Tomographic method for resolving the Galatic binaries:

including multiple interferometers and antenna patterns

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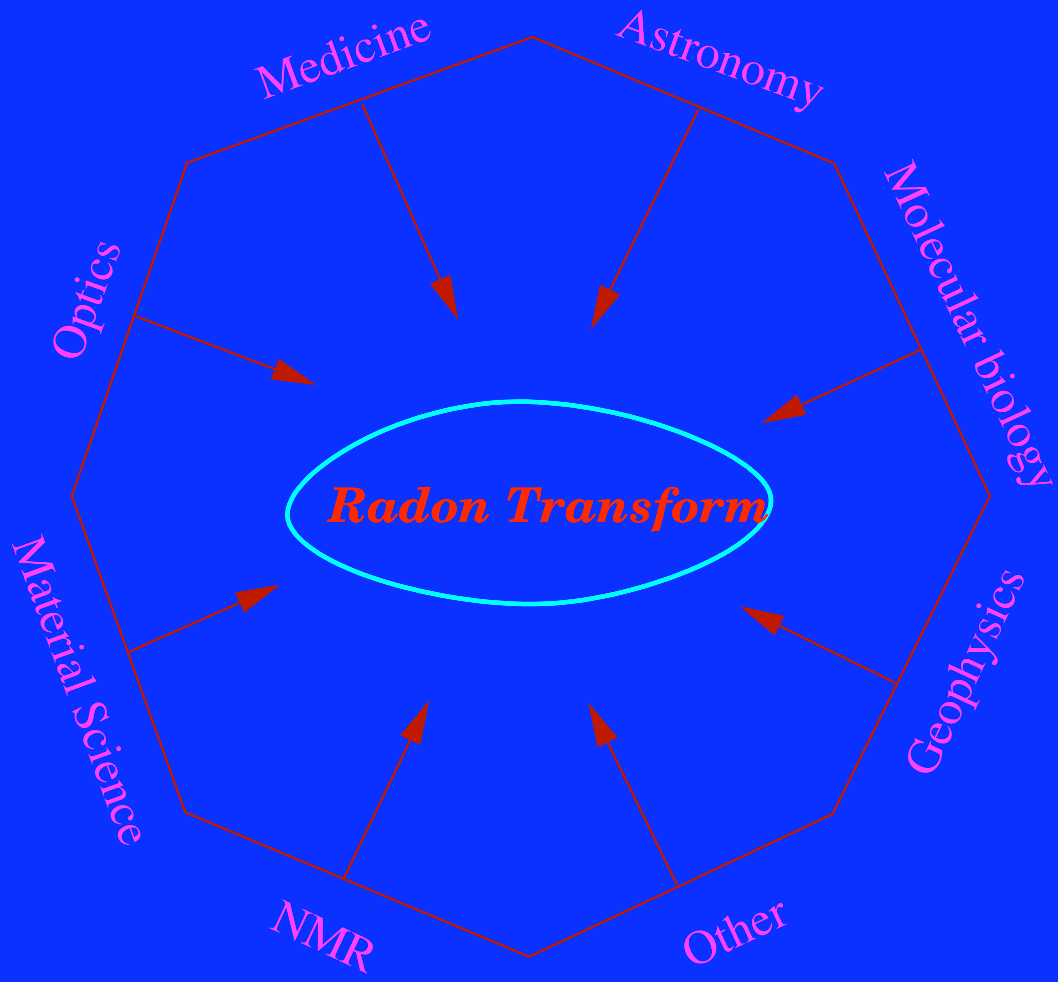
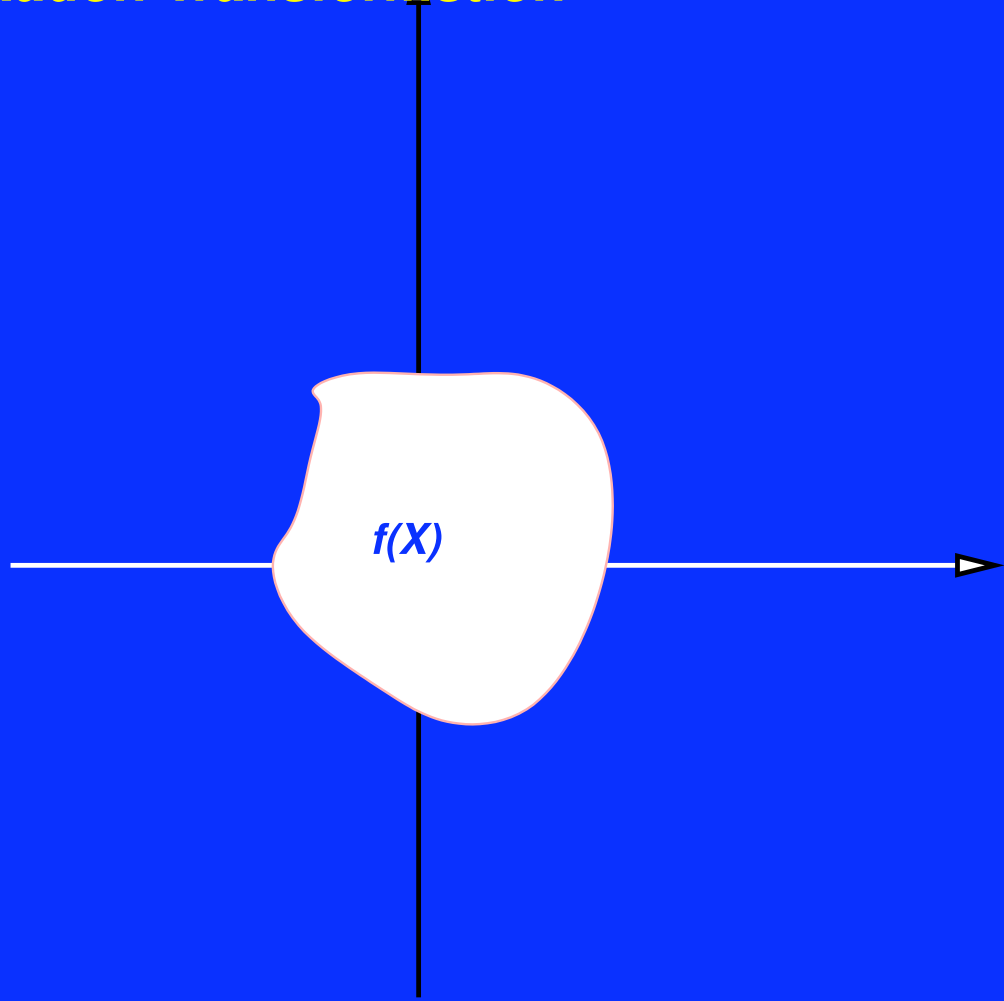
Tomography and Radon transform

* Radon transfrom is the mathematical foundation of tomographic method such as CAT–Scan etc. . . These applications have led to developments of several inverion methods for the Radon transfrom

* Here we use Radon transfrom inversion method for Identification of Galatic binaries



Radon Transformstion

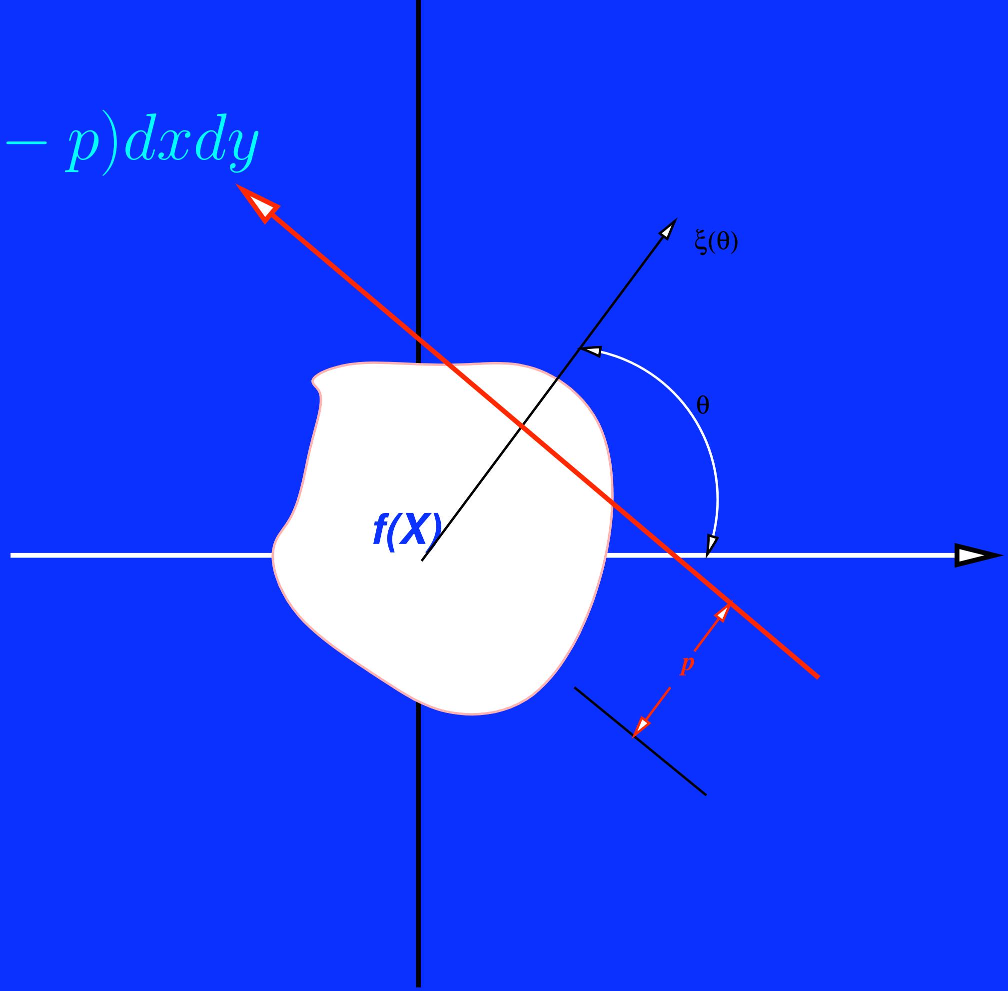


Radon Transformstion

$$F(p,\xi) = \int f(x,y)\delta(x\cdot\vec{\xi} - p)dxdy$$

X-Ray beam in the case of

CAT-Scan



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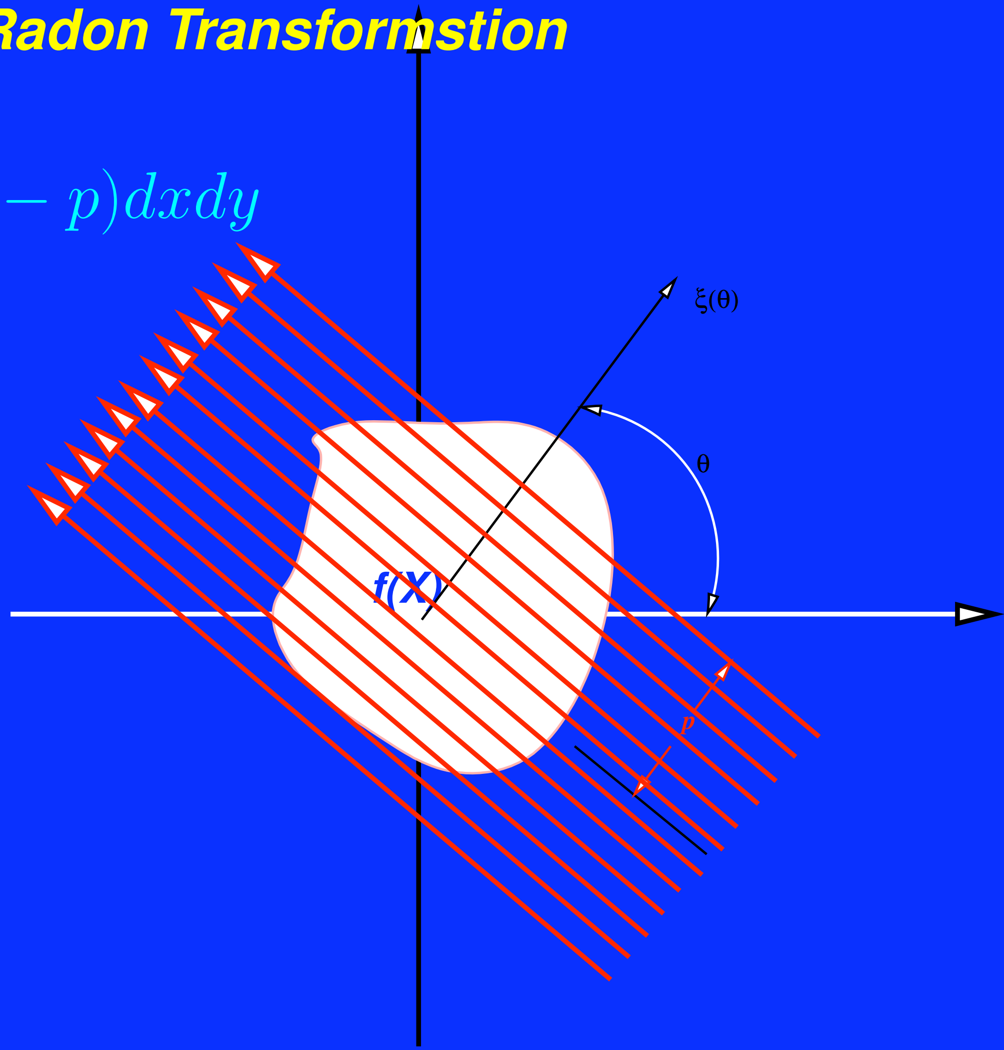

