments by various electronic methods. While a full treatment of all bidders considering resale would thus not be appropriate for the domain of online marketplaces, it is still possible that the sellers themselves act as speculators and submit bids in anticipation of reselling the purchased items later. While theoretically plausible, speculation for resale does not seem to be empirically prevalent in either of the two datasets considered in Section V: over 99 percent of the bidders are not observed selling anything in the same category within a month, and of the several thousand sellers observed, only about 2.5 percent submitted at least one bid per month within the entire category, all together affecting less than three percent of the auctions.

Even if private bidders bid while taking the presence of speculators into account, the main qualitative predictions of the theory remain unchanged for the domain of eBay because any resale on eBay is delayed by several days (median auction last a week and it takes time to physically receive a purchased object), and its possibility hence does not affect the already-listed objects offered in the near future – usually in matters of minutes. Instead, the anticipation of speculators bidding for resale would make ρ, the long-term rate of type-incidence, endogenous in the model.

Finally, even if speculators were able to resell instantaneously as may be the case in marketplaces other than eBay, their presence would not necessarily change the equilibrium behavior of the private bidders. In their recent paper, Garratt and Tröger (2004) show that in a simple setting of one auction and a future potential-resale period, second-price sealed-bid auctions are "weakly speculation-proof" in that the speculator bidding zero and never winning is supported in equilibrium. If their result extends to the infinite-horizon setting, the bidding strategy characterized by Propositions 1 and 2 remains an equilibrium bidding strategy of the private buyers. Garratt and Tröger also show that there exist other equilibria, in which the speculator submits a positive bid and thus makes it beneficial for low-valuation bidders to abstain from the auction and wait for the resale. Hence, there may be other equilibria besides the one characterized in Proposition 1, in which low-valuation bidders reduce their bids because of the presence of the speculator. Since only positive bids will be considered in the empirical test, these other equilibria do not constitute an alternative explanation for the effects documented in Section V.

Multi-period look-ahead model

When bidders are able to see A >1 periods ahead, the forward-seeing information states are of the form (Φ,Ω) where $\Phi=(\varphi_1,...,\varphi_A)$ and $\Omega=(\omega_1,\omega_2,...,\omega_A)$. Two empirically relevant summary statistics of (Φ,Ω) will be considered: $H(\Omega)$ = the number of auctions in the next hour implied by Ω, and $w(\Phi,\Omega)$ = waiting time until the first future auction that sells a product of the same type as the current product. $H(\Omega)$ is relevant because eBay auctions ending in the next hour are highlighted in red, and $H(\Omega)$ is thus very easy to discern at a glance. $w(\Phi,\Omega)$ is relevant because it is invariant to the way consumers actually use the eBay website, i.e. whether they search for the all listings in the category or for listings of their specific product-type only. Given these definitions, the same arguments as in the proofs of Propositions 1 and 2 can be used to show that there is a Markov Perfect equilibrium pure strategy $b(\varphi_0,\Phi,\Omega,v)$, with the following properties:

Corollary 1: When bidders see A periods ahead, the equilibrium bidding function $b(\varphi_0,\Phi,\Omega,v)$ has the following properties:

1) $b(1,\Phi^1,\Omega^1,v)\le b(1,\Phi^0,\Omega^0,v)$ for all Φ^i,Ω^i such that (Φ^1,Ω^1) has one additional listing of any type in the next hour and is otherwise the same as (Φ^0,Ω^0), so $H(\Omega^1)>H(\Omega^0)$
2) $b(0,\Phi,\Omega,v)=0<b(1,\Phi^1,\Omega,v)<b(1,\Phi^0,\Omega,v)<v$ for all v>0 and for all $\Phi^1\succ\Phi^0$, where $\Phi^1\succ\Phi^0$ is defined by $\varphi_a^1\ge\varphi_a^0$ for all a, $\varphi_b^1>\varphi_b^0$ for some b.

   In particular, the inequality holds for all $\Phi^1\succ\Phi^0$ such that $\Phi^0=0$ and $\Phi^1\ge 0$.

3) $b\left(1,\Phi^1,\Omega^1,v\right)<b\left(1,\Phi^0,\Omega^0,v\right)$ for all Φ^i,Ω^i such that $w\left(\Phi^1,\Omega^1\right)<w\left(\Phi^0,\Omega^0\right)$ and the continuation of the $\left(\Phi^0,\Omega^0\right)$ sequence is the same as the continuation of the $\left(\Phi^1,\Omega^1\right)$ sequence after $w\left(\Phi^0,\Omega^0\right)$.

4) Δb_a decreases in with *a*,

where $\Delta b_a = b\left(1,\Phi_a^0,\Omega,v\right)-b\left(1,\Phi_a^1,\Omega,v\right)$ and $\varphi_{a,j}^i = \begin{cases} 0 \text{ when } j<a \\ i \text{ when } j=a \\ \varphi_{a,j}^{-i} \text{when } j>a \end{cases}$

The first two claims are direct analogues of 1) and 2) in Proposition 2 and they follow from the fact that even a chance of bidding on a desired product in the future always gives the bidder strictly more surplus than certainty of bidding on an undesired product. The last two claims need a little more explanation: Claim 3) states that bids decrease as the desired product becomes available earlier in calendar time, and is thus a time-based analogue of claim 1) that follows from discounting and the fact that attrition is linked to time. Claim 4) goes a step further and examines how the magnitude of the bid-decrement shown to be positive in 1) changes as the first occurrence of the desired product-type gets more distant in the sequence, keeping timing the same. The decrement becomes smaller as the future recedes into the distance - an immediate consequence of discounting and the chance of attrition.

To construct statements about average differences in bids for empirical testing, it is necessary to average over the parts of (Φ,Ω) kept constant in each claim of the Corollary 1. Let $\overline{b}\left(x,v\right)=E\left[b\left(1,\Phi,\Omega,v\right)\mid x\right]$ stand for the expected bid of a bidder with valuation *v* conditional on *x*, with the expectation over all of the components of the states that vary as *x* remains constant. Then, as long as future timing Ω is independent of future types Φ and the continuation of the sequence independent of its beginning, the claims in Corollary 1 average out to testable predictions:

Model predictions: If bidders act consistently with the proposed model, the following four relationships will hold for all valuations *v>0* and for all desired types *k* =1…K:

1) $\overline{b}\left(H,v\right)$ decreases in number of auctions in the next hour *H(Ω)*
2) $\overline{b}\left(\Phi^1,v\right)<\overline{b}\left(\Phi^0,v\right)$ for all $\Phi^1 \succ \Phi^0$ such that $\Phi^0=0$ and $\Phi^1 \geq 0$, so $\overline{b}\left(\Phi,v\right)$ decreases in the indicator function $\mathrm{I}\Phi^1$ of Φ^1.
3) $\overline{b}\left(w,v\right)$ increases in waiting time until the same type *w*
4) $\Delta\overline{b}\left(a,v\right)$ decreases in a, where $\Delta\overline{b}_a=\overline{b}\left(\Phi_a^0,v\right)-\overline{b}\left(\Phi_a^1,v\right)$ and $\varphi_{a,j}^i=\begin{cases}0 \text{ when } j<a\\ i \text{ when } j=a\end{cases}$

   So $\overline{b}\left(\Phi,v\right)$ "decreases" in the indicator function $\mathrm{I}\Phi_a^1$ of Φ_a^1, as shown in 2), and the decrease is attenuated by *a*.

