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TECHNICAL REPORT:

CALCULATION OF OPTICAL PROPERTIES FOR A VENETIAN BLIND TYPE OF SHADING DEVICE

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1. INTRODUCTION

Venetian is a set of procedures for calculation of optical properties for a Venetian blind type of shading devices. These calculations are performed for ultraviolet (UV), visible (VIS) and near infra-red (NIR) wavelengths (UV/VIS/NIR), jointly referred to as solar or SOL range in this document, and also separately for far infra-red range, referred to as FIR in this document.

The calculations of SOL properties (transmittance, reflectance at the front and at the back side of the blind) are done for the set of incident radiation and outgoing radiation angles, and the results are recorded in two BTDF (forward and back transmittance) and two BRDF matrices (forward and back reflectance). BTDF/BRDF stands for Bi-directional Transmittance/Reflectance Distribution Functions. The calculations in SOL range are done for the discrete wavelengths, whose spacing depends on the quality of the spectral data available.

These matrices are later used in the calculation of the overall glazing system solar-optical properties using procedure detailed in Klems, J.H. 1994a. and Klems, J.H. 1994b. Venetian blind is treated as just one layer in the glazing system, albeit having both specular and diffuse transmission/reflection properties, as opposed to “simple” transparent layer, which is assumed to have purely specular properties.

Venetian also calculates FIR properties of the venetian blind: FIR transmittance, emissivity at the front and at the back side of the blind. These properties are diffuse values. These properties are calculated for the single band comprising entire FIR range from 2.5 μm to 50 μm.

2. INPUT DATA FOR VENETIAN BLINDS PROCEDURE

Input data for Venetian blinds procedure can be divided into four categories: slat geometry data, slat optical properties, angle coordinates and calculation parameters (see Table **1**). Slat geometry data and slat optical properties describe the blind, while angular coordinates define all incident and outgoing directions, which must be the same for each layer in the glazing system. Calculation parameters define which calculation method will be used and whether View Factor (VF) values should be calculated and stored in memory, or previously stored VF values would be used.

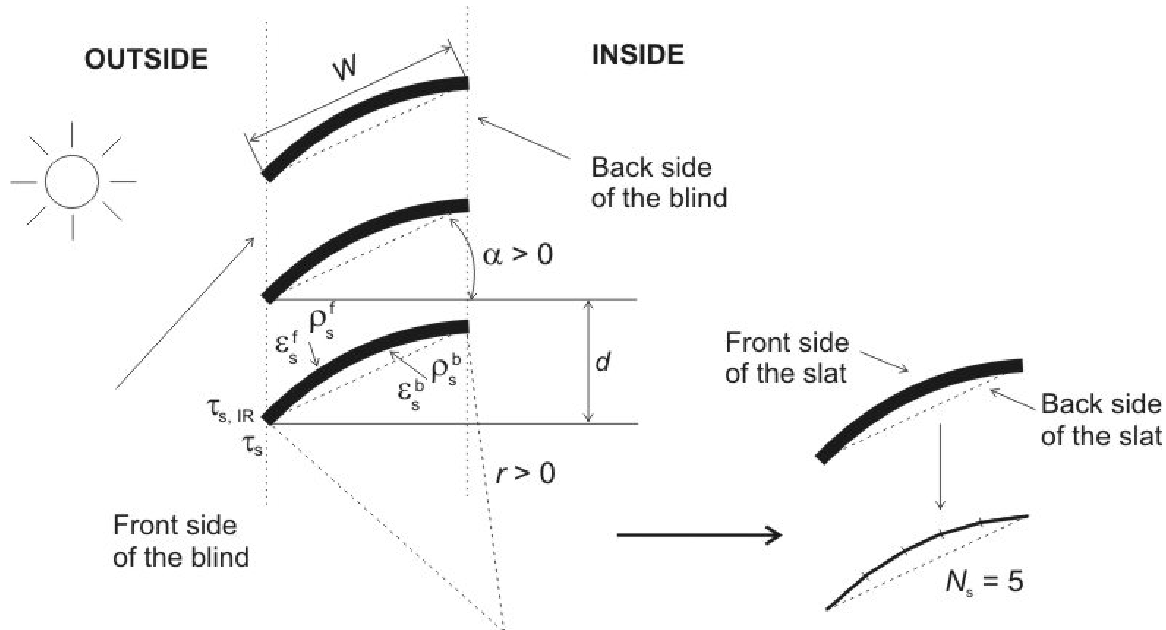

Figure 1. Slat geometry and optical properties

2.1. Slat Geometry Data

Figure **1**. describes slat geometry data which defines the geometry of the Venetian blind. It shows a 2-D cross-section of the slats, perpendicular to the plane of the window. It also shows definitions for "front" and "back" side of the Venetian blind, as well as for "front" and "back" surface of each slat, which will be used throughout this document.

Table **1** shows definitions and ranges for all input parameters needed for Venetian's calculations.

Table 1. Definition of Terms for Venetian Blinds

Type of data	Designator	Description	Unit	Limitations
Slat geometry data	W	Width of the slat (without curvature)	m	$W > 0$
	d	Distance between adjacent slats	m	$d > 0$
	α	Slat tilt angle	°	$-90° \leq \alpha \leq 90°$
	r	Radius of slat curvature	m	$r > W / 2$
	N_{seg}	Number of slat segments		$N_s > 0$
Slat material properties	ρ_s^f	Front reflectance		$0 < \rho_s^f < 1$
	ρ_s^b	Back reflectance		$0 < \rho_s^b < 1$
	τ_s	Transmittance		$0 < \tau_s < 1$
	ε^f	Front emissivity FIR		$0 < \varepsilon^f < 1$
	ε^b	Back emissivity FIR		$0 < \varepsilon^b < 1$
	τ_{IR}	Transmittance FIR		$0 < \tau_{IR} < 1$
Angle coordinates	N_θ	Number of angles θ		$N_\theta > 0$
	θ_i	Array of θ values (i = 1, … , $N_{\theta)}$	°	$0° \leq \theta_i < 90°$
	$N_{\varphi,i}$	Number of φ angles for each θ_i		
	φ_i	Array of φ values for each θ_i	°	$0° \leq \varphi_i < 360°$

Slat tilt angle α has positive values for the slat configuration shown in Figure **1** (dashed line "above horizon"), and negative for configurations in which cross-section of the slat lies below defined horizontal line. Value α = 0 corresponds to horizontal slats (open blind). For describing flat slats, curvature radius r = 0 is used. Curvature radius r can have both positive and negative values. Positive r corresponds to a curved slat, such as the one in Figure **1**, i.e. configuration in which the slats are curved towards the outside, and negative r means that the slats are curved towards the inside. For the purpose of calculations, slats are divided into N_{seg} smaller segments, where more segments provide better accuracy. For curved slats, cross-section is treated as piece-wise linear, with N_{seg} linear segments (see Figure **1**). The optimal number of slats was determined to be N_{seg} = 5, which gives good compromise between accuracy and number of calculations.

2.2. Properties of Slat Material

Slat material properties are divided into two groups: **SOL** properties (used for calculations in SOL range, covering wavelengths from 0.3 μm to 2.5 μm) and **FIR** properties, used for calculations in far infra-red range, covering wavelengths from 2.5 μm to 50 μm).

SOL properties of the slat are transmittance of the slat – τ_s, and reflectance of the slat at the front and back surface of the slat (Figure **1**) – ρ_s^f and ρ_s^b, respectively. SOL properties are normally given for discrete values of wavelengths, providing spectral distribution.

FIR properties are: FIR transmittance of the slat material in FIR range – τ_{IR}, emissivity at the front surface and at the back surface of the slat in FIR range – ε^f and ε^b, respectively. FIR properties are normally prescribed for a single band covering entire FIR range.

2.3. Definition of Angle Coordinates

Figure **2** shows two coordinate systems: *xyz* (we will refer to it as a "forward" system) and $x^r y^r z^r$ ("reversed" or "reflected" system).

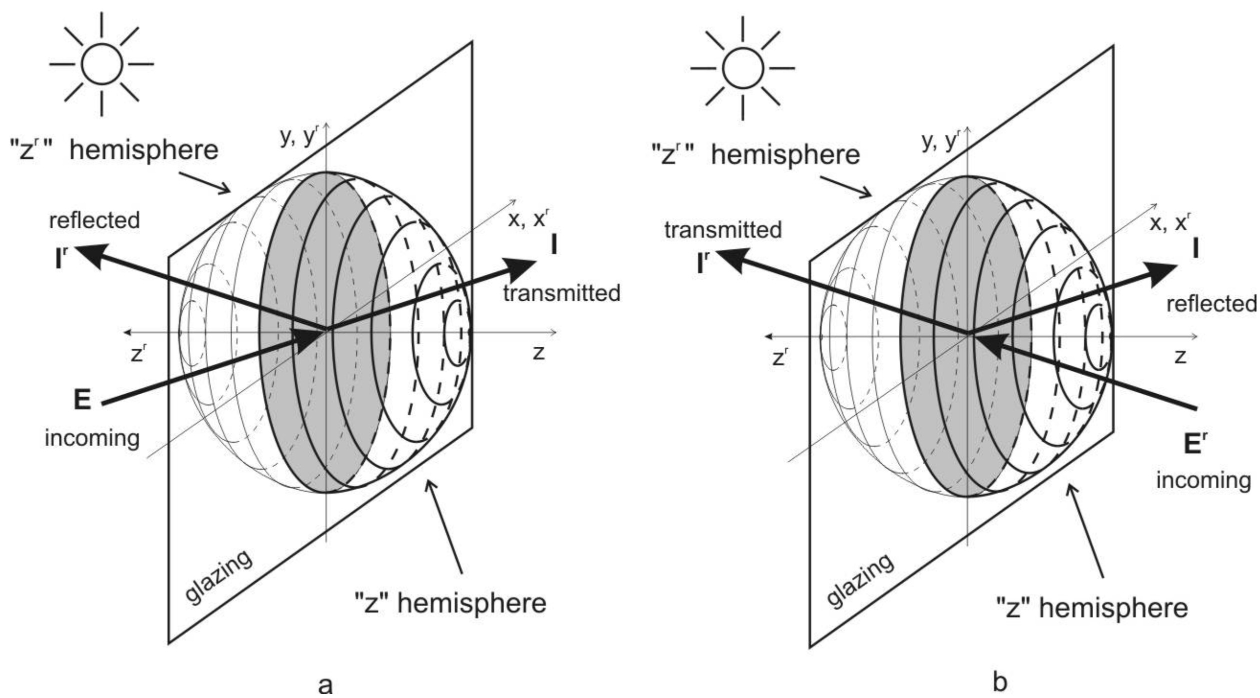

Figure 2. Two hemispheres, with corresponding coordinate systems

Axis *z* points towards the indoor side, while axis z^r points towards the outdoor side. These two axes define 2 hemispheres. All directions that correspond to radiation that travels forward ("left-to-right" with regards to the Venetian blind, where 'left' is outdoor side and 'right' is indoor side) are defined in *xyz* coordinate system, using "*z*" hemisphere (incident and transmitted radiation – *E* and *I* in Figure **2**a, or reflected radiation *I* in Figure **2**b), and directions that correspond to backward-going radiation ("right-to-left" or toward the outdoor side) are defined in the reflected $x^r y^r z^r$ coordinate system, using "z^r" hemisphere (reflected radiation I^r in Figure **2**a, or incident and transmitted radiation E^r and I^r in Figure **2**b).

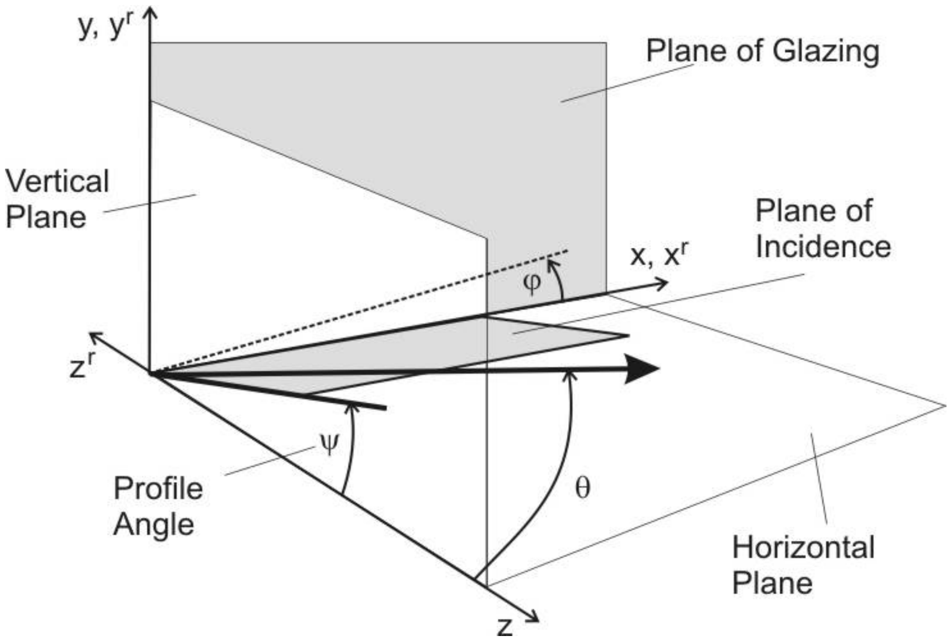

Figure 3. Angles definitions – forward-going light

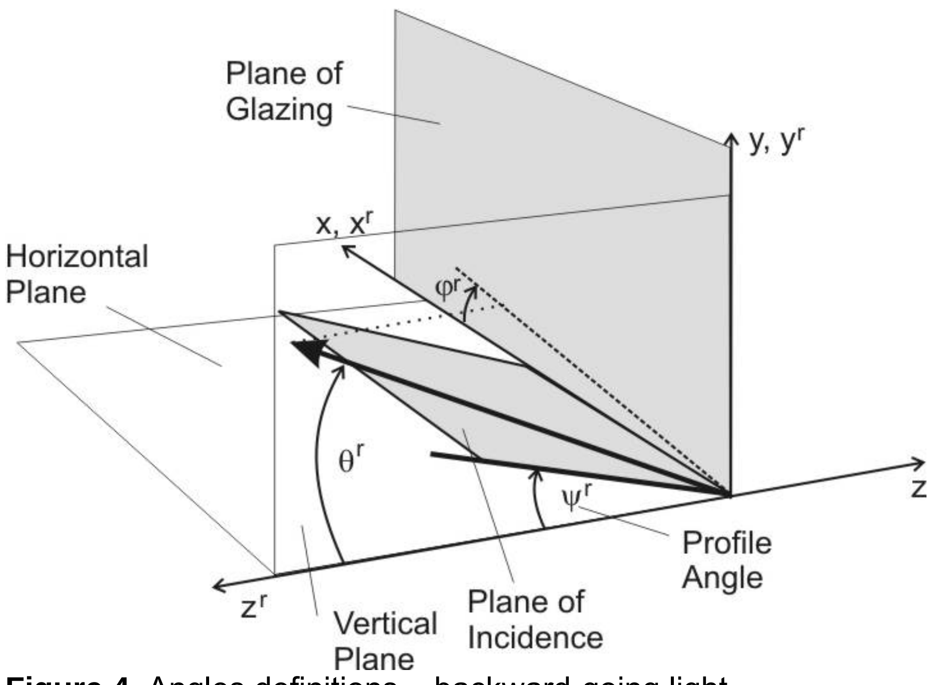

Figure 4. Angles definitions – backward-going light

Directions of interest are defined using angular coordinates θ (latitude angle) and φ (azimuth angle) for forward-going radiation (Figure **3**), and θ^r and φ^r for backward going radiation (Figure **4**). Latitude angle θ is the angle between z axis and the direction of interest, φ is the angle between *x* axis and *x0y* projection of the direction of interest.

Angle θ^r is the angle between z^r axis and the direction of interest, and angle φ^r is in fact equal to φ, because *x* and *y* axes are the same in both coordinate systems. To illustrate this: $\theta = 0°$ lies on the *z* axis; all directions defined as ($\theta > 0°$, $\varphi = 0°$) lie in the horizontal (*x0z*) plane; all directions defined as ($\theta > 0°$, $\varphi = 90°$) lie in the vertical (*y0z*) plane etc. Values of θ and φ are defined within following limits:

$$\begin{aligned} 0° &\le \theta < 90° \\ 0° &\le \varphi < 360° \end{aligned} \qquad (1)$$

Same restrictions apply to θ^r and φ^r.

Figure **5** shows an example with directions of interest in planar projection of the "*z*" hemisphere in *x0y* plane, with *z* axis pointing towards the viewer. The diameters of the circles representing θ angles are growing with θ value. Value of angle φ grows in positive (counter-clockwise) direction. Numbers 1 – 49, shown in Figure 5, correspond to a set of pre-defined bins, defined by 7 θ angles and 8 φ angles. This set of 49 bins had been given only as an example to illustrate the concept, since full angular set consists of 145 bins. In this example, direction 15 (or D_{15}) is defined as ($\theta_{15} = 45°$, $\varphi_{15} = 225°$). For backward directions, Figure **5** can also be used: axes x^r and y^r are oriented the same as axes *x* and *y*, and axis z^r points away from the viewer. In this frame of reference, numbers in Figure **5** correspond to backward-going directions.

