Fermi National Accelerator Laboratory

FERMILAB-Lecture-76/01-THY/EXP
March 1976

Lectures on Weak Interactions*

C. QUIGG
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

*Given in the Fermilab Academic Lecture Series, February, 1976.

Operated by Universities Research Association Inc. under contract with the Energy Research and Development Administration

INTRODUCTION

These lectures are intended to provide an elementary introduction to the theory of weak interactions, particularly as it pertains to phenomena studied in Fermilab experiments. High-energy neutrino scattering is emphasized, whereas the voluminous literature on nuclear β-decay (a principal concern of the adolescence of the subject) is utterly neglected. The newly-discovered neutral currents are treated at some length, and I discuss (but only operationally) such conjectural entities as the intermediate bosons. Unified (gauge) theories of weak and electromagnetic interactions lurk near at hand.

I place a high premium on carrying through calculations. For all the subtle and beautiful concepts in which modern gauge field theories abound, the low-order (tree) diagrams for processes of practical and pedagogical interest lie within the computational ability of all. Consequently, I will perform calculations in detail instead of merely quoting results.

There exist numerous introductions to the subject which are of a high pedagogical standard. I am partial to those that served as my introduction:

E.D. Commins, Weak Interactions (New York: McGraw-Hill, 1973).

L.B. Okun', Weak Interaction of Elementary Particles, (Oxford: Pergamon, 1965).

J.D. Jackson, in Elementary Particle Physics and Field Theory, vol. 1 of the 1962 Brandeis Summer Institute Lectures, ed. K.W. Ford (New York: Benjamin, 1963), p. 263.

There is also the encyclopedic treatise, Theory of Weak Interactions in Particle Physics, by R.E. Marshak, Riazuddin, and C.P. Ryan (New York: Wiley-Interscience, 1969). Regrettably, all these works predate the recent renascence of interest in weak-interaction phenomena inspired by experiments with high-energy neutrino beams at CERN, ANL, BNL, and Fermilab. I shall therefore try to concentrate on new phenomena and leave to the textbooks extended discussions of the old.

Two shortcomings of lectures by an amateur are easy to foresee and difficult to eliminate. The first is the inevitable lack of the sort of perspective which an expert would bring to the subject. Second, my innocence of the literature means that useful and deserved citations to original work may be too frequently overlooked. For these defects I must ask the reader's indulgence. Credit for any evidence of good taste and insight must go to Ben Lee and especially Helen Quinn, who have given me the benefit of their experience in many conversations.

The lectures have been confined almost exclusively to the weak interactions of leptons. The very interesting practical case of neutrino scattering on nucleons can be treated, at least in the parton model, by transcription of the results presented here. In a perfect world, that transcription would have been included in these lectures. However, a workmanlike treatment would have required roughly doubling the length of these notes. I have excused myself from this effort on the ground that much of the material to be covered can be found elsewhere.

Notation and Conventions

I use the Pauli metric. A four-vector is

$$A_\mu = (\vec{A}, iA_0), \tag{1}$$

and the scalar product is

$$A\cdot B = \vec{A}\cdot\vec{B} - A_0 B_0. \tag{2}$$

The Dirac γ-matrices are Hermitian:

$$\vec{\gamma} = \begin{pmatrix} 0 & -i\vec{\sigma} \\ i\vec{\sigma} & 0 \end{pmatrix}, \tag{3}$$

where the Pauli matrices are

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \tag{4}$$

$$\gamma_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{5}$$

and

$$\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \tag{6}$$
$$= \frac{1}{4!}\,\varepsilon_{\kappa\lambda\mu\nu}\,\gamma_\kappa\gamma_\lambda\gamma_\mu\gamma_\nu.$$

The spin tensor is

$$\sigma_{\mu\nu} = (1/2i)(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu). \tag{7}$$

The γ-matrices satisfy the usual anticommutation relations,

$$\{\gamma_\mu, \gamma_\nu\} = \gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}, \qquad (8)$$
$$\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5.$$

A free particle with momentum p and spin s is described by the positive-energy spinor $u_s(p)$. The four-vector s satisfies $s \cdot p = 0$, $s \cdot s = 1$. In the rest frame of the particle, it is the polarization vector $s = (\vec{s}, 0)$. The positive-energy spinor satisfies the Dirac equation

$$(m + i\gamma \cdot p)\, u_s(p) = 0. \qquad (9)$$

The adjoint spinor $\bar{u}_s(p) = u_s(p)^\dagger \gamma_4$ satisfies

$$\bar{u}_s(p)\,(m + i\gamma \cdot p) = 0. \qquad (10)$$

The negative-energy solutions $v_s(p)$ and $\bar{v}_s(p) = v_s(p)^\dagger \gamma_4$ correspond to antiparticles. For them, the Dirac equation is

$$(-m + i\gamma \cdot p)\, v_s(p) = 0 \qquad (11)$$

or

$$\bar{v}_s(p)\,(-m + i\gamma \cdot p) = 0. \qquad (12)$$

The spinors are normalized so that

$$\bar{u}_\lambda(p)\, u_\mu(p) = 2m\,\delta_{\lambda\mu} \qquad (13)$$

$$\bar{v}_\lambda(p)\, v_\mu(p) = -2m\,\delta_{\lambda\mu} \qquad (14)$$

$$\bar{u}_\lambda(p)\, v_\mu(p) = 0 = \bar{v}_\lambda(p)\, u_\mu(p), \qquad (15)$$

where now $\lambda, \mu = \pm 1$ represent the eigenvalues of the helicity operator $i\gamma_5\, \gamma\cdot s$ (defined to be 2x helicity for convenience.) The projection operators which occur in trace calculations are

$$\Lambda^{(+)}(p) = \sum_\lambda u_\lambda(p)\, \bar{u}_\lambda(p) = m - i\gamma\cdot p\,, \qquad (16)$$

$$\Lambda^{(-)}(p) = \sum_\lambda v_\lambda(p)\, \bar{v}_\lambda(p) = -m - i\gamma\cdot p\,, \qquad (17)$$

$$\Lambda^{(+)}_\lambda(p) = u_\lambda(p)\, \bar{u}_\lambda(p) = (m - i\gamma\cdot p)\frac{(1 + i\lambda\gamma_5\, \gamma\cdot s)}{2}, \qquad (18)$$

$$\Lambda^{(-)}_\lambda(p) = v_\lambda(p)\, \bar{v}_\lambda(p) = (-m - i\gamma\cdot p)\frac{(1 + i\lambda\gamma_5\, \gamma\cdot s)}{2}. \qquad (19)$$