These four predictions can now be taken to the data to test whether and how much actual bidders see forward. By construction, if the bidders see only the *H* summary of the near future auctions, then prediction 1) will hold but the other predictions will not because they all rely on the bidders seeing types of the future products. Finally, when the bidders do not see forward, none of the predictions will hold.

V. Empirical Evidence: Analysis of eBay data

Section III proposed a model of forward-looking bidding with public information about both timing and objects of future auctions, and Section IV discussed the robustness of the model's qualitative predictions about the relationship between current bids and both object-types Φ and ending-times Ω of near-future auctions. This section uses the predictions highlighted in the end of Section IV to construct a statistical test that uses actual eBay bidding data, and attempts to reject the proposed model in favor of one of two simpler models: the model with forward-seeing of only the type-independent summary statistic $H(\Omega)$, and a model without any forward-seeing, i.e. without the information about the near future influencing current bids. Before describing the test and its results, it is necessary to describe some key features of the available data.

Data

EBay kindly provided two datasets, each corresponding to a different product-category: MP3 players and movies on DVD. Both categories involve differentiated mass-produced consumer goods usually bought for private use, so consumer preferences are likely to be well-approximated by the IPV framework underlying the model. As discussed in Section IV, most bidders do not engage in selling and hence are likely to have private valuations for the products. Moreover, consumers are likely to have unit demand for a specific model of an MP3 player or for a specific title of a movie, so the unit-demand assumption also fits the product domains.

Data from any online-auction site that only facilitates the communication between sellers is bound to come without definitive identification of each item sold. To match each listing to a product type (movie title or player model), researchers have to rely on the item description written by the sellers. Given the large volume of the data, automated procedures need to be used to "read" and classify the descriptions, and some classification error is inevitable, especially considering that no two objects sold on eBay are exactly alike. In both datasets used here, a word-matching algorithm classified about 80 percent of the listings as likely selling a known product-type,[2] but the resulting classification is still only approximate. To refine the classification, a few outlier auctions of each type were removed from the data because their final prices were too far out of line with the bulk of their type, suggesting that they probably actually sell something else than a single unit of the type as indicated by the word-matching algorithm.[3]

While both datasets are similar to each other in many respects, they differ slightly; the similarities are discussed first. Each dataset contains all submitted bids in each recorded auction as well as information about each listing including its timing and a text description of the item sold. The bidding data captures all the proxy bids made, including the winning bid which remains undisclosed in reality.[4] Individual bidders and sellers are tracked over time with unique identification numbers. All auctions that involve reserve prices or bid-cancellations are eliminated from the analysis because their modeling is beyond the scope of this paper.

The MP3-player dataset is constructed to capture all the auctions for the top thirty types (models) of MP3 players held during a 4-month period in the beginning of 2001.The top 30 types account for about 91% of the identified products and 70% of all listings. The final sample contains 6,967 auctions. Both Buy-It-Now (BIN) auctions and simple auctions are recorded in the MP3-player dataset. The minority (23 percent) of the BIN auctions that ended early at the BIN price level were excluded from the analysis because modeling the use of the BIN option is

---

[2] In the MP3 player category, 21 percent of all auction listings remained unclassified, either because their description was insufficient for identification ("new cool mp3 player for sale") or because they do not belong to the product category at all ("napster t-shirt" or "128mb sandisk memory card").

[3] Removing top and bottom three percent of all bids on each type is sufficient to eliminate all bids that are either multiples of the median price on the type, probably indicating an undetected bundle, or that are less than 10 percent of the type's median price, indicating a listing that is just an accessory or that is problematic for reasons unobserved by the analyst.

[4] Please see www.ebay.com for a definition of proxy bids and a thorough description of the bidding rules.

beyond the scope of this paper, and because their early termination makes them not useful as forward-seen future options. The remaining BIN auctions that either reverted to simple auctions or remained unsold were retained in the analysis.[5] Of the resulting 6,967 auctions used in the analysis, about 50 percent were originally started as BIN auctions. Listing all of the auctions were 2663 unique sellers. Participating in the 4852 (70 percent) auctions that received bids were 15,574 unique bidders, 3.2 per auction on average. Almost half of the bidders participated in multiple auctions, raising the average number of unique bidders in an auction to 7.5, median of 7.

The movie-dataset is constructed to capture all simple auctions for thirty popular titles in October 2002, where popularity was judged using bestseller lists.[6] BIN auctions were not recorded in the movie-data. There dataset contains 4864 auctions listed by 1607 unique sellers. Participating in the 3384 (69 percent) of auctions that received bids were 7,445 unique bidders, 2.2 per auction on average. About a third of the bidders participated in multiple auctions, resulting in an average number of unique bidders per auction of 3.7, median 3. The movie-market thus involves much lower bidder-competition than the MP3-player market.

Some preliminary evidence that the model is consistent with actual bidding behavior can be gleaned from simple summary statistics of the data: Most eventual winners won only one unit within the data-period (93% in MP3-players and 87% in movies). Yet, a substantial number of bidders participated in more than one auction (43% in MP3-players and 33% in movies). One alternative explanation of multi-auction bidding can be ruled out right away: It does not seem that the multi-auction bidders simply submitted a very low bid initially to learn about the auction

[5] On eBay, the BIN option disappears when a bid lower than the BIN price is made, and the auction reverts to a simple auction. Therefore, BIN auctions that reverted can be treated as simple auctions by the bidders. BIN auctions that remained without any bids have had at least partial option-value to the bidders, so they are also retained as parts of the auction-stream.

[6] I would like to thank Uri Simonsohn for selecting the popular titles, processing the movie dataset, and manually spot-checking the item-descriptions to ascertain which listing sold which movie.

process or their true valuation, and only later raised their bid to their "full" willingness to pay. Instead, of the 2276 bidders who bid on the same movie-title at least twice in a row, only 49 percent submitted a higher second bid. The corresponding figure among the 4543 MP3-player bidders who bid on the same player-model at least twice in a row is 59 percent. A more precise test of the proposed model based on the empirical relationship between bids and properties of near-future auctions is described next.