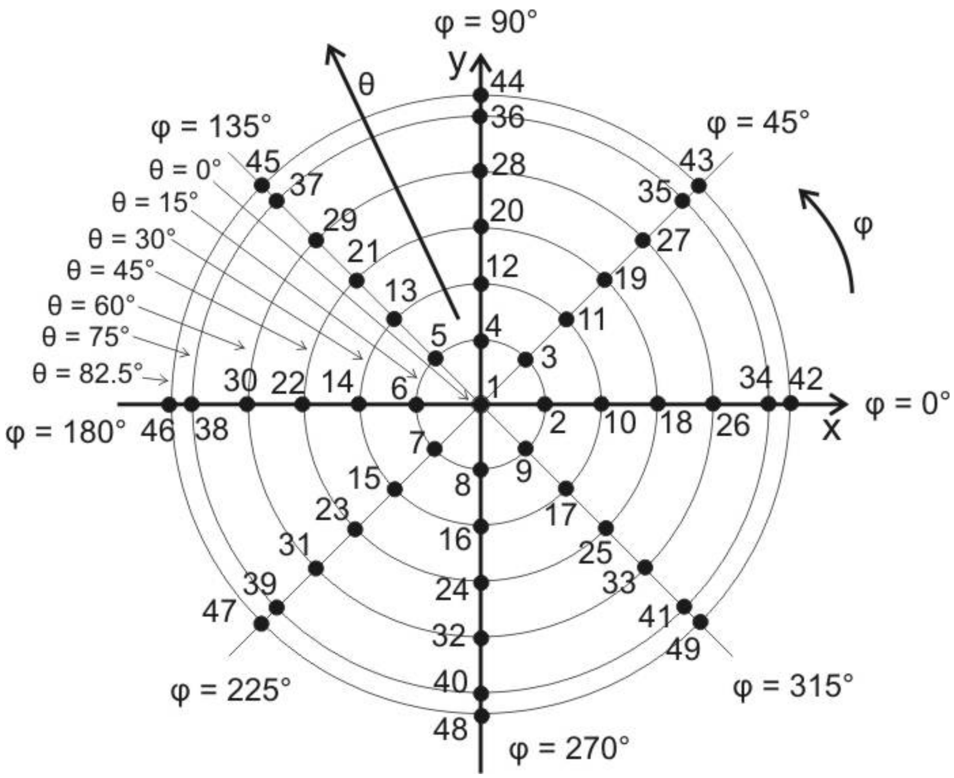

Figure **5.** Projection of "z" hemisphere in *x0y* plane, viewed from the indoor side

Each direction *i*, in forward or reversed coordinates, is determined by its angular coordinates, as (θ_i, φ_i) or $(\theta^r_i, \varphi^r_i)$, where i = 1, … , N_{dir} (number of directions).

The calculations for Venetian blinds are essentially done in 2-D space, ignoring edge effects in x and y directions. Transition from 3D to 2D space is carried out through introduction of profile angles ψ (for forward-going radiation) and ψ^r (for backward-going radiation). Profile angle is the angle between the plane of incidence and horizontal plane, when projected on a vertical plane. Profile angles for forward- and backward-going directions are shown in Figure **3** and Figure **4**, respectively. The connection between angular coordinates and profile angle in general is given by:

$$\tan\psi = \sin\varphi \cdot \tan\theta \qquad (2)$$

One of the consequences of using angular coordinates defined in two coordinate systems as shown in Figure **3** and Figure **4** is that incident directions are being defined in the “z” hemisphere. For example, incident direction coming from the outdoor side from above the horizontal plane will actually be defined in the “z” hemisphere. Therefore, associated profile angle for this direction calculated using equation (2) will have a negative value. For incident radiation, profile angles will be defined as:

$$\begin{aligned} \psi_i &= -\arctan\left(\sin\varphi_i \cdot \tan\theta_i\right) \\ \psi_i^r &= -\arctan\left(\sin\varphi_i^r \cdot \tan\theta_i^r\right) \end{aligned} \qquad (3)$$

where ψ_i is profile angle of the i-th incident direction for forward-going radiation, and ψ_i^r is profile angle of the i-th incident direction, for backward-going radiation.

For outgoing angles, profile angles will be defined as:

$$\begin{aligned} \psi_j &= \arctan\left(\sin\varphi_j \cdot \tan\theta_j\right) \\ \psi_j^r &= \arctan\left(\sin\varphi_j^r \cdot \tan\theta_j^r\right) \end{aligned} \qquad (4)$$

where ψ_j is profile angle of the j-th outgoing direction for forward-going radiation, and ψ_j^r is profile angle of the j-th outgoing direction, for backward-going radiation.

All 2D calculations are performed using these profile angles, and results are stored in resulting bi-directional matrices (BTDF/BRDF matrices) in places that correspond to the particular (θ, φ) bin and its associated profile angle.

Figure **6** shows examples of incident and outgoing profile angles, for forward- and backward-going radiation.

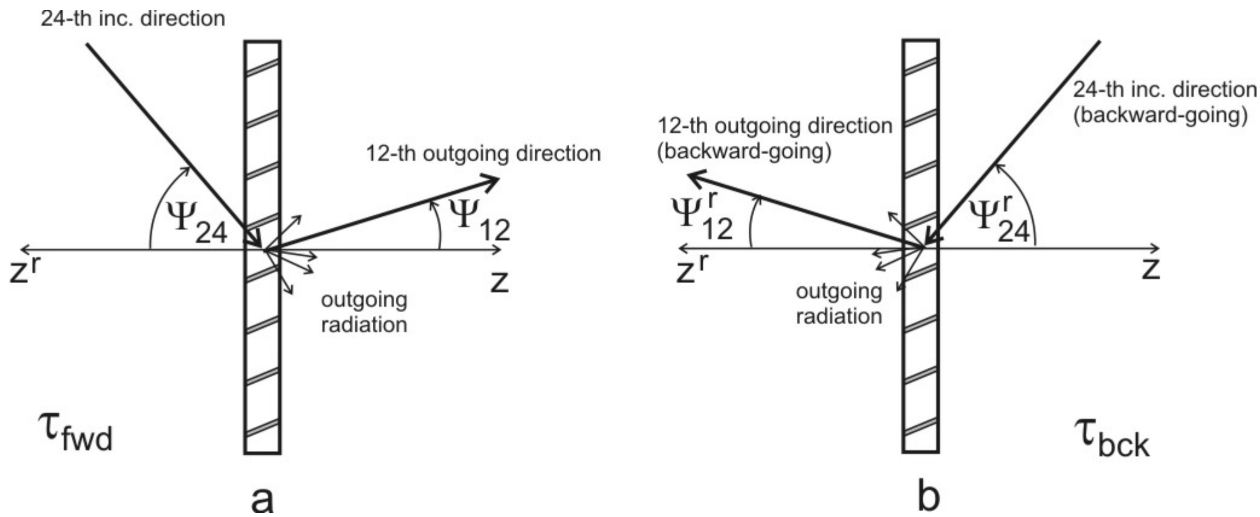

Figure 6. Profile angles for incident and outgoing light *[Note that particular incident and outgoing directions (i.e., 24^{th} incident and 12^{th} outgoing directions) are given for illustrative purposes as an example]*

2.4. Irradiance and Outgoing Radiance in 2-D Space

All incident and outgoing directions are defined in 3-D space using angular coordinates (θ, φ). However, incident irradiance and outgoing radiance can be calculated for actual 3-D directions based on corresponding profile angles, ψ, thus allowing all calculations to be essentially performed in 2-D space, using vertical cross-section of the Venetian blind, as indicated in Section 2.3.

Bi-directional scattered properties are defined with assumed unit irradiance at the plane of glazing (Figure **7**, Entry point). For a given incident direction *i*, irradiance at a flat segment *p* of the slat is given by:

$$E_{p,i} = \frac{1}{\cos\theta_i}\left|\left(\mathbf{s}_i \cdot \mathbf{n}_p\right)\right| \cdot \frac{A'_p}{A_p} \qquad (5)$$

where:

$\mathbf{s}_i$ - unit vector of incident direction *i*

$\mathbf{n}_p$ - normal vector for segment *p*

A_p - area (length in 2D) of segment *p*

A'_p - irradiated area (length in 2D) of segment *p*

$\frac{1}{\cos\theta_i}$ - flux of incident radiation, coming from direction *i*, which results in unit irradiance at the front side of the blind

Vector $\mathbf{s}_j$ is a 3D vector, defined by θ and φ angles of the incident direction *j*. However, the two dot products in equation (5) can be rearranged:

$$\frac{1}{\cos\theta_i}\cdot\left|\left(\mathbf{s}_i\cdot\mathbf{n}_p\right)\right| = \frac{1}{\cos\theta_i}\left(\cos\beta_p\sin\theta_i\sin\varphi_i - \sin\beta_p\cos\theta_i\right) = \\ = \cos\beta_p\tan\theta_i\sin\varphi_i - \sin\beta_p = \cos\beta_p\tan\psi_i - \sin\beta_p \tag{6}$$

where β_p is the angle between segment normal $\mathbf{n}_p$ and y axis (which is equal to tilt angle of segment p with respect to z axis), and ψ_i is the profile angle of incident direction with respect to the z^r axis (Figure **8**). Therefore, equation (5) has an equivalent 2D form:

$$E_{p,i} = (\cos\beta_p\tan\psi_i - \sin\beta_p)\cdot\frac{A'_p}{A_p} \tag{7}$$

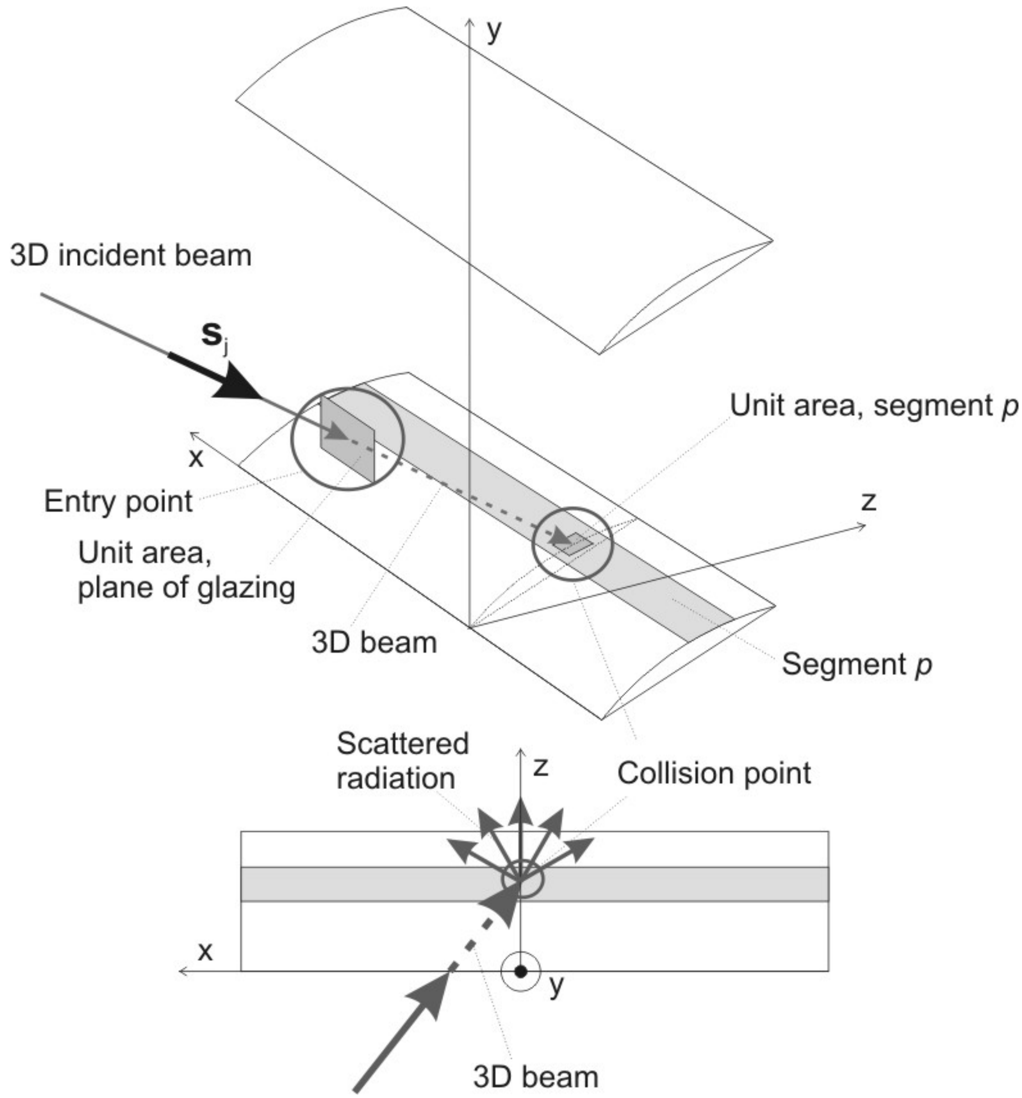

Figure 7. 3D representation and top view of incident radiation beam

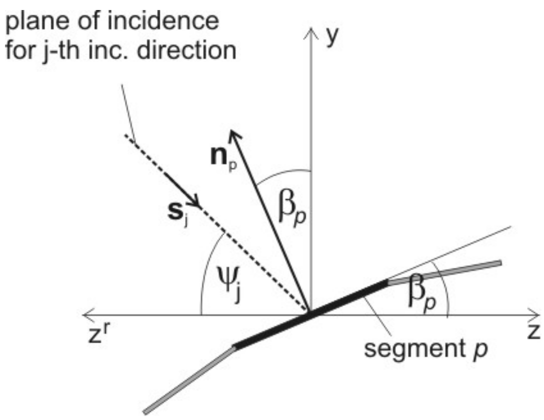

Figure 8. Angles in vertical plane used in calculation of irradiance

An irradiated segment is assumed to emit energy equally in each direction (which is basic assumption in purely diffuse model). For segment *p*, total emitted radiosity is $J_{p,i}$ (Figure **9**). Outgoing radiance, which is a result of incident radiation coming from incident direction *i* is given as $\frac{J_{p,i}}{\pi}$. Radiant intensity $\iota_{p,j}$ (radiant energy transmitted per unit time and per unit solid angle), coming from segment *p* at outgoing direction *j*, is given as:

$$\iota_{p,j} = \frac{J_{p,i}}{\pi} \cdot A_p \cdot \left(\mathbf{s}_j \cdot \mathbf{n}_p\right) \tag{8}$$

Total contribution of this segment to outgoing radiance for outgoing direction *k* with respect to plane of glazing is given by:

$$I_{p,j} = \frac{J_{p,i}}{\pi} \cdot \left(\mathbf{s}_j \cdot \mathbf{n}_p\right) \cdot \frac{1}{\cos\theta_j} \tag{9}$$

where:

$\mathbf{s}_j$ - unit vector of outgoing direction *j*

$\mathbf{n}_p$ - normal vector for segment *p*

$J_{p,i}$ - outgoing radiosity emerging from slat segment *p,* as a result of incident radiation from incident direction *i*

A_p - area (length in 2D) of segment *p*

θ_j - latitude angle of outgoing direction *j*

As shown in the case of irradiance (Eq. 6), this formula can be rearranged into a 2D form:

$$I_{p,j} = \frac{J_{p,i}}{\pi} \cdot \left(\cos\beta_p \tan\psi_j - \sin\beta_p\right) \tag{10}$$

where β_p is the angle between segment normal $\mathbf{n}_p$ and *y* axis (or tilt angle of segment *p*), and ψ_j is the profile angle of outgoing direction *j* with respect to *z* axis (Figure **9**).

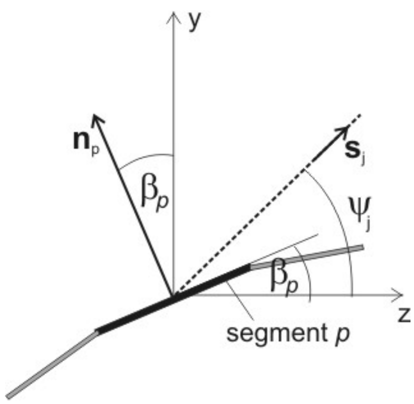

Figure 9. Angles in vertical plane used in calculation of outgoing radiance

2D formulas for calculation of incident irradiances and outgoing radiances are significant because they allow reduction of calculation iterations, as it is sufficient to carry out a single calculation for all incident/outgoing directions that have the same combination of profile angles.

3. MATHEMATICAL MODELS FOR CALCULATION OF SOLAR-OPTICAL PROPERTIES OF A VENETIAN BLIND LAYER

This chapter provides information about mathematical models used in calculating solar-optical properties of Venetian blinds. Two different models are presented in this section; a) ISO 15099, Section 7 (ISO 15099) method and b) Bi-Directional method.

ISO 15099 method calculates integrated properties, transmittance, front reflectance and back reflectance. Bi-Directional method calculates transmittance and front and back reflectance for each incident and outgoing angle (when integrated over all angles, Bi-Directional method should produce same integrated values as ISO 15099 method). In FIR range, which will be presented in Chapter 4, both methods calculate integrated IR transmittance and front and back emissivity for a single band comprising entire FIR range.