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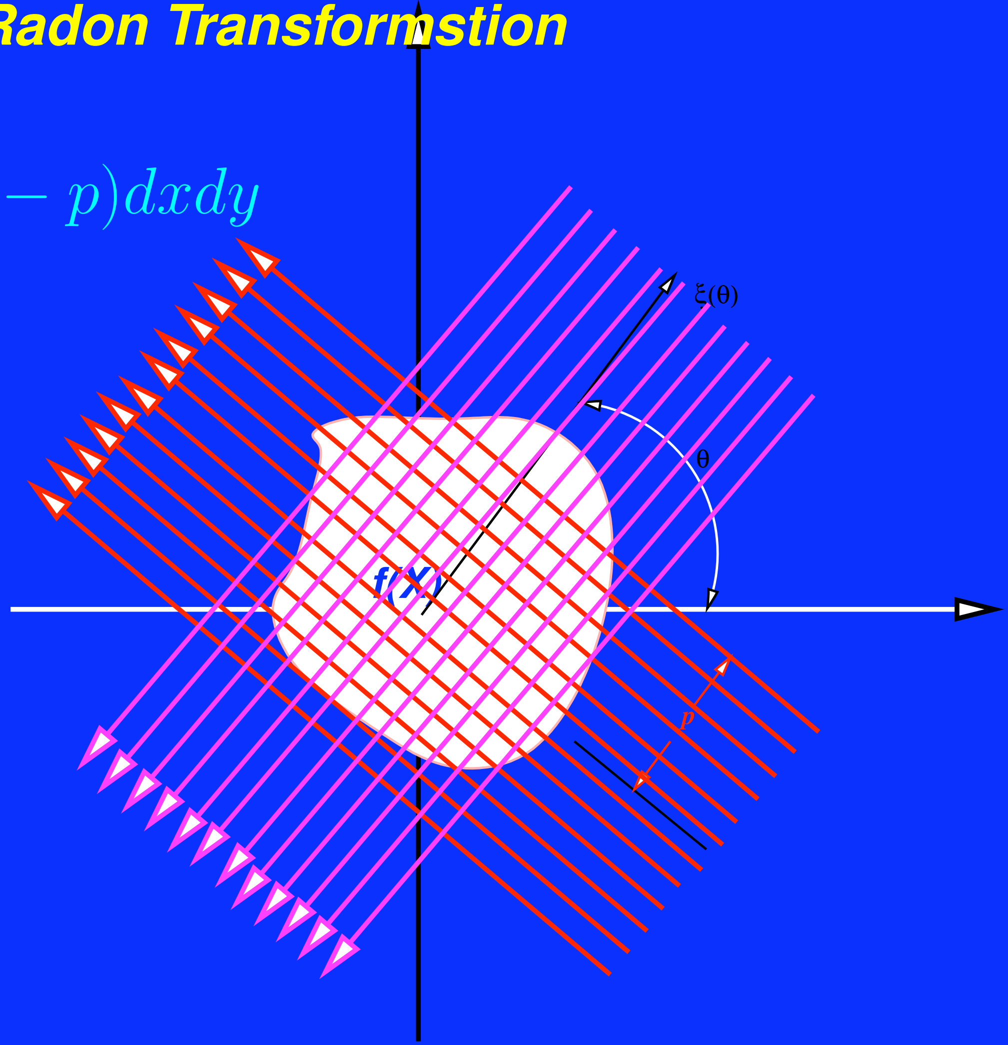

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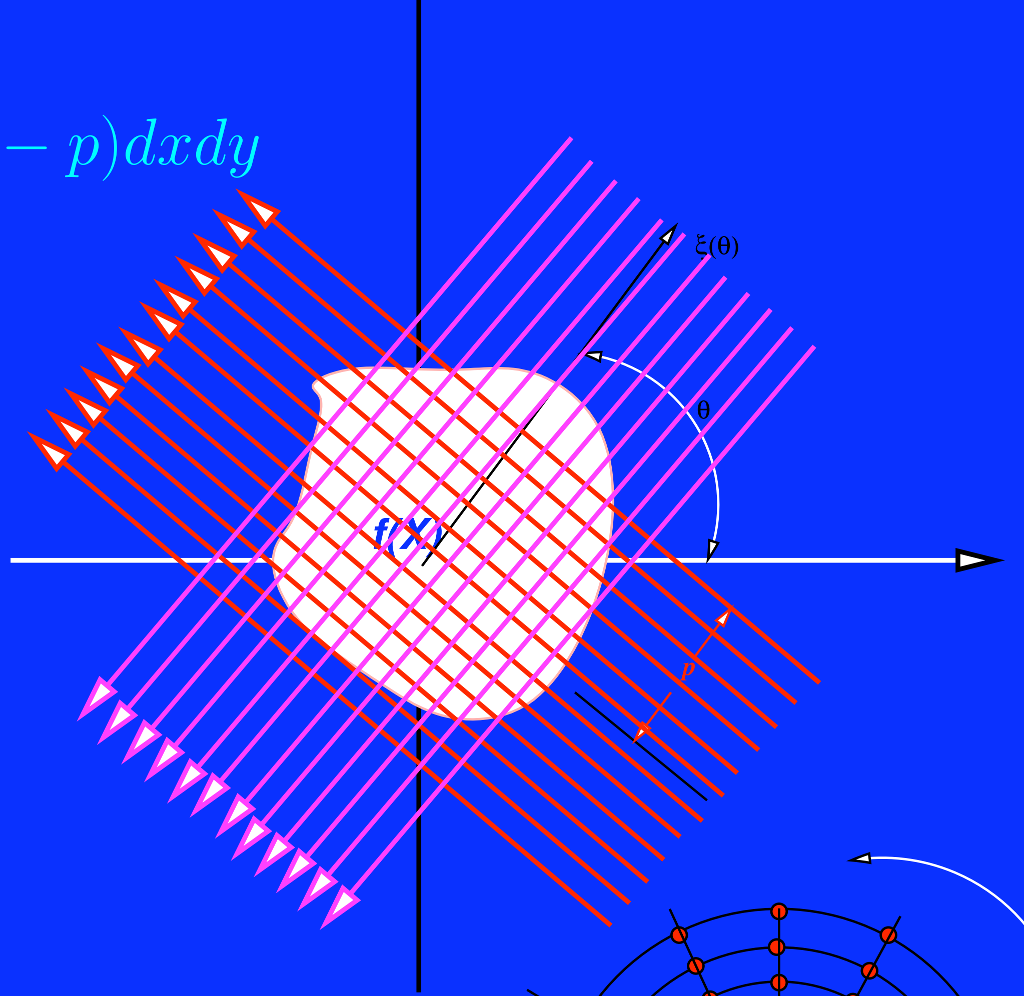
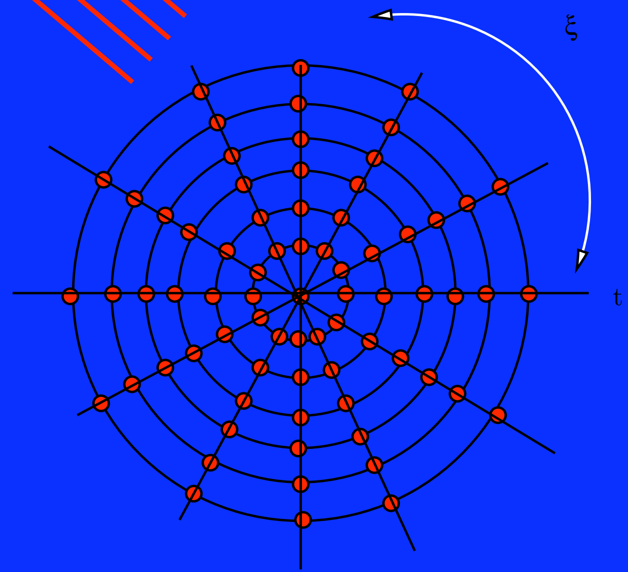


This is a new representation: where integrals are known for all the points on the circle, for all circles.



Relation to the Fourier Transform

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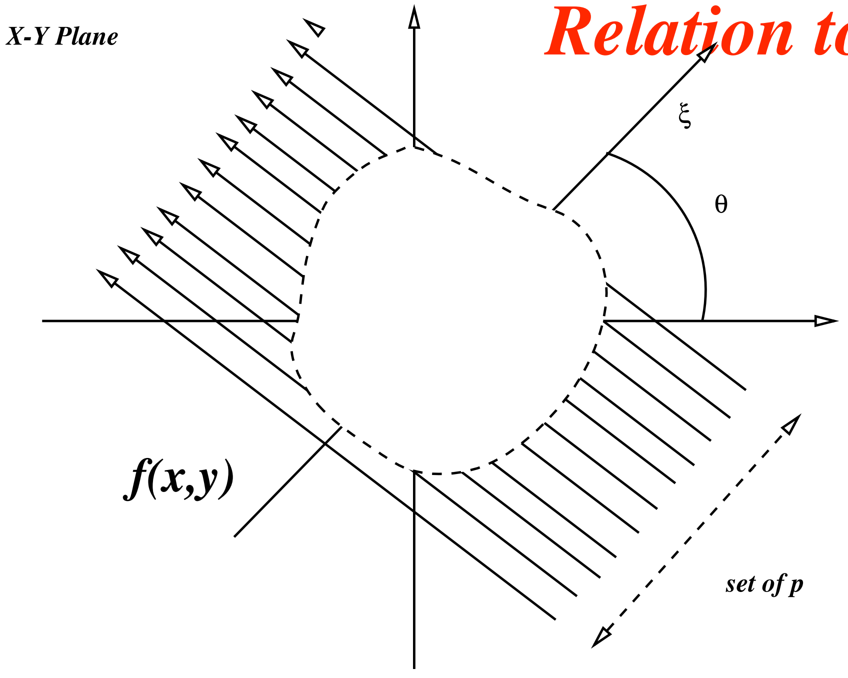
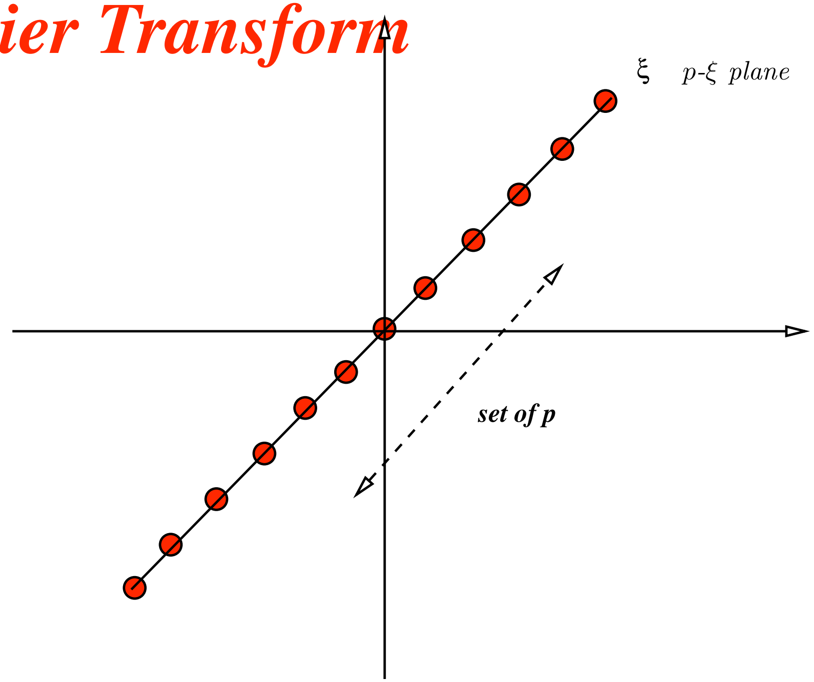
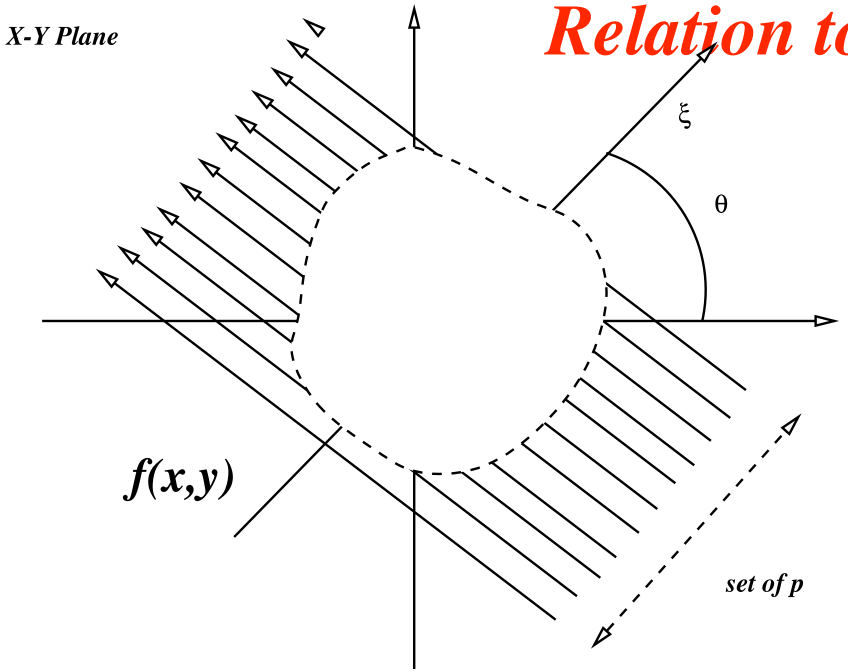


Function in R^n



Function in (p, ξ) (Radon Domain)