Econometric test

The dataset is a biased sample of willingness-to-bid because on eBay, a bid can only be submitted if it exceeds the highest bid at the moment. Therefore, the dataset contains relatively more high bids and relatively fewer low bids than a random sample of willingness-to-bid modeled by the sealed-bid abstraction $\overline{b}(x,v)$. While many latent bids may be truncated by the eBay ascending-auction procedure, two bids in every auction are always recorded – the highest and the second-highest bid in each auction. Therefore, the first- and second-order statistics of the population distribution of bids $\overline{b}(x,v)$ conditional on x, $\overline{b}_{(1)}(x)$ and $\overline{b}_{(2)}(x)$ respectively, are observed in the data without any bias. Since all the model predictions concerning $\overline{b}(x,v)$ are true for all v and valuations are by definition independent of the near-future details, the predictions will be true for the order-statistics of $\overline{b}(x,v)$ as well, and $\overline{b}(x,v)$ can thus be replaced with $\overline{b}_{(1)}(x)$ or $\overline{b}_{(2)}(x)$ in all the model predictions of Section IV.[7] The following linear regression can then be used to test the qualitative predictions about the relationship between bids and both object-types Φ and ending-times Ω of near-future auctions:

[7] Note that this approach does not require the knowledge of private valuations because the predictions concern the impact of commonly-known forward-seen types rather than the impact of privately known valuations.

$$b_{(m),i} = \alpha_{m,k(i)} + \beta_m H_i + \gamma_m x_{i,k(i)} + \theta_m z_i + \varepsilon_{m,i} \tag{4}$$

where *i* indexes auctions, *k* indexes types, and *m* is the order of the bid-statistic, $\alpha_{m,k(i)}$ is the type-specific fixed-effect, β_m is the effect of the type-independent forward-seeing variable H_i, γ_m is the effect of forward-seeing variables specific to auction *i*'s type *k(i)*: $x_{i,k(i)} \in \left\{ \mathrm{I}\Phi^1_{i,k}, w_{i,k}, \left(\mathrm{I}\Phi^1_{1,i,k}, \mathrm{I}\Phi^1_{2,i,k} ..., \mathrm{I}\Phi^1_{A,i,k} \right) \right\}$, z_i is a vector of control-variables specific to the auction *i*, and $\varepsilon_{m,i}$ is mean-zero error.

Consistent estimates of all parameters can be obtained by Ordinary Least Squares. Since the three different specifications of $x_{i,k(i)}$ are at least partially correlated, three separate specifications of (4) were estimated for both levels of *m* and both datasets. To improve the theoretical quality of the linear approximation implicit in (4), the analysis of the MP3-player dataset was further split into two separate analyses because of the high variance in the price across the types. The median type sold for a median price of \$105, so the players were split into 15 "low-priced" players with median prices of less than \$100, and 15 "high-priced" players with median prices above \$100. In each level-specific analysis, the players corresponding to the other price-level are retained as part of the auction-stream, lumped together into the 31-st "other" type. The control variables z_i are discussed next.

Since order-statistics of the bidding distribution are used as dependent variables, the most important control variable is the number of current competing bidders, which varies from auction to auction, and which clearly increases any order-statistic ceteris paribus. The number of bidders in an auction is not accurately observed because of the truncation issue, but the number of observed unique bidders is probably highly correlated with the true number, and is used throughout. In specifying z_i in equation (4), a non-parametric specification using a separate dummy for each number of observed bidders was considered, but it did not contribute much more than a simple linear effect used in the final analysis.

Besides the influence implied by the proposed model, including current competition also controls for the following alternative explanation of a potential negative correlation between the near-future desirability and order-statistics of current bids: If the bidders, instead of acting sequentially as proposed, randomly chose only one auction of their product-type to bid in and subsequently acted myopically by bidding their valuation, more auctions in a short time-period would imply *both* a more desirable near future, and fewer bidders per auction - hence a lower order-statistic of the bids. Therefore, there would arise a mechanical negative correlation between near-future desirability and the order-statistic of the current bids, and including the current number of bidders as a control is critical to rule out this explanation.

Other control-variables included in z_i were a measure of seller reputation[8] shown by Resnick et al (2002), Wilcox(2000) and many others to have a positive impact on bids, a dummy variable for the description of the unit containing words like “new” or “mint” as a coarse measure of within-type vertical differentiation of the products, and (not included by eBay in the movie dataset) seller-controlled differentiation indicators of the listing itself like “photo included”, “bold-type listing”, or “gallery listing”.

Since only the highest and the second highest bid in each auction are used in the analysis and eBay does not allow a bidder to outbid herself, the two order-statistics correspond to bids submitted by different people, and each is the highest bid in the auction submitted by its respective bidder. The analysis thus resolves the issue of “multiple bidding”, i.e. the fact that

[8] Seller feedback score used in the MP3-player analysis was not included in the movie-dataset, so a dummy indicating whether the seller was an “eBay Top Seller” was used instead to capture the effect of reputation.

some eBay bidders submit multiple bids in the same auction, by retaining the highest bid for each bidder as “the” bid of that bidder in the auction. Multiple bidding remains an unresolved curiosity of eBay because it cannot constitute equilibrium behavior even if the bidders have a common value (as shown by Bajari & Hortacsu 2003). Instead, multiple bidding has been linked to bidder inexperience by both Roth & Ockenfels (2002) and by Wilcox (2000), and explained at least partially as a “naïve English” bidding scheme that ignores the eBay proxy system and instead bids as if the eBay auctions were open-outcry English auctions and bidders paid their bids. Capturing multiple bidding is beyond the scope of the present model because of the assumed sealed-bid abstraction, but both datasets used here suggest that multiple bidding is, in fact, not very prevalent (71 percent of movie-bidders and 46 percent of MP3-player bidders never engage in multi-bidding) and it is also negatively correlated with bidder experience as suggested by previous findings.

In both datasets, special care was taken to exclude bids obviously not made by private bidders modeled by the theory. In the MP3-player data, bids made by sellers (about 2 percent) as well as bids made by bidders who won more that one unit within the data-period (about 12 percent of highest and 7 percent of second-highest) were eliminated, resulting in 4068 highest bids and 4099 second-highest bids. Among the movie-auctions, around a hundred (3 percent) of both highest and second-highest bids were eliminated because of being made by a seller or by a bidder who won multiple units of the same title, and another approximately hundred bids were eliminated because they were made by bidders who bid on too many types, resulting in 3114 highest and 2433 second-highest bids. Please see Table 1 for summary statistics of all the variables in the final datasets used in estimation.

Results

In both datasets and according to both order-statistics of bids, bidders seem to engage in at least one form of forward-seeing bidding. The two datasets are discussed in turn, Table 2 presents the parameter estimates for movies, Table 3 for MP3 players. In the movie-dataset, all type-specific forward-seeing variables have coefficients γ_m consistent with the theory: waiting times until the next auction of the same type increase bids, the same type offered in the next 5 auctions decreases bids, and the impact of another offering in the near future decreases with the number of intervening auctions. The coefficient β_m on the type-independent variable (number of auctions in the next hour) is not significant but generally negative as predicted. Interestingly, the effects are smaller for highest bids than for second-highest bids. The measured effects on second-highest bids seem quite large, and they have the added relevance of essentially capturing the effects on price[9]. With the average price in the category around $10, the same movie offered in the immediately following auction leads to an average price-reduction of 72 cents, and the same movie offered at least once within the next five auctions reduces price by about 31 cents. All control-variable parameters are significant and have the anticipated signs.

Bids on high-priced MP3-players exhibit large and significant β_m and γ_m consistent with the theory: more auctions in the next hour reduce current bids, waiting times until the next auction of the same type increase bids, the same type offered in the next 5 auctions decreases bids, and the impact of another offering in the near future decreases with the number of intervening auctions. In the subsample of bids on low-priced players, β_m is still as predicted by theory, but γ_m are not significant. One explanation for this difference is that on lower-priced

[9] Second-highest bids are just a constant increment different from prices, so their analysis is the same as the analysis of price conditional on there being at least two bidders in the auction.

players, detailed examination of the near future is not worth the effort, and bidders find it sufficient to just glance at the red ink and account for the number of auctions ending in the next hour. This explanation is not ruled out by the above case of movies (even cheaper than low-priced players) because there tend to be many more bidders in the MP3-player auctions than in the movie-auctions (median 7 versus 3), and such increased competition exponentially reduces the expected future surplus.