3.5. Basic Naming Conventions

Quantities, symbols, directions and their notation are presented in Table **2**. Directions of interest are shown in Figure **10**

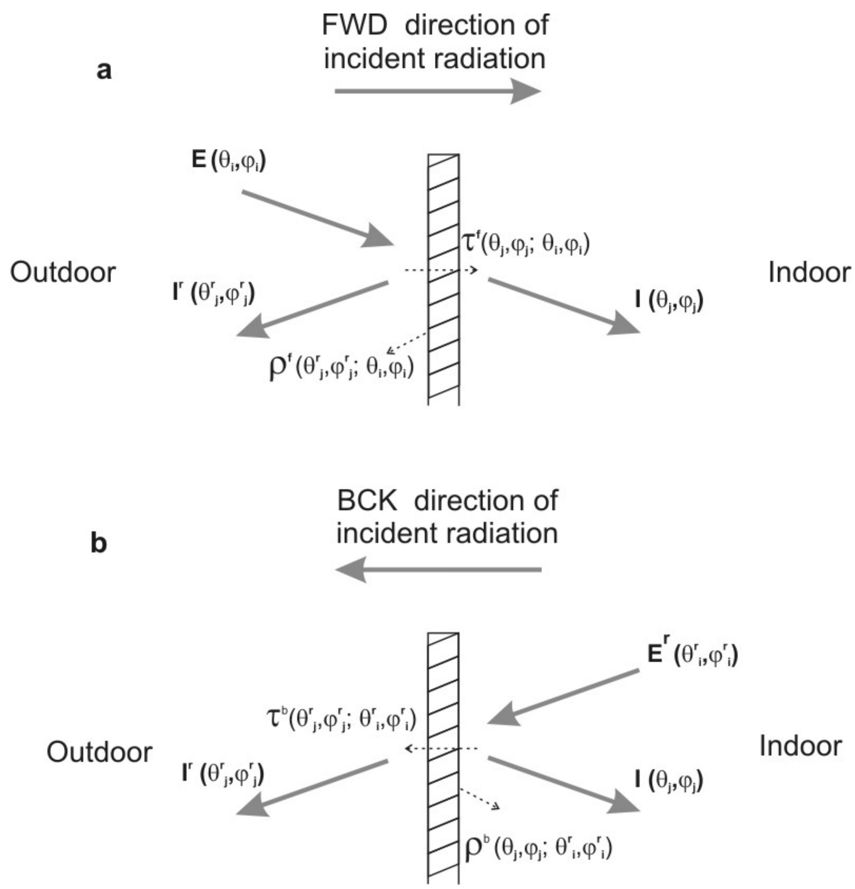

Figure 10. Incident irradiance, reflected and transmitted radiance, for 2 directions of incident radiation: a) forward-going (FWD) and b) backward-going (BCK)

Table 2. Symbols and indexes used

Symbol (index):	Meaning:
r	"in reversed coordinates", used for vector quantities defined in $x^r y^r z^r$ system (in z^r hemisphere)
f	index indicating front side of the slat segment and/or Venetian blind
b	index indicating back side of the slat segment and/or Venetian blind
E	1. incident irradiance – for FWD going light only; 2. in ISO model, *E* denotes total irradiance arriving at a slat segment
E^r	incident irradiance in reversed coordinates – for BCK going light only
I	radiance – always in FWD (L2R) direction: - transmitted part of *E* - reflected part of E^r
I^r	radiance – always in BCK (R2L) direction: - transmitted part of E^r - reflected part of *E* ;
J	radiosity

3.6. ISO 15099 Method

ISO 15099, Section 7 outlines methods for calculating optical properties of Venetian blinds (ISO. 2003). These methods are based on following assumptions:

1. Diffuse radiation transmitted or reflected by the solar shading device is assumed to remain diffuse ("dif-dif" transmission/reflection).

2. Beam radiation transmitted or reflected by the solar shading device is considered in two parts:
   - undisturbed part (specular transmission – "dir-dir" transmission)
   - disturbed part, approximated as anisotropic diffuse (Lambertian – "dir-dif" transmission/reflection)

This chapter describes in detail how all three types of blind's optical properties ("dir-dir", "dir-dif" and "dif-dif") are calculated. These methods can be used both for flat and curved Venetian blind slats.

3.6.1. Dir-Dir Portion

Dir-Dir transmittance is defined as portion of incident beam light that goes undisturbed through the shading device. It can be defined as (Figure **11** – FWD going incident beam):

$$\tau_{dir-dir} = \frac{J_{dir-dir}}{J_{inc}} \tag{11}$$

where J_{inc} is the incident external radiosity, and $J_{dir-dir}$ is part of the incident radiation that reaches the back side (imaginary surface) of the blind without interaction with slat material (i.e., undisturbed part).

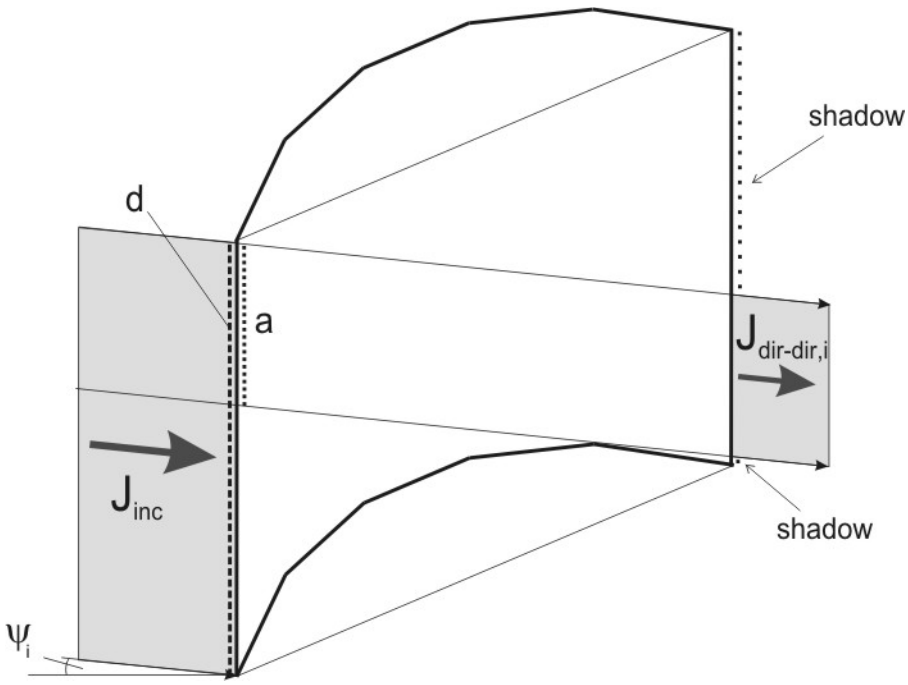

Figure 11. Direct-direct transmission

Dir-Dir portion depends on the profile angle of the incident light beam. Following notation from Figure **11**, Dir-dir portion of transmittance for incident direction *i* can be expressed geometrically as:

$$\tau_{dir-dir,i} = \frac{a}{d} \tag{12}$$

where *d* is the distance between two adjacent slats.

3.6.2. Dif-Dif Portion

Dif-Dif transmittance and reflectance are used in cases where incident and outgoing radiation is diffuse. It can be defined as ratio of outgoing and incident radiant energy. Figure **12** explains Dif-Dif transmittance and reflectance in general, for forward-going incident radiation:

$$\tau_{dif-dif} = \frac{J_{out}}{J_{inc}}, \quad \rho_{dif-dif} = \frac{J^r_{out}}{J_{inc}} \tag{13}$$

where J_{inc} is the incident external radiosity, J_{out} and J^r_{out} are portions of incident radiation that leave from the back and the front side of the blind, respectively, after a series of diffuse Lambertian reflections off the slat surfaces.

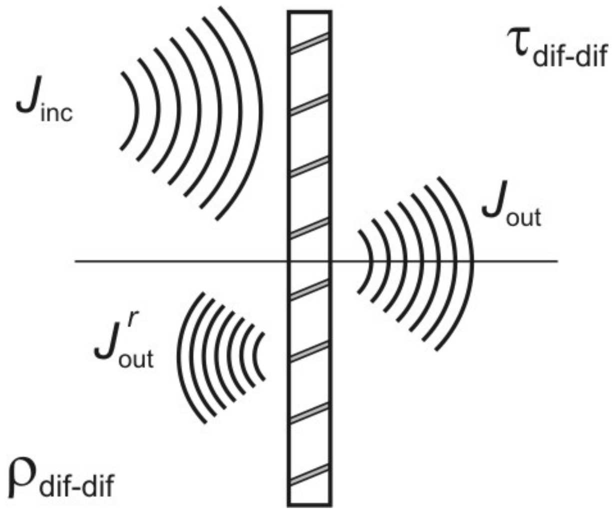

Figure 12. Dif-Dif Propagation

If a single slat enclosure, defined by the two adjacent slats, is considered, diffuse transmittance at the front side can be defined as the ratio of outgoing radiance J_{out}, leaving the back opening of the enclosure, and incident irradiance E reaching the front opening of the enclosure (Figure **13**). If unit irradiance at the front side of the blind is assumed, transmittance becomes equal to outgoing radiance, leaving the back opening.

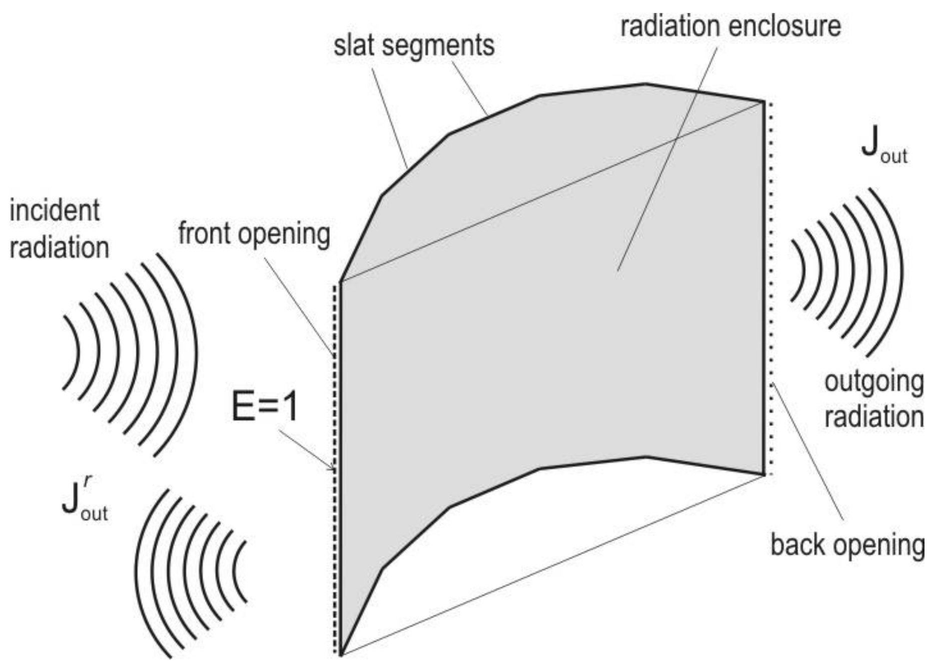

Figure 13. Radiation enclosure formed by two adjacent slats

Similarly, diffuse reflectance at the front side can be expressed as J^r_{out}, outgoing radiance, leaving the front opening.

Radiation enclosure shown in Figure **13** (also referred to as “slat enclosure”) is formed by flat segments of the slats, and virtual segments that represent front and back opening. Slat surfaces are assumed to scatter incident radiation diffusely, and have uniform temperatures. For such enclosure, a set of energy balance equations is formed in order to calculate the outgoing radiance at front and back openings.

The procedure is based on a model, shown in Figure **14**, which considers two adjacent slats divided into N_{seg} equal parts (default N_{seg} = 5) and forming "slat enclosure", together with front opening ("segment" 0) and back opening ("segment" N_{seg+1}, i.e. segment 6). Every segment may have different surface optical properties, which is given through different values of surface reflectance and transmittance: $\rho_{f,i}$ and $\rho_{b,i}$ (diffuse reflectance of the material at the front and back side of the slat segment *i*) and τ_i (diffuse transmittance of the material of slat segment *i*). "Special" segments – front and back opening – are defined with zero reflectances and unit transmittance: $\tau_0 = \tau_6 = 1$, $\rho_{f,0} = \rho_{b,0} = \rho_{f,6} = \rho_{b,6} = 0$.

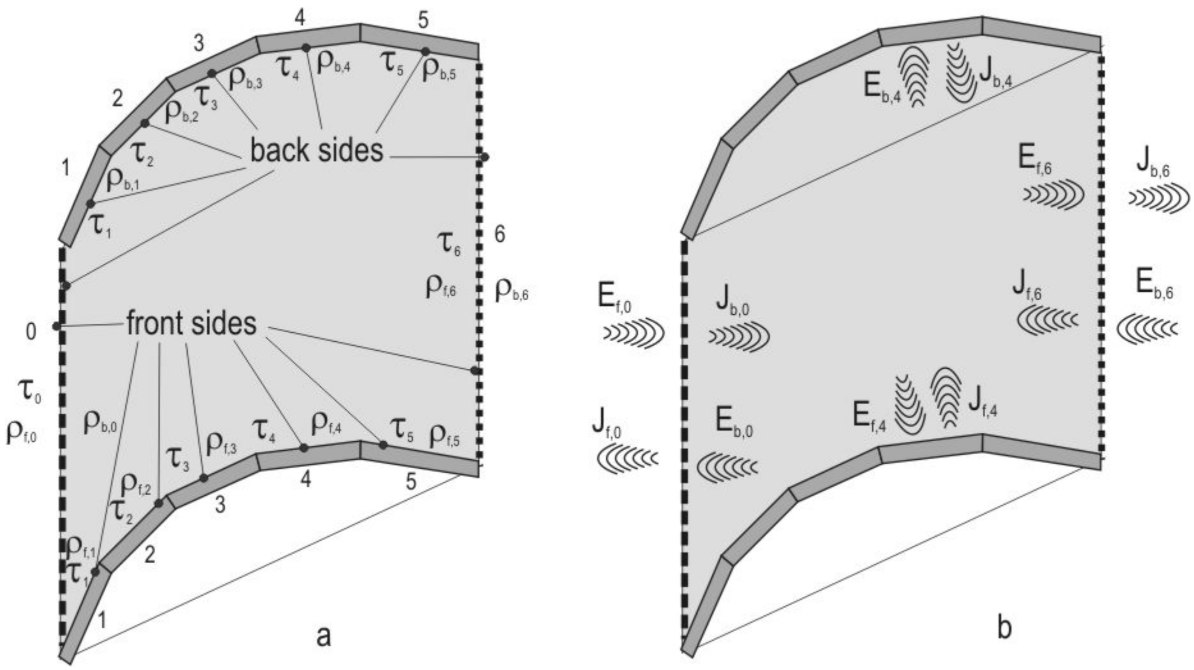

Figure 14. Segments of the enclosure and their indexing

Following naming convention will be used in energy balance equations (Figure **14**):

f – indicates front surface of enclosure segment

b – indicates back surface of enclosure segment

$J_{f,i}$ – radiosity leaving the front surface of enclosure segment *i*

$J_{b,i}$ – radiosity leaving the back surface of enclosure segment *i*

$E_{f,i}$ – irradiance at the front surface of enclosure segment *i*

$E_{b,i}$ – irradiance at the back surface of enclosure segment *i*

Using this notation, the radiant outgoing fluxes (i.e. radiosities) – $J_{f,i}$ and $J_{b,i}$ for each enclosure segment, is given by the following set of equations:

$$J_{f,0} = \rho_{f,0}E_{f,0} + \tau_{b,0}E_{b,0} = E_{b,0} \quad (\rho_{f,0} = 0; \tau_{b,0} = 1) \tag{14}$$

$$J_{f,1} = \rho_{f,1}E_{f,1} + \tau_{b,1}E_{b,1} \tag{15}$$

$$J_{f,2} = \rho_{f,2}E_{f,2} + \tau_{b,2}E_{b,2} \tag{16}$$

$$J_{f,3} = \rho_{f,3}E_{f,3} + \tau_{b,3}E_{b,3} \tag{17}$$

$$J_{f,4} = \rho_{f,4}E_{f,4} + \tau_{b,4}E_{b,4} \tag{18}$$

$$J_{f,5} = \rho_{f,5}E_{f,5} + \tau_{b,5}E_{b,5} \tag{19}$$

$$J_{f,6} = \rho_{f,6}E_{f,6} + \tau_{b,6}E_{b,6} = E_{b,6} \quad (\rho_{f,6} = 0\ ;\ \tau_{b,6} = 1) \tag{20}$$

$$J_{b,0} = \rho_{b,0}E_{b,0} + \tau_{f,0}E_{f,0} = E_{f,0} \quad (\rho_{b,0} = 0; \tau_{f,0} = 1) \tag{21}$$

$$J_{b,1} = \rho_{b,1}E_{b,1} + \tau_{f,1}E_{f,1} \tag{22}$$

$$J_{b,2} = \rho_{b,2}E_{f,2} + \tau_{f,2}E_{f,2} \tag{23}$$

$$J_{b,3} = \rho_{b,3}E_{b,3} + \tau_{f,3}E_{f,3} \tag{24}$$

$$J_{b,4} = \rho_{b,4}E_{b,4} + \tau_{f,4}E_{f,4} \tag{25}$$

$$J_{b,5} = \rho_{b,5}E_{b,5} + \tau_{f,5}E_{f,5} \tag{26}$$

$$J_{b,6} = \rho_{b,6}E_{b,6} + \tau_{f,6}E_{f,6} = E_{f,6} \quad (\rho_{b,6} = 0\ ;\ \tau_{f,6} = 1) \tag{27}$$

It should be noted that, for front transmittance and reflectance, irradiance on the back surface of boundary of a shading device from indoor environment – $E_{b,6}$ is set to zero ($E_{b,6}$ = 0) due to lack of radiosity from the internal environment. At the same time, irradiance on the front surface of boundary between shading device and external environment – $E_{f,0}$ is set to 1.

After introducing these assumptions, equations (20) and (21) become:

$$J_{f,6} = 0 \tag{28}$$

$$J_{b,0} = 1 \tag{29}$$

On the other hand, relation between radiosities from each segment and irradiances at any other segment can be defined by means of *view factors*. View factor represents a fraction of energy leaving one surface, that arrives at a second surface.

For surfaces with finite area, view factors are generally defined by:

$$F_{i\to j} = \frac{1}{A_i}\int_{A_i}\int_{A_j}\frac{\cos\theta_i \cos\theta_j}{\pi S^2}\,dA_i dA_j \tag{30}$$

where S is a distance from a point on surface A_j to a point on surface A_i. The angles θ_j and θ_k are measured between the line S and the normal to the surface. $F_{i\to j}$ represents "view factor between segments *i* and *j*". Figure **15** illustrates terms from equation (30),

for two surfaces in a radiation enclosure:

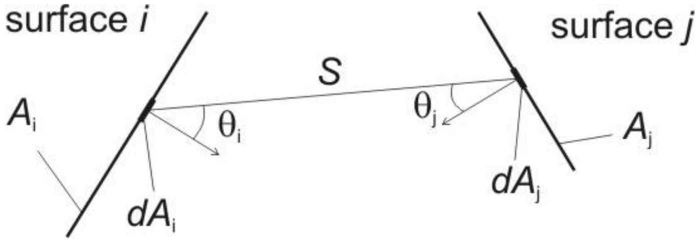

Figure 15. Cross section of two segments in radiation enclosure

View factors between two flat segments in a 2-D radiation enclosure can be calculated based on lengths in a vertical cross-section of the enclosure, according to the *cross-string* rule:

$$F_{i\to j} = \frac{r_{12} + r_{21} - (r_{11} + r_{22})}{2L_i} \tag{31}$$

where L_i is the length of segment *i*, and *r* are distances between endpoints of segments *i* and *j*.