The following rules are often helpful in the evaluation of traces:

$$\mathrm{tr}\,[\gamma_\mu] = 0 \qquad (20)$$

$$\mathrm{tr}\,[\text{odd number of } \gamma\text{-matrices}] = 0 \qquad (21)$$

$$\mathrm{tr}\,(I) = 4 \qquad (22)$$

$$\mathrm{tr}\,(\gamma\cdot a\ \gamma\cdot b) = 4\, a\cdot b \qquad (23)$$

$$\mathrm{tr}(\gamma\cdot a\ \gamma\cdot b\ \gamma\cdot c\ \gamma\cdot d) = 4\,[(a\cdot b)(c\cdot d) - (a\cdot c)(b\cdot d) + (a\cdot d)(b\cdot c)] \qquad (24)$$

$$\mathrm{tr}\,(\gamma_5) = 0 \qquad (25)$$

$$\mathrm{tr}\,(\gamma_5\ \gamma\cdot a\ \gamma\cdot b) = 0 \qquad (26)$$

$$\mathrm{tr}(\gamma_5\ \gamma\cdot a\ \gamma\cdot b\ \gamma\cdot c\ \gamma\cdot d) = 4\,\varepsilon_{\kappa\lambda\mu\nu}\, a_\kappa b_\lambda c_\mu d_\nu . \qquad (27)$$

It is sometimes useful to have an explicit form for the spinor. The positive energy spinor with momentum p along the positive z-axis and helicity λ is

$$u_\lambda(p\hat{z}) = \sqrt{E+M}\begin{pmatrix} \chi_\lambda \\ \frac{2\lambda p}{E+M}\chi_\lambda \end{pmatrix}, \qquad (28)$$

where $\chi_{+\frac{1}{2}} = \begin{pmatrix}1\\0\end{pmatrix}$ and $\chi_{-\frac{1}{2}} \begin{pmatrix}0\\1\end{pmatrix}$. The operators $\frac{1}{2}(1\pm\gamma_5)$ are spin projection operators (in the limit $M\to 0$). Thus,

$$\tfrac{1}{2}(1+\gamma_5)\, u_\pm(p) = \sqrt{E+M}\,\frac{\left(1 \mp \frac{p}{E+M}\right)}{2}\begin{pmatrix}\chi_\pm \\ -\chi_\pm\end{pmatrix} \qquad (29)$$

and

$$\tfrac{1}{2}(1-\gamma_5)\, u_\pm(p) = \sqrt{E+M}\,\frac{\left(1 \pm \frac{p}{E+M}\right)}{2}\begin{pmatrix}\chi_\pm \\ \chi_\pm\end{pmatrix} \qquad (30)$$

become for massless particles ($\frac{p}{E+m} \to 1$)

$$\tfrac{1}{2}(1+\gamma_5)\,u_\pm(p) = \begin{pmatrix} 0 \\ u_-(p) \end{pmatrix} \tag{31}$$

and

$$\tfrac{1}{2}(1-\gamma_5)\,u_\pm(p) = \begin{pmatrix} u_+(p) \\ 0 \end{pmatrix}. \tag{32}$$

A spinor with momentum $\vec{p}\,'$ ($\hat{p}'\cdot\hat{z} = \cos\theta$) is obtained by a rotation about the y-axis as

$$u_\lambda(\vec{p}\,') = e^{-i\sigma_2\theta/2}\, u_\lambda(p\hat{z})$$

$$= \sqrt{E+m}\begin{pmatrix} \alpha\cos\frac{\theta}{2} - \beta\sin\frac{\theta}{2} \\ \alpha\sin\frac{\theta}{2} + \beta\cos\frac{\theta}{2} \\ \left(\alpha\cos\frac{\theta}{2} - \beta\sin\frac{\theta}{2}\right)\frac{2\lambda p'}{E+m} \\ \left(\alpha\sin\frac{\theta}{2} + \beta\cos\frac{\theta}{2}\right)\frac{2\lambda p'}{E+m} \end{pmatrix}, \tag{33}$$

where $\alpha = 1\ (0)$, $\beta = 0\ (1)$ for helicity $+\frac{1}{2}\ (-\frac{1}{2})$.

<u>Exercise</u>. Verify that $\frac{1}{2}(1\pm\gamma_5)$ act as helicity projection operators for the positive-energy spinor (33).

Tensor identities which are useful in the evaluation of matrix elements are these:

$$\begin{aligned} \varepsilon_{\alpha\lambda\mu\nu}\,\varepsilon_{\alpha\rho\sigma\tau} = {} & \delta_{\lambda\rho}(\delta_{\mu\sigma}\delta_{\nu\tau} - \delta_{\mu\tau}\delta_{\nu\sigma}) \\ & - \delta_{\lambda\sigma}(\delta_{\mu\rho}\delta_{\nu\tau} - \delta_{\mu\tau}\delta_{\rho\sigma}) \\ & + \delta_{\lambda\tau}(\delta_{\nu\sigma}\delta_{\mu\rho} - \delta_{\mu\sigma}\delta_{\nu\rho}) \end{aligned} \tag{34}$$

$$\varepsilon_{\alpha\beta\mu\nu}\,\varepsilon_{\alpha\beta\sigma\tau} = 2\left(\bar{\delta}_{\mu\sigma}\,\bar{\delta}_{\nu\tau} - \bar{\delta}_{\mu\tau}\,\bar{\delta}_{\nu\sigma}\right) \tag{35}$$

$$\varepsilon_{\alpha\beta\sigma\nu}\,\varepsilon_{\alpha\beta\sigma\tau} = 6\,\bar{\delta}_{\nu\tau} \tag{36}$$

$$\varepsilon_{\alpha\beta\sigma\tau}\,\varepsilon_{\alpha\beta\sigma\tau} = 24. \tag{37}$$

<u>Four-Fermion Theory of Neutrino-Electron Scattering</u>

We begin our exploration of weak interaction phenomena with an investigation of the consequences of the local four-fermion interaction in simple situations. Let us briefly consider the predictions of various space-time forms of the interaction. Then we shall examine more fully the properties of the "correct" choice, the V-A structure.