Another interesting property of bidding on the low-priced players comes from one of the model-extensions outlined in Section IV, namely from the model involving bidders desiring multiple types. When the analysis is focused on multi-type bidders and each bidder's desired types are identified as all of the 30 types on which that bidder ever submitted a valid bid, a regression analogous to (4) reveals that bidders submitting $\overline{b}_{(1)}$ and $\overline{b}_{(2)}$ on low-priced MP3 players significantly decrease their bids by \$2-\$3 whenever a high-priced type they desire is available within the next five auctions (details not reported). The converse is not true for high bidders on high-priced players and, as predicted, this model-extension is also not empirically supported in the movie dataset. These results illustrate the richness of information contained in forward-seeing behavior, but their further development is beyond the scope of this paper.

The β_m effects of the number of auctions in the next hour are of similarly small magnitude in both MP3-player sub-samples: doubling of the number of auctions ending in the next hour is associated with approximately 2 percent reduction in prices[10]. On the other hand, the type-specific effects γ_m on high-priced players are substantial: On roughly \$180 items, the same product being available within the next 5 auctions reduces prices by \$8 (4.4 percent) on average, \$10 (5.6 percent) when the same product is available in the immediately subsequent

[10] log(2) β_m /mean(price) is -0.025 in low-priced and -0.019 in high-priced players.

auction. Analogously, delaying the next offering of the same product by a mere hour from the average of 53 minutes is correlated with an increase in bids of over $2. Note that all these estimates are actually conservative because they suffer from the errors-in-variables problem and are hence biased towards zero. All control-variable parameters have the anticipated signs, but unlike in the movie dataset, some variables are insignificant, notably most of the listing-variables under seller control like photo, bold and gallery. The insignificance does not imply that these instruments are of no value because their usage is endogenous.

V. Discussion

This paper studies what happens when the role of an auction changes from selling unique objects at Sotheby's to driving large sequential markets for consumer durables on eBay and other online auction sites. In such markets, seemingly independent auctions become linked through the demand-side strategies. When participating in a sequence of auctions for substitutes, rational forward-looking bidders reduce their bids in anticipation of future auctions offering the same products. When details of some future offerings are already common knowledge as near-future offerings are on eBay, rational forward-looking bidders base their bids on the available information. This paper proposed a new model of such forward-looking bidding, compared its empirical predictions to the predictions of two nested models of simplified bidding behavior, and found strong support for the predicted behavior in actual eBay data from two different product-categories – movies on DVD and MP3-players.

The model departs from previous models of sequential auctions by assuming that bidders know not only the type of the current product they are bidding on, but also what and when will be sold next. Then, under a wide variety of assumptions, there exists an intuitive pure-strategy Markov-perfect equilibrium of the game among the bidders, and the equilibrium bidding strategy can be qualitatively characterized. All rational bidders reduce their current bids when there are more near-future auctions in general, as well as when more of those auctions offer units of the good specifically desired by the bidders and when those units are offered sooner. These findings contribute to the auction-theory literature, and they have obvious relevance to bidders in sequential auctions on eBay or elsewhere.

In both eBay product-categories studied, a test of the model predictions rejects the alternative simpler model without forward-seeing. Moreover, in both categories, at least some bidders seem to take detailed information about the near future into account, leading to price-reductions between three and seven percent whenever the same type of good is available in the next five auctions. The empirical evidence therefore strongly suggests that the bidders behave consistently with the proposed model, and that such behavior does have a sizeable economic impact on the seller revenues.

The findings give focus to future modelers of online auction marketplaces by providing a fairly high lower bound on the sophistication of eBay bidders: eBay bidders look beyond a single auction as they should, and they seem to systematically take what they see into account. Therefore, bids in online auctions cannot be simply interpreted as unbiased signals of the bidders' item-valuations as if each auction were isolated. The eBay markets for MP3 players and DVD movies are examples of large internet-auction markets, in which such stand-alone analysis of individual auctions would be inappropriate. Instead, this paper demonstrates that individual auctions need to be interpreted and analyzed within their context of other auctions selling similar objects, and provides a model that can be used to achieve such analysis. The model implies that observed bids are actually always negatively biased measures of true valuations because winning right away involves an additional opportunity cost arising from not participating in future auctions for the same good.

Because the present results only provide a lower bound on bidder sophistication, there is a lot of room for further empirical modeling of buyer behavior in sequential auction marketplaces. For example, it may be possible to build structural estimation methodologies to infer properties of the underlying distribution of valuations from the observed distribution of bids, and thus learn about the properties of demand. Forward-seeing behavior clearly contains information about within-product intertemporal elasticity of demand, but an extension of the model may be used to measure cross-product elasticity of demand as well: When bidders desire multiple types A and B, the change in bid on type A as a result of changing near-future availability of type B is fundamentally related to cross-price elasticity of demand. Moreover, the randomly varying nature of the near-future offerings is akin to consumers facing different substitutes every time they participate in the marketplace while everything is clearly observed by the analyst. Therefore, future modelers may be able to learn a lot about marketplace substitution patterns – one of the main goals of demand-estimation – from sequential auction-data with forward-seeing bidders.

The findings reported here may also have impact on seller strategies, leading to yet more future research. There has been a dramatic increase in the use of auctions and one could ask whether the inter-auction competition analyzed in this paper will limit the extent to which auctions are used in the future. Understanding the supply side of the auction-driven markets is an interesting topic for future research that may shed light on this question of the scope of auction-driven markets. Throughout this paper, the seller was assumed to be exogenous, but allowing for strategic selling may both qualitatively and quantitatively change the bidder's strategy. A companion paper (Zeithammer 2004) provides the first model of such strategic sellers facing forward-looking buyers described here. This first "full-equilibrium" model demonstrates that the forward-looking effects documented here can persist even when the sellers are also strategic. Moreover, it seems that equilibrium forward-looking bidding strategies automatically reduce the bid-decrement when the sellers' profitability of selling in the auction marketplaces diminishes, and the existence of the forward-looking and forward-seeing strategies thus does not necessarily reduce the scope of auction-marketplaces as trading institutions for substitutes (please see Zeithammer 2004 for details).

It is interesting to contrast this demand-side of the implied market with a demand-side of a traditional posted-price market. In both marketplaces, consumers engage in sequential search. The difference is not just in the increased rapidity and built-in non-reversibility of the implied "search" in the auction marketplace. This paper demonstrates that in the online-auction marketplace, useful information about the other (future) purchase opportunities is available, and this information enters current observed demand, effectively making the underlying "search" less sequential and more simultaneous by integrating information about several purchase-opportunities. This paper provides the first step, but more research is clearly needed to see exactly how consumers should and do cope with this new shopping environment, what is the role of seller strategic behavior, and what are the overall economic efficiency and distributional implications of sequential-auction marketplaces like eBay.