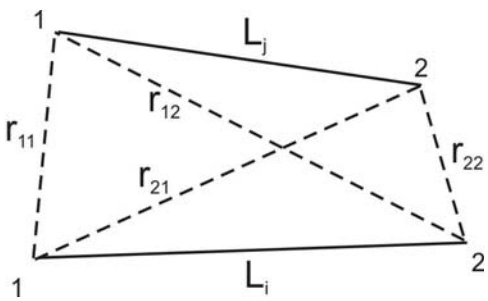

Figure 16. Illustration of the Cross String Rule for Calculating View Factors

Two view factors exist for two given segments *i* and *j*: view factor from *i* to *j* - $F_{i\to j}$, and view factor from *j* to *i* - $F_{i\to j}$. These two values are related as:

$$L_i \cdot F_{i\to j} = L_j \cdot F_{j\to i} \tag{32}$$

In case there is a shadowing, caused by a third, blocking surface (as in Figure **17**), view factor between two segments can be calculated by dividing each segment into a finite number *n* of sub-segments. If a pair of sub-segments *p* and *q* (where one sub-segment belongs to segment *i*, and the other belongs to segment *j*) are mutually visible, which means that line r_{pq} (which is a line between midpoints of these sib-segments) does not intersect the blocking segment (Figure **17**a), view factor between *p* and *q* is calculated using equation (30). Otherwise, $F_{p\to q}$ is set to zero (Figure **17**b). This is repeated for each pair of sub-segments belonging to segments *i* and *j*. Finally, view factor between segments *i* and *j* is obtained as:

$$F_{i\to j} = \sum_{p=1}^{n}\sum_{q=1}^{n} F_{p\to q} \tag{33}$$

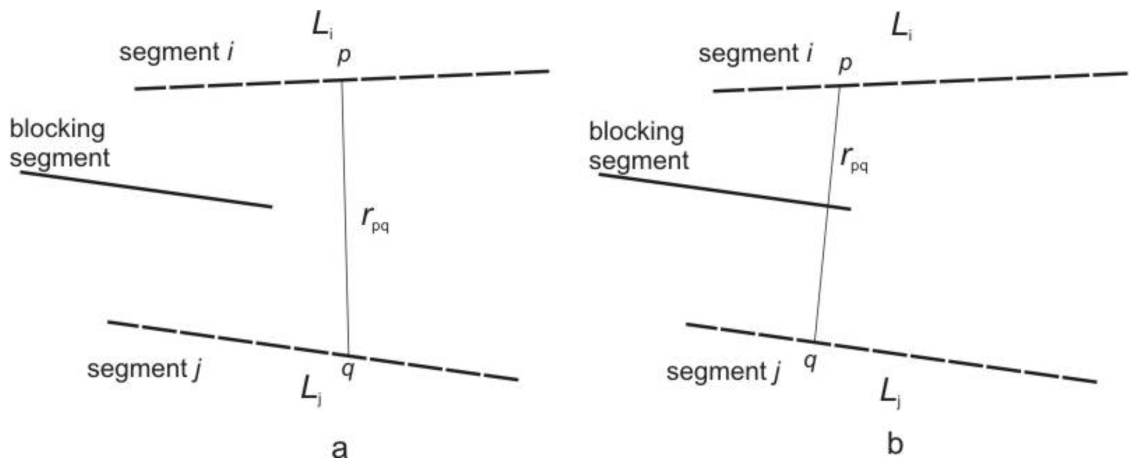

Figure 17. Third surface shadowing: a) “un-shadowed” sub-segments, b) “shadowed” sub-segments

In case of a slat enclosure shown in Figure **14**, view factors between segment surfaces are given as:

$$F_{a,b\to c,d} \tag{34}$$

where *a* and *c* indicate front (f) or back (b) side of each segment, and *b* and *d* are indices of the two segments. For example, view factor between back side of segment 1 of the upper slat, and front side of segment 3 of the lower slat is given as $F_{f,1\to b,3}$. Non-zero view factors exist only between segment surfaces that are facing the interior of the enclosure. View factors from the front side of segment 0 (front opening segment) to any other segment of the enclosure are zero: $F_{f,0\to f,i} = F_{f,0\to b,i} = 0$. Same applies to the back side of segment 6 (back opening segment): $F_{b,6\to f,i} = F_{b,6\to b,i} = 0$.

After introducing view factors, irradiances – $E_{f,i}$ and $E_{b,i}$ can be defined as:

$$E_{f,0} = 1 \tag{35}$$

$$E_{f,1} = J_{f,0}F_{f,0\to f,1} + J_{f,1}F_{f,1\to f,1} + \ldots + J_{f,6}F_{f,6\to f,1} + J_{b,0}F_{b,0\to f,1} + J_{b,1}F_{b,1\to f,1} + \ldots + J_{b,6}F_{b,6\to f,1} \tag{36}$$

$$E_{f,2} = J_{f,0}F_{f,0\to f,2} + J_{f,1}F_{f,1\to f,2} + \ldots + J_{f,6}F_{f,6\to f,2} + J_{b,0}F_{b,0\to f,2} + J_{b,1}F_{b,1\to f,2} + \ldots + J_{b,6}F_{b,6\to f,2} \tag{37}$$

$$E_{f,3} = J_{f,0}F_{f,0\to f,3} + J_{f,1}F_{f,1\to f,3} + \ldots + J_{f,6}F_{f,6\to f,3} + J_{b,0}F_{b,0\to f,3} + J_{b,1}F_{b,1\to f,3} + \ldots + J_{b,6}F_{b,6\to f,3} \tag{38}$$

$$E_{f,4} = J_{f,0}F_{f,0\to f,4} + J_{f,1}F_{f,1\to f,4} + \ldots + J_{f,6}F_{f,6\to f,4} + J_{b,0}F_{b,0\to f,4} + J_{b,1}F_{b,1\to f,4} + \ldots + J_{b,6}F_{b,6\to f,4} \tag{39}$$

$$E_{f,5} = J_{f,0}F_{f,0\to f,5} + J_{f,1}F_{f,1\to f,5} + \ldots + J_{f,6}F_{f,6\to f,5} + J_{b,0}F_{b,0\to f,5} + J_{b,1}F_{b,1\to f,5} + \ldots + J_{b,6}F_{b,6\to f,5} \tag{40}$$

$$E_{f,6} = J_{f,0}F_{f,0\to f,6} + J_{f,1}F_{f,1\to f,6} + \ldots + J_{f,6}F_{f,6\to f,6} + J_{b,0}F_{b,0\to f,6} + J_{b,1}F_{b,1\to f,6} + \ldots + J_{b,6}F_{b,6\to f,6} \tag{41}$$

$$E_{b,0} = J_{b,0}F_{b,0\to b,0} + J_{b,1}F_{b,1\to b,0} + \ldots + J_{b,6}F_{b,6\to b,0} + J_{f,0}F_{f,0\to b,0} + J_{f,1}F_{f,1\to b,0} + \ldots + J_{f,6}F_{f,6\to b,0} \tag{42}$$

$$E_{b,1} = J_{b,0}F_{b,0\to b,1} + J_{b,1}F_{b,1\to b,1} + \ldots + J_{b,6}F_{b,6\to b,1} + J_{f,0}F_{f,0\to b,1} + J_{f,1}F_{f,1\to b,1} + \ldots + J_{f,6}F_{f,6\to b,1} \tag{43}$$

$$E_{b,2} = J_{b,0}F_{b,0\to b,2} + J_{b,1}F_{b,1\to b,2} + \ldots + J_{b,6}F_{b,6\to b,2} + J_{f,0}F_{f,0\to b,2} + J_{f,1}F_{f,1\to b,2} + \ldots + J_{f,6}F_{f,6\to b,2} \tag{44}$$

$$E_{b,3} = J_{b,0}F_{b,0\to b,3} + J_{b,1}F_{b,1\to b,3} + \ldots + J_{b,6}F_{b,6\to b,3} + J_{f,0}F_{f,0\to b,3} + J_{f,1}F_{f,1\to b,3} + \ldots + J_{f,6}F_{f,6\to b,3} \quad (45)$$

$$E_{b,4} = J_{b,0}F_{b,0\to b,4} + J_{b,1}F_{b,1\to b,4} + \ldots + J_{b,6}F_{b,6\to b,4} + J_{f,0}F_{f,0\to b,4} + J_{f,1}F_{f,1\to b,4} + \ldots + J_{f,6}F_{f,6\to b,4} \quad (46)$$

$$E_{b,5} = J_{b,0}F_{b,0\to b,5} + J_{b,1}F_{b,2\to b,5} + \ldots + J_{b,6}F_{b,6\to b,5} + J_{f,0}F_{f,0\to b,5} + J_{f,1}F_{f,1\to b,5} + \ldots + J_{f,6}F_{f,6\to b,5} \quad (47)$$

$$E_{b,6} = 0 \quad (48)$$

The system of equations (35) – (48) defines irradiances ($E_{f,i}$ and $E_{b,i}$), in term of radiosities and associated view factors.

General expressions for the irradiances $E_{f,i}$ and $E_{b,i}$ can be derived from the energy balance, shown in equations (36) – (47):

$$E_{f,i} = \sum_{\substack{k=0 \\ k\neq i}}^{N_{seg}+1} J_{f,k}F_{f,k\to f,i} + \sum_{k=0}^{N_{seg}+1} J_{b,k}F_{b,k\to f,i} \quad (49)$$

$$E_{b,i} = \sum_{\substack{k=0 \\ k\neq i}}^{N_{seg}+1} J_{b,k}F_{b,k\to b,i} + \sum_{k=0}^{N_{seg}+1} J_{f,k}F_{f,k\to b,i} \quad (50)$$

where N_{seg} is number of slat segments. Note the exclusion of terms for k=i in left-hand sums in equations (49) and (50). This is done because there is no transfer of radiation if source surface is the same as target surface.

Applying expressions for the radiosities $J_{f,i}$ and $J_{b,i}$, given in equations (14) – (27), and substituting them into equations 36 and 37, the irradiances $E_{f,i}$ and $E_{b,i}$ become:

$$E_{f,i} = \sum_{\substack{k=0 \\ k\neq i}}^{N_{seg}+1} (\rho_{f,k}E_{f,k} + \tau_{b,k}E_{b,k})F_{f,k\to f,i} + \sum_{k=0}^{N_{seg}+1} (\rho_{b,k}E_{b,k} + \tau_{f,k}E_{f,k})F_{b,k\to f,i} \quad (51)$$

$$E_{b,i} = \sum_{\substack{k=0 \\ k\neq i}}^{N_{seg}+1} (\rho_{b,k}E_{b,k} + \tau_{f,k}E_{f,k})F_{b,k\to b,i} + \sum_{k=0}^{N_{seg}+1} (\rho_{f,k}E_{f,k} + \tau_{b,k}E_{b,k})F_{f,k\to b,i} \quad (52)$$

Where the first term in the equation (51) is the amount of radiant energy, transmitted through the back surface of the lower slat's segment *k* and reflected from the front surface of that same segment, while the second term in that equation is the amount of radiant energy, transmitted through the front surface of the upper slat's segment *k* and reflected from the back surface of the same segment, which combined reaches the front surface of the lower slat's segment *i*. Correspondingly, the first term in equation (52) is the amount of radiant energy, transmitted through the front surface of the upper slat's segment *k* and reflected from the back surface of that same segment, while the second term in that equation is the amount of radiant energy, transmitted through the back surface of the lower slat's segment *k* and reflected from the front surface of the same segment, which combined reaches the back surface of upper slat's segment *i*. In the example in Figure **18**, *i* =2 and *k* = 4.

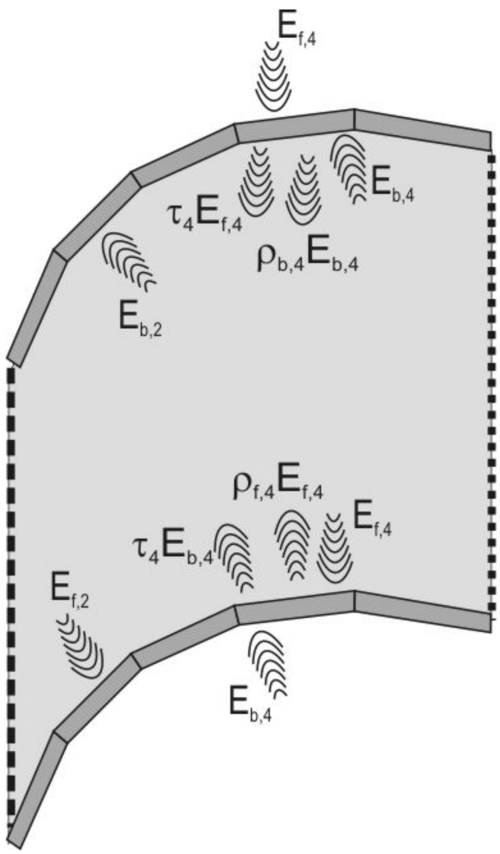

Figure 18. Example of Radiation Exchange Between Segments 2 and 4 (equations 51 and 52)

After solving 2 x (N_{seg} + 1) system of equations, expanded from equations (51) and (52), the resulting front transmittance is calculated as:

$$\tau^f_{dif-dif} = \frac{E_{f,6}}{E_{f,0}} = \frac{E_{f,6}}{1} = E_{f,6} \tag{53}$$

Resulting front reflectance becomes:

$$\rho^f_{dif-dif} = \frac{E_{b,0}}{E_{f,0}} = \frac{E_{b,0}}{1} = E_{b,0} \tag{54}$$

Note that the results in equations (53) and (54) represent Dif-Dif optical properties for FWD case. For BCK case, certain adjustments have to be made. Equations (28) and (29) become:

$$J_{f,6} = J_{ex} = 1 \Rightarrow E_{b,6} = 1 \tag{55}$$

$$J_{b,0} = 0 \tag{56}$$

After solving the system for BCK case, resulting back transmittance becomes:

$$\tau^b{}_{dif-dif} = \frac{E_{b,0}}{E_{b,6}} = \frac{E_{b,0}}{1} = E_{b,0} \tag{57}$$

Resulting BCK reflectance becomes:

$$\rho^{b}{}_{dif-dif} = \frac{E_{f,6}}{E_{b,6}} = \frac{E_{f,6}}{1} = E_{f,6} \tag{58}$$

3.6.3. Dir-Dif Portion

In contrast to the diffuse radiation coming from all directions uniformly in Dif-Dif portion of radiation, presented in section 3.6.2. Dir-Dif means that there is a "beam" incident radiation source that produces diffuse transmitted and reflected radiance (Figure **19**). Note that Dir-Dif properties depend on the profile angle of the incident radiation.

Basically, Dir-Dif properties can be obtained using system of equations defined for Dif-Dif case. However, since now there is a "beam" input instead of diffuse input, certain modifications have to be made – modifications of view factors between front side of the front opening segment ("0th" segment) and all other segments within slat enclosure, in case of forward-going incident radiation (incident radiation coming from the outdoor environment), and back opening segment ("N_{seg}+1st" segment, in the default case, 6th" segment) and all other segments in the slat enclosure, in case of backward-going incident radiation (incident radiation coming from the indoor environment).

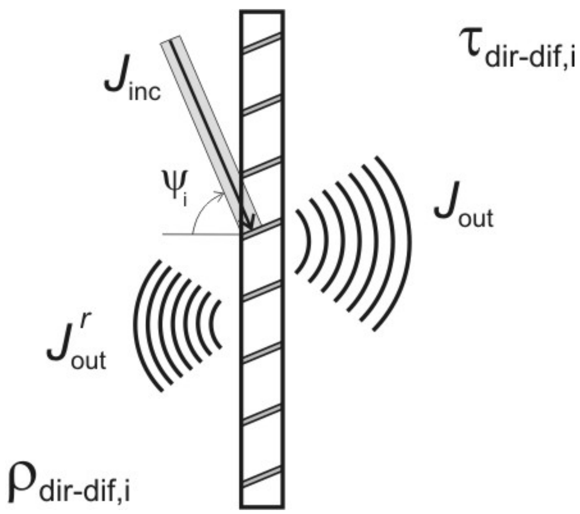

Figure 19. Dir-Dif propagation

As in the case of dif-dif propagation, unit irradiance at the front side of the blind is assumed. View factors, used in Equations (51) and (52), which are used for calculations of Dif-Dif properties will mostly be used here as well since from the moment after direct beam hits any surface, it is reflected fully diffusely. The exception to this are imaginary surfaces 0 and N_{seg}+1 (front and back opening of the blind), since they represent beam radiation source at the outer side and the diffuse view factors between the front surface of segment 0 and other slat segments in case of forward-going incident radiation (or back surface of segment 6 and other slat segments in case of backward-going incident radiation) cannot be used. Instead, it is necessary to determine which part of the incident beam radiation hits each slat segment, and use those values instead of corresponding view factors. We will call these quantities "Beam view factors", or *BF*.

Figure **20**. shows an example of 5 segment slats (default). Incident radiation hits front sides of slat segments 1, 2 and partially 3 (dotted line). *BF* for those surfaces are defined as fractions of incident radiation that hit the segment of interest. In our example, *BF* of aforementioned slat segment surfaces are equal to the ratio of lengths of b1, b2 and b3, respectively (dashed lines) to the slat distance *d*.