Relation to the Fourier Transform



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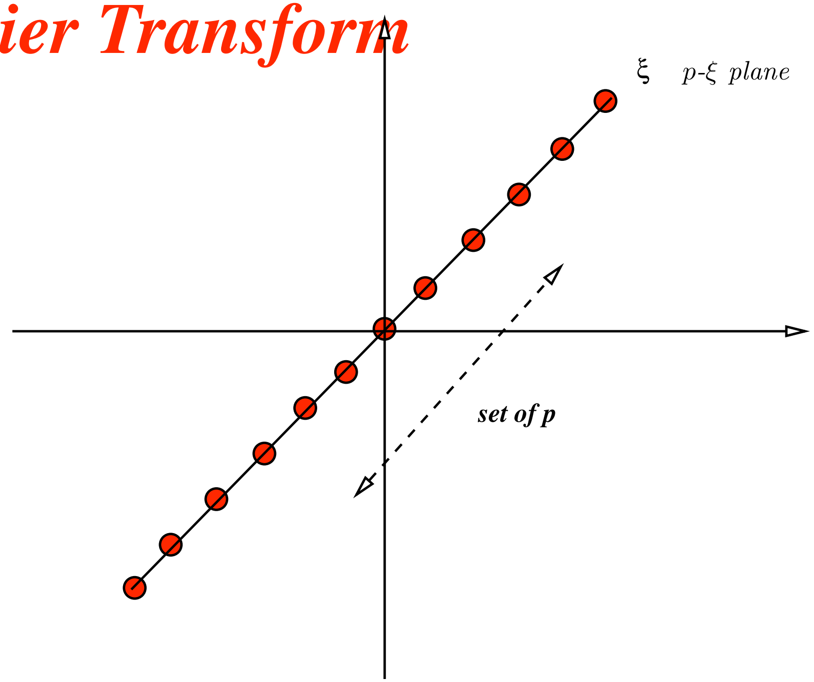

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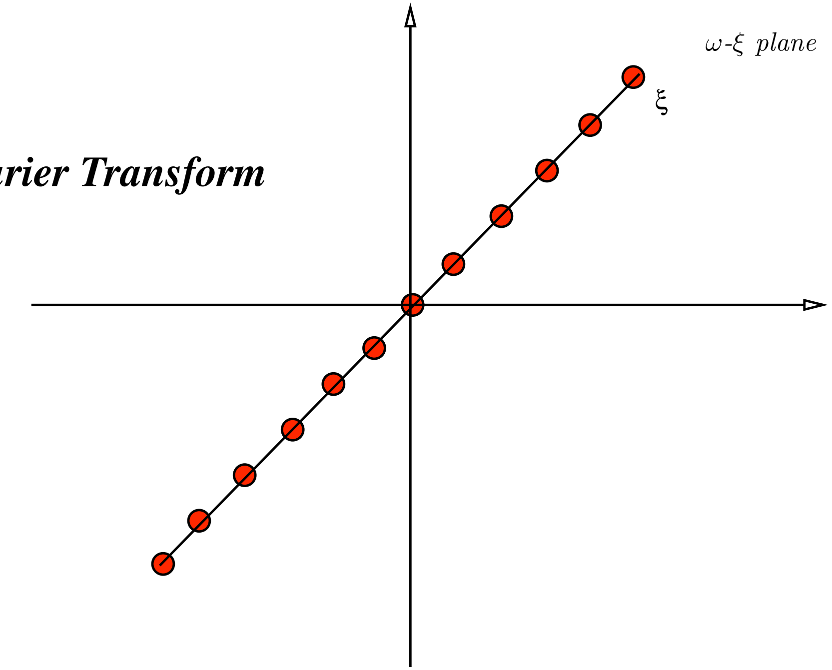
1-D Fourier Transform



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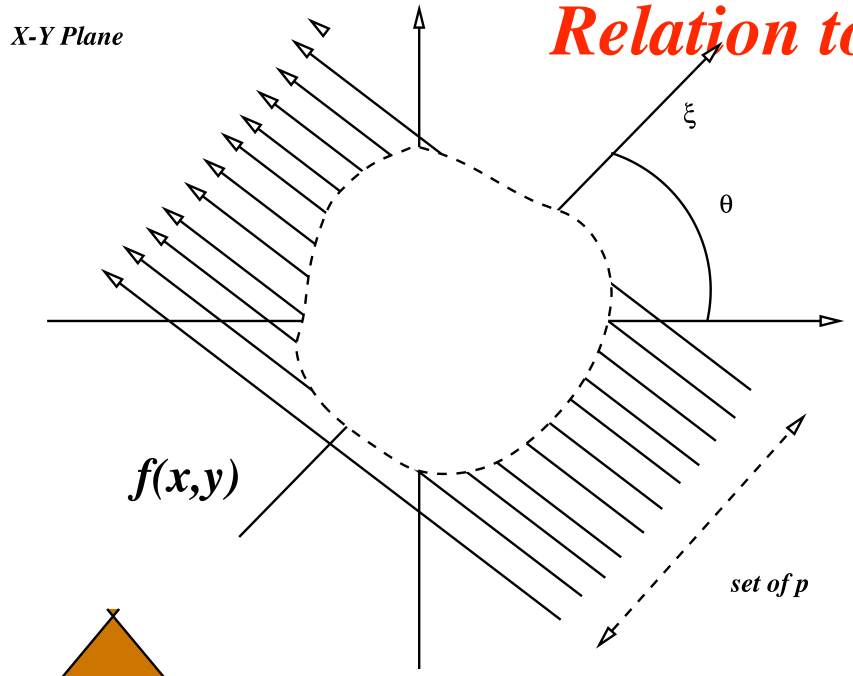

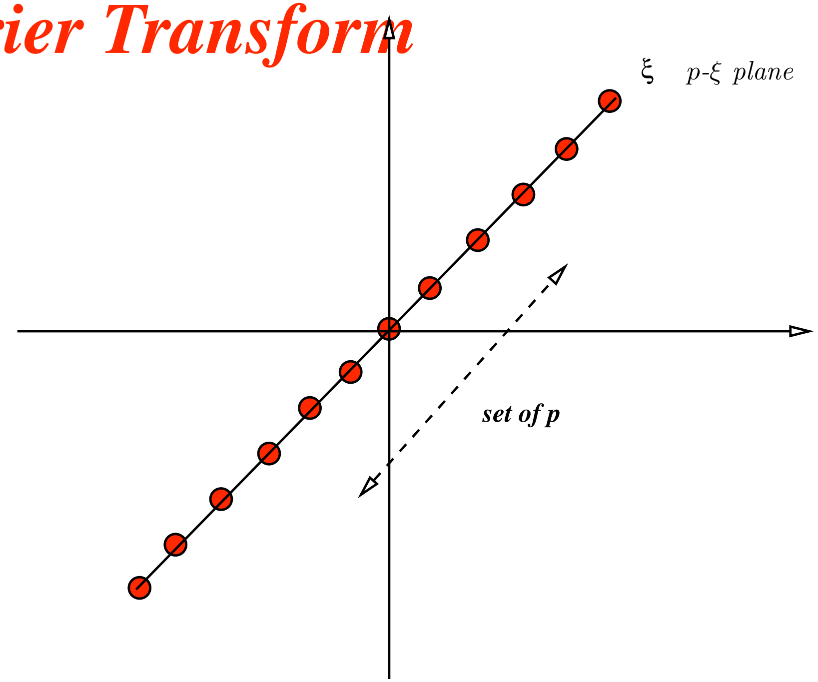

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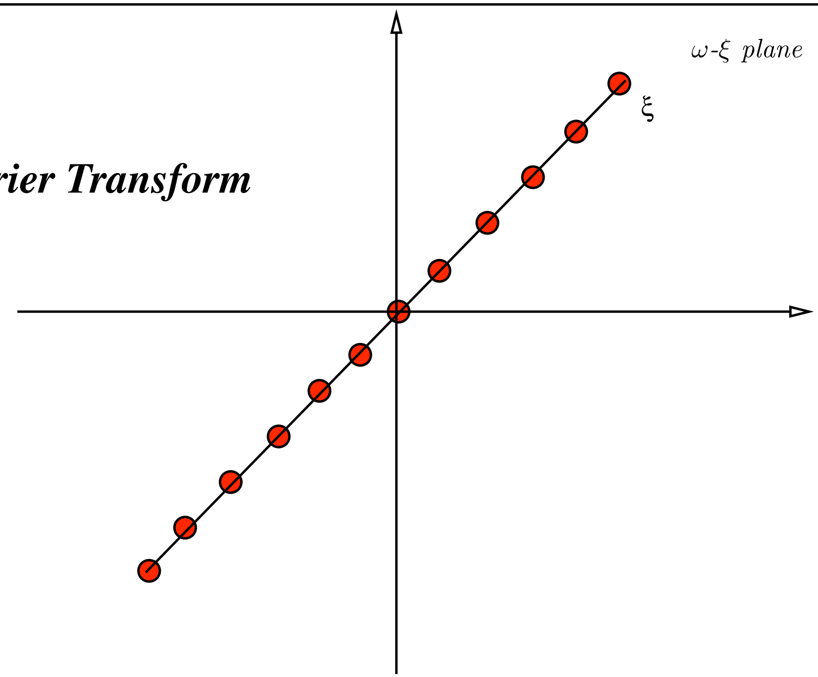
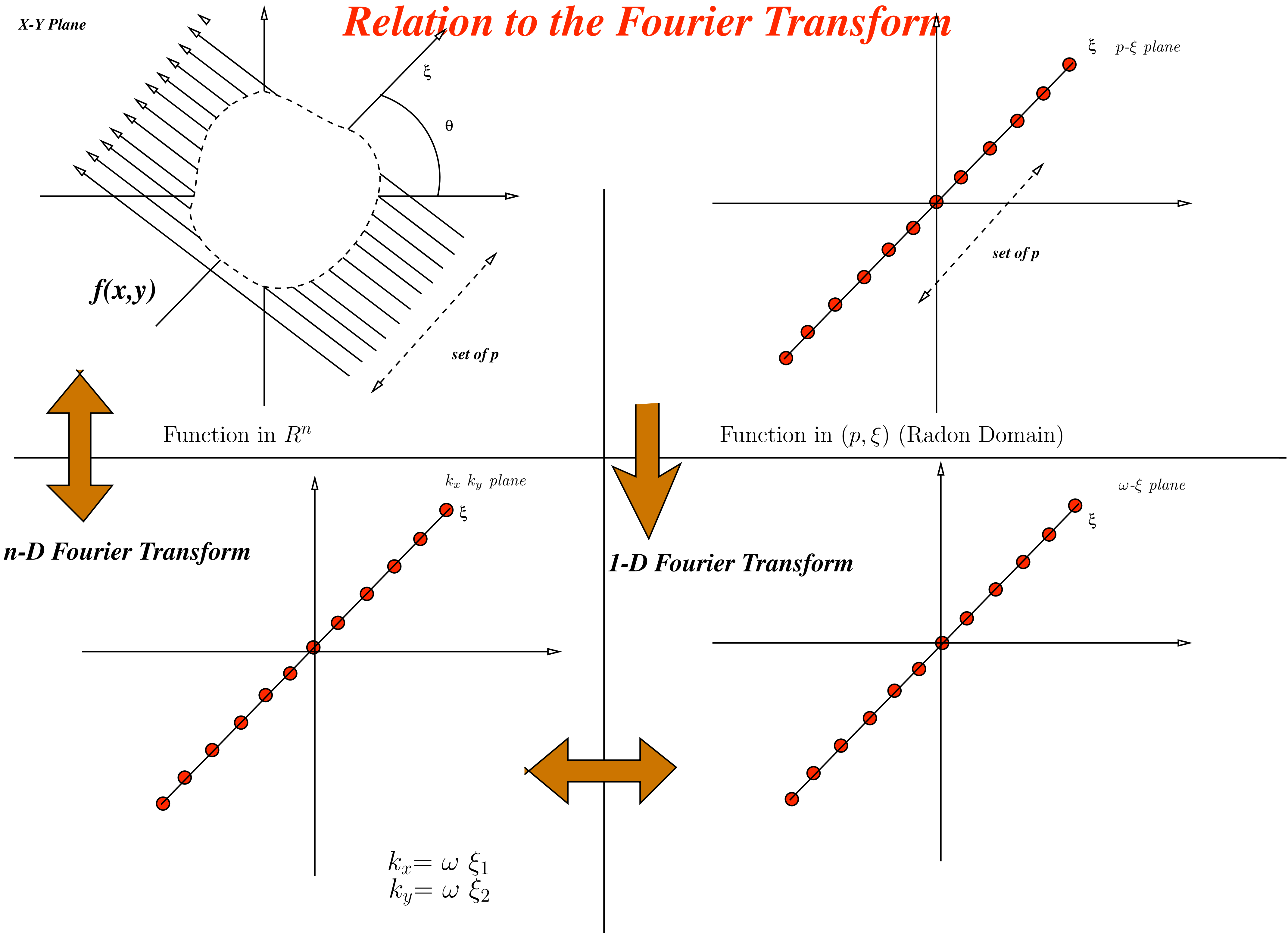