Table 1: Summary statistics

	Movies				Low-priced MP3 players				High-priced MP3 players			
	Highest bid (N=3113)		Second highest bid (N=2431)		Highest bid (N=1693)		Second highest bid (N=1646)		Highest bid (N=2372)		Second highest bid (N=2451)	
	Mean	*Std*	Mean	*Std*	Mean	*Std*	Mean	*Std*	Mean	*Std*	Mean	*Std*
log (Seller Reputation + 6)					4.803	*1.742*	4.709	*1.715*	4.431	*1.417*	4.463	*1.402*
top-seller dummy	0.466	*0.499*	0.466	*0.499*								
photo-listing dummy					0.745	*0.436*	0.765	*0.424*	0.699	*0.459*	0.699	*0.459*
bold-listing dummy					0.034	*0.182*	0.037	*0.189*	0.045	*0.208*	0.047	*0.211*
gallery-listing dummy					0.094	*0.292*	0.095	*0.294*	0.069	*0.254*	0.068	*0.252*
new dummy	0.441	*0.497*	0.429	*0.495*	0.569	*0.495*	0.554	*0.497*	0.667	*0.471*	0.668	*0.471*
current competition (# unique bidders)	3.683	*2.455*	4.450	*2.238*	7.080	*3.969*	7.845	*3.493*	9.341	*4.084*	9.659	*3.862*
log(# auctions next hour+1)	2.403	*0.671*	2.366	*0.669*	1.469	*0.599*	1.478	*0.590*	1.580	*0.586*	1.576	*0.586*
log(time until next+1)	3.179	*0.885*	3.270	*0.807*	4.047	*0.880*	4.043	*0.882*	3.714	*1.040*	3.698	*1.043*
dummy (same type next 5)	0.178	*0.383*	0.172	*0.377*	0.299	*0.458*	0.291	*0.454*	0.497	*0.500*	0.505	*0.500*
dummy (same type 1 auction from now)	0.065	*0.246*	0.056	*0.231*	0.129	*0.335*	0.123	*0.329*	0.223	*0.416*	0.236	*0.425*
dummy (same type 2 auctions from now)	0.027	*0.161*	0.027	*0.163*	0.047	*0.212*	0.048	*0.214*	0.116	*0.320*	0.116	*0.320*
dummy (same type 3 auctions from now)	0.028	*0.165*	0.026	*0.159*	0.048	*0.215*	0.050	*0.218*	0.073	*0.261*	0.068	*0.252*
dummy (same type 4 auctions from now)	0.037	*0.188*	0.037	*0.189*	0.039	*0.194*	0.037	*0.188*	0.051	*0.219*	0.047	*0.212*
dummy (same type 5 auctions from now)	0.023	*0.148*	0.025	*0.156*	0.035	*0.185*	0.033	*0.180*	0.035	*0.184*	0.038	*0.190*

Notes to Table 1:

In the movie-data, the shares of each type range fairly continuously from 1.5% to 14.1% (Black Hawk Down).

In the MP3-player data, each both low- and high-priced players have dominant products, KB Gear Jamp3 (27%) in the low-priced category and Diamond Rio 500 (48%) in the high-priced category. The shares of the remaining products are all below 10% and decline fairly continuously to 1% for the 30-th product.

Table 2: Estimation results: movies

	Highest bid		2nd highest bid		Highest bid		2nd highest bid		Highest bid		2nd highest bid	
Variable	Estimate	*(t-stat)*	Estimate	*(t-stat)*	Estimate	*(t-stat)*	Estimate	*(t-stat)*	Estimate	*(t-stat)*	Estimate	*(t-stat)*
α (30 type-specific dummies)	suppressed for parsimony (mean 7.92, standard deviation 1.39, minimum 5.40, maximum 10.9)											
θ (top-seller dummy)	0.645	*(9.30)*	0.575	*(8.51)*	0.637	*(9.33)*	0.561	*(8.41)*	0.628	*(9.18)*	0.553	*(8.31)*
θ (new dummy)	0.756	*(10.63)*	0.826	*(11.87)*	0.747	*(10.70)*	0.817	*(11.93)*	0.748	*(10.71)*	0.825	*(12.05)*
θ (current competition)	0.128	*(8.81)*	0.094	*(6.20)*	0.127	*(8.89)*	0.096	*(6.38)*	0.125	*(8.71)*	0.094	*(6.26)*
β (log (# next hour+1))	-0.045	*-(0.83)*	0.033	*(0.63)*	-0.087	*-(1.74)*	-0.039	*-(0.80)*	-0.084	*-(1.68)*	-0.039	*-(0.81)*
γ (log time until next)	0.06	*(2.35)*	0.108	*(4.07)*								
γ (same type next 5 auctions)					-0.17	*-(1.92)*	-0.313	*-(3.51)*				
γ (same type 1 a. from now)									-0.348	*-(2.52)*	-0.722	*-(5.00)*
γ (same type 2 a. from now)									-0.433	*-(2.10)*	-0.385	*-(1.93)*
γ (same type 3 a. from now)									0.136	*(0.67)*	0.098	*(0.48)*
γ (same type 4 a. from now)									-0.051	*-(0.29)*	-0.182	*-(1.05)*
γ (same type 5 a. from now)									0.07	*(0.31)*	0.025	*(0.12)*
	N=3017	$R^2=0.42$	N=2356	$R^2=0.53$	N=3113	$R^2=0.42$	N=2431	$R^2=0.53$	N=3113	$R^2=0.42$	N=2431	$R^2=0.53$