The most general matrix element for the charged-current νe interaction, free of derivative couplings, is

$$\mathcal{M} = \sum_{i=1}^{5} \mathcal{M}_i = \sum_i \bar{\nu}\, \mathcal{O}_i\, e\ \bar{e}\, \mathcal{O}_i\, (C_i + C_i'\gamma_5)\, \nu,$$

where the operators $\mathcal{O}_i$ are

Scalar	1
Pseudoscalar	γ_5
Tensor	$\sigma_{\mu\nu}$
Vector	γ_μ
Axial vector	$\gamma_\mu\gamma_5$

Knowing that neutrino sources of interest in nature yield left-handed neutrinos, we may set $C_i = C_i' = 1$ without any significant loss of generality In addition, it will be convenient to employ the combinations $V \pm A$, which contribute incoherently to matrix elements squared. Let us now compute the cross section which arises from each interaction in turn. While much of this exercise is only of academic interest for charged current interactions, we shall find it a useful reference when we discuss neutral current interactions, for which the space-time structure has not yet been established.

$\nu e \to \nu e$ and $\bar{\nu} e \to \bar{\nu} e$

Notation and kinematics. $P_\mu \equiv (iE, P_z, P_x, P_y)$

ν_{in} : q_1

ν_{out} : q_2

e_{in} : p_1

e_{out} : p_2

In LAB frame,

$$(p_1+q_1)^2 = -s = -m_e^2 - 2m_eE$$

$$q_1 = (iE, E, 0)$$
$$p_1 = (im_e, 0, 0)$$
$$p_2 = (iE', P'\cos\theta_L, P'\sin\theta_L)$$
$$q_2 = (i(E+m_e-E'), E-P'\cos\theta_L, -P'\sin\theta_L)$$

$$E' = \sqrt{P'^2+m_e^2} \equiv yE$$

$$p_1\cdot q_1 = -m_eE \qquad p_2\cdot q_2 = -m_eE$$

$$p_1\cdot q_2 = -m_e(E+m_e-E') \approx -m_eE(1-y) \qquad q_1\cdot p_2 = -m_e(E+m_e-E') \approx -m_eE(1-y)$$

$$p_1\cdot p_2 = -m_eE' = -m_eEy$$

$$q_1\cdot q_2 = -m_e(E'-m_e) \approx -m_eEy$$

$$p_2 \cdot q_2 = -m_e E = E p' \cos\theta_L - p'^2 + E'^2 - E'(E+m)$$

$$\therefore \underline{E(E' - p'\cos\theta_L) = m(E+m-E')}$$

$$q_1 \cdot p_2 = -E(E' - p'\cos\theta_L) = -m(E+m-E')$$

$$q_1 \cdot q_2 = \cancel{E^2} - E p'\cos\theta_L - \cancel{E^2} - mE + EE'$$

$$= -E(E' - p'\cos\theta_L) - mE$$

$$= -m(E'-m) = p_1 \cdot p_2 + m^2 \checkmark$$

In CM frame

$$p^* \equiv p_{cm} = mE/\sqrt{s} = \left(\frac{m^2E^2}{2mE+m^2}\right)^{1/2} = \left(\frac{mE}{2+m/E}\right)^{1/2}$$

$$\simeq \sqrt{s}/2$$

$$\omega \equiv \sqrt{p^{*2}+m^2}$$

$$p_1^* = (i\omega, -p^*, 0); \quad p_2^* = (i\omega, p^*\cos\theta, p^*\sin\theta)$$

$$q_1^* = (ip^*, p^*, 0); \quad q_2^* = (ip^*, -p^*\cos\theta, -p^*\sin\theta)$$

$$p_1 \cdot q_1 = p_2 \cdot q_2 = -p(p+\omega) \approx -2p^2$$

$$p_1 \cdot q_2 = p_2 \cdot q_1 = p^2\cos\theta - p\omega \approx -p^2(1-\cos\theta)$$

$$p_1 \cdot p_2 = -p^2\cos\theta - \omega^2 \approx -p^2(1+\cos\theta)$$

$$q_1 \cdot q_2 = -p^2(1+\cos\theta)$$

$$p = \tfrac{1}{2} m_e E$$

(i) Scalar Interaction: $\nu e \to \nu e$

$$-i\mathcal{M} = \bar{u}(\nu, q_2)\, u(e, p_1)\, \bar{u}(e, p_2)(1+\gamma_5)\, u(\nu, q_1)$$

$$|\mathcal{M}|^2 = \mathrm{tr}\left[u(e,p_1)\bar{u}(e,p_1)u(\nu,q_2)\bar{u}(\nu,q_2)\right] \times$$
$$\times\ \mathrm{tr}\left[(1+\gamma_5)u(\nu,q_1)\bar{u}(\nu,q_1)(1-\gamma_5)u(e,p_2)\bar{u}(e,p_2)\right]$$

first trace: $\mathrm{tr}[(m - i\gamma\cdot p_1)(-i\gamma\cdot q_2)] = -4\, p_1\cdot q_2$

second trace:

$$\mathrm{tr}[(1+\gamma_5)(-i\gamma\cdot q_1)(1-\gamma_5)(m - i\gamma\cdot p_2)]$$
$$= 2\,\mathrm{tr}[(1+\gamma_5)(-i\gamma\cdot q_1)(m - i\gamma\cdot p_2)] = -8\, q_1\cdot p_2$$

$$|\mathcal{M}|^2 = 32\, p_1\cdot q_2\; q_1\cdot p_2$$

Average over initial electron spins:

$$\overline{|\mathcal{M}|^2} = 16\, p_1\cdot q_2\; q_1\cdot p_2$$

$$\frac{d\sigma}{d\Omega_{cm}} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 s} = \frac{\overline{|\mathcal{M}|^2}}{256\pi^2 p^2} = \frac{p^2(1-\cos\theta)^2}{16\pi^2}$$

$$= \frac{mE}{32\pi^2}(1-\cos\theta)^2.$$

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega = \frac{mE}{16\pi}\underbrace{\int_{-1}^{1} dz\,(1-z)^2}_{8/3}$$

$$= mE/6\pi$$

Notice $(1-\cos\theta) = 2(1-y)$, $y = E'/E$

$$\frac{d\sigma}{dy} = 4\pi \frac{d\sigma}{d\Omega} = \frac{\overline{|M|^2}}{32\pi m E}$$

$$= \frac{mE\,(1-y)^2}{2\pi}$$

$$\sigma = \int_0^1 dy\, \frac{d\sigma}{dy} = \frac{mE}{6\pi}$$

(ii) Scalar interaction $\bar\nu e \to \bar\nu e$

$$-iM = \bar v(\nu, q_1)\, u(e, p_1)\, \bar u(e, p_2)(1+\gamma_5)\, v(\nu, q_2)$$

$$|M|^2 = \mathrm{tr}\left[(m - i\gamma\cdot p_1)(-i\gamma\cdot q_1)\right] 2\,\mathrm{tr}\left[(1+\gamma_5)(-i\gamma\cdot q_2)(m - i\gamma\cdot p_2)\right]$$