n-D Fourier Transform

1-D Fourier Transform

Relation to the Fourier Transform



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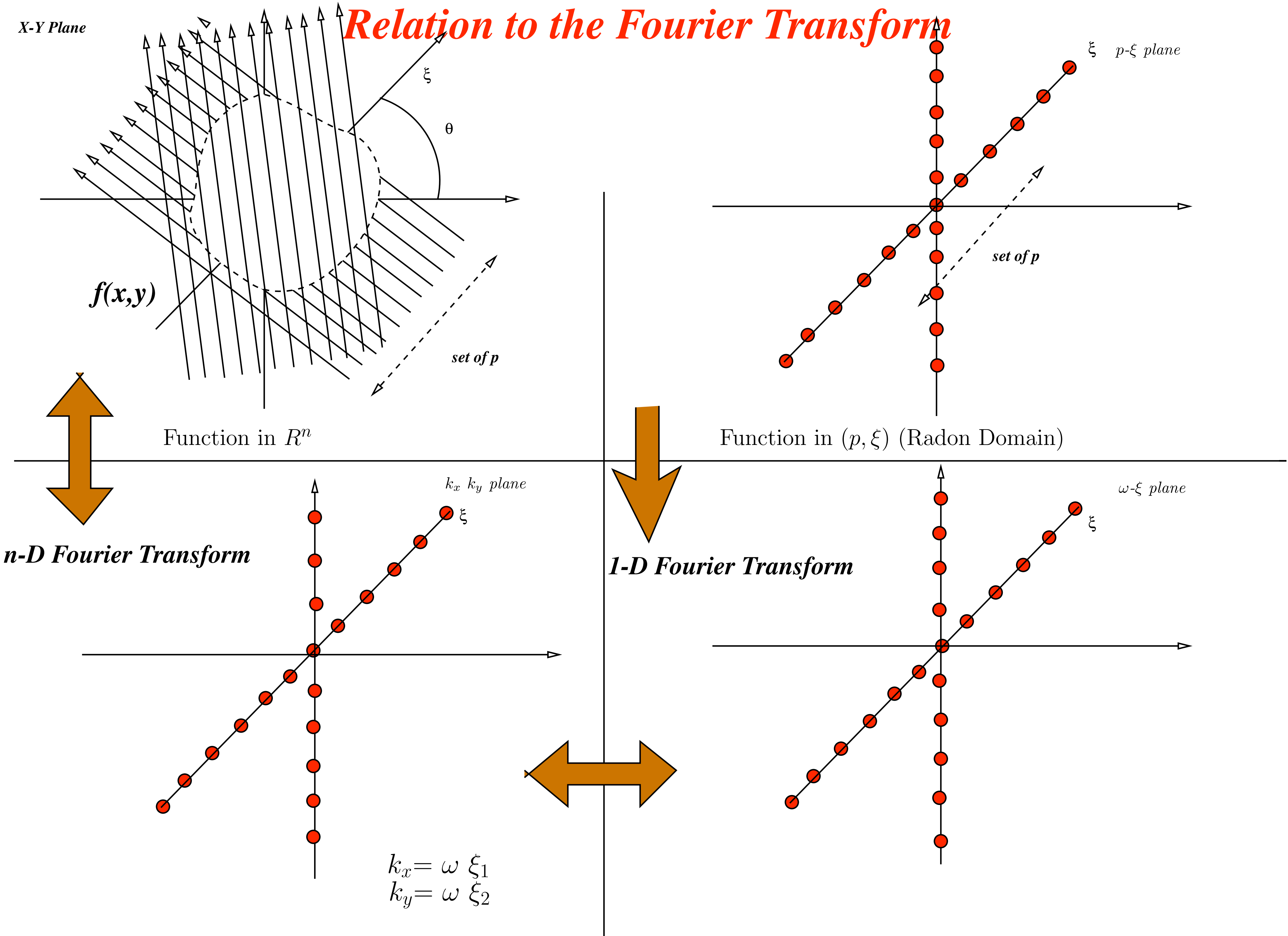

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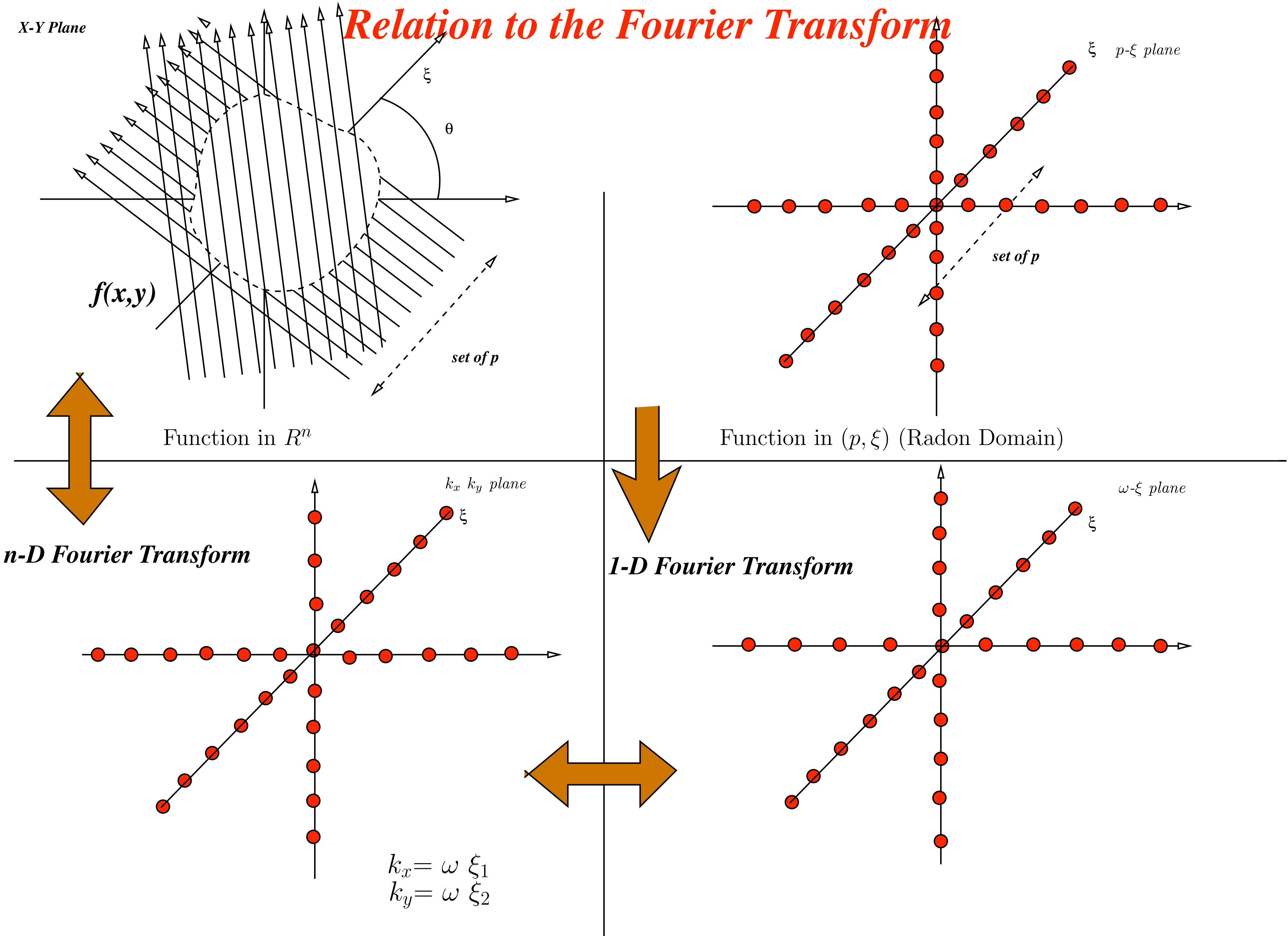

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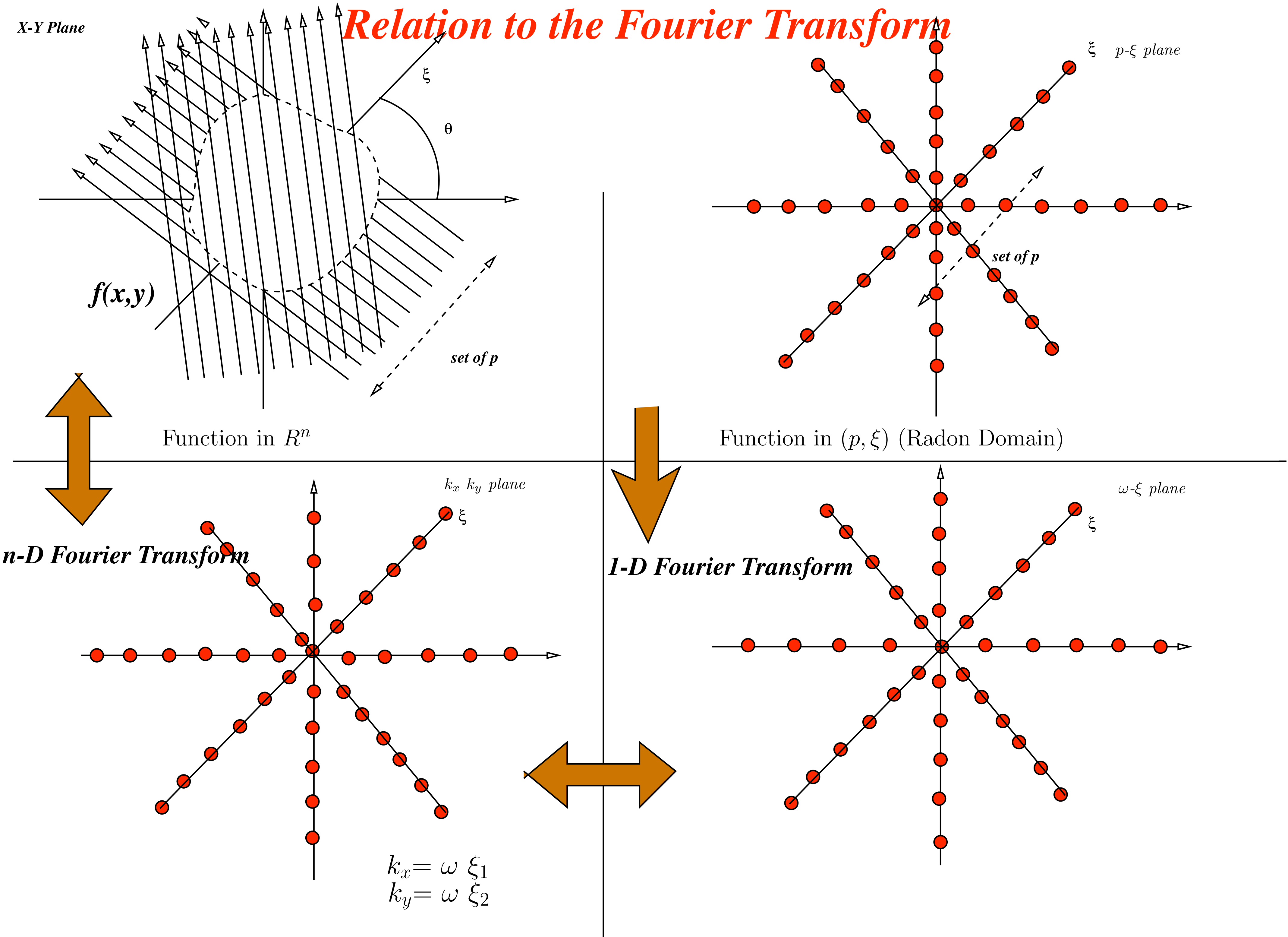

LISA Response as Radon transform

- ☞ Gravitational Wave detector are omni-directional:
  LISA responds to the GW signal from all over the sky
  $h(t) = \int d\theta d\phi K(\theta, \phi, t)$

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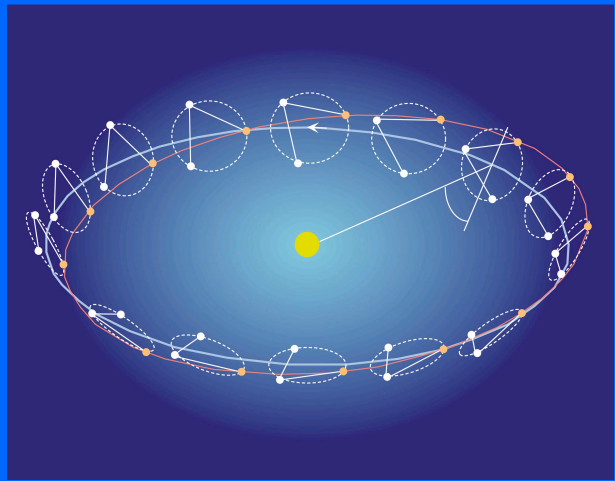
- ☞ Gravitational Wave detector are omni-directional: LISA responds to the GW signal from all over the sky $h(t) = \int d\theta d\phi K(\theta, \phi, t)$

- ☞ LISA motion around the Sun introduces Doppler modulation: i.e:

If a source is at sky position (θ, ϕ)
$\Omega t \longrightarrow \Omega \left[t + \Phi_D(\theta, \phi, t)\right]$
with
$\Phi_D = R \sin\theta \cos(\phi - \omega t)$



This implies the integration not on $t = constant$ on the (θ, ϕ, t) space but on the surface $t + \Phi_D(\theta, \phi, t)$ plane.