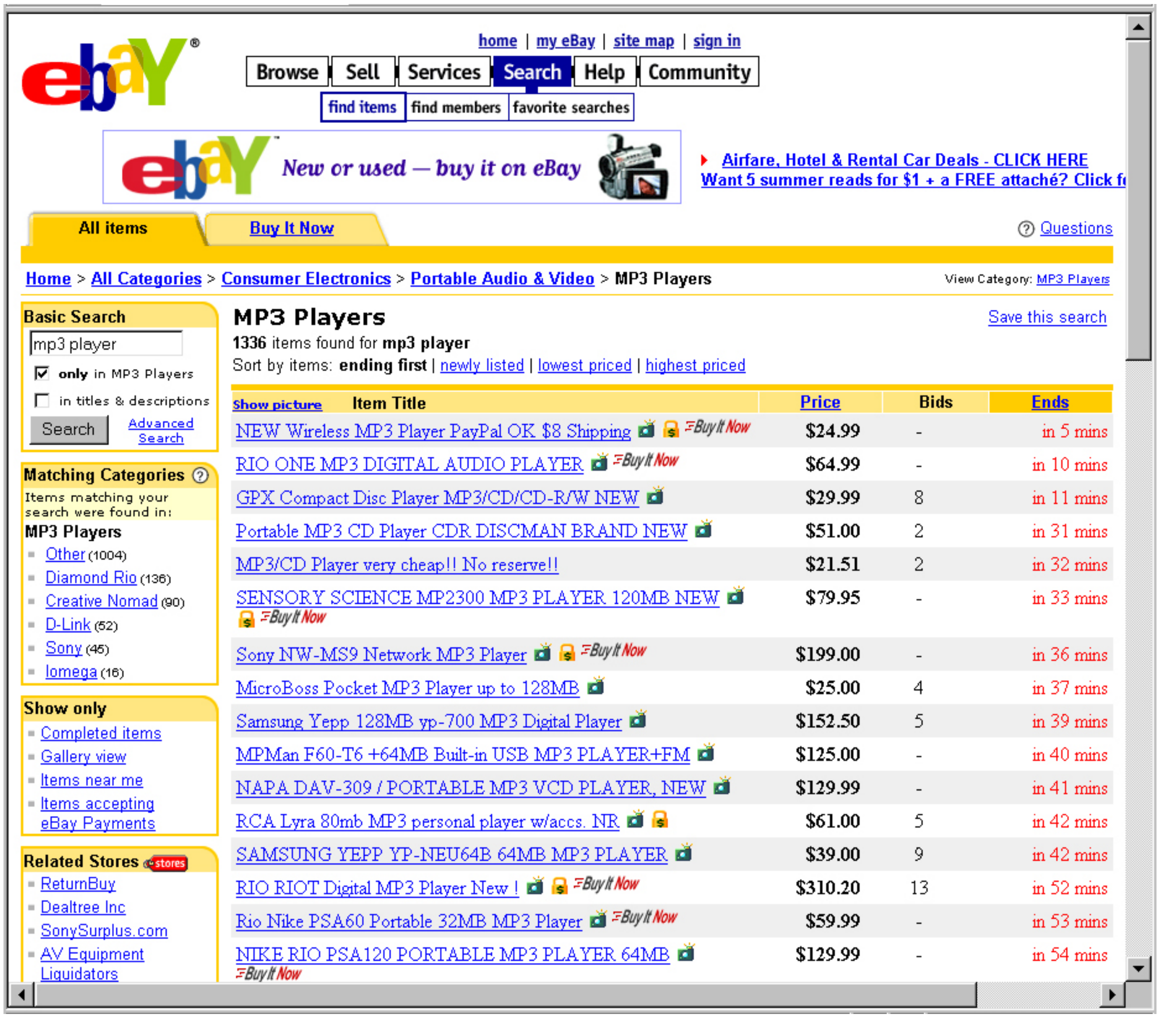
Note to Table 2: 30 movie-title fixed-effects are suppressed for parsimony. The first model has a slightly smaller sample-size because calculating time until next auction of the same type for every type requires a longer forward-seeing horizon and hence there is more truncation of forward-seeing information in the end of the data-period.

Table 3: Estimation results: MP3 players

Low-price players:	**Highest bid**		**2nd highest bid**		**Highest bid**		**2nd highest bid**		**Highest bid**		**2nd highest bid**	
Variable	Estimate	*(t-stat)*	Estimate	*(t-stat)*	Estimate	*(t-stat)*	Estimate	*(t-stat)*	Estimate	*(t-stat)*	Estimate	*(t-stat)*
α (15 type-specific dummies)	suppressed for parsimony (mean 65-70, standard deviation 19-20, minimum 40, max 103-108)											
θ (log (Seller Reputation+6))	0.869	*(2.92)*	0.038	*(0.16)*	0.803	*(2.79)*	0.041	*(0.17)*	0.799	*(2.77)*	0.037	*(0.16)*
θ (photo-listing dummy)	1.855	*(1.63)*	0.207	*(0.23)*	2.04	*(1.83)*	0.428	*(0.48)*	2.065	*(1.85)*	0.483	*(0.53)*
θ (bold-listing dummy)	-1.318	*-(0.53)*	-1.673	*-(0.88)*	-0.996	*-(0.41)*	-1.664	*-(0.89)*	-0.996	*-(0.40)*	-1.605	*-(0.86)*
θ (gallery-listing dummy)	4.633	*(3.03)*	4.322	*(3.56)*	4.339	*(2.90)*	4.092	*(3.43)*	4.331	*(2.89)*	4.088	*(3.42)*
θ (new dummy)	2.951	*(3.03)*	4.264	*(5.53)*	3.215	*(3.36)*	4.378	*(5.75)*	3.217	*(3.36)*	4.369	*(5.73)*
θ (current competition)	0.28	*(2.46)*	0.537	*(5.37)*	0.287	*(2.57)*	0.544	*(5.51)*	0.286	*(2.56)*	0.544	*(5.51)*
β (# next hour)	-2.578	*-(3.48)*	-2.392	*-(4.00)*	-2.587	*-(3.69)*	-2.576	*-(4.55)*	-2.596	*-(3.69)*	-2.596	*-(4.57)*
γ (log time until next)	0.048	*(0.15)*	0.176	*(0.70)*								
γ (same type next 5 auctions)					-0.965	*-(1.00)*	-0.358	*-(0.46)*				
γ (same type 1 a. from now)									-0.454	*-(0.34)*	0.296	*(0.27)*
γ (same type 2 a. from now)									-1.376	*-(0.68)*	-0.359	*-(0.23)*
γ (same type 3 a. from now)									-1.436	*-(0.73)*	-0.221	*-(0.14)*
γ (same type 4 a. from now)									-1.586	*-(0.73)*	-1.914	*-(1.07)*
γ (same type 5 a. from now)									-0.88	*-(0.39)*	-1.046	*-(0.56)*
	N=1645	R^2=0.63	N=1600	R^2=0.73	N=1693	R^2=0.63	N=1646	R^2=0.72	N=1693	R^2=0.63	N=1646	R^2=0.72
High-price players:												
Variable	Estimate	*(t-stat)*	Estimate	*(t-stat)*	Estimate	*(t-stat)*	Estimate	*(t-stat)*	Estimate	*(t-stat)*	Estimate	*(t-stat)*
α (15 type-specific dummies)	suppressed for parsimony (mean 186-171, standard deviation 57-58, minimum 99-114, max 316-334)											
θ (top-seller dummy)	0.729	*(1.67)*	1.332	*(3.41)*	0.816	*(1.89)*	1.405	*(3.60)*	0.787	*(1.82)*	1.392	*(3.57)*
θ (photo-listing dummy)	0.009	*(0.01)*	0.977	*(0.85)*	-0.162	*-(0.13)*	0.746	*(0.65)*	-0.102	*-(0.08)*	0.852	*(0.74)*
θ (bold-listing dummy)	4.427	*(1.50)*	2.597	*(1.00)*	4.369	*(1.49)*	2.898	*(1.12)*	4.176	*(1.42)*	2.693	*(1.04)*
θ (gallery-listing dummy)	1.088	*(0.45)*	-1.11	*-(0.51)*	0.966	*(0.40)*	-1.228	*-(0.57)*	1.146	*(0.48)*	-1.171	*-(0.54)*
θ (new dummy)	7.112	*(5.07)*	7.131	*(5.65)*	7.259	*(5.21)*	7.112	*(5.62)*	7.117	*(5.10)*	6.918	*(5.47)*
θ (current competition)	0.582	*(3.90)*	0.646	*(4.64)*	0.536	*(3.62)*	0.634	*(4.55)*	0.538	*(3.64)*	0.64	*(4.60)*
β (# next hour)	-5.057	*-(4.58)*	-3.542	*-(3.61)*	-6.755	*-(6.61)*	-5.783	*-(6.35)*	-6.577	*-(6.41)*	-5.522	*-(6.05)*
γ (log time until next)	2.617	*(5.51)*	3.232	*(7.73)*								
γ (same type next 5 auctions)					-7.441	*-(4.89)*	-8.172	*-(6.07)*				
γ (same type 1 a. from now)									-8.83	*-(4.95)*	-10.248	*-(6.57)*
γ (same type 2 a. from now)									-7.624	*-(3.55)*	-7.696	*-(4.04)*
γ (same type 3 a. from now)									-8.364	*-(3.37)*	-8.959	*-(3.97)*
γ (same type 4 a. from now)									-3.214	*-(1.13)*	-4.832	*-(1.86)*
γ (same type 5 a. from now)									-4.023	*-(1.22)*	-1.389	*-(0.49)*
	N=2317	R^2=0.86	N=2393	R^2=0.88	N=2372	R^2=0.85	N=2451	R^2=0.87	N=1693	R^2=0.85	N=1646	R^2=0.87

Note to Table 3: Please also see note to Table 2 for an explanation of different sample-sizes.