This is simply $q_1 \leftrightarrow q_2$ on νe result

$$\overline{|M|^2} = 16\, p_1\cdot q_1\, p_2\cdot q_2$$

$$\frac{d\sigma}{dy} = \frac{mE}{2\pi}$$

$$\sigma = \frac{mE}{2\pi}$$

(iii) Pseudoscalar interaction $\nu e \to \nu e$

$$-iM = \bar u(\nu, q_2)\, \gamma_5\, u(e, p_1)\, \bar u(e, p_2)\, \gamma_5 (1+\gamma_5)\, u(\nu, q_1)$$

$$|M|^2 = \mathrm{tr}\left[\gamma_5 (m - i\gamma\cdot p_1)(-\gamma_5)(-i\gamma\cdot q_2)\right] \mathrm{tr}\left[(1+\gamma_5)(-i\gamma\cdot q_1)(1-\gamma_5)(m - i\gamma\cdot p_2)\right]$$

= same as scalar interaction

(iv) Pseudoscalar interaction $\bar\nu e \to \bar\nu e$

Same as scalar interaction

(v) Tensor coupling, $\nu e \to \nu e$

$$-iM = \bar{u}(\nu, q_2)\,\sigma_{\mu\nu}\,u(e, p_1)\,\bar{u}(e, p_2)\,\sigma_{\mu\nu}(1+\gamma_5)\,u(\nu, q_1)$$

$$|M|^2 = 64\,[2\,q_1\cdot q_2\,p_1\cdot p_2 + 2\,q_1\cdot p_1\,q_2\cdot p_2 - q_1\cdot p_2\,q_2\cdot p_1]$$

$$= 64\,m^2E^2\,(1+y)^2$$

$$\frac{d\sigma}{dy} = \frac{mE}{\pi}(1+y)^2$$

$$\sigma = \frac{7}{3}\frac{mE}{\pi}$$

(vi) Tensor coupling, $\bar{\nu} e \to \bar{\nu} e$

$$-iM = v(\nu, q_1)\,\sigma_{\mu\nu}\,u(e, p_1)\,\bar{u}(e, p_2)\,\sigma_{\mu\nu}(1+\gamma_5)\,v(\nu, q_2)$$

$$= 64\,[2\,q_1\cdot q_2\,p_1\cdot p_2 + 2\,p_1\cdot q_2\,p_2\cdot q_1 - q_2\cdot p_2\,p_1\cdot q_1]$$

$$= 64\,m^2E^2(1-2y)^2$$

$$\frac{d\sigma}{dy} = \frac{mE}{\pi}(1-2y)^2$$

$$\sigma = \frac{mE}{3\pi}$$

(vii) S-T Interference, $\nu e \to \nu e$

$$-iM = \bar{u}(\nu, q_2)\,u(e, p_1)\,\bar{u}(e, p_2)(1+\gamma_5)\,u(\nu, q_1)$$
$$+ \bar{u}(\nu, q_2)\,\sigma_{\mu\nu}\,u(e, p_1)\,\bar{u}(e, p_2)\,\sigma_{\mu\nu}(1+\gamma_5)\,u(\nu, q_1)$$

Interference terms =

$$\bar{u}(\nu,q_2)\,u(e,p_1)\,\bar{u}(e,p_2)\,\sigma_{\mu\nu}\,u(\nu,q_2)\times$$

$$\times\ \bar{u}(e,p_2)(1+\gamma_5)\,u(\nu,q_1)\,\bar{u}(\nu,q_1)(1-\gamma_5)\,\sigma_{\mu\nu}\,u(e,p_2)$$

$$+\ \bar{u}(\nu,q_2)\,\sigma_{\mu\nu}\,u(e,p_1)\,\bar{u}(e,p_1)\,u(\nu,q_2)\times$$

$$\times\ \bar{u}(e,p_2)\,\sigma_{\mu\nu}(1+\gamma_5)\,u(\nu,q_1)\,\bar{u}(\nu,q_1)(1-\gamma_5)\,u(e,p_2)$$

$$= \mathrm{tr}[(m-i\gamma\cdot p_1)\,\sigma_{\mu\nu}\,(-i\gamma\cdot q_2)]\,2\,\mathrm{tr}[(1+\gamma_5)(-i\gamma\cdot q_1)\,\sigma_{\mu\nu}\,(m-i\gamma\cdot p_2)]$$

$$+\ \mathrm{tr}[\sigma_{\mu\nu}(m-i\gamma\cdot p_1)(-i\gamma\cdot q_2)]\,2\,\mathrm{tr}[\sigma_{\mu\nu}(1+\gamma_5)(-i\gamma\cdot q_1)(m-i\gamma\cdot p_2)]$$

$$\sigma_{\mu\nu} = \frac{\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu}{2i} = -i(\gamma_\mu\gamma_\nu-\delta_{\mu\nu})$$

$$\mathrm{tr}[\gamma_\mu\gamma_\nu(\gamma\cdot p_1)(\gamma\cdot q_2)] = 4[\cancel{p_1\cdot q_2\,\delta_{\mu\nu}} - p_{1\mu}q_{2\nu} + p_{1\nu}q_{2\mu}]$$

$$-\mathrm{tr}[\delta_{\mu\nu}\,\gamma\cdot p_1\,\gamma\cdot q_2] = \cancel{-4\,p_1\cdot q_2\,\delta_{\mu\nu}}$$

$$\mathrm{tr}[\sigma_{\mu\nu}(\gamma\cdot p_1)(\gamma\cdot p_2)] = 4(q_{2\mu}p_{1\nu} - p_{1\mu}q_{2\nu})$$

Therefore, $|M|^2 = 64(p_{2\mu}q_{1\nu} - q_{1\mu}p_{2\nu})(q_{2\mu}p_{1\nu} - p_{1\mu}q_{2\nu})$

$$= 128(p_1\cdot q_1\;p_2\cdot q_2 - p_1\cdot p_2\;q_1\cdot q_2)$$

$$\frac{d\sigma_{ST}}{dy} = \frac{2mE}{\pi}(1+y)(1-y)$$

$$\sigma_{ST} = \frac{2mE}{\pi}\int_0^1 dy\,(1-y^2) = \frac{4mE}{3\pi}$$

(A)

(viii) ST interference, $\bar{\nu} e \to \bar{\nu} e$.