$h(t) = \int d\theta d\phi K(\theta, \phi, t + \Phi_D)$

Mohanty and Nayak, *Phys. Rev.* D 73, 083006 (2006)

In the case of LISA the surfaces are not planes. We do a coordinate transformation from (θ, ϕ) to a set of new Euclidean like coordinate system

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sqrt{2}R \sin\theta \cos\phi \\ \sqrt{2}R \sin\theta \sin\phi \\ \sqrt{2}(t + \Phi_D) \end{pmatrix}$$

And the surface becomes a plane satisfying the condition

$$t = \xi \cdot x$$

where

$$\xi = \frac{1}{\sqrt{2}}(-\cos\omega t,\ -\sin\omega t,\ 1)$$

And the signal becomes

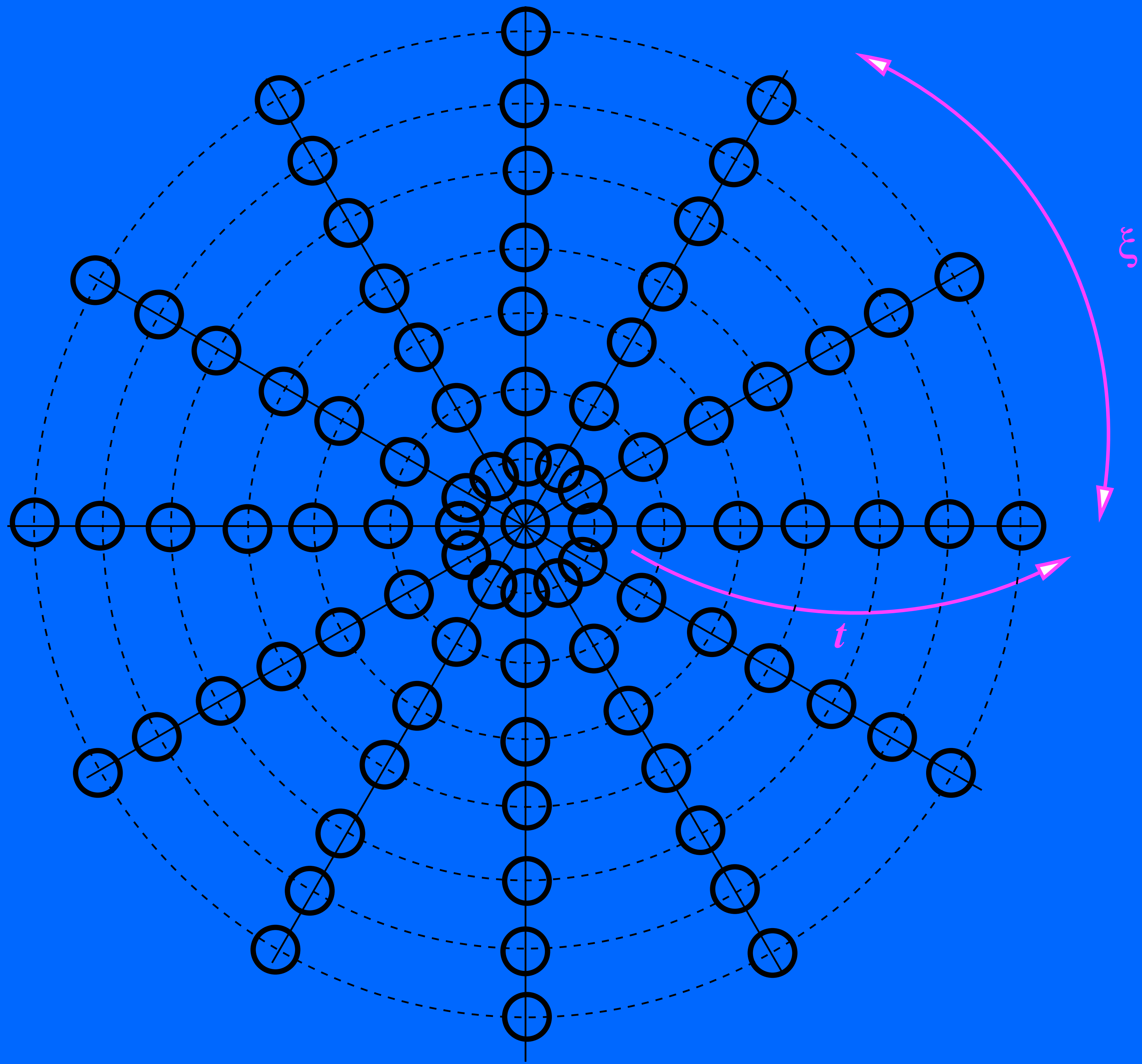
$$h(t) = \int K(x)\delta\left(t - \xi \cdot x\right) dx$$

$K(x)$is signal from binaries in the new coordinate system.

☞ This Show that the LISA response is Radon transform of Galatic binary signal

☞ However, there is one major difference, in Standard Radon transform t andξ are independent of each other and span the complete domain where the signal defined. In our caseξ is function of time. This leads to what is know as missing projections. i.e. we do not have all th realizations of Radon Transform.

Missing Projection ?



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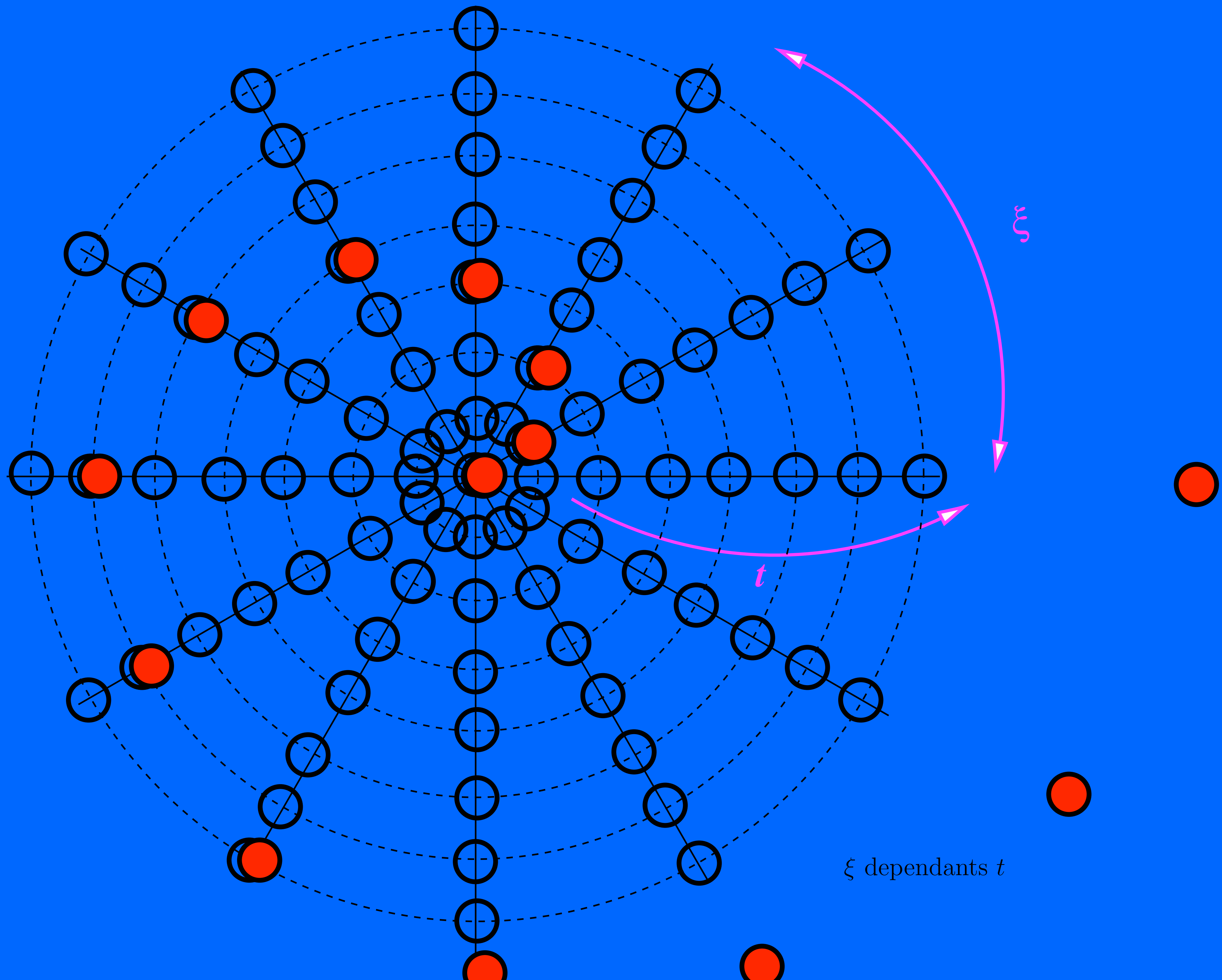

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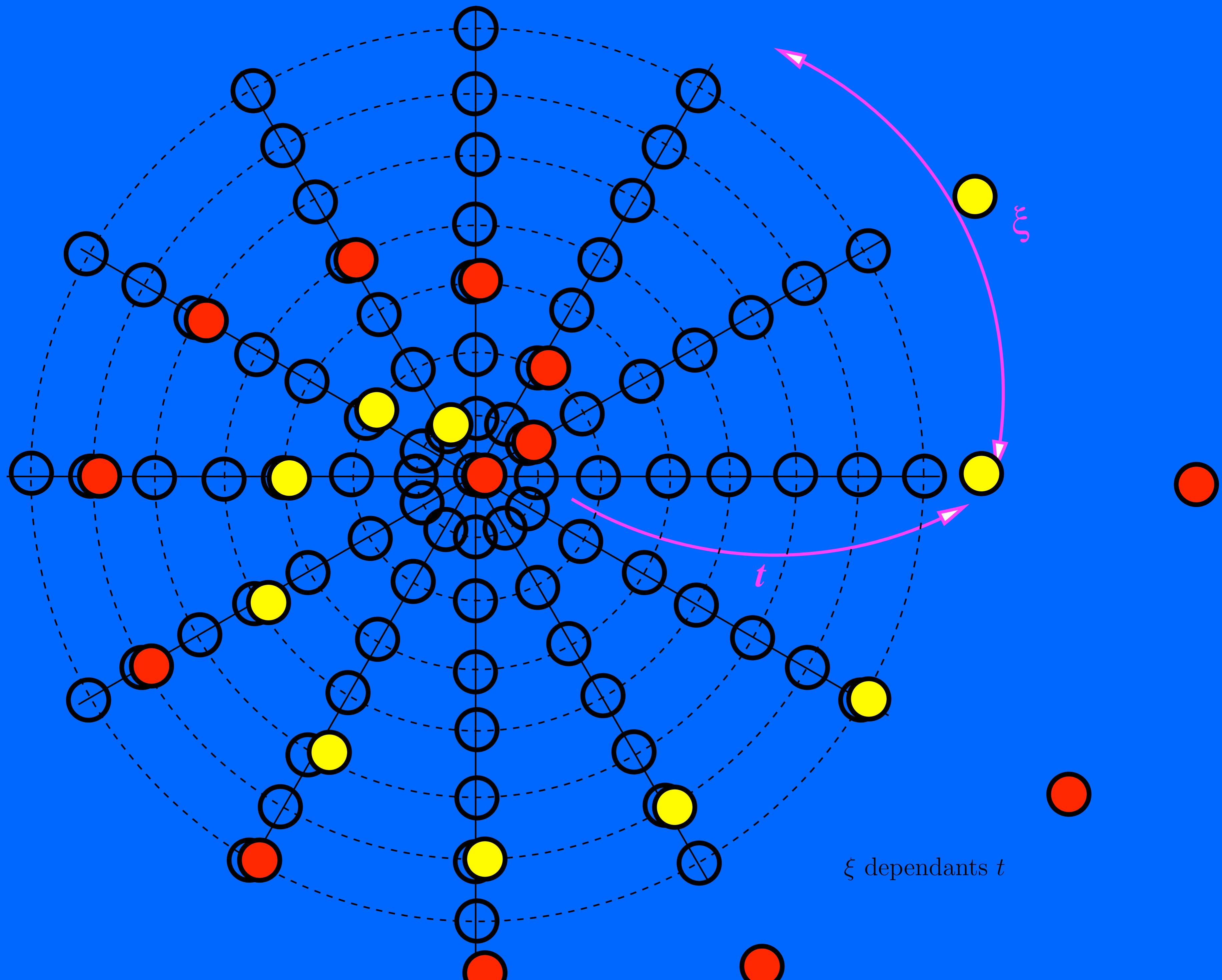
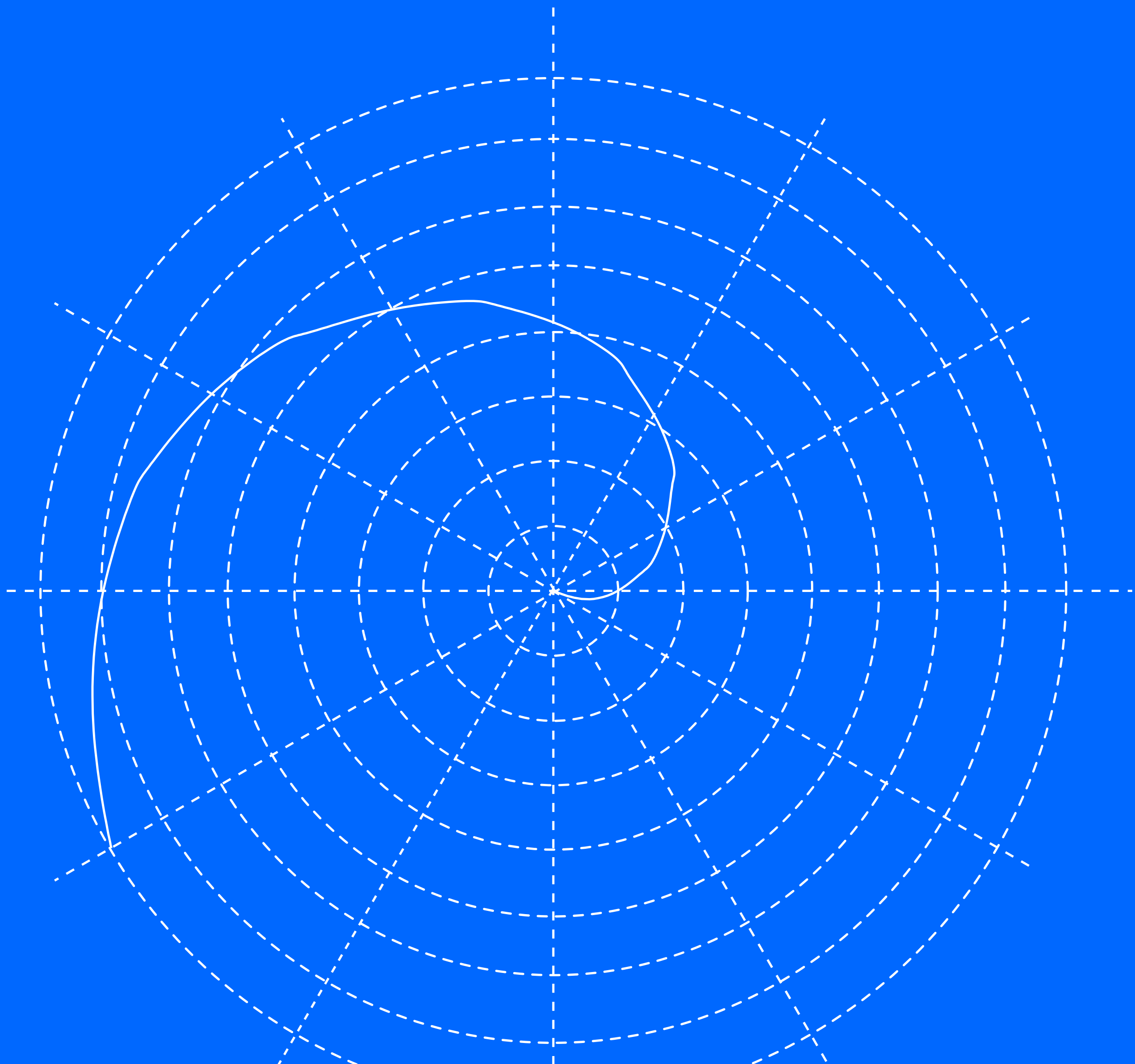