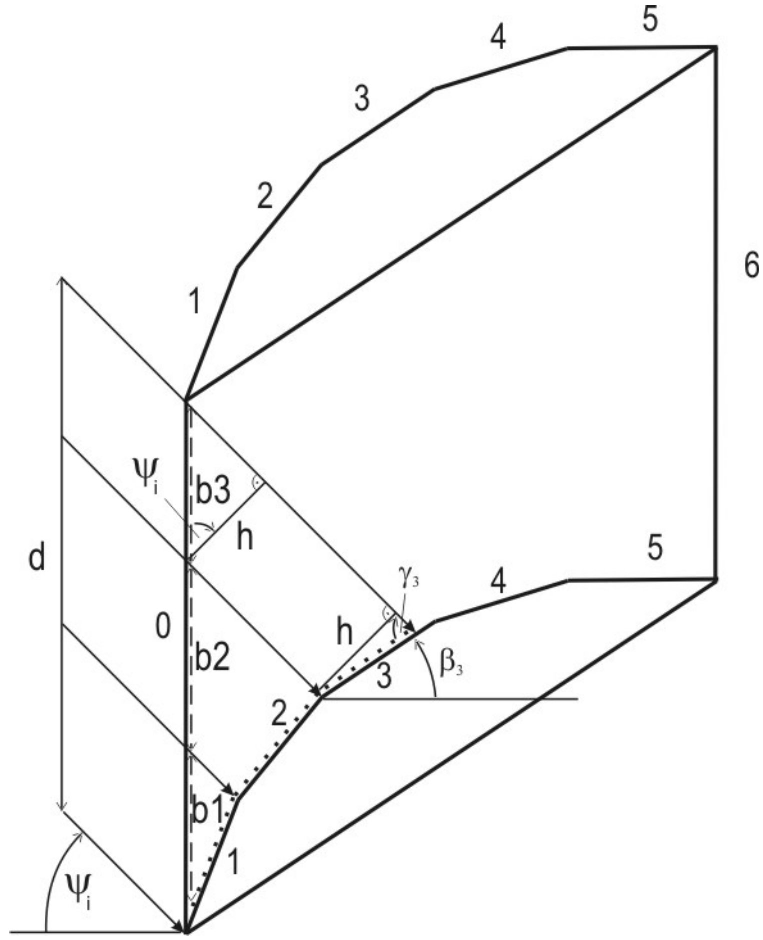

Figure 20. Calculation of Beam view factors

BF for the front surface of segment 3 is given as:

$$BF_{f,3} = \frac{h}{d \cdot \cos\psi_i} \qquad (59)$$

Where:

$$h = A'_3 \cdot \sin\gamma_3 \qquad (60)$$

$$\gamma_3 = \beta_3 + \psi_i \qquad (61)$$

A'_3 – length of segment 3 which is directly irradiated

β_3 – tilt angle of segment 3 (with respect to horizontal plane)

ψ_i – profile angle of incident direction *i*

d – slat spacing

f – indicates front surface of the segment

Finally:

$$BF_{f,3} = \frac{A_3' \cdot \sin(\beta_3 + \psi_i)}{d \cdot \cos\psi_i} \tag{62}$$

*BF*s for front surfaces of segments 1 and 2 ($BF_{f,1}$ and $BF_{f,2}$) can be calculated the same way. The rest of the slat segments shown in Figure **20** are not directly irradiated, so remaining *BF*s (including *BF*s for the back sides of segments 1-5) are equal to zero:

$$BF_{f,4} = BF_{f,5} = BF_{b,1} = BF_{b,2} = BF_{b,3} = BF_{b,4} = BF_{b,5} = 0 \tag{63}$$

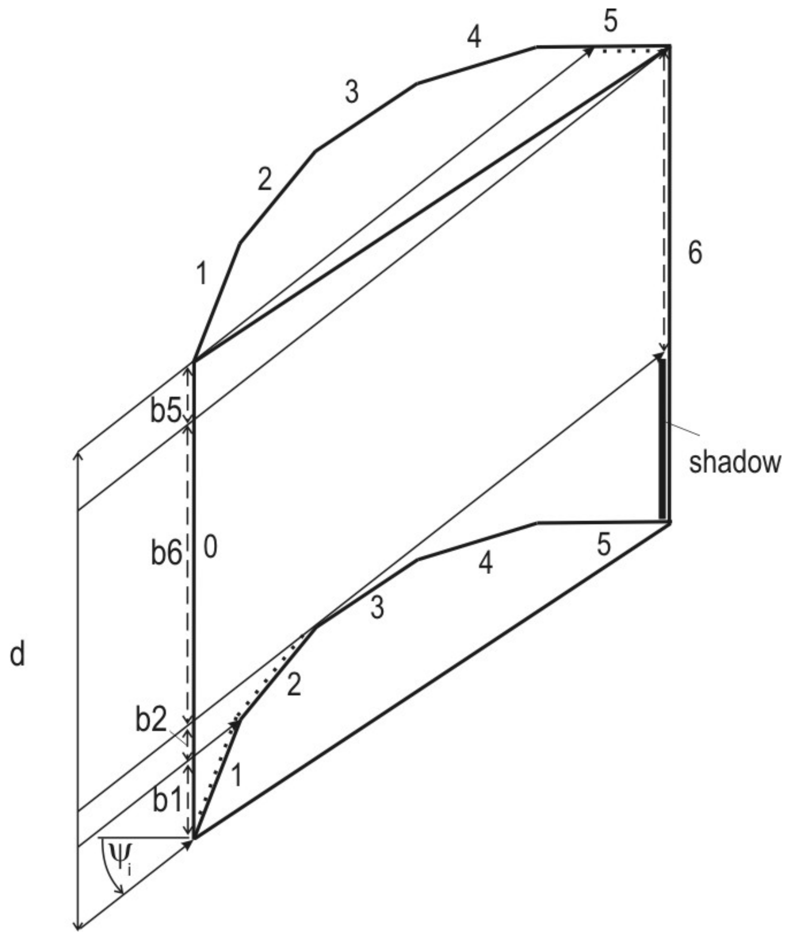

Figure 21. Beam view factors and Dir-Dir transmittance

For extreme negative profile angles of incident light, some of the upper slat segments are irradiated. As a result, some of *BF*s for back surfaces of segments ($BF_{b,\ i}$) will have non-zero values. This is the case shown in Figure **21**: non-zero *BF* are $BF_{f,1}$, $BF_{f,2}$ and $BF_{b,5}$. Note that according to Figure **21**, a part of the incident beam passes through the blind without hitting the slats. Ratio of b6 to slat distance *d* corresponds to dir-dir portion of transmitted light – $\tau_{dir\text{-}dir}$, as defined in section 3.6.1:

$$\tau_{dir-dir} = \frac{b6}{d} \tag{64}$$

Sum of all *BF* and $\tau_{dir,dir}$ must equal 1:

$$\sum_{i=1}^{N_{seg}} BF_{f,i} + \sum_{i=1}^{N_{seg}} BF_{b,i} + \tau_{dir-dir} = 1 \tag{65}$$

After *BF* are calculated for all slat segments using formulas similar to equation (62), in case of forward-going incident light these BF are used to substitute certain view factors *F* in equations (51-52):

$$F_{f,0->f,i} = BF_{f,i} \ , \ F_{f,0->b,i} = BF_{b,i} \quad , \ i = 1,...,N_{seg} \tag{66}$$

As in the case of dif-dif portion, unit irradiance at the front side of the blind is assumed: $E_{f,0}$ = 1, with no incidence at the back side: $E_{b,6}$ = 0. After making these changes, the system of equations is solved for $E_{f,i}$ and $E_{b,i}$. The resulting Dir-Dif transmittance for the front side of the blind is then calculated for *i*-th incident direction as:

$$\tau^{f}{}_{dir-dif,i} = \frac{E_{f,6}}{E_{f,0}} = \frac{E_{f,6}}{1} = E_{f,6} \tag{67}$$

Resulting front-side reflectance becomes:

$$\rho^{f}{}_{dir-dif,i} = \frac{E_{b,0}}{E_{f,0}} = \frac{E_{b,0}}{1} = E_{b,0} \,. \tag{68}$$

In case of backward-going incident light, BF are calculated using the same geometry approach at the back side of the blind. View factors in are then replaced as:

$$F_{b,6->f,i} = BF_{f,i} \ , \ F_{b,6->b,i} = BF_{b,i} \quad , \ i = 1,...,N_{seg} \tag{69}$$

Unit irradiance at the back side of the blind is assumed: $E_{f,0}$ = 0, $E_{b,6}$ = 1. After solving the modified system of equations, dir-dif transmittance for i^r-th incident direction at the back-side is given as:

$$\tau^{b}{}_{dir-dif,i^r} = \frac{E_{b,0}}{E_{b,6}} = \frac{E_{b,0}}{1} = E_{b,0} \tag{70}$$

Back-side reflectance becomes:

$$\rho^{b}{}_{dir-dif,i^r} = \frac{E_{f,6}}{E_{b,6}} = \frac{E_{f,6}}{1} = E_{f,6} \,. \tag{71}$$

3.7. Bi-Directional Method

3.7.1. Introduction

This section presents an overview of the bi-directional properties (transmittances and reflectances) calculation for the optical layer which can be specular and diffuse at the same time (i.e., directly transmitted portion in venetian blind is considered specular, while dif-dif and dir-dif are diffuse). These properties are defined for each combination of incident and outgoing directions, and they form BTDF and BDRF matrices - square matrices, with equal number of columns and rows defined by the total number of different angular coordinates (θ,φ), described in section 2.3. These matrices describe single specular and diffusing layer and they are used for calculation of the SOL properties of the glazing system that contains one or more of these layers, and which is described in detail in Klems, J.H. 1994a. and Klems, J.H. 1994b. It should be noted that for purely specular optical layers, all non-diagonal terms are zero (diffuse terms), producing diagonal matrices. For partially or fully diffuse layers, the non-diagonal terms in the bi-directional matrices can be non-zero.

In the case of forward-going incident radiation, *E*, coming from i-th direction, following equations apply for forward-going transmitted radiation in direction *j*, *I*, and backward-going reflected radiation, I^r in direction j^r (see Figure **10**a):

$$\begin{aligned} I(\theta_j,\varphi_j) &= \tau^f(\theta_j,\varphi_j;\theta_i,\varphi_i)E(\theta_i,\varphi_i) \\ I^r(\theta_j^r,\varphi_j^r) &= \rho^f(\theta_j^r,\varphi_j^r;\theta_i,\varphi_i)E(\theta_i,\varphi_i) \end{aligned} \qquad (72)$$

After introducing matrix notations, we can define **TAU_F** – a BTDF matrix that contains bi-directional transmittances for **FWD** going incident radiation:

$$\mathbf{I} = \mathbf{TAU\_F} \times \mathbf{E}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \cdots \\ \\ I_N \end{bmatrix} = \begin{bmatrix} \tau^f_{1,1} & \tau^f_{1,2} & \tau^f_{1,3} & \cdots & \tau^f_{1,N} \\ \tau^f_{2,1} & \tau^f_{2,2} & \tau^f_{2,3} & \cdots & \tau^f_{2,N} \\ \tau^f_{3,1} & \tau^f_{2,2} & \tau^f_{3,3} & \cdots & \tau^f_{3,N} \\ \cdots & \cdots & \cdots & & \cdots \\ \\ \tau^f_{N,1} & \tau^f_{N,2} & \tau^f_{N,3} & \cdots & \tau^f_{N,N} \end{bmatrix} \times \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \cdots \\ \\ E_N \end{bmatrix} \qquad (73)$$

where $\tau^f_{j,i}$ is short notation used for the front bidirectional transmittance $\tau^f(\theta_j,\varphi_j;\theta_i,\varphi_i)$ (radiation coming from direction *i*, and transmitted in direction *j*), and *N* is the number of incident/outgoing directions defined as (θ, φ) pairs (in our case, shown in Figure **5**, N = 49).

For reflected radiation, we have **RHO_F** – a BRDF matrix that contains bi-directional reflectances for **FWD** going incident radiation:

$$\mathbf{I^r} = \mathbf{RHO\_F} \times \mathbf{E}$$

$$\begin{bmatrix} I^r_1 \\ I^r_2 \\ I^r_3 \\ \cdots \\ \\ I^r_N \end{bmatrix} = \begin{bmatrix} \rho^f_{1,1} & \rho^f_{1,2} & \rho^f_{1,3} & \cdots & \rho^f_{1,N} \\ \rho^f_{2,1} & \rho^f_{2,2} & \rho^f_{2,3} & \cdots & \rho^f_{2,N} \\ \rho^f_{3,1} & \rho^f_{3,2} & \rho^f_{3,3} & \cdots & \rho^f_{3,N} \\ \cdots & \cdots & \cdots & & \cdots \\ \\ \rho^f_{N,1} & \rho^f_{N,2} & \rho^f_{N,3} & \cdots & \rho^f_{N,N} \end{bmatrix} \times \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \cdots \\ \\ E_N \end{bmatrix} \qquad (74)$$

where $\rho^f_{j,i}$ is short notation used for the front bidirectional reflectance $\rho^f(\theta_j^r,\varphi_j^r;\theta_i,\varphi_i)$ (radiation coming from direction *i*, and reflected in direction j^r, which is in reversed coordinates, in z^r hemisphere), and *N* is the number of incident/outgoing directions.

As for the backward-going incident radiation, E^r, following equations apply for backward-going transmitted radiation, I^r and forward-going reflected radiation, I, (see Figure **10**b):

$$
\begin{aligned}
I^r\left(\theta_j^r,\varphi_j^r\right) &= \tau^b\left(\theta_j^r,\varphi_j^r;\theta_i^r,\varphi_i^r\right)E^r\left(\theta_i^r,\varphi_i^r\right) \\
I\left(\theta_j,\varphi_j\right) &= \rho^b\left(\theta_j,\varphi_j;\theta_i^r,\varphi_i^r\right)E^r\left(\theta_i^r,\varphi_i^r\right)
\end{aligned}
\tag{75}
$$

After introducing matrix notations, we can define **TAU_B** – a BTDF matrix that contains bi-directional transmittances for **BCK** going incident radiation:

$$
\mathbf{I^r} = \mathbf{TAU\_B} \times \mathbf{E^r}
$$

$$
\begin{bmatrix} I_1^r \\ I_2^r \\ I_3^r \\ \dots \\ \\ I_N^r \end{bmatrix} =
\begin{bmatrix}
\tau_{1,1}^b & \tau_{1,2}^b & \tau_{1,3}^b & \dots & \tau_{1,N}^b \\
\tau_{2,1}^b & \tau_{2,2}^b & \tau_{2,3}^b & \dots & \tau_{2,N}^b \\
\tau_{3,1}^b & \tau_{2,2}^b & \tau_{3,3}^b & \dots & \tau_{3,N}^b \\
\dots & \dots & \dots & & \dots \\
\\
\tau_{N,1}^b & \tau_{N,2}^b & \tau_{N,3}^b & \dots & \tau_{N,N}^b
\end{bmatrix} \times
\begin{bmatrix} E_1^r \\ E_2^r \\ E_3^r \\ \dots \\ \\ E_N^r \end{bmatrix}
\tag{76}
$$

where $\tau_{j,i}^b$ represents bidirectional transmittance at the back side $\tau^b\left(\theta_j^r,\varphi_j^r;\theta_i^r,\varphi_i^r\right)$ (radiation coming from direction i^r, and transmitted in direction j^r, both of which are defined in reversed coordinates, in z^r hemisphere), and N is the number of incident/outgoing directions defined as (θ,φ) pairs (in our case, N = 49).

For reflected radiation, we have **RHO_B** – a BRDF matrix that contains bi-directional reflectances for **BCK** going incident radiation:

$$
\mathbf{I} = \mathbf{RHO\_B} \times \mathbf{E^r}
$$

$$
\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \dots \\ \\ I_N \end{bmatrix} =
\begin{bmatrix}
\rho_{1,1}^b & \rho_{1,2}^b & \rho_{1,3}^b & \dots & \rho_{1,N}^b \\
\rho_{2,1}^b & \rho_{2,2}^b & \rho_{2,3}^b & \dots & \rho_{2,N}^b \\
\rho_{3,1}^b & \rho_{3,2}^b & \rho_{3,3}^b & \dots & \rho_{3,N}^b \\
\dots & \dots & \dots & & \dots \\
\\
\rho_{N,1}^b & \rho_{N,2}^b & \rho_{N,3}^b & \dots & \rho_{N,N}^b
\end{bmatrix} \times
\begin{bmatrix} E_1^r \\ E_2^r \\ E_3^r \\ \dots \\ \\ E_N^r \end{bmatrix}
\tag{77}
$$

where $\rho_{j,i}^b$ represents bidirectional reflectance at the back side $\rho^b\left(\theta_j,\varphi_j;\theta_i^r,\varphi_i^r\right)$ (radiation coming from direction i^r, which is given in reversed coordinates, in z^r hemisphere, and reflected in direction j, which is defined by "regular" coordinates, in z hemisphere), and N is the number of incident/outgoing directions.

It is important to note that BTDF (Tau) matrices should be represented as sums of two matrices which correspond to two transfer mechanisms (undisturbed, or directly transmitted, dubbed "dir" and Lambertian diffuse reflections from slats, dubbed "dif"), as shown in equations (78) and (79), which describe **TAU_F** and **TAU_B** matrices, defined in equations (73) and (76), respectively:

$$\mathbf{TAU\_F} = \mathbf{TAU\_F\_dir} + \mathbf{TAU\_F\_dif} \tag{78}$$

$$\mathbf{TAU\_B} = \mathbf{TAU\_B\_dir} + \mathbf{TAU\_B\_dif} \tag{79}$$

On the other hand, reflected radiation contains only "dif" part.

3.7.2. Directly Transmitted Radiation

Specular portion (τ_{dir}^{f} for FWD going incident radiation, or τ_{dir}^{b} for BCK going incident radiation) is a portion of incident radiation that travels through the blind without interaction with the slat material. It is calculated for each incident profile angle, using the Dir-Dir methodology, explained in Section 3.6.1. These values are affected only by the geometry of the blind and incident profile angle - slat material properties have no influence. After calculating portion of light that goes undisturbed through the venetian blind for a certain incident profile angle ψ_i, which corresponds to i-th incident direction of forward-going radiation, this value is divided by Lambda, and placed in appropriate place in a diagonal **TAU_F_dir** matrix:

$$\tau_{i,i\,dir}^{f} = \frac{\tau_{dir-dir,i}^{f}}{\Lambda_i}, \; i = 1,\ldots,N \tag{80}$$

Lambda in equation (80) represents propagation operators – elements of a matrix which transforms radiance vectors emerging from one layer into irradiance vectors incident on the next layer. Lambda values are geometrical quantities associated with the partitioning of solid angle, as described in Klems, J.H. 1994a. and Klems, J.H. 1994b. Each $\Delta\Omega$ bin has a corresponding Lambda value. They can be expressed as:

$$\Lambda_i = \frac{1}{2}\left(\sin^2\theta_i^{hi} - \sin^2\theta_i^{lo}\right)\Delta\varphi_i, \; i = 1,\ldots,N_\theta \tag{81}$$

If directions are equally spaced along each strip defined by θ angle, we have:

$$\Lambda_i = \frac{1}{2}\left(\sin^2\theta_i^{hi} - \sin^2\theta_i^{lo}\right)\frac{2\pi}{N_{\varphi_i}}, \; i = 1,\ldots,N_\theta \tag{82}$$

where $N\varphi_i$ is the number of directions in one θ strip.