$q_1 \longleftrightarrow q_2$ on νe

$$|M|^2 = 128 (p_2\cdot q_1\, q_2\cdot p_1 - p_1\cdot p_2\, q_1\cdot q_2)$$

$$= 128 m^2 E^2 (1-2y)$$

$$\frac{d\sigma}{dy} = \frac{2mE}{\pi}(1-2y)$$

$$\sigma = 0$$

(ix) V-A, $\nu e \to \nu e$

$$-iM = \bar{u}(\nu, q_2)\gamma_\mu(1+\gamma_5)u(e,p_1)\bar{u}(e,p_2)\gamma_\mu(1+\gamma_5)u(\nu,q_1)$$

$$|M|^2 = \mathrm{tr}[\gamma_\mu(1+\gamma_5)(m-i\gamma\cdot p_1)(1-\gamma_5)\gamma_\nu(-i\gamma\cdot q_2)] \times$$

$$\times\, \mathrm{tr}[\gamma_\mu(1+\gamma_5)(-i\gamma\cdot q_1)(1-\gamma_5)\gamma_\nu(m-i\gamma\cdot p_2)]$$

$$A_{\mu\nu} = +2\,\mathrm{tr}[(1\mp\gamma_5)\gamma_\mu(m-i\gamma\cdot p_1)\gamma_\nu(-i\gamma\cdot q_2)]$$

$$= -8[q_{1\mu}p_{2\nu} - \delta_{\mu\nu}\, q_1\cdot p_2 + p_{2\mu}q_{1\nu}] \pm 8\,\varepsilon_{\alpha\beta\gamma\delta}\,\delta_{\alpha\mu}\,q_{1\beta}\,\delta_{\gamma\nu}\,p_{2\delta}$$

$$B_{\mu\nu} = 2\,\mathrm{tr}[(1-\gamma_5)\gamma_\mu(-i\gamma\cdot q_1)\gamma_\nu(m-i\gamma\cdot p_2)]$$

$$= -8[p_{1\mu}q_{2\nu} - \delta_{\mu\nu}\, p_1\cdot q_2 + q_{2\mu}p_{1\nu}] + 8\,\varepsilon_{\lambda\rho\sigma\tau}\,\delta_{\lambda\mu}\,p_{1\rho}\,\delta_{\sigma\nu}\,q_{2\tau}$$

$$A_{\mu\nu} B_{\mu\nu} = 64 \left[2 p_1 \cdot q_1 \; p_2 \cdot q_2 + 2 p_1 \cdot p_2 \; q_1 \cdot q_2 \right]$$

$$\left.\begin{array}{l} - 64\, \varepsilon_{\lambda\rho\sigma\tau} \left[q_{1\lambda} p_{1\rho} p_{2\sigma} q_{2\tau} + p_{2\lambda} p_{1\rho} q_{1\sigma} q_{2\tau} \right] \\ \mp 64\, \varepsilon_{\alpha\beta\gamma\delta} \left[p_{1\alpha} q_{1\beta} q_{2\gamma} p_{2\delta} + q_{2\alpha} q_{1\beta} p_{1\gamma} p_{2\delta} \right] \end{array} \right\} \begin{array}{l} \text{vanish for} \\ 2 \to 2 \\ \text{process} \end{array}$$

$$\pm 64\, \varepsilon_{\alpha\beta\gamma\delta}\, \varepsilon_{\lambda\rho\sigma\tau}\, \delta_{\alpha\lambda}\, \delta_{\gamma\sigma}\, q_{1\beta}\, p_{2\delta}\, p_{1\rho}\, q_{2\tau}$$

Last term $= \pm 128 \left(\delta_{\beta\rho} \delta_{\delta\tau} - \delta_{\beta\tau} \delta_{\delta\rho} \right) q_{1\beta} p_{2\delta} p_{1\rho} q_{2\tau}$

$= \pm 128 \left(p_1 \cdot q_1 \; p_2 \cdot q_2 - p_1 \cdot p_2 \; q_1 \cdot q_2 \right)$

$$|m|^2 = 256 (p_1 \cdot q_1)(p_2 \cdot q_2)$$

$$= 256\, m^2 E^2$$

$$\frac{d\sigma}{dy} = \frac{4mE}{\pi} \qquad \sigma = \frac{4mE}{\pi}$$

(x). V−A, $\bar{\nu} e \to \bar{\nu} e$

$q_1 \leftrightarrow q_2$ on νe.

$$|m|^2 = 256 (p_1 \cdot q_2)(q_1 \cdot p_2)$$

$$= 256\, m^2 E^2 (1-y)^2$$

$$\frac{d\sigma}{dy} = \frac{4mE}{\pi} (1-y)^2$$

$$\sigma = \frac{4mE}{3\pi}$$

(xi) V+A, $\nu e \to \nu e$

$$-i\mathcal{M} = \bar{u}(\nu, q_2)\gamma_\mu(1-\gamma_5)u(e,p_1)\ \bar{u}(e,p_2)\gamma_\mu(1+\gamma_5)u(\nu,q_1)$$

Same as evaluation of V−A, except use red sign

$$|\mathcal{M}|^2 = 256(p_1\cdot p_2)(q_1\cdot q_2)$$
$$= 256\, m^2E^2 y^2$$
$$\frac{d\sigma}{dy} = \frac{4mE}{\pi} y^2$$
$$\sigma = \frac{4mE}{3\pi}$$

(xii) V+A, $\bar{\nu} e \to \bar{\nu} e$

~~from~~ . $q_1 \leftrightarrow q_2$ on νe

$$\therefore \frac{d\sigma}{dy} = \frac{4mE}{\pi} y^2$$
$$\sigma = \frac{4mE}{3\pi}$$

We summarize our results for

$$\frac{d\Sigma}{dy} = \frac{d\sigma}{dy}(SPT) + \frac{d\sigma}{dy}(V\text{-}A) + \frac{d\sigma}{dy}(V+A)$$

in Tabular form.

Coupling	νe	$\bar{\nu} e$
S, P	$\frac{1}{2}(1-y)^2$	$\frac{1}{2}$
T	$(1+y)^2$	$(1-2y)^2$
ST	$2(1+y)(1-y)$	$2(1-2y)$
V-A	4	$4(1-y)^2$
V+A	$4y^2$	$4y^2$

Entries are $(\pi/mE)\, d\sigma/dy$

Notice the following "confusion theorem": Any combination of V-A and V+A can be reproduced by a suitably chosen combination of STP.