Appendix 1: eBay screen (MP3 players)



Appendix 2: Proofs of propositions

Proof of Proposition 1: First, consider $b(0,\varphi_1,\omega_1,v)$: Since the expected surplus function is obviously positive, and bidding any positive amount on a personally worthless object yields a negative current-period payoff, any positive bid is dominated by a zero bid. Second, consider $b(1,\varphi_1,\omega_1,v)$: As long as $\dfrac{\partial S(1,\varphi_1,\omega_1,v\,|\,c_0)}{\partial c_0} > -\dfrac{1}{(\lambda\delta)^{\omega_1}}$, a solution to the optimal bidding problem and characterized by first-order condition in (2) because the problem is concave at the solution to (2). Moreover, the solution to (2) is unique for every φ_1, ω_1 and v because for all c_0 :

$\frac{\partial}{\partial c_0}\left(v - (\lambda\delta)^{\omega_1} S(1, \varphi_1, \omega_1, v \mid c_0)\right) < 1$, and since $v - (\lambda\delta)^{\omega_1} S(1, \varphi_1, \omega_1, v \mid c_0 = 0) > 0$ and

$v - (\lambda\delta)^{\omega_1} S(1, \varphi_1, \omega_1, v \mid c_0 = 1) < 1$, it follows by continuity of S and the Intermediate Value Theorem, there is exactly one $b(1, \varphi_1, \omega_1, v)$ that satisfies (2). Note that $\frac{\partial S(1, \varphi_1, \omega_1, v \mid c_0)}{\partial c_0} < 0$ because bids will increase in valuations and valuations of today's losers persist until tomorrow, so facing higher competition today implies higher competition tomorrow, and hence lower expected future surplus. To show that this effect is indeed uniformly bounded, $\frac{\partial S(1, \varphi_1, \omega_1, v \mid c_0)}{\partial c_0} > -\frac{1}{(\lambda\delta)^{\omega_1}}$, let the steady-state order-statistics of the current competition be $Y_{(1)} > Y_{(2)} > ... > Y_{(N-1)}$, and note that $\left|\frac{\partial S(1, \varphi_1, \omega_1, v \mid c_0)}{\partial c_0}\right| < \left|\frac{\partial \tilde{S}\left(1, 1, \omega_1, 1 \mid Y_{(1)} = c_0\right)}{\partial c_0}\right|$, where $\tilde{S}(1, \varphi_1, \omega_1, v \mid c_0)$ is the future surplus conditional on $Y_{(2)}$ surviving until the next auction for sure (as opposed to with probability λ^{ω_1}). The impact of today's winning bid on tomorrow's competition is clearly the highest when today's highest loser survives for sure, and the impact increases in the chance of winning tomorrow's auction because of attrition and discounting, and hence is bounded by the impact on the highest possible bidder v=1. Let z be the expected maximum bid of new entrant(s). Then, $\tilde{S}(1, 1, \omega_1, 1 \mid c_0)$ can be evaluated as

$\tilde{S}(1, 1, \omega_1, 1 \mid c_0) = E_{\varphi_2, \omega_2}\left[1 - E(c_1 \mid c_0)\right] = E_{\varphi_2, \omega_2}\left[1 - E\left(\max\left(Y_{(2)}, z\right) \mid Y_{(1)} = c_0\right)\right]$, and it is clear that the impact of c_0 on $\tilde{S}(1, 1, \omega_1, 1 \mid c_0)$ is limited by the slope of the expectation of the second order-statistic in the first-order statistic, which is less than unity for any distribution by elementary statistics: $\left|\frac{\partial \tilde{S}\left(1, 1, \omega_1, 1 \mid Y_{(1)} = c_0\right)}{\partial c_0}\right| < \left|\frac{\partial E\left(Y_{(2)} \mid Y_{(1)} = c_0\right)}{\partial c_0}\right| < 1$. This concludes the proof that there is a well-defined pure-strategy characterized by (2). Equation (3) is the Bellman equation that a steady-state strategy must satisfy to be perfect. The assumption of no memory allows future surpluses to depend only on future information. *QED*

Proof of Proposition 2:

The fact that $b(1,\varphi_1,\omega_1,v)$ increases in ω_1, follows immediately from the optimal bidding condition (2). The second claim that $0<b(1,1,\omega_1,v)<b(1,0,\omega_1,v)<v$ for all v>0 requires multiple steps to prove and hinges on the fact that bidding on a desired type always gives the bidder at least as much surplus, ceteris paribus, as bidding on an undesired type.

The claim that $b(1,\varphi_1,\omega_1,v)<v$ follows from the obvious fact that S is positive. The claim that $0<b(1,\varphi_1,\omega_1,v)$ follows from the fact that $S(\varphi_0,\varphi_1,\omega_1,v\,|\,c_0)<v$, which is always true because the bidder cannot get more utility than that from getting a unit of the desired type for free, for sure, and immediately, and hence receiving a surplus of v. The central claim of the first part of Proposition 2 will be shown in two steps: 1) for every c_0 $S(1,0,\omega_1,v\,|\,c_0)<S(1,1,\omega_1,v\,|\,c_0)$ and 2) to show 1) implies $b(1,0,\omega_1,v)>b(1,1,\omega_1,v)$.

To show 1), it is instructive to write down the four Bellman equations characterizing the steady-state expected-surplus functions in all possible combinations of current and future desirability states, keeping timing ω_1=ω_2=1 constant and suppressing it from all equations, and hence focusing on $S(\varphi_0,\varphi_1,v\,|\,c_0)$ and $G(c_1\,|\,c_0,\varphi_0,\varphi_1,\varphi_2)$. This is WLOG because the claim is for a given fixed ω_1, and ω_2 only adds another integral to all RHS below:

$$S(1,1,v\,|\,c_0)=E_{\varphi_2}\left[\int^{b(1,\varphi_2,v)}(v-c_1)\,dG(c_1\,|\,c_0,1,1,\varphi_2)+\lambda\delta\int_{b(1,\varphi_2,v)}S(1,\varphi_2,v\,|\,c_1)\,dG(c_1\,|\,c_0,1,1,\varphi_2)\right]$$

$$S(0,1,v\,|\,c_0)=E_{\varphi_2}\left[\int^{b(1,\varphi_2,v)}(v-c_1)\,dG(c_1\,|\,c_0,0,1,\varphi_2)+\lambda\delta\int_{b(1,\varphi_2,v)}S(1,\varphi_2,v\,|\,c_1)\,dG(c_1\,|\,c_0,0,1,\varphi_2)\right]$$

$$S(1,0,v\,|\,c_0)=E_{\varphi_2}\left[\lambda\delta\int S(0,\varphi_2,v\,|\,c_1)\,dG(c_1\,|\,c_0,1,0,\varphi_2)\right]$$

$$S(0,0,v\,|\,c_0)=E_{\varphi_2}\left[\lambda\delta\int S(0,\varphi_2,v\,|\,c_1)\,dG(c_1\,|\,c_0,0,0,\varphi_2)\right]$$

The G distribution arises from all the surviving losers of the current auctions as well as from all the new entrants, and since the number of each is random, G is not simple to evaluate. However, it will always be true that the expected competition after a desired type is sold today is always slightly weaker than when today's type is not desired because a trade means that the highest competing bidder certainly exited the bidder pool while no trade means that the highest competing bidder only exited the bidder pool with probability *(1-λ)*.

