A Centennial: Calculus with Infinitesimals since Levi-Cività

Detlef Laugwitz
Darmstadt

Extended Abstract

During the 1890's Tullio Levi-Cività considered ordered fields of 'numbers' defined by generalized formal power series. Though his main objective was the justification of Veronese's Foundations of Geometry, Levi-Cività also applied these fields successfully to a rudimentary approach at a calculus involving infinitely small numbers. Taken up by Hans Hahn in 1907, the fields marked toe beginning of a modern algebra of ordered structures.

More recently, the approach was extended to an easily accessible version of nonstandard analysis, suitable for the purposes of numerical computation. Given some standard theory (as of arithmetic or analysis), adjoin a new symbol Ω. For any formula or sentence form $S(.)$ of the standard theory, such that $S(n)$ is a true statement for every sufficiently large standard natural number n, $S(\Omega)$ will, by definition, be a true statement of the nonstandard theory. In particular, $\Omega > n_0$ is a true statement for each standard natural n_0: Ω is infinitely large, and $\omega = \Omega^{-1}$ is a positive infinitesimal. Power series in ω (or in some other, possibly 'very small' infinitesimal) represent elements of the Levi-Cività fields. The extension of standard functions and standard analysis is immediate. Moreover, the extended calculus also covers non-standard functions representing distributions.