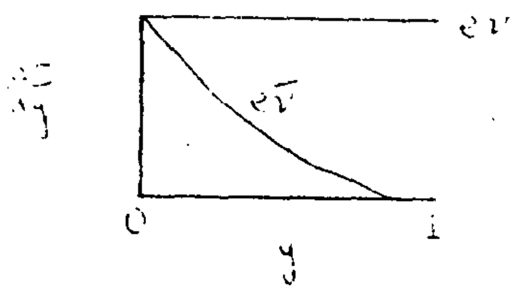
For the physically interesting V-A case, we obtain predictions for νe scattering by multiplying $G^2/2 \times$ our computed cross section. $G^2 = 5.27 \times 10^{-38}\ cm^2/GeV^2$

<u>νe</u>

$$\frac{d\sigma}{dy} = \frac{2mEG^2}{\pi} \qquad \sigma = \frac{2mEG^2}{\pi} = 1.72 \times 10^{-41}\ cm^2 \times \left(\frac{E}{1\,GeV}\right)$$

$\bar{\nu} e$

$$\frac{d\sigma}{dy} = \frac{2mEG^2}{\pi}(1-y)^2 \qquad \sigma = \frac{2mEG^2}{3\pi} = 0.571 \times 10^{-41}\ cm^2 \times \left(\frac{E}{1\,GeV}\right).$$



An aside: the reaction $\nu_\mu e^- \to \mu^- \nu_e$
$$\begin{matrix} q_1 & p_1 & & p_2 & q_2 \end{matrix}$$

Our definitions are as in the case of νe elastic scattering, except that the final muon energy is $E' = (p'^2 + \mu^2)^{1/2}$, with μ the muon mass. The invariants are

$$q_1 \cdot p_1 = -m_e E$$
$$q_2 \cdot p_2 = -m_e E + (\mu^2 - m_e^2)/2$$
$$p_1 \cdot p_2 = -m_e E'$$
$$p_1 \cdot q_2 = -m_e (E + m_e - E')$$
$$q_1 \cdot q_2 = -m_e E' + (\mu^2 + m^2)/2$$
$$p_2 \cdot q_1 = -e(E - E') - (\mu^2 + m^2)/2$$

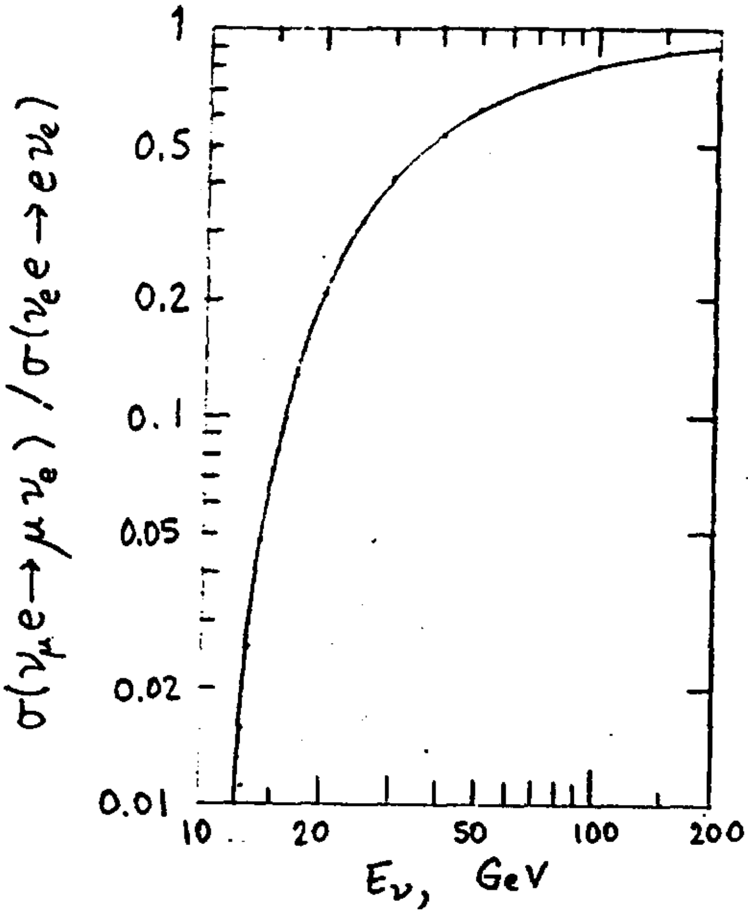
$$\frac{d\sigma}{d\Omega} = \frac{\overline{|m|^2}}{64\pi^2 s} \frac{p'_{cm}}{p_{cm}}$$

$$\frac{p'_{cm}}{p_{cm}} = \frac{m^2 + 2mE - \mu^2}{m^2 + 2mE - m^2} = 1 - \frac{(\mu^2 - m^2)}{2mE}$$

$$\frac{d\sigma}{dy} = \frac{G^2}{2} \frac{128\,(p_1 \cdot q_1)(p_2 \cdot q_2)}{16\pi \cdot 2mE} \left[1 - \frac{(\mu^2 - m^2)}{2mE}\right]$$
$$= 2G^2 mE \left[1 - \frac{(\mu^2 - m^2)}{2mE}\right]^2$$

$$\frac{d\sigma(\nu_\mu e \to \mu \nu_e)/dy}{d\sigma(\nu_e e \to e \nu_e)/dy} = \left[1 - \frac{(\mu^2 - m^2)}{2mE}\right]^2$$

22B



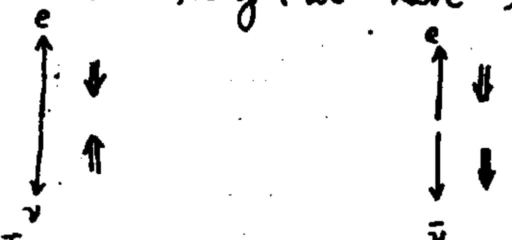
1
0.5
0.2
0.1
0.05
0.02
0.01
10
20
50
100
200
E_ν, GeV
$\sigma(\nu_\mu e \to \mu \nu_e) / \sigma(\nu_e e \to e \nu_e)$

Origin of the Factor $\frac{1}{3}$:

neutrinos have helicity $-\frac{1}{2}$
antineutrinos " " $+\frac{1}{2}$

$J_z = 0$ $\qquad$ $J_z = +1$

backward scattering (we have labelled this $\cos\theta = 0, y = 1$) gives



$J_z = 0$ $\qquad$ $J_z = -1$

allowed! $\qquad$ forbidden!