Lambda values are stored in a matrix. Indexes *lo* and *hi* in equation (81) stand for "lower" and "higher" bounding angles of $\Delta\Omega$ bins. Figure **22** shows one $\Delta\Omega$ bin with corresponding bounding angles (using planar projection of the hemisphere).

The "z" hemisphere is divided into N_θ spherical strips (first of which is actually a spherical cap). Each strip is defined by the two boundary angles: θ^{lo} and θ^{hi}. Relation between these boundary angles and corresponding θ angle is:

$$\begin{aligned} &\theta_1^{lo} = 0°; \quad \theta_{N_\theta}^{hi} = 90°; \\ &\theta_i^{lo} = \theta_{i-1}^{hi} = 2\theta_i - \theta_i^{hi}, \ i = N_\theta, ..., 2; \end{aligned} \tag{83}$$

Note that bounding angles should be calculated from the highest θ angle towards the lowest θ.

Figure **23** shows one spherical strip, an area bordered with "low" and "high" boundary angles.

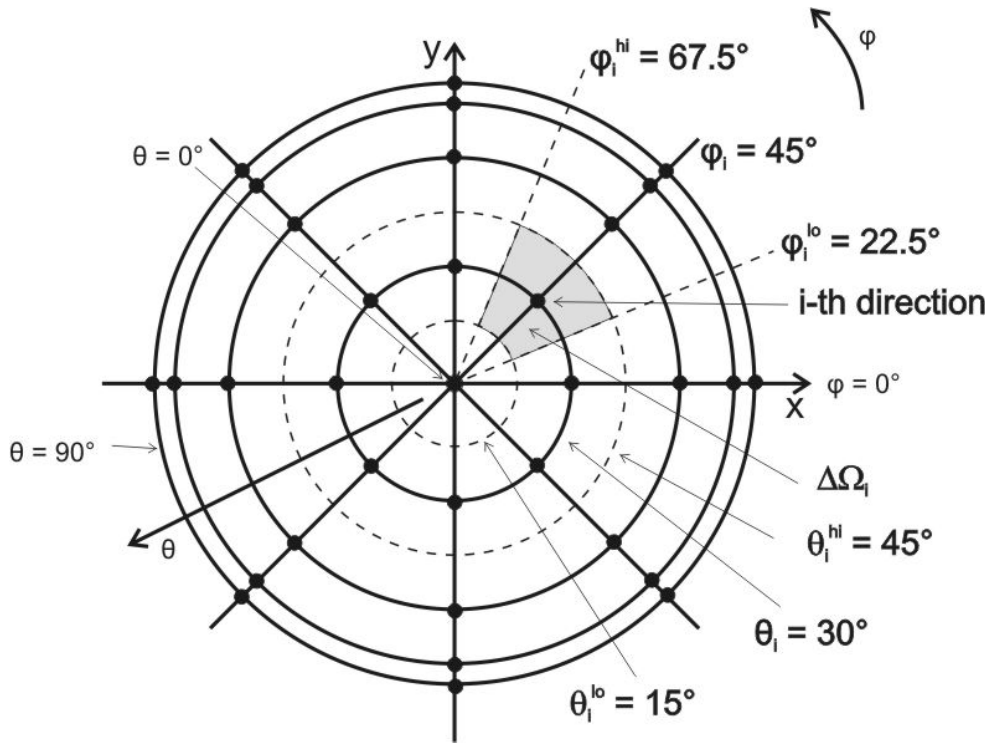

Figure 22. Angle Definitions for Calculation of Lambda values

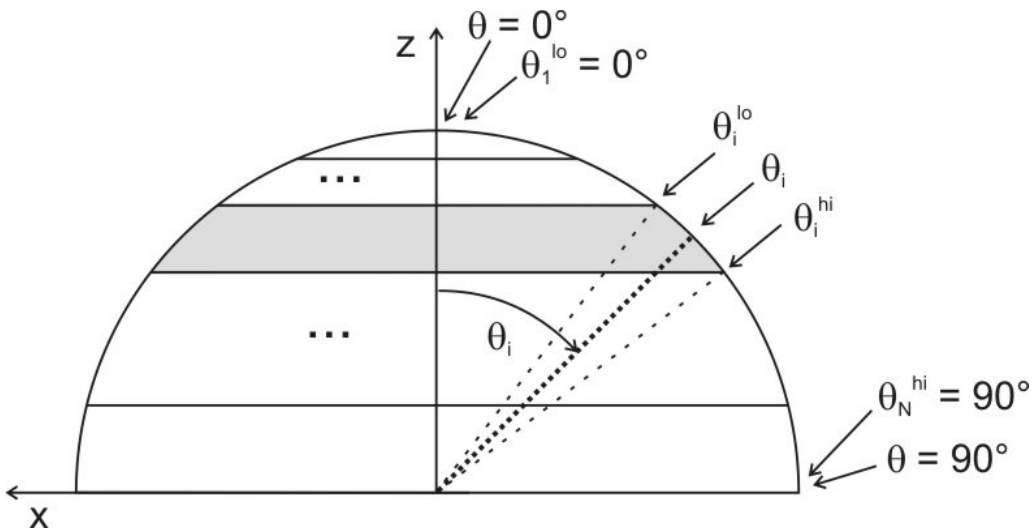

Figure 23. Theta limits which form a spherical strip ("z" hemisphere)

In our example shown in Figure **22** i=3, and Λ_3 would be:

$$\Lambda_3 = \frac{1}{2}\left(\sin^2\theta_3^{hi} - \sin^2\theta_3^{lo}\right)\frac{2\pi}{N_{\varphi_3}} = \frac{1}{2}\left(\sin^2\frac{\pi}{4} - \sin^2\frac{\pi}{12}\right)\cdot\frac{2\pi}{8} = \frac{\pi}{8}\left(\sin^2\frac{\pi}{4} - \sin^2\frac{\pi}{12}\right) = 0.17 \quad (84)$$

3.7.3. Diffusely Transmitted and Reflected Radiation

Diffuse (or "dif") transmitted or reflected portion of the incident radiation is a portion of the incident radiation that arrives at slat surface(s), and leaves the blind at the other, or at the same side, after a series of Lambertian reflections off slat surfaces.

In all calculations of SOL range diffuse properties of the Venetian blind, SOL properties of the slat material are used (τ_s, ρ_s^f and ρ_s^b).

Two different methods for calculations of diffuse portion of solar-optical properties in SOL range are presented; a) Uniform Diffuse method and b) Directional Diffuse method.

Uniform-Diffuse Method

This method utilizes Dir-Dif methodology, presented in section 3.6.2.

For each incident direction *i*, or to be more precise, for each incident direction's profile angle ψ_i, $\tau_{dir-dif,i}$ and $\rho_{dir-dif,i}$ are calculated twice, for FWD and BCK going incident radiation. $\tau_{dir-dif,i}$ and $\rho_{dir-dif,i}$ are obtained after solving systems of equations, defined by equations (51-52) presented in Section 3.6.2.

These results are hemispherical values, averaged across the hemisphere. Therefore, they need to be divided by π before placing in BTDF/BRDF matrices.

Note that this calculation method gives uniform distribution of diffuse values for one incident profile angle ψ_i. This results in **RHO** matrices, **TAU_F_dif** and **TAU_B_dif** matrices having equal values within any column.

When "dir" and "dif" results are summed, **TAU_F** from equation (73) becomes:

$$\mathbf{TAU\_F} = \begin{bmatrix} \frac{\tau^f_{dir-dir,1}}{\Lambda_1} + \frac{\tau^f_{dir-dif,1}}{\pi} & \frac{\tau^f_{dir-dif,2}}{\pi} & \ldots & \frac{\tau^f_{dir-dif,N}}{\pi} \\ \frac{\tau^f_{dir-dif,1}}{\pi} & \frac{\tau^f_{dir-dir,2}}{\Lambda_2} + \frac{\tau^f_{dir-dif,2}}{\pi} & \ldots & \frac{\tau^f_{dir-dif,N}}{\pi} \\ & & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\tau^f_{dir-dif,1}}{\pi} & \frac{\tau^f_{dir-dif,2}}{\pi} & \ldots & \frac{\tau^f_{dir-dir,N}}{\Lambda_N} + \frac{\tau^f_{dir-dif,N}}{\pi} \end{bmatrix} \tag{85}$$

Figure **24** shows how TAU_F matrix is formed.

Elements of **RHO_F** matrix from equation (74) are:

$$\rho^f_{j,i} = \frac{\rho^f_{dir-dif,i}}{\pi}, \; i,j = 1,\ldots,N \tag{86}$$

The **RHO_F** is:

$$\mathbf{RHO\_F} = \begin{bmatrix} \frac{\rho^f{}_{dir-dif,1}}{\pi} & \frac{\rho^f{}_{dir-dif,2}}{\pi} & \ldots & \frac{\rho^f{}_{dir-dif,N}}{\pi} \\ \frac{\rho^f{}_{dir-dif,1}}{\pi} & \frac{\rho^f{}_{dir-dif,2}}{\pi} & \ldots & \frac{\rho^f{}_{dir-dif,N}}{\pi} \\ & & \ldots & \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\rho^f{}_{dir-dif,1}}{\pi} & \frac{\rho^f{}_{dir-dif,2}}{\pi} & \ldots & \frac{\rho^f{}_{dir-dif,N}}{\pi} \end{bmatrix} \tag{87}$$

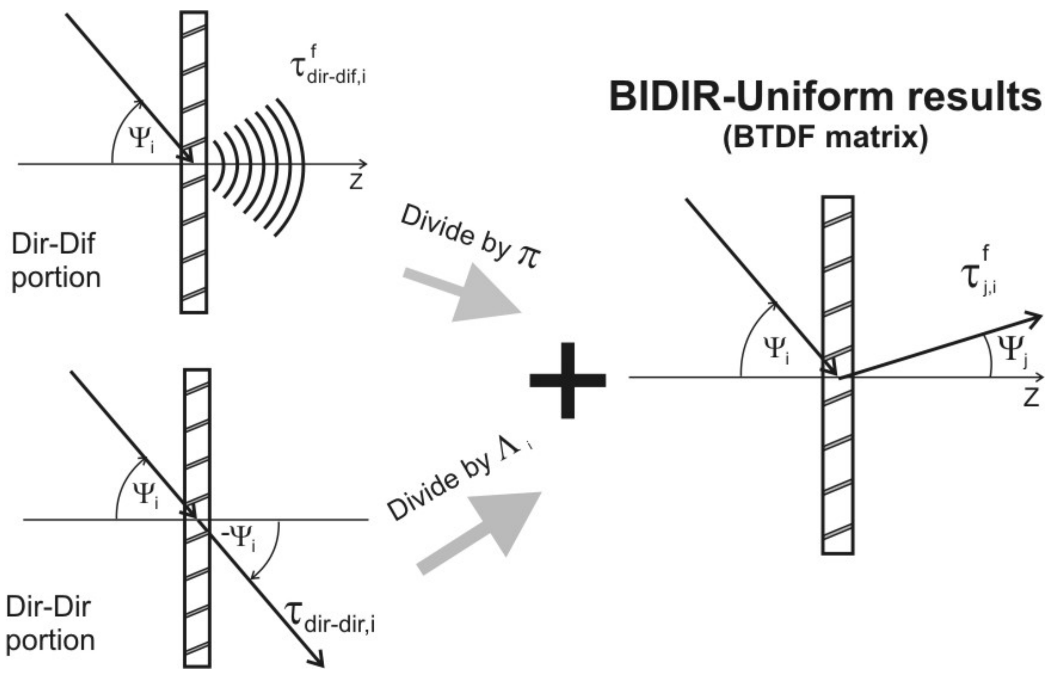

Figure 24. Overview of BiDir – Uniform Diffuse calculation method (FWD transmittance)

Matrices for backward radiation direction are formed by following the same pattern, only in reversed coordinates. Calculations of $\tau_{dir-dir,i}$, $\tau_{dir-dif,i}$ and $\rho_{dir-dif,i}$ are performed for BCK direction of incident radiation, and stored in appropriate result matrices in a previously described manner.

More examples with elements of BSDF matrices, based on a full angular set of 145 directions, are available in Appendix A.

Directional-Diffuse Method

Unlike Uniform-Diffuse Method, which gives uniform distribution of diffusely transmitted and reflected radiance across all outgoing directions for a given incident direction, Directional-Diffuse method calculates diffuse component for each outgoing direction taking into account which segments are "visible" and which segments are "shielded" by other slat, when considered at the outgoing angle. In other words, the procedure considers cut-off angle and adjusts diffuse radiation intensity based on that cut-of angle. This approach provides more realistic angular distribution of the diffuse radiation. However, in the average sense, integrated, or hemispherical value of diffuse transmittance or reflectance of an isolated layer, such as venetian blind, should be the same, regardless of the approach taken. The improved accuracy comes from the fact that the matrix method for determining solar-optical properties of multiple layer systems, consisting of specular, diffuse, or partially diffuse layers, described in Klems, J.H. 1994a. and Klems, J.H. 1994b, considers individual outgoing directions in the process of calculating the system properties, so having more accurate angular distribution of the diffuse radiation improves the accuracy of the method.

Figure **25** shows how outgoing radiance (and hence transmittance) changes with outgoing profile angle, for a simple case of a flat-slat Venetian blind. It shows a situation in which the blind is irradiated by light that arrives at an incident profile angle ψ_{in}. The incident radiation is diffusely reflected off the both slats. When the radiation reaches indoor facing imaginary plane, four different bands can be identified for outgoing directions which lay in a vertical plane, starting from the lowest outgoing angle of -90° ($-\pi/2$):

1) Increasing radiance – the total outgoing radiance increases with ψ_{out}, from zero to $I_{max,1}$, achieved at outgoing angle ψ_{max1}, as larger part of the upper slat becomes visible
2) Decreasing radiance, as angle between outgoing direction and upper slat normal vector decreases, from ψ_{max1} to zero at slat tilt angle ψ_{slat}
3) Increasing radiance – when ψ_{out} increases past slat tilt angle ψ_{slat}, lower slat is no longer visible, and lower slat becomes completely visible, outgoing radiance increases from zero to $I_{max,2}$, which is achieved at outgoing angle ψ_{max2}
4) Decreasing radiance – as outgoing angle increases past ψ_{max2}, decrease of total visible part of the lower slat outweights the decrease of angle between outgoing direction and normal vector of the lower slat in terms of an overall contribuiton to outgoing radiance, which results in decreasing of the total outgoing radiance from $I_{max,2}$, to zero for $\psi_{out} = -90°$ ($-\pi/2$).

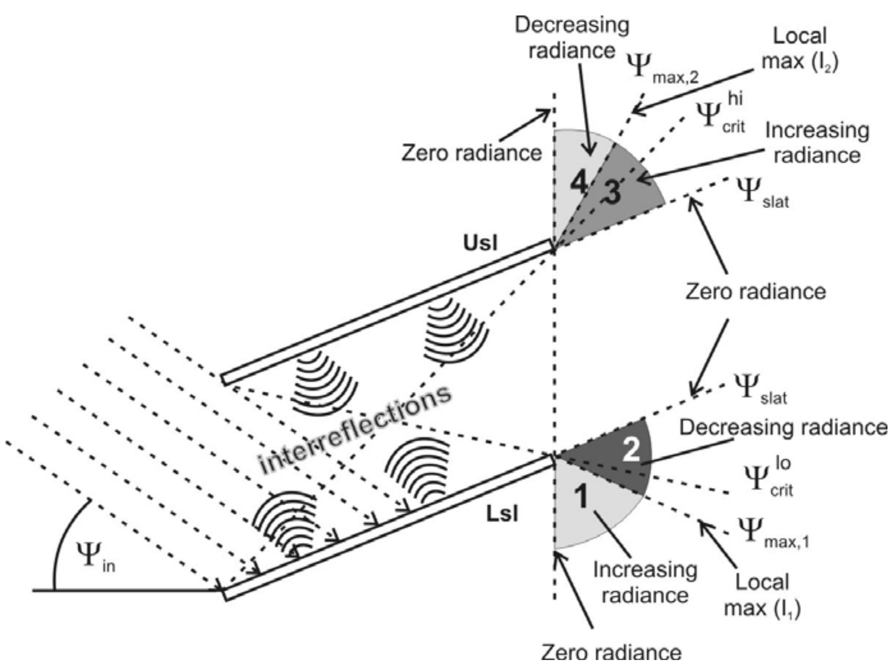

Figure 25. Illustration of directional-diffuse transmittance

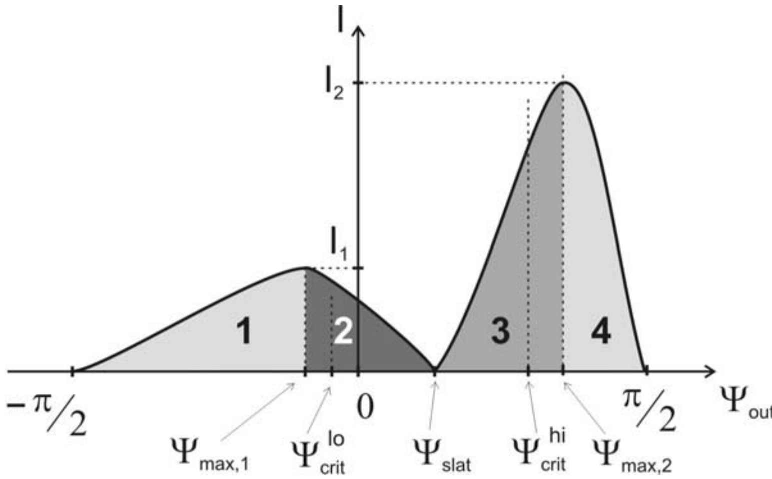

Figure 26. Outgoing radiance vs. outgoing profile angle for directional-diffuse transmittance in vertical outgoing plane

Figure **26** shows the plot of outgoing radiance against outgoing profile angle, for the example from Figure **25**. The shape of the curve is highly influenced by several factors: incident direction, overall geometry of the slat (width, distance, curvature, tilt) and optical properties of slat material (at both sides).