Therefore, $G(c_1 \mid c_0, 0, \varphi_1, \varphi_2) < G(c_1 \mid c_0, 1, \varphi_1, \varphi_2)$, and since S decreases in c as shown in the proof to Proposition 1, $S(0, \varphi_1, v \mid c_0) < S(1, \varphi_1, v \mid c_0)$. On the other hand, $G(c_1 \mid c_0, 1, 1, \varphi_2) = G(c_1 \mid c_0, 1, 0, \varphi_2)$ because the bidding function is increasing in v, and so φ_1 has no differential impact on the kind of bidders likely to survive from the past period 0. Therefore, I can write the key difference between surpluses as:

$$S(1,1,v \mid c_0) - S(1,0,v \mid c_0) = E_{\varphi_2}\left[\int^{b(1,\varphi_2,v)} \left(v - c_1 - S(0,\varphi_2,v \mid c_1)\right) dG(c_1 \mid c_0, \varphi_2)\right] +$$

$$+\lambda\delta \int_{b(1,\varphi_2,v)} \left[S(1,\varphi_2,v \mid c_1) - S(0,\varphi_2,v \mid c_1)\right] dG(c_1 \mid c_0, \varphi_2)$$

, and this difference is positive because $S(0, \varphi_1, v \mid c_0) < S(1, \varphi_1, v \mid c_0)$ and because $\left[v - c_1 - S(1, \varphi_2, v \mid c_1)\right] > 0$ for all $c_1 < b(1, \varphi_2, v)$ which follows from the single-crossing property discussed in the proof of Proposition 1. Step 2) also follows from the single-crossing property of the first-order condition: Since $\left[v - c_1 - S(1, \varphi_2, v \mid c_1) = 0\right]$ has a unique solution and $\left[v - 0 - S(1, \varphi_2, v \mid 0) > 0\right]$, $b_0 = b(1,0,v)$ implies $\left[v - b_0 - S(1,1,v \mid b_0) < 0\right]$ and hence the point b_1 such that $\left[v - b_1 - S(1,1,v \mid b_1) = 0\right]$ must lie to the left of b_0.

The fact that $b(1, \varphi_1, \omega_1, v)$ decreases in ρ follows from differentiation of the Bellman equation (3) after writing the expectation $E_{\varphi_2}[.]$ as $\rho(\varphi_2=1)+(1-\rho)(\varphi_2=0)$ and noting by an argument analogous to the one above that $S(\varphi_0, 0, \omega_1, v \mid c_0) < S(\varphi_0, 1, \omega_1, v \mid c_0)$, i.e. whatever today's type, it is always better to face a desired type tomorrow than not. Since higher ρ increases the chance of $\varphi_2=1$, the result follows. *QED*

References

Ariely, Dan and Simonson, Itamar (2003), "Buying, Bidding, Playing, or Competing? Value Assessment and Decision Dynamics in Online Auctions," *Journal of Consumer Psychology*, vol. 13, 113-123.

Arora, Ashish and Xu, Hao and Padman, Rema and Vogt, William (2002), "Optimal Bidding in Sequential Online Auctions," working paper, Heinz School of Public Policy Management, Carnegie Mellon University.

Ashenfelter, Orley (1989), "How Auctions Work for Wine and Art," *Journal of Economic Perspectives*, vol. 3, No. 3, 23-36.

Bajari, Patrick and Hortacsu, Ali (2003), "Winner's Curse, Reserve Prices, and Endogenous Entry: Empirical Insights from eBay Auctions," *RAND Journal of Economics*, vol. 34, No. 2.

Dholakia, Utpal (Paul) and Soltysinski, Kerry (2001), "Coveted or Overlooked? The Psychology of Bidding for Comparable Listings in Digital Auctions," *Marketing Letters*, vol. 12:3, 225-237.

Engelbrecht-Wiggans, Richard (1994), "Sequential Auctions of Stochastically Equivalent Objects," *Economic Letters*, vol. 44, 87-90.

Engelbrecht-Wiggans, Richard (1993), "Optimal Auctions Revisited," *Games and Economic Behavior*, vol. 5, 227-239.

Gale, Ian L. and Hausch, Donald B. (1994), "Bottom-fishing and declining prices in sequential auctions," *Games and Economic Behavior*, vol. 7, 318-331.

Garratt, Rod and Tröger, Thomas (2004), "Speculation in Standard Auctions with Resale," UCSB working paper.

Jofre-Bonet, Mireia and Pesendorfer, Martin (2003), "Estimation of a Dynamic Auction Game," *Econometrica*, vol. 71, 1443-1489.

Klemperer, Paul (1999), "Auction Theory: A Guide to the Literature," *Journal of Economic Surveys*, vol. 13, 227-286.

Milgrom, Paul R. and Weber, Robert J. (1982), "A Theory of Auctions and Competitive Bidding," *Econometrica*, vol. 50, 1089-1122.

Milgrom, Paul R. and Weber, Robert J. (1999), "A Theory of Auctions and Competitive Bidding II," in *The Economic Theory of Auctions*, Paul Klemperer (ed.), Edward Elgar Publishing.

Resnick, Paul, Zeckhauser, Richard, Swanson, John, and Lockwood, Kate (2002), "The Value of Reputation on eBay: A Controlled Experiment," working paper, University of Michigan.

Roth, Al and Ockenfels, Axel (2002), "Last-Minute Bidding and the Rules for Ending Second-Price Auctions: Evidence from eBay and Amazon Auctions on the Internet," *American Economic Review*, Vol. 92, No. 4, pp. 1093-1103.

Oren, Shmuel S. and Rothkopf, Michael H. (1975), "Optimal Bidding in Sequential Auctions," *Operations Research*, Vol. 23, No. 6, 1080-1090.

Weber, Robert J. (1983), "Multi-object auctions," in *Auctions, Bidding and Contracting: Uses and Theory*, eds. Engelbrecht-Wiggans, Shubik and Stark, New York University Press.

Wilcox, Ronald T. (2000), "Experts and Amateurs: The Role of Experience in Internet Auctions", *Marketing Letters*, Vol. 11(4), 363-74.

Zeithammer, Robert (2004), "An Equilibrium Model of a Dynamic Auction Marketplace," working paper, Graduate School of Business, University of Chicago.