<u>The unitarity question</u>

$\nu e \to \nu e$ is isotropic; pure s-wave scattering. Partial-wave unitarity requires

$$\sigma < \frac{1}{2}\frac{\pi}{p^2}.$$

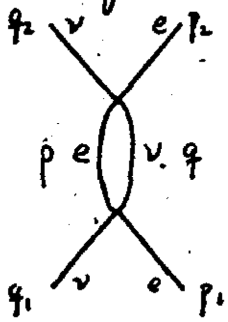
but we computed

$$\sigma \approx 4G^2 p^2/\pi,$$ which satisfies the constraint only for

$$p^4 < \pi^2/8G^2,$$

i.e. for $p < 309$ GeV/c. This means $s < 3.81 \times 10^5$ GeV^2, and $E_\nu < 3.73 \times 10^8$ GeV.

Could the violation of unitarity be remedied in higher orders? In second order, we may consider the diagram



for which

$$-i\mathcal{M} = \frac{G^2}{2}\frac{1}{(2\pi)^4}\int d^4q\; \bar{u}(e,p_2)\gamma_\mu(1+\gamma_5)\frac{(-i\gamma\cdot q)}{i\,q^2}\gamma(1+\gamma_5)$$

$$u(e,p_1)\,\bar{u}(\nu,q_2)\,\gamma_\mu(1+\gamma_5)\frac{(m-i\gamma\cdot p)}{i(p^2+m^2)}\gamma_\nu(1+\gamma_5)u(\nu,q_1)$$

$$\propto \int\frac{d^4q}{pq} \sim \int\frac{q^3\,dq}{q^2} \sim q^2, \text{ quadratically divergent.}$$

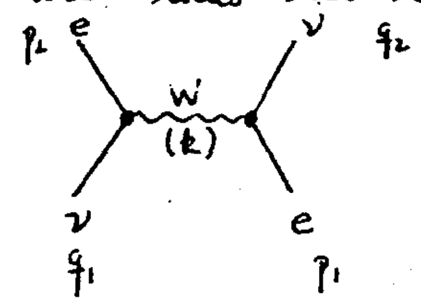
The divergence of the point-coupling theory grows more severe in each order of perturbation theory. We may try to arrange a constant cross section at high energies by assuming (in analogy with QED) that the weak interaction is mediated by vector boson exchange. The intermediate boson W must have these three properties:

(1) It carries charge ± 1.

(2) It must be rather massive, to reproduce the short range of the weak force.

(3) Its parity is indefinite.

We are thus led to compute



At each vertex, we shall have the coupling

$$g = (GM_W^2/\sqrt{2})^{1/2}.$$

The W-boson propagator is

$$\frac{\delta_{\mu\nu} + \frac{k_\mu k_\nu}{M_W^2}}{i(k^2 + M_W^2)}.$$

The matrix element for $\nu e \to \nu e$ is

$$-i\mathcal{M} = \frac{GM_W^2}{\sqrt{2}}\, \bar{u}(e, p_2)\gamma_\mu(1+\gamma_5)u(\nu, q_1)$$

$$\cdot \frac{\delta_{\mu\nu} + \frac{k_\mu k_\nu}{M_W^2}}{i(k^2+M_W^2)}\quad \bar{u}(\nu, q_2)\gamma_\nu(1+\gamma_5)u(e, p_1),$$

with $k = p_2 - q_1 = p_1 - q_2$. By substituting $k_\mu = p_{2\mu} - q_{1\mu}$ and $k_\nu = p_{1\nu} - q_{2\nu}$ and using the Dirac equation, we find that in this instance the $k_\mu k_\nu / M_W^2$ term in the propagator contributes only $O(m_e^2/M_W^2)$, and can be safely neglected. What remains is simply the amplitude we encountered in the point coupling theory times $M_W^2/(k^2+M_W^2)$. Without further computation, we have

$$\frac{d\sigma}{dy} = \frac{2mEG^2}{\pi\ (1+k^2/M_W^2)^2}$$

$$k^2 = -m_e^2 - 2p_2\cdot q_1 = +2mE(1-y)$$

$$\frac{d\sigma}{dy} = \frac{2mEG^2}{\pi\left[1+\frac{2mE}{M_W^2}(1-y)\right]^2}$$

$$\sigma = \frac{2mEG^2}{\pi} \underbrace{\int_0^1 dy \left[1 + \frac{2mE}{M_W^2}(1-y)\right]^{-2}}$$

$$\int_1^{1+2mE/M_W^2} \frac{M_W^2}{2mE}\, dZ \;\; Z^{-2}$$

$$= \frac{M_W^2}{2mE}\left[1 - \frac{1}{1+2mE/M_W^2}\right]$$

$$= \frac{1}{1+2mE/M_W^2}$$

$$\sigma = \frac{2mEG^2}{\pi(1+2mE/M_W^2)}$$ approaches a constant value at high energies.

$$\lim_{E\to\infty} \sigma = \frac{G^2 M_W^2}{\pi}$$

This is a great improvement over the point-coupling theory, but a problem remains: the s-wave amplitude violates partial-wave unitarity. To see this we write

$$f(\theta) = \left(\frac{2d\sigma}{d\Omega}\right)^{1/2} = \frac{\sqrt{2}\,Gp}{\pi}\left[1 + \frac{2p^2}{M_W^2}(1-\cos\theta)\right]^{-1}$$

$$= \frac{1}{p}\sum_{J=0}^{\infty} (J+1/2)\, P_J(\cos\theta)\, \mathcal{M}_J \, .$$

The s-wave amplitude is

$$\mathcal{M}_0 = \frac{\sqrt{2}\, G p^2}{\pi} \int_{-1}^{1} d(\cos\theta)\, \left[1 + \frac{2p^2}{M_W^2}(1-\cos\theta)\right]^{-1}$$

Let

$$w = 1 + \frac{2p^2}{M_W^2}(1-\cos\theta)$$

$$dw = -\frac{2p^2}{M_W^2}\, d(\cos\theta)$$

$$\mathcal{M}_0 = \frac{\sqrt{2}\, G p^2}{\pi} \int_{1}^{1+4p^2/M_W^2} dw\, \frac{M_W^2}{2p^2}\, w^{-1}$$

$$= \frac{G M_W^2}{\sqrt{2}\,\pi} \log\left(1 + 4p^2/M_W^2\right)$$

$$= \frac{G M_W^2}{\sqrt{2}\,\pi} \log\left(1 + 2mE/M_W^2\right)$$

The constraint of partial-wave unitarity is

$$\mathcal{M}_0 < 1,$$

which implies

$$E < \frac{M_W^2}{2m}\left(\exp\left[\frac{\sqrt{2}\,\pi}{G M_W^2}\right] - 1\right).$$