Approximations needed for applying Direct Fourier method:



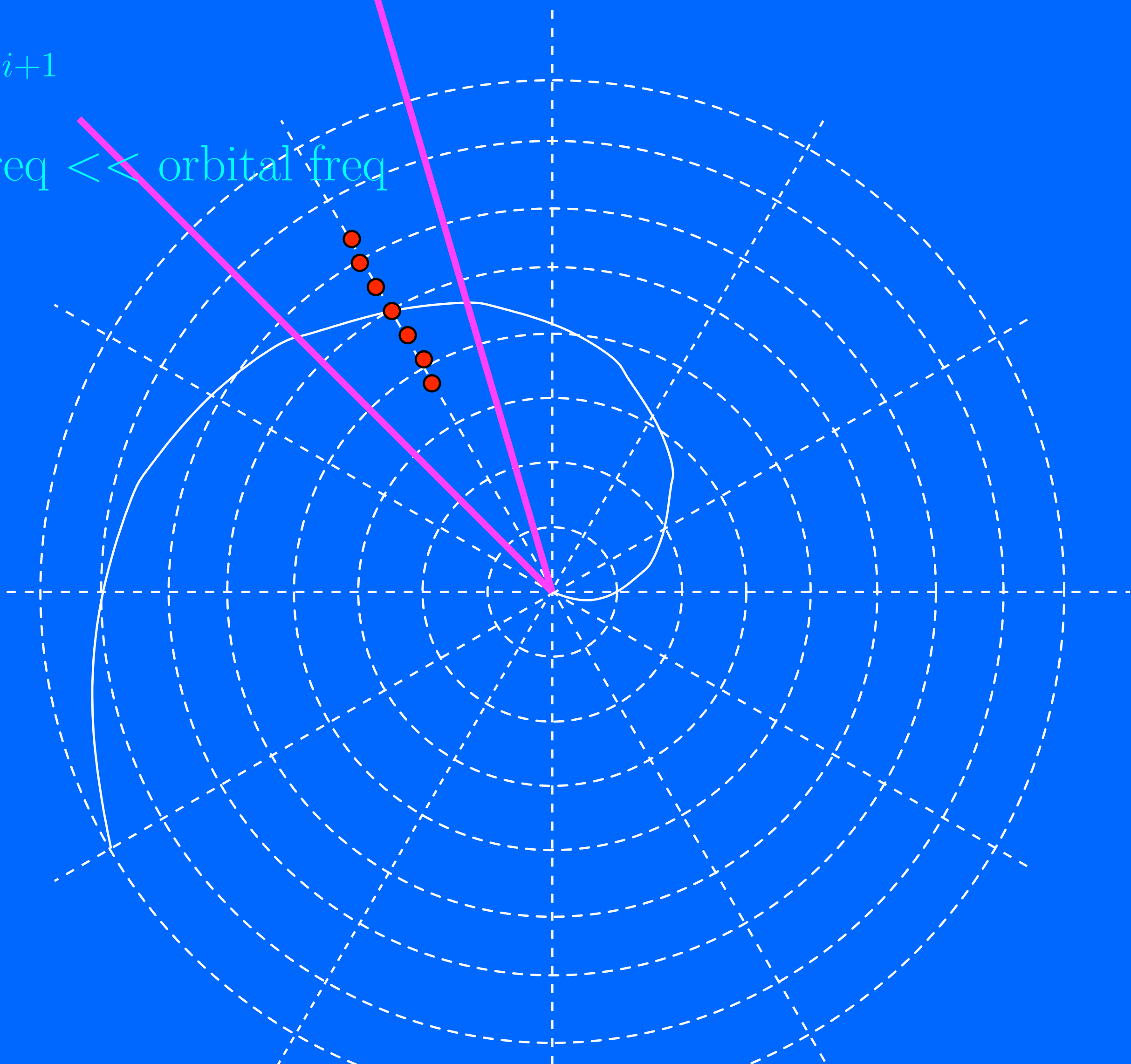
Approximations needed for applying

Direct Fourier method:

Assume ξ constant in a small time interval

$\Delta = t_i$ to t_{i+1}

sampling freq << orbital freq



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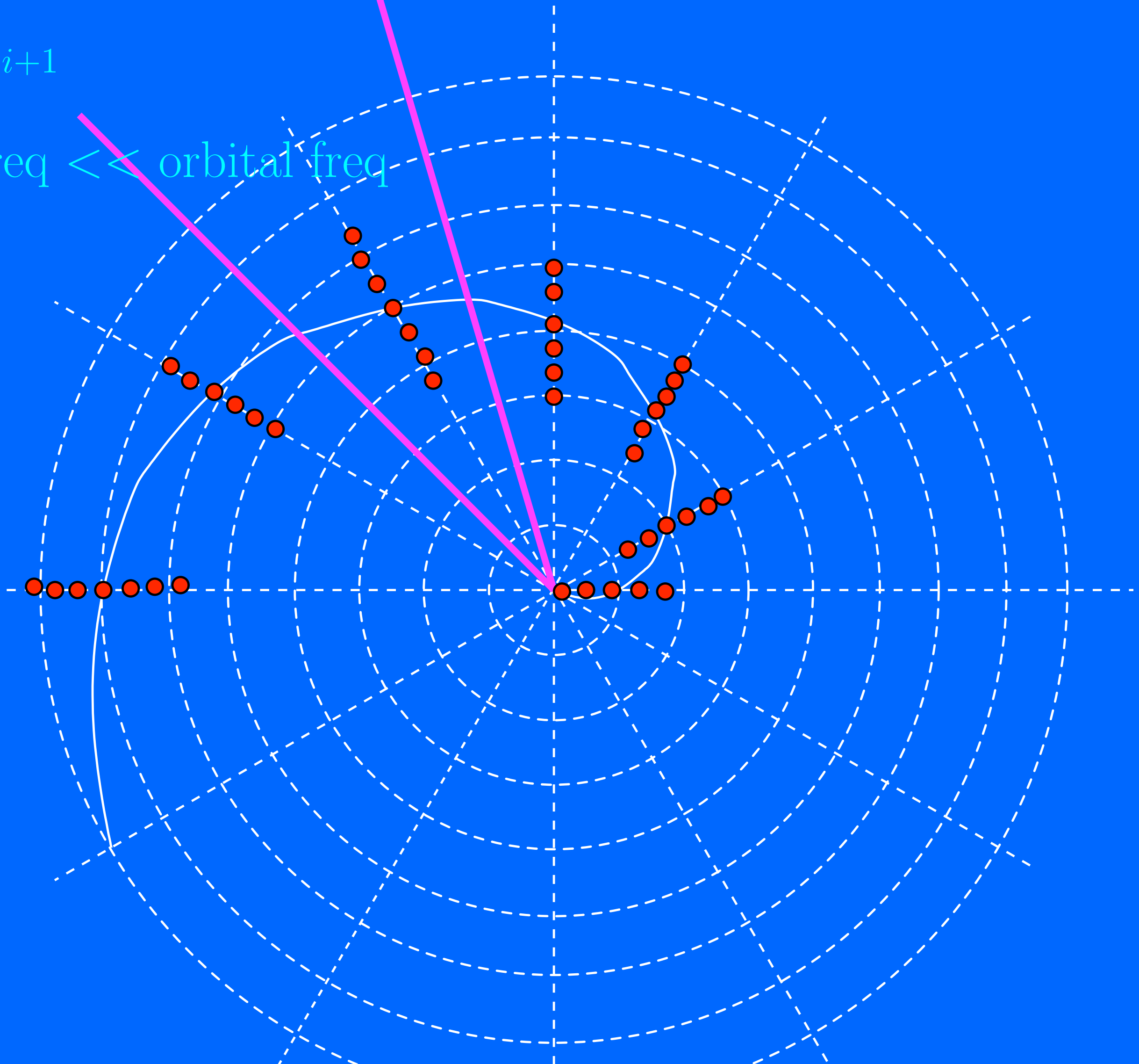

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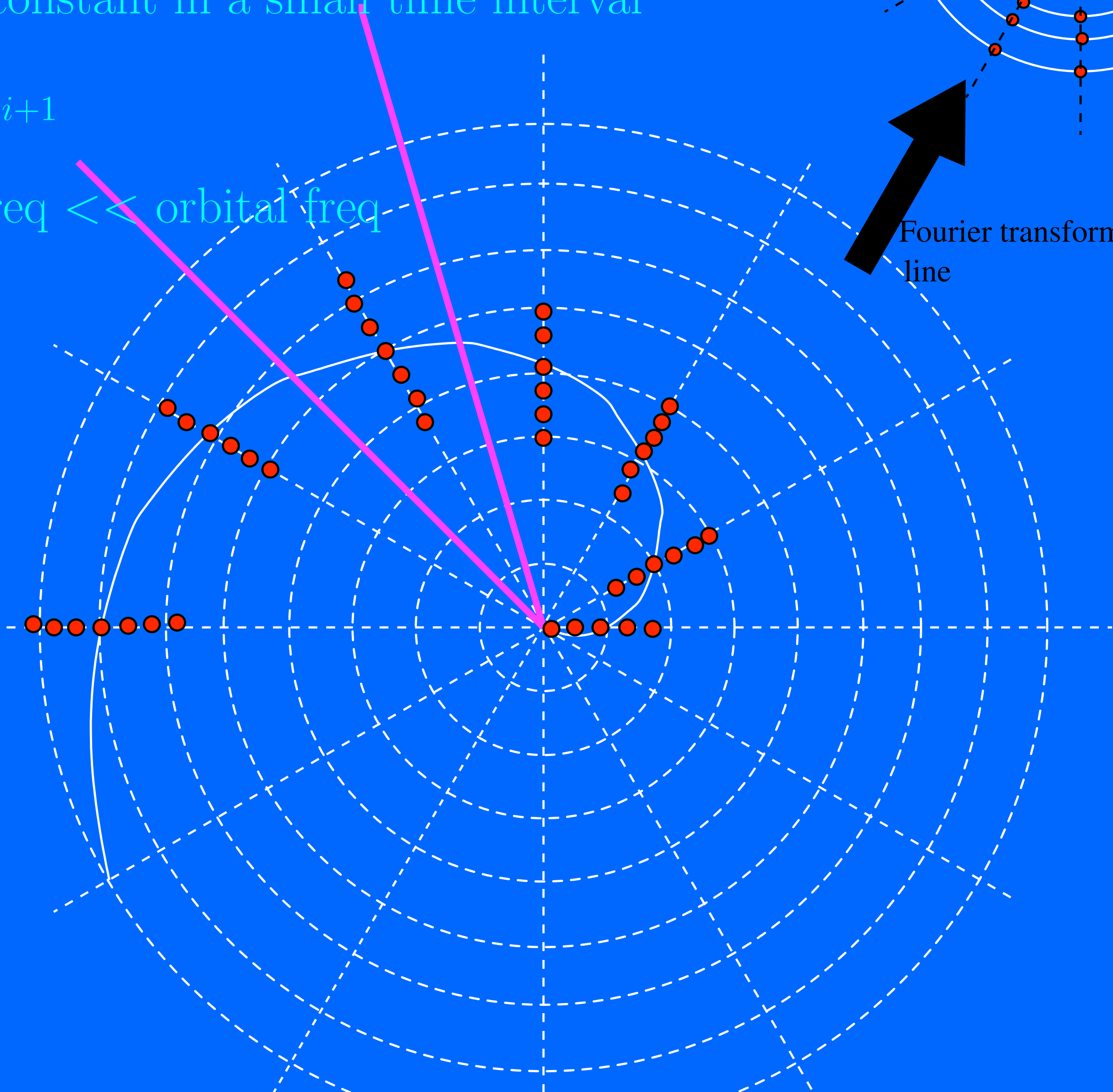
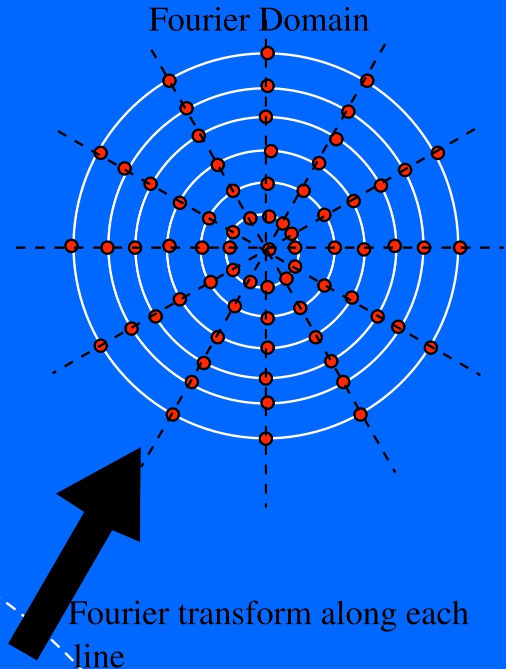