Although individual basic energy balance equations are essentially the same as the ones used in Section 3.6.2, equations (35-48), there are certain differences in the resulting equation systems:

- Unlike the "slat enclosure" model (Figure **14**), there are no "virtual" segments for front and back opening; therefore, dimension of this system is reduced by 2.
- System of equations is formed and solved for radiosities leaving slat segments, instead of irradiances arriving at slat segments.
- Resulting outgoing radiation is calculated as a sum of contributions of each segment of the two adjacent slats, using radiosities leaving each slat segment instead of irradiance reaching the "virtual" segment "6", used in Uniform diffuse approach.
- Although the physical meaning is similar, this system uses a slightly different notation for slat segments, which is more appropriate for the directional-diffuse approach described in this section.

The system of energy-balance equations in Directional-Diffuse method has the following form (see also Klems, J.H. 2004)**:**

- for each segment p of the lower slat (*Lsl*):

$$\sum_q \left[\delta_{pq} - \left(r_p^f \cdot F_{p,q} + t_p \cdot F_{p',q}\right)\right] \cdot J_{q,i} = \left(r_p^f \cdot E_{p,i} + t_p \cdot E_{p',i}\right) \tag{88}$$

- for each segment p' of the upper slat (*Usl*):

$$\sum_q \left[\delta_{p'q} - \left(r_{p'}^b \cdot F_{p',q} + t_{p'} \cdot F_{p',q}\right)\right] \cdot J_{q,i} = \left(r_{p'}^b \cdot E_{p',i} + t_{p'} \cdot E_{p,i}\right) \tag{89}$$

where:

$$\delta_{pq} = \begin{cases} 1, & p = q \\ 0, & p \neq q \end{cases}$$

q - any slat segment (of both *Lsl* and *Usl*)

p, p' - slat segment of the lower and upper slat, respectively

i- indicates *i*-th incident direction

r_p^f, r_p^b - reflectance of the front and back side of slat segment *p*, respectively

t_p - transmittance of slat segment *p*

$F_{p,q}$ - view factor from segment *p* towards *q*

$E_{p,i}$ - irradiance incident on a slat segment *p*, radiation coming from incident direction *i*

$J_{q,i}$ - outgoing radiosity from slat segment *q*, which comes from incident radiation coming from incident direction *i*

Front Transmittance:

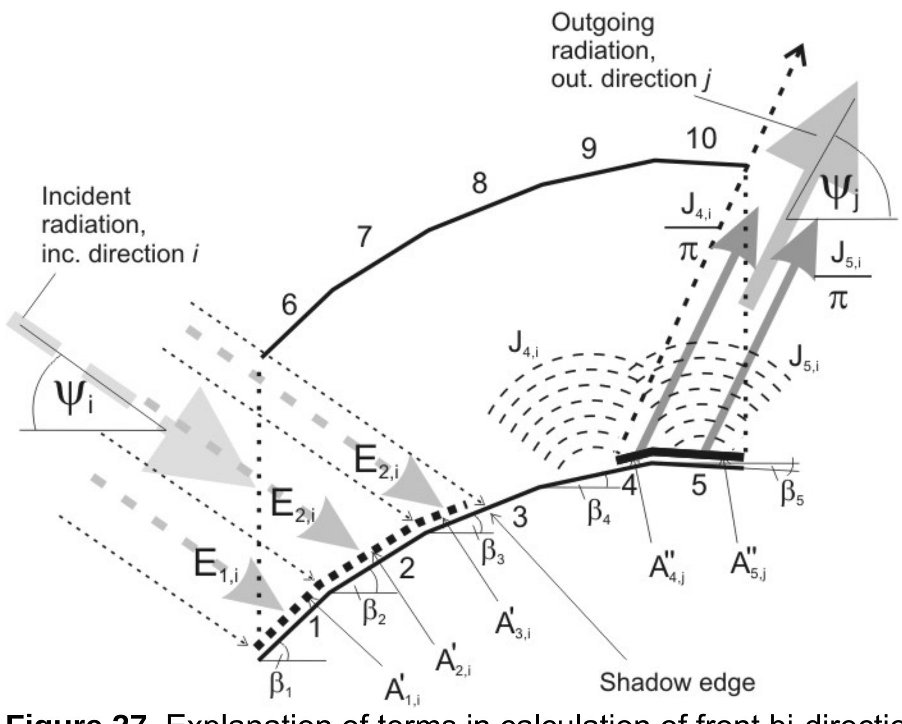

Figure 27. Explanation of terms in calculation of front bi-directional transmittance

Figure 27 explains terms in calculation of front bi-directional transmittance, for forward-going incident direction *i* and forward-going outgoing direction *j*. These directions are transformed into profile angles ψ_i and ψ_j, respectively. Thick dotted line represents irradiated parts of the lower slat, while thick full line marks parts of the lower slat visible in the outgoing direction.

As previously shown in section 2.4, irradiances on each slat segment p (caused by radiation from incident direction i), needed for energy-balance equations, can be calculated in 2D space, using:

$$E_{p,i} = (\cos\beta_p \tan\psi_i - \sin\beta_p) \cdot \frac{A'_{p,i}}{A_p} \tag{90}$$

where:

β_p - tilt angle of slat segment p

ψ_i - profile angle of incident direction i

$A'_{p,i}$ - area (length in 2D) of segment p that is irradiated by radiation coming from incident direction i

A_p - area of slat segment p (length in 2D)

With regards to equation (90), following values correspond to our example shown in Figure 27:

$$\begin{aligned} &A'_{1,i} = A_1, \\ &A'_{2,i} = A_2, \\ &A'_{3,i} < A_3, \\ &A'_{p,i} = 0, \quad p > 3 \end{aligned} \tag{91}$$

In order to calculate front transmittance for a given outgoing direction (profile angle), it is necessary to solve set of equations defined by equations (88-89), with irradiances E calculated using Equation (7). After that, it is necessary to determine which slat segments (or parts of certain slat segments) are visible from that particular outgoing direction, as shown in Figure **27**.

Contribution of each slat to total outgoing radiance in the outgoing direction k is given by equation (10). Total outgoing transmittance is calculated as an averaged sum of these contributions, with regards to the area of back opening:

$$\tau^f_{j,i} = \frac{1}{A_d \cos\theta_j} \sum_p \left[\left(\cos\beta_p \cdot \tan\psi_j - \sin\beta_p \right) A''_{p,j} \frac{J_{p,i}}{\pi} \right] \tag{92}$$

where:

A_d - area of slat opening (slat distance in 2D)

θ_j - latitude angle of outgoing direction j

β_p - tilt angle of slat segment p

ψ_j - profile angle of outgoing direction j

$J_{p,i}$ - outgoing radiosity from slat segment p (as a result of irradiance at all slat segments, coming from incident direction i) – solution of set of equations (88-89)

$A''_{p,j}$ - area (length in 2D) of segment p that is visible from outgoing direction j

With regards to equation (92), following values would be used in our example:

$$A''_{5,j} = A_5,$$
$$A''_{4,j} < A_4, \qquad (93)$$
$$A''_{p,j} = 0, \quad p \in \{1,2,3,6,7,8,9,10\}$$

As shown in equation (92), transmitted part is calculated as a sum of contributions (diffuse radiosities divided by π) of each slat segment visible from outgoing direction *j*, and this is calculated for each outgoing direction. This results in a non-uniform distribution of results, unlike the distribution of results, present in Uniform-diffuse method.

This calculation can be summarized for front-side transmittance as:

- calculate incident and outgoing profile angles: ψ_i (forward-going) and ψ_j (forward-going), respectively
- determine visible areas $A'_{p,i}$ for each slat segment *p* (from incident direction *i*)
- calculate irradiance $E_{p,i}$ of each slat segment *p*
- calculate radiosities $J_{p,i}$ for each slat segment *p*, by solving set of equations (88-89)
- determine parts $A''_{p,j}$ of each slat segment *p*, visible from the outgoing direction *j*.
- use equation (92) to calculate front bi-directional transmittance.

These steps are repeated for each pair of (ψ_j, ψ_i), and results of these calculations are stored in appropriate places in **TAU_F_dif** bi-directional matrix.

Front Reflectance:

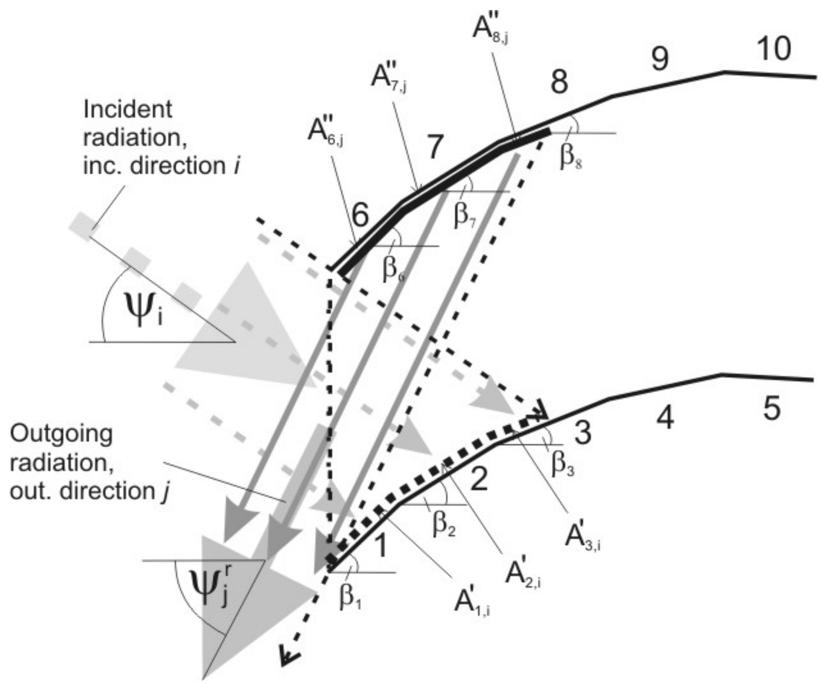

Figure 28. Explanation of terms in calculation of front bi-directional reflectance

Front bi-directional reflectance is calculated for forward-going incident direction *i* and backward-going outgoing direction *j* (Figure **28**), as:

$$\rho_{j,i}^{f} = \frac{1}{A_d} \sum_{p} \left(\cos \beta_p \tan \psi_j^r - \sin \beta_p \right) A_{p,j}'' \frac{J_{p,i}}{\pi} \tag{94}$$

Calculation of **RHO_F** bi-directional matrix is done using the same procedure as shown for **TAU_F_dif** bi-directional matrix.

Back-side transmittances and reflectances are calculated after applying the same procedure shown for front-side transmittances and reflectances to the backward-going incident directions. The resulting bi-directional matrices are **TAU_B_dif** and **RHO_B.**

Again, directly transmitted values (**TAU_F_dir** and **TAU_B_dir**) are added to matrices **TAU_F_dif** and **TAU_B_dif** to form full set of bid-directional matrices **TAU_F** and **TAU_B**.

3.8. Direct to Hemispherical Solar-optical Properties

Direct to hemispherical solar-optical properties (transmittances and reflectances) of the blind are integrated properties calculated from bi-directional results for each incident angle or direction. These properties represent transmitted/reflected radiation integrated over outgoing hemisphere, which is a result of incident directional radiation (a beam), coming from a certain incident direction (Figure **29**)..

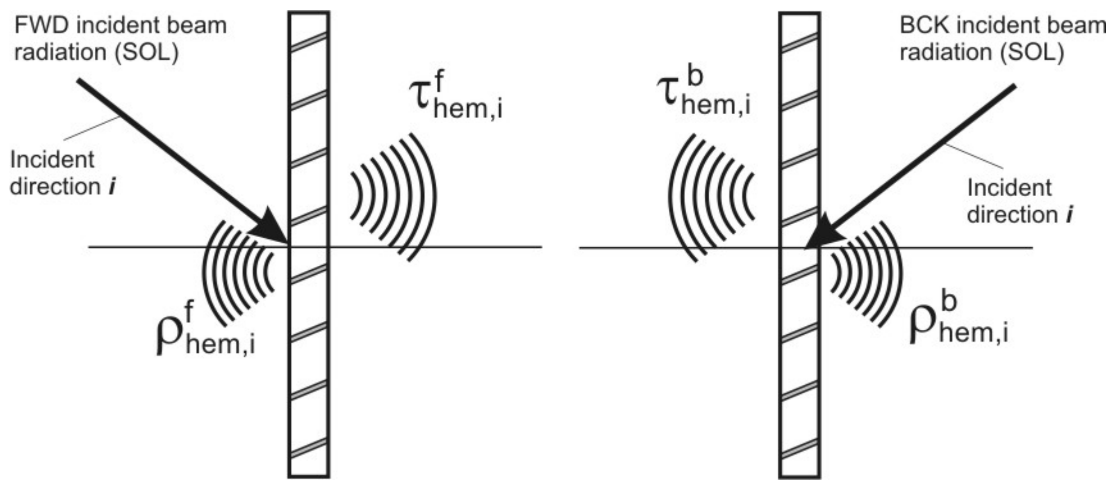

Figure 29. Directional-to-hemispherical properties at front and back side of the Venetian blind

Due to inherent asymmetry of the geometry and slat material properties of the blind with respect to incident angles, integrated transmittances and reflectances are calculated separately for the front and for the back side of the blind. Thus, we define 4 different sets of direct to hemispherical properties:

- Hemispherical transmittances at the front side of the blind, $\tau^{f}_{hem,i}$
- Hemispherical transmittances at the back side of the blind, $\tau^{b}_{hem,i}$
- Hemispherical reflectances at the front side of the blind, $\rho^{f}_{hem,i}$
- Hemispherical reflectances at the back side of the blind, $\rho^{b}_{hem,i}$

As equation (73) shows, an element of bi-directional transmittance matrix for the front side of the blind **TAU_F** is defined as $\tau^{f}_{j,i}$ – radiation coming from direction *i*, and transmitted in direction *j* – (*i*,*j* = 1, … , *N*, where *N* is the number of incident/outgoing directions). The same matrix format is used in other three matrices with bi-directional properties (**TAU_B**, **RHO_F**, and **RHO_B**).

Direct-to-Hemispherical transmittance (front or back) for *i*-th incident direction $\tau^{f}_{hem,i}$ and $\tau^{b}_{hem,i}$ are defined as hemispherical average of bi-directional transmittances, over an outgoing hemisphere:

$$\tau_{hem,i} = \frac{\int\limits_{\substack{outgoing\\hemisphere}} \tau_{j,i} \cos\theta_j \, d\omega_j}{\int\limits_{\substack{outgoing\\hemisphere}} \cos\theta_j \, d\omega_j} \tag{95}$$

Where for $\tau^{f}_{hem,i}$, $\tau^{f}_{j,i}$ is used and for $\tau^{b}_{hem,i}$, $\tau^{b}_{j,i}$ is used. Solid angle of the outgoing hemisphere ω can be expressed in terms of θ and φ:

$$d\omega_j = \frac{1}{2\pi} \sin\theta_j d\theta_j d\varphi_j \tag{96}$$

After re-arranging:

$$\tau_{hem,i} = \frac{1}{\pi} \int_{\theta_j=0}^{\pi/2} \int_{\varphi_j=0}^{2\pi} \tau_{j,i} \cos\theta_j \sin\theta_j \, d\theta_j \, d\varphi_j \tag{97}$$

For a finite number of directions, a summation formula is used:

$$\tau_{hem,i} = \frac{\sum_{j=1}^{N} \tau_{j,i} \left(\sin^2 \theta_j^{hi} - \sin^2 \theta_j^{lo} \right) \cdot \Delta\varphi_j}{2} \tag{98}$$

where N is number of (incident and outgoing) directions. This again applies to both front and back directions. This can be written as:

$$\tau_{hem,i}^{f} = \sum_{j=1}^{N} \tau^{f}{}_{j,i} \Lambda_j \tag{99}$$

$$\tau_{hem,i}^{b} = \sum_{j=1}^{N} \tau^{b}{}_{j,i} \Lambda_j \tag{100}$$

where ${\tau_{j,i}}^{f}$ and ${\tau_{j,i}}^{b}$ correspond to $\tau_{j,i}$ elements of **TAU_F** or **TAU_B** matrix, and Λ_j is a Lambda value that corresponds to *j*-th outgoing direction, calculated using equation (81). Essentially, front hemispherical transmittance for *i*-th incident direction is obtained by averaged summation of the *i*-th column of **TAU_F** matrix.

Similarly, front/back hemispherical reflectances are obtained using:

$$\rho_{hem,i} = \frac{\sum_{j=1}^{N} \rho_{j,i} \left(\sin^2 \theta_j^{hi} - \sin^2 \theta_j^{lo} \right) \cdot \Delta\varphi_j}{2} \tag{101}$$

This can be written as:

$$\rho_{hem,i}^{f} = \sum_{j=1}^{N} \rho^{f}{}_{j,i} \Lambda_j \tag{102}$$

$$\rho_{hem,i}^{b} = \sum_{j=1}^{N} \rho^{b}{}_{j,i} \Lambda_j \tag{103}$$

3.9. Hemispherical to Hemispherical Solar-optical Properties

Hemispherical to hemispherical (also referred to as *diffuse*) solar-optical properties (transmittances and reflectances) of the venetian blind layer are full integrated quantities, giving scalar result independent from incident or outgoing directions. These scalar quantities are calculated from bi-directional results as well, by performing double integration, and they represent the amount of transmitted/reflected diffuse radiation as a result of diffuse incident radiation (Figure **30**).