Signal From Binary

When the formalism was first developed we used only one state of Polarization, i.e:

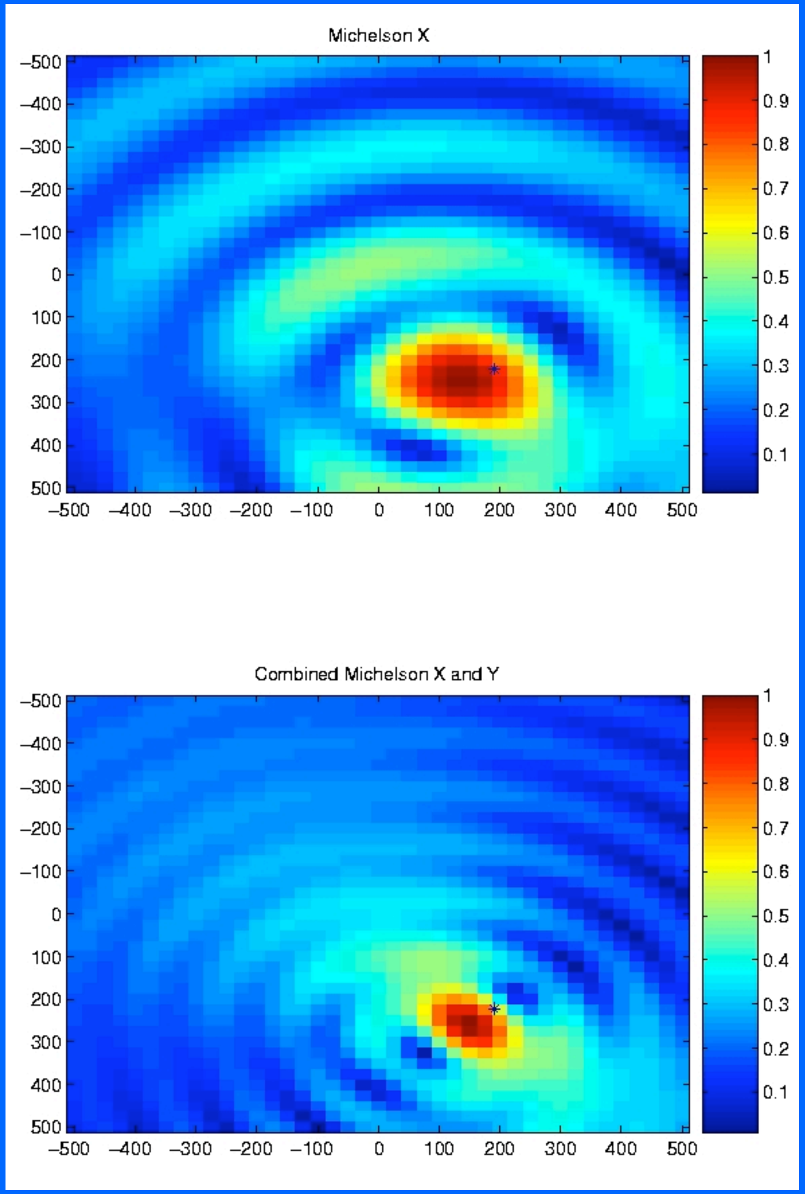
$$h(\theta,\phi,t) = \exp(2\pi i\Omega t)$$

Here we use the realistic source we have:

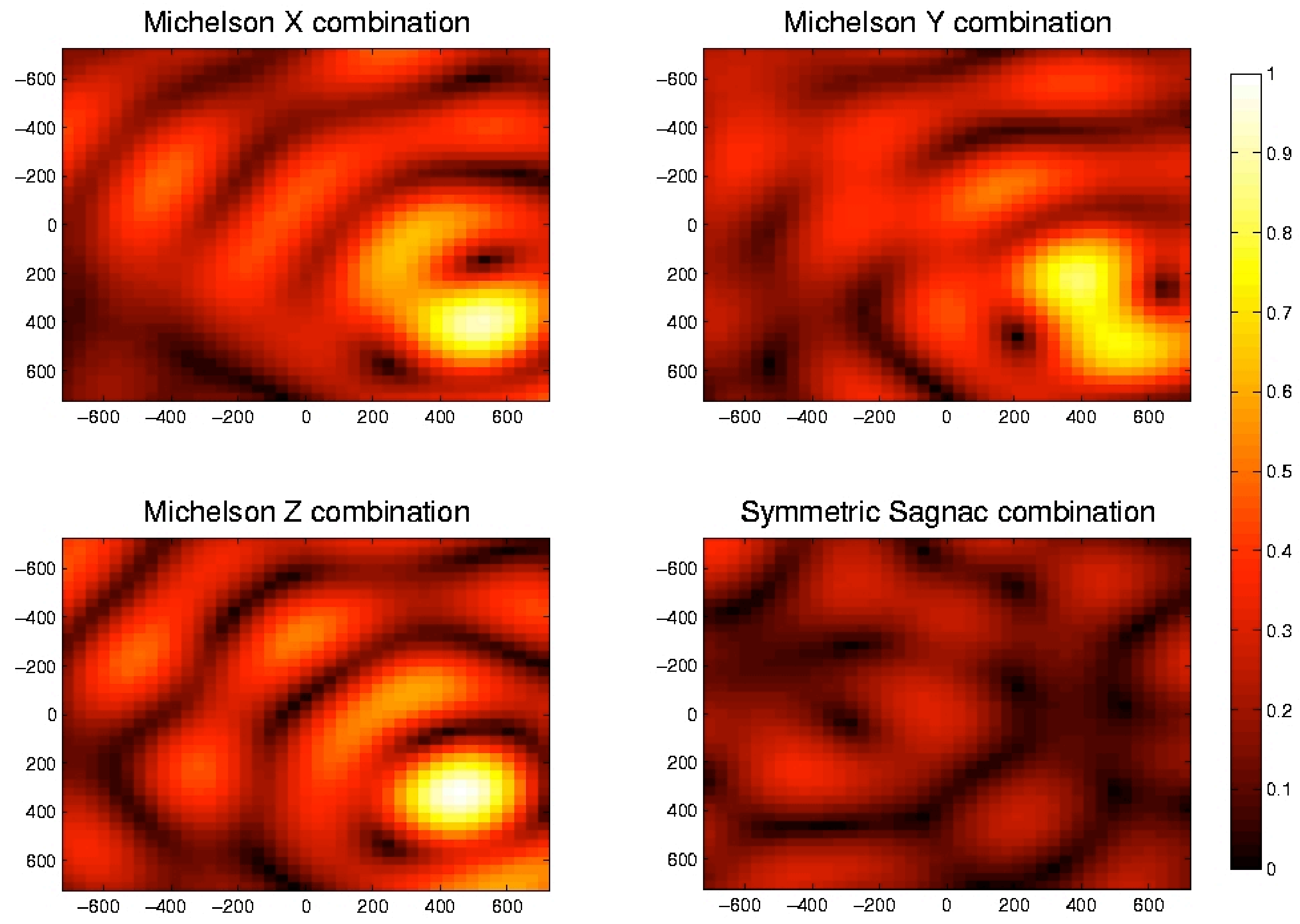
$$h^I(\theta,\phi,t) = h_+(\theta,\phi,t)F_+^I(\theta,\phi,t) + h_\times(\theta,\phi,t)F_\times^I(\theta,\phi,t)$$

- ☞ It can be shown tha the response is still a Radon transform.
- ☞ We can also understand how to generalize to arbitrary detector motion.

Correlating two detector output:

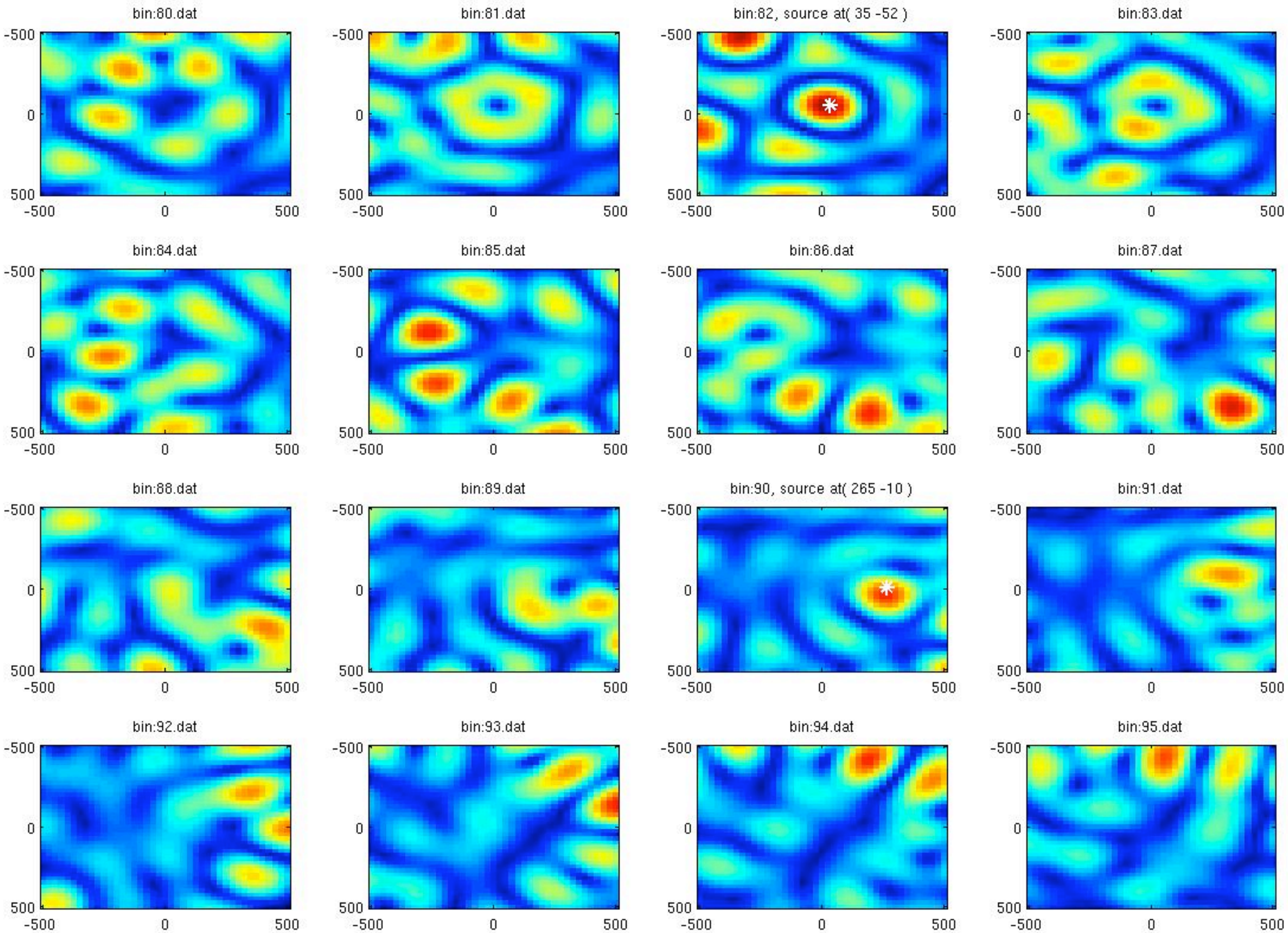


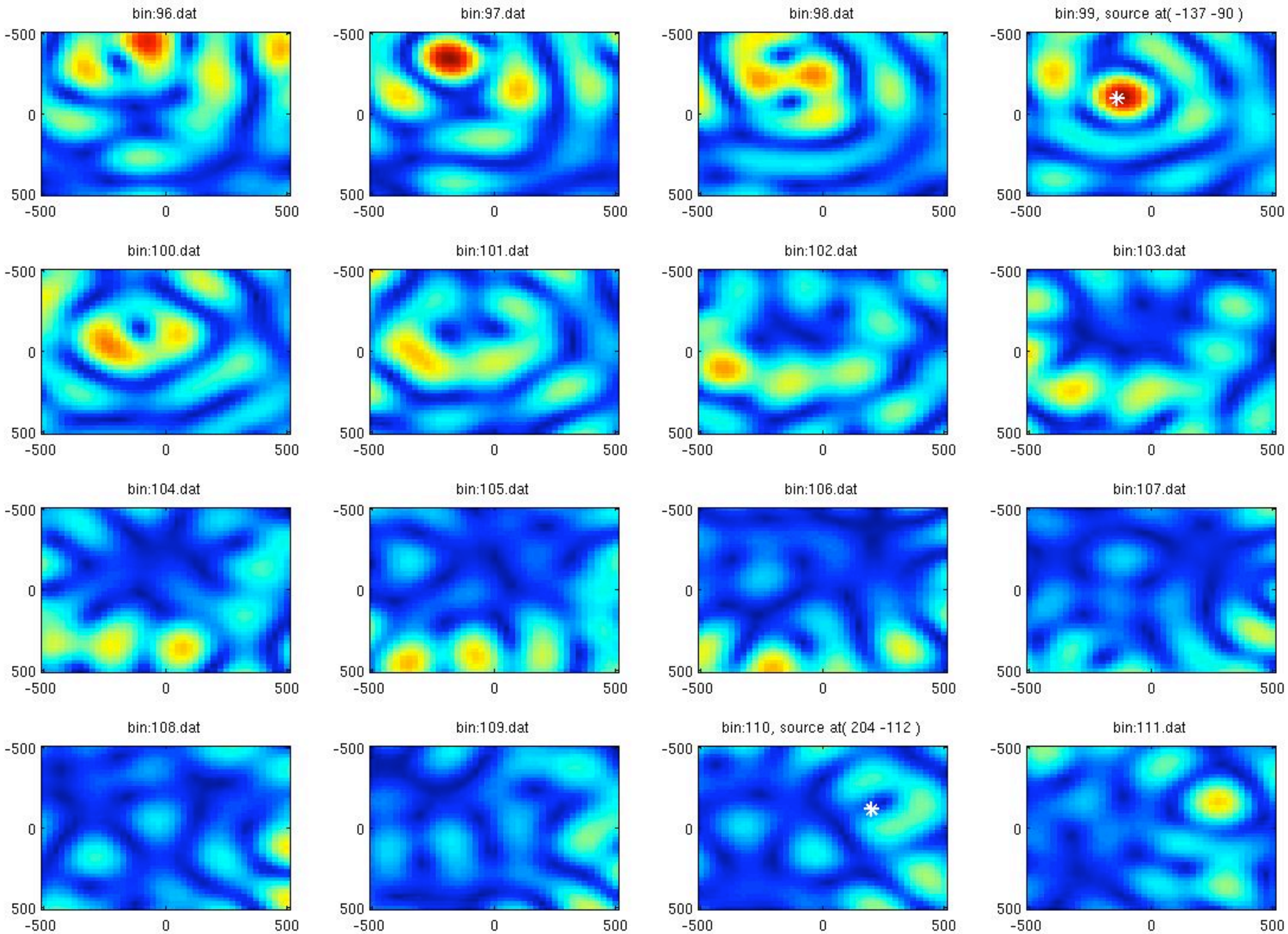
Noise estimation from Symmetric Sagnac



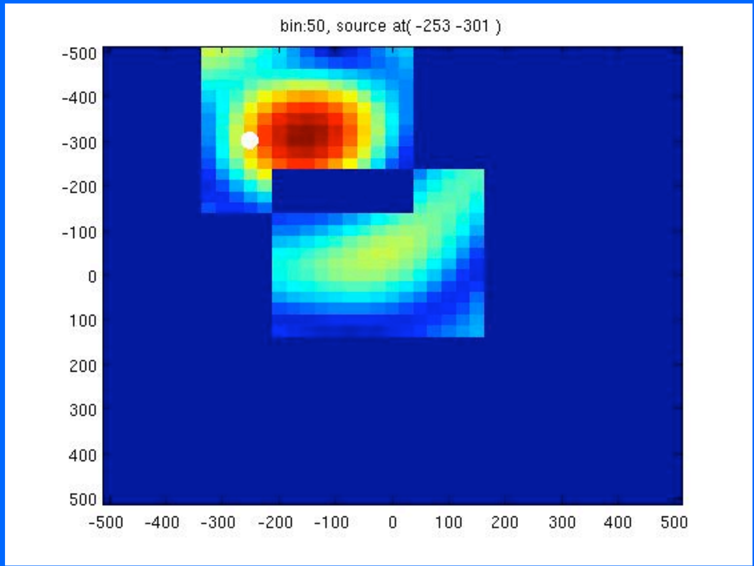
Simulation with a distribution of binaries

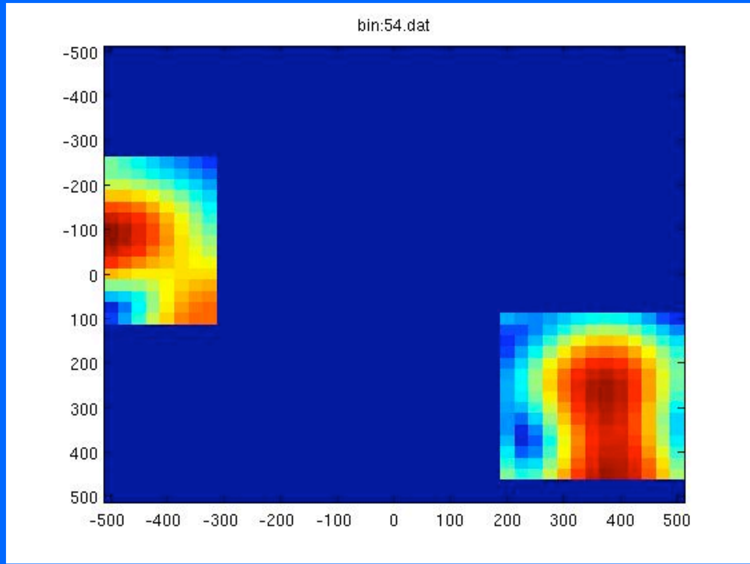
- ☞ Signal is generated for a distribution of binaries with 100 binaries.
- ☞ Sky locations are generated Randomly.
- ☞ distribution along the frequency is generated from 2 mHz with various separations.
- ☞ Reconstruction was done for 200 Frequency bin starting from 2 mHz.


bin:80.dat
bin:81.dat
bin:82, source at(35 -52)
bin:83.dat
bin:84.dat
bin:85.dat
bin:86.dat
bin:87.dat
bin:88.dat
bin:89.dat
bin:90, source at(265 -10)
bin:91.dat
bin:92.dat
bin:93.dat
bin:94.dat
bin:95.dat


bin:96.dat
bin:97.dat
bin:98.dat
bin:99, source at(-137 -90)
bin:100.dat
bin:101.dat
bin:102.dat
bin:103.dat
bin:104.dat
bin:105.dat
bin:106.dat
bin:107.dat
bin:108.dat
bin:109.dat
bin:110, source at(204 -112)
bin:111.dat

Simple detection method:





A modified CLEAN method has been developed Hayama, please see the poster

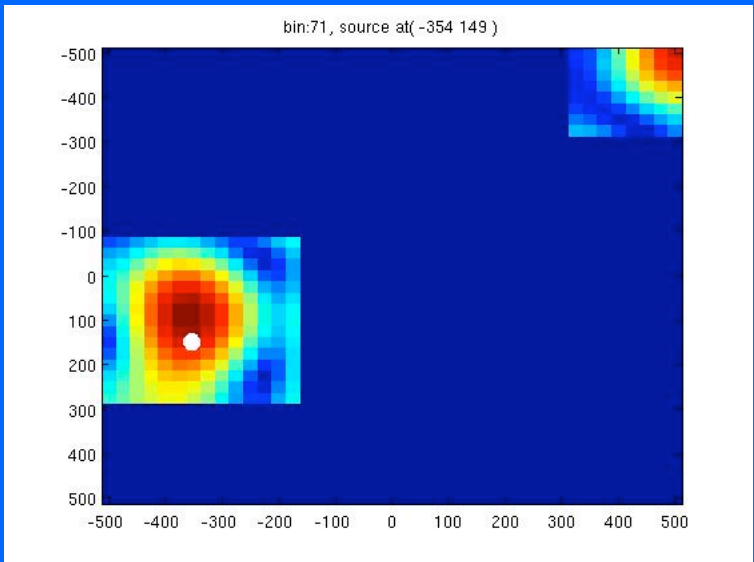
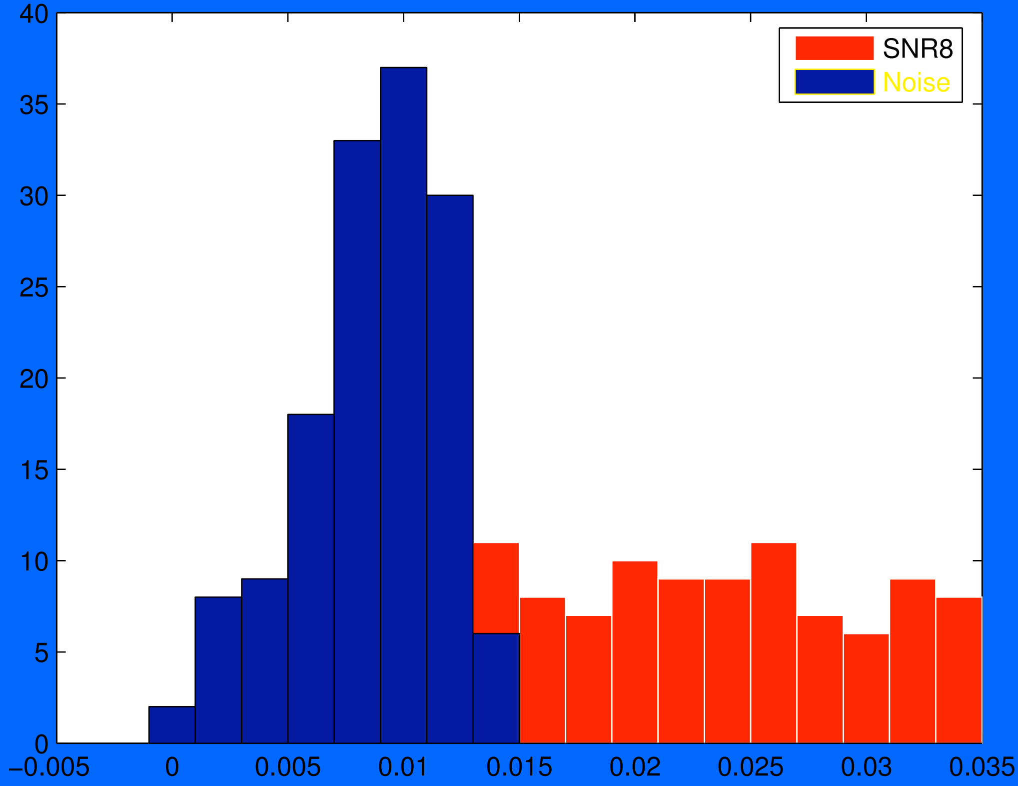
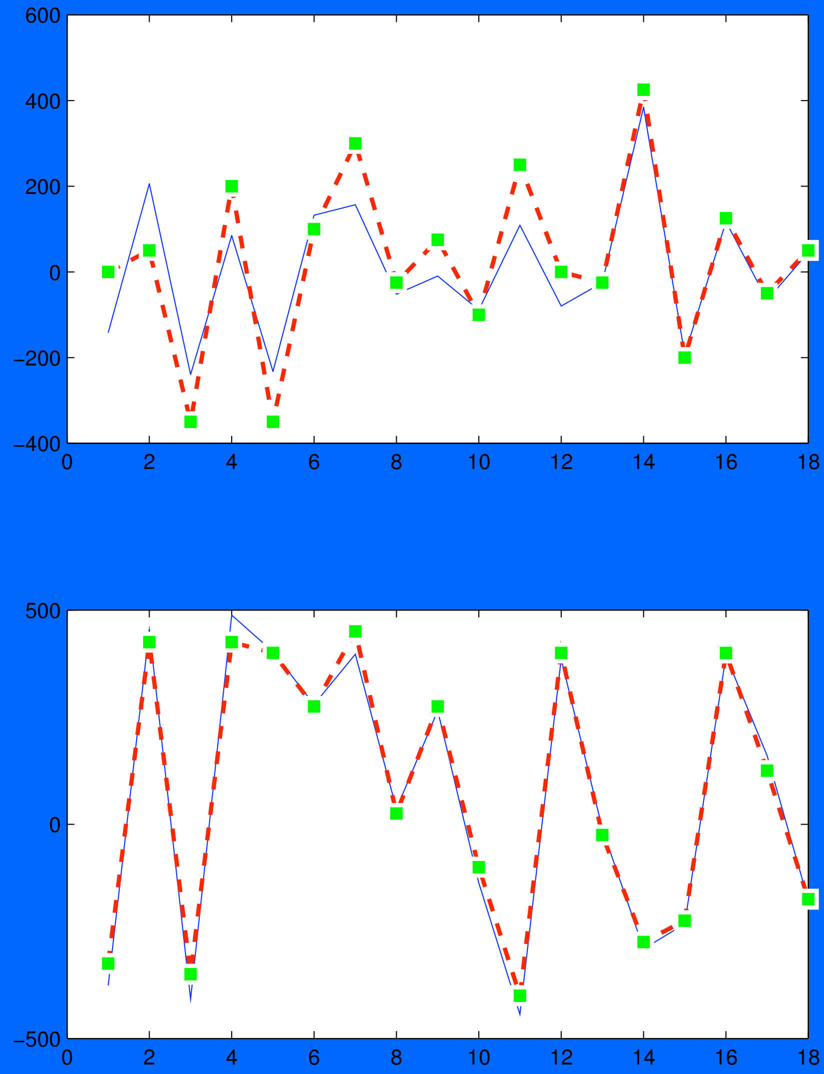


Figure 1: Histogram of reconstructed image around the source location for SNR=8



Error in estimated Sky position:



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☞ Amplitude modulation may be deconvolve from the reconstructed image to obtain the polarization state, work under progress.