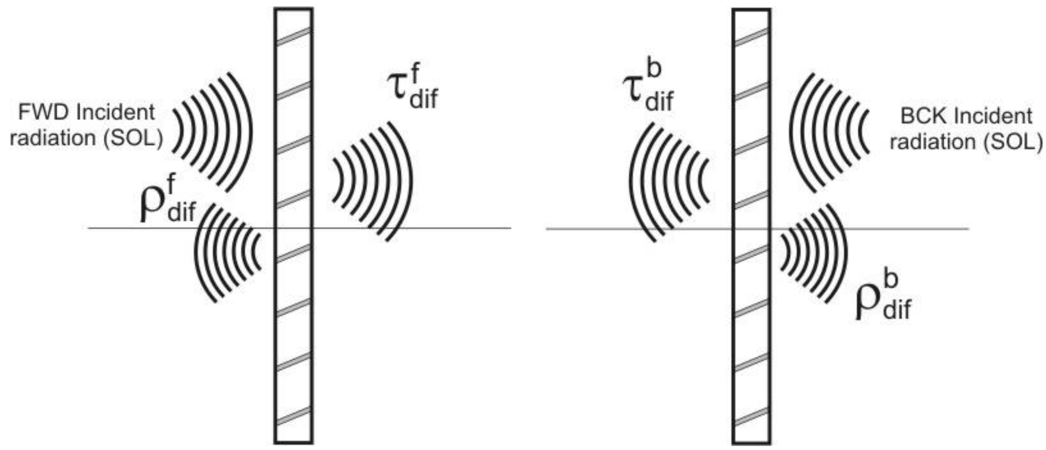

Figure 30. Diffuse (hemispherical-to-hemispherical) properties at front and back side of the Venetian blind

Hemispherical to hemispherical properties are easily calculated from directional to hemispherical properties, as their integral sum over all incident angles. Transmittance is calculated as:

$$\tau_{dif} = \int\limits_{\substack{incident\\hemisphere}} \tau_{hem,i} \cdot \cos\theta_i \, d\omega_i \tag{104}$$

where $\tau_{hem,\,i}$ is direct to hemispherical transmittance for *i*-th incident direction, and ω_i is unit solid angle of the hemisphere at incident side of the blind.

After rearranging:

$$\tau_{dif} = \frac{1}{2\pi} \int\limits_{\theta_i=0}^{\pi/2} \int\limits_{\varphi_i=0}^{2\pi} \tau_{hem,i} \cdot \cos\theta_i \sin\theta_i \, d\theta_i \, d\varphi_i \tag{105}$$

For a finite number of directions, *N*, a summation formula is used:

$$\tau_{dif} = \frac{1}{\pi} \sum_{i=1}^{N} \tau_{hem,i} \cdot \Lambda_i \tag{106}$$

where Λ_i is a Lambda value that corresponds to *i*-th incident direction. This applies to both front and back directions:

$$\tau_{dif}^{f} = \frac{1}{\pi}\sum_{i=1}^{N}\tau_{hem,i}^{f}\cdot\Lambda_i \tag{107}$$

$$\tau_{dif}^{b} = \frac{1}{\pi}\sum_{i=1}^{N}\tau_{hem,i}^{b}\cdot\Lambda_i \tag{108}$$

where $\tau_{hem,i}^{f}$ and $\tau_{hem,i}^{b}$ represent front and back directional to hemispherical transmittance for *i*-th incident direction, calculated using equations (99) and (100), respectively. Note that τ_{dif}^{f} must be equal to τ_{dif}^{b}, so it is sufficient to calculate diffuse transmittance for one side of the blind only.

Similarly, front and back diffuse reflectances are calculated as:

$$\rho_{dif}^{f} = \frac{1}{\pi}\sum_{i=1}^{N}\rho_{hem,i}^{f}\cdot\Lambda_i \tag{109}$$

$$\rho_{dif}^{b} = \frac{1}{\pi}\sum_{i=1}^{N}\rho_{hem,i}^{b}\cdot\Lambda_i \tag{110}$$

where $\rho_{hem,i}^{f}$ and $\rho_{hem,i}^{b}$ are front and back directional to hemispherical reflectance for *i*-th incident direction, calculated using equations (102) and (103), respectively.

4. MATHEMATICAL MODELS FOR CALCULATION OF FAR INFRA-RED (FIR) PROPERTIES OF A VENETIAN BLIND

4.1. Introduction

Optical properties of the Venetian blind in far infra-red (FIR) range are:

- IR transmittance, τ_{IR}
- IR emissivity of the front side of the blind, ε_f
- IR emissivity of the back side of the blind, ε_b

These are hemispherical-to-hemispherical, which means that they are not angle dependent. In addition to material properties, which are purely diffuse, just like in SOL range, in FIR range the incident radiation is also always diffuse, which means that calculation of layer properties are done using only dif-dif procedure. The transmittance and reflectance are ratios of outgoing diffuse radiation with incident diffuse radiation and they are a single scalar value for forward and backward directions. Note that τ_{IR} must be equal for both directions, front and back:

$$\tau_{IR} = \tau_f = \tau_b \tag{111}$$

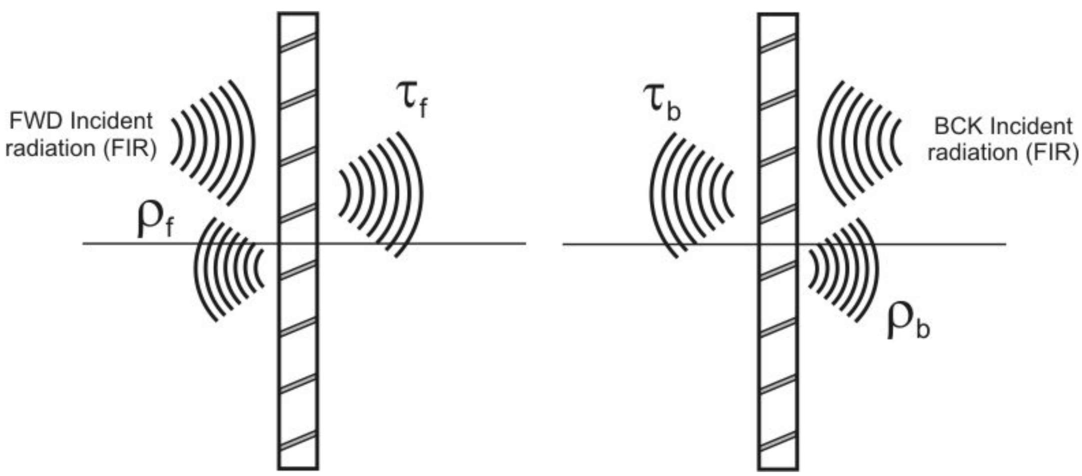

Figure 31. FIR properties (front and back) of the Venetian blind

Following notations from Figure **31**, appropriate values for emissivities can be obtained using:

$$\begin{aligned}
\tau_{IR} &= \tau_{IR}^f = \tau_{IR}^b \\
\varepsilon^f &= 1 - \tau_{IR} - \rho_{IR}^f \\
\varepsilon^b &= 1 - \tau_{IR} - \rho_{IR}^b
\end{aligned} \tag{112}$$

In these calculations, single band FIR range properties of the slat material are used (τ_{IR}, ρ_{IR}^f and ρ_{IR}^b).

Transmittance and front and back reflectance in FIR range of the Venetian blind layer can be calculated using either a) ISO 15099 Dif-Dif methodology; b) BTDF/BRDF uniform diffuse method and then doing hemispherical-to-hemispherical integration, presented in Section 3.9; or c) BTDF/BRDF directional diffuse method and then doing hemispherical-to hemispherical integration. Each of these methods is expected to produce identical final results.

4.2. ISO15099 Method

ISO15099 FIR method is a direct implementation of Dif-Dif calculation procedure from Section 3.6.2. The calculations are done for a single band using FIR properties of the slat material.

4.3. Bi-Directional Methods

These methods use BTDF/BRDF matrices, obtained using either Uniform Diffuse or Directional Diffuse bi-directional calculation method. This calculation is done for a single set of FIR slat material properties.

Bidirectional results are integrated over the two hemispheres (in which all incident and outgoing directions are defined), resulting in three hemispherical FIR values: τ, ρ^f and ρ^b. These integrations are done the same way as in the case of hemispherical to hemispherical solar optical properties, as shown in Section 3.9. Transmittance, τ, can be calculated from either front or back transmittance matrix, using equation (107) or (108). Reflectances, ρ, can be calculated from front and back reflectance matrices, using equations (109) and (110).

APPENDIX A – INDEXING OF INCIDENT AND OUTGOING DIRECTIONS IN BSDF MATRICES

Following figures illustrate the indexing of incident and outgoing directions in BSDF matrices. Full angular basis, which consists of 145 directions, was assumed. Full angular basis contains a normal direction, plus 8 strips defined by a latitude angle θ, with each θ_i strip divided into $N\varphi_i$ equal bins (i = 1, … , 8), as shown in Table A 1.

Table A 1: Number of directions per one θ strip

Theta index i	θ_i	$N\varphi_i$	$\Delta\varphi_i$
1	0°	1	360°
2	10°	8	45°
3	20°	16	22.5°
4	30°	20	18°
5	40°	24	15°
6	50°	24	15°
7	60°	24	15°
8	70°	16	22.5°
9	82.5°	12	30°

Directions are numbered 1-145, with direction 1 being normal direction (θ=0°), followed by directions from each strip, beginning with $\varphi = 0°$, and ending with $\varphi = (N\varphi_i - 1) \cdot \Delta\varphi_i$.

For example:

- direction 31 is defined by (θ = 30°, φ = 90°);
- direction 41 is defined by (θ = 30°, φ = 270°);
- direction 100 is defined by (θ = 60°, φ = 90°);
- direction 112 is defined by (θ = 60°, φ = 270°);

Dimension of four BSDF matrices for full angular basis is 145.

Figure **A 1** describes an element of front BTDF matrix **TAU_F**, placed in 112th column of the 31st row. This element represents a ratio of forward-going radiance, leaving the blind at outgoing direction 31, to incident forward-going irradiance, coming from incident direction 112. Note that both incident and outgoing directions are defined in *z* hemisphere. Figure **A 2** and Figure **A 3** show elements **TAU_F**(41,112) and **TAU_F**(31,31), respectively.

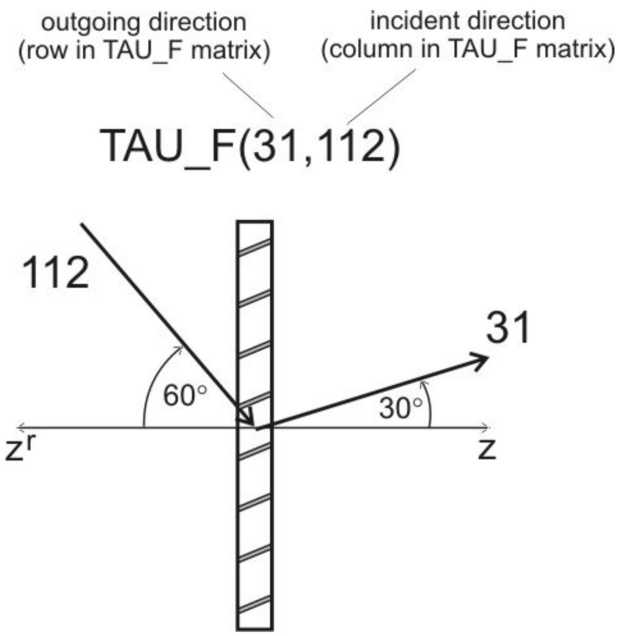

Figure A 1. Front bi-directional transmittance, inc. direction 112, out. dir. 31

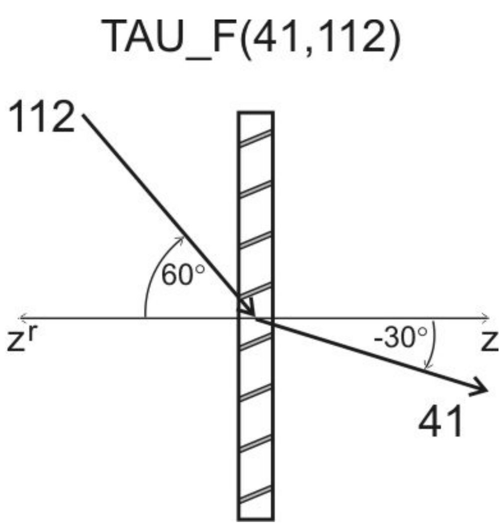

Figure A 2. Front bi-directional transmittance, inc. direction 112, out. dir. 41

Figure **A 4** shows a member of front reflectance matrix – **RHO_F**(31,112). In this case, outgoing direction 31^r is defined in z^r hemisphere.

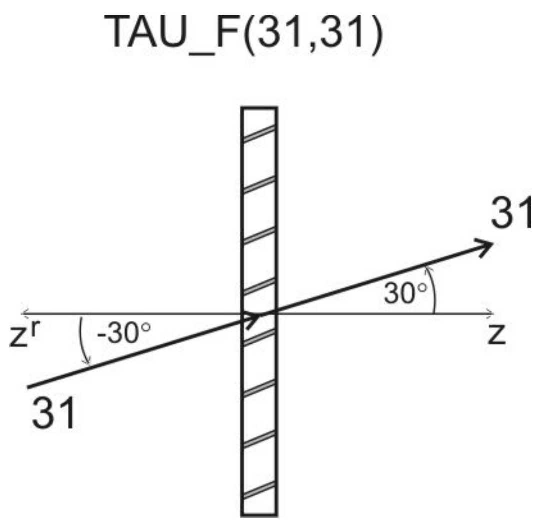

Figure A 3. Front bi-directional transmittance, inc. direction 31, out. dir. 31

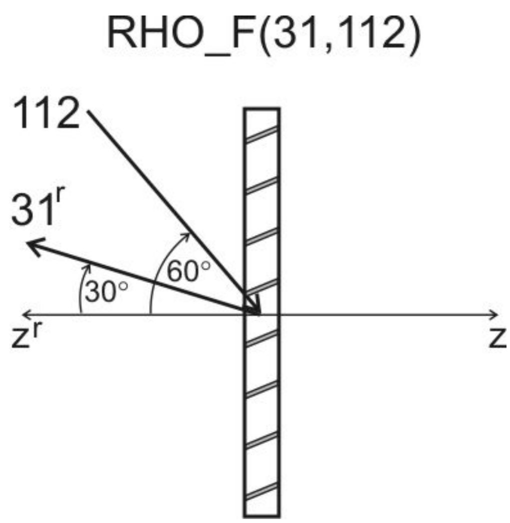

Figure A 4. Front bi-directional reflectance, inc. direction 112, out. dir. 31^r

Figure **A 5** shows a member of back transmittance matrix – **TAU_B**(31,112). In this case, both incident and outgoing directions (112^r and 31^r , respectively) are defined in z^r hemisphere.

Figure **A 6** shows a member of back reflectance matrix – **RHO_B**(100,112). Incident direction 112^r is defined in z^r hemisphere, while outgoing direction 100 is defined in z hemisphere.

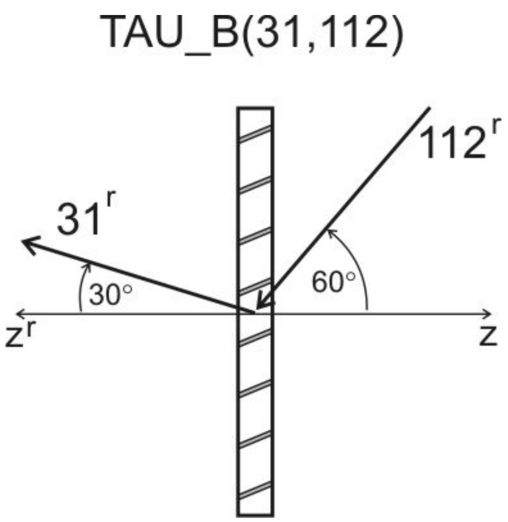

Figure A 5. Back bi-directional transmittance, inc. direction 112^r, out. dir. 31^r

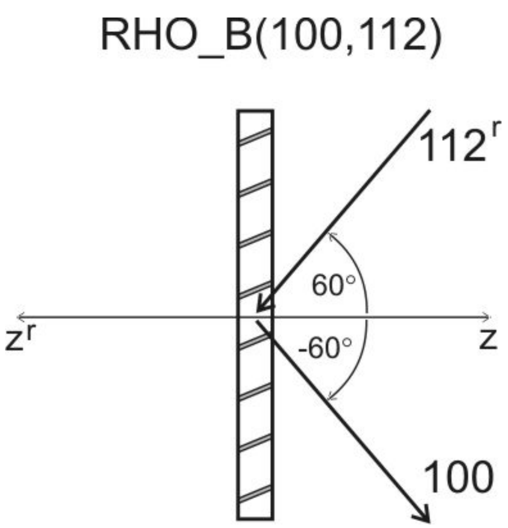

Figure A 6. Back bi-directional reflectance, inc. direction 112^r, out. dir. 100

REFERENCES

Klems, J.H. 1994a. A New Method for Predicting the Solar Heat Gain of Complex Fenestration Systems I. Overview and Derivation of the Matrix Layer Calculation. ASHRAE Transactions V.100, Pt. 1

Klems, J.H. 1994b. A New Method for Predicting the Solar Heat Gain of Complex Fenestration Systems II. Detailed Description of the Matrix Layer Calculation *ASHRAE Transactions* V.100, Pt. 1

ISO. 2003. "ISO 15099: Thermal Performance of Windows, Doors and Shading Devices – Detailed calculations." *International Standardization Organization*. First edition. November, 2003.

Klems, J.H. 2004. Detailed Equations for Connecting a 2D Blind Model with the Bidirectional Calculation, internal memo.

Klems, J.H. 2005. Calculating Outgoing Radiance in the 2D Venetian Blind Model, internal memo, rev.2.

ASHRAE. 2001. Handbook of Fundamentals, *American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.* June 2001.

IEA. 2000. Daylight in Buildings, A report of IEA SHC Task 21/ ECBCS Annex 29. *International Energy Agency*. July 2000.