Notice that in the limit $M_W \to \infty$ we recover the point-coupling result,

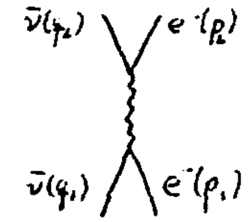
$$E < \pi / G m \sqrt{2}$$

These violations arise at incredibly high energies:

M_W	E_{MAX}
45	1.09×10^{88} GeV
50	3.97×10^{72}
100	3.49×10^{23}
1000	4.54×10^{8}
10^4	3.74×10^{8}
∞	3.73×10^{8}

Could this mild unitarity violation be conquered in higher orders? No, the $k_\mu k_\nu / M_W^2$ term in the W propagator makes our theory non-renormalizable and gives rise to problems in higher order. In addition, as we shall see, the introduction of the W causes new problems of its own. We shall identify these and see how they can be corrected later on. For the moment, let us continue to investigate the consequences of the W.

In $\bar{\nu}e$ scattering, the W appears in the s-channel, so the differential cross section is



$$\frac{d\sigma}{dy} = \frac{M_W^4}{[M_W^2 + (p_1+q_1)^2]^2} \cdot \frac{d\sigma}{dy}(\text{point coupling})$$

$$\simeq \frac{2mEG^2(1-y)^2}{\pi\left[1 - \frac{2mE}{M_W^2}\right]^2}, \qquad \sigma = \frac{2mEG^2}{3\pi\left[1 - \frac{2mE}{M_W^2}\right]^2} \xrightarrow[E\to\infty]{} G^2 M_W^2 / 6\pi m E$$

Leptonic Decay of $W^{\pm}$

$\uparrow e^-(p)$

$\downarrow \bar{\nu}(q)$

In the W^- rest frame,

$$p = \left(\tfrac{1}{2}M_W \sin\theta,\ 0,\ \tfrac{1}{2}M_W\cos\theta,\ \tfrac{1}{2}M_W\right)$$
$$q = \left(-\tfrac{1}{2}M_W \sin\theta,\ 0,\ -\tfrac{1}{2}M_W\cos\theta,\ \tfrac{1}{2}M_W\right)$$

The polarization of the decaying W is $\varepsilon = (\vec{\varepsilon}, 0)$. We neglect m_e everywhere. The matrix element is

$$-i\mathcal{M} = \bar{u}(e,p)\,\gamma_\mu(1+\gamma_5)\,v(\nu,q)\,\varepsilon_\mu \quad (GM_W^2/\sqrt{2})^{1/2}.$$

$$|\mathcal{M}|^2 = -\mathrm{tr}\left[\gamma\cdot\varepsilon\,(1+\gamma_5)(-i\gamma\cdot q)(1-\gamma_5)\,\gamma\cdot\varepsilon^*(-i\gamma\cdot p)\right]\ GM_W^2/\sqrt{2}$$
$$= +2\,\mathrm{tr}\left[(1-\gamma_5)\,\gamma\cdot\varepsilon\ \gamma\cdot q\ \gamma\cdot\varepsilon^*\ \gamma\cdot p\right]\ GM_W^2/\sqrt{2}$$
$$= +8\left\{\varepsilon\cdot q\ \varepsilon^*\cdot p - \varepsilon\cdot\varepsilon^*\ p\cdot q + \varepsilon\cdot p\ \varepsilon^*\cdot q - \epsilon_{\lambda\mu\nu\rho}\,\varepsilon_\lambda q_\mu \varepsilon^*_\nu p_\rho\right\}\frac{GM_W^2}{\sqrt{2}}$$

Longitudinal Polarization. $\varepsilon = (0,0,1,0) = \varepsilon^*$, so final term vanishes.

$$|\mathcal{M}|^2 = +8\left\{-\frac{M_W^2}{4}\cos^2\theta + \frac{M_W^2}{2} - \frac{M_W^2}{4}\cos^2\theta\right\}\frac{GM_W^2}{\sqrt{2}}$$
$$= +4M_W^2\,(1-\cos^2\theta)\ GM_W^2/\sqrt{2}$$

$$\frac{dW}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2 M_W} = \frac{G\,M_W^3}{16\pi^2\sqrt{2}}\,(1-\cos^2\theta)^*$$

$$W = \int d\Omega\,\frac{dW}{d\Omega} = \frac{GM_W^3}{8\pi\sqrt{2}}\int_{-1}^{1} dz\,(1-z^2)$$
$$= \frac{GM_W^3}{6\pi\sqrt{2}} = 6.64\times 10^{17}\ \mathrm{sec}^{-1}\left(\frac{M_W}{1\,\mathrm{GeV}}\right)^3$$

$$= 4.37 \times 10^{-4} \text{ MeV} \left(\frac{M_W}{1\,\text{GeV}/c^2}\right)^3$$

Example: $M_W = 37.3$ GeV : $W(W^- \to e^- \nu) = 22.15$ MeV

6 94 keV

Helicity + 1 : $\varepsilon = \frac{1}{\sqrt{2}}(1, i, 0, 0)$.

$$|m|^2 = \frac{8}{\cancel{4}}\left\{-\frac{M_W^2}{8}\sin^2\theta + \frac{M_W^2}{2} - \frac{M_W^2}{8}\sin^2\theta - \underbrace{\varepsilon_{\lambda\mu\nu\rho}\,\varepsilon_\lambda q_\mu \varepsilon^*_\nu p_\rho}\right\}\frac{GM_W^2}{\sqrt{2}}$$

$$= \frac{M_W^2}{\cancel{4}}\left\{-\varepsilon_1 q_3 \varepsilon_2^* p_4 + \varepsilon_1 q_4 \varepsilon_2^* p_3 + \varepsilon_2 q_3 \varepsilon_1^* p_4 - \varepsilon_2 q_4 \varepsilon_1^* p_3\right\}$$

$$= \frac{M_W^2}{8}\left\{+\cos\theta + \cos\theta + \cos\theta + \cos\theta\right\}$$

$$= \frac{M_W^2 \cos\theta}{2}$$

$$|m|^2 = 8M_W^2\left[\frac{1}{2} - \underbrace{\frac{1}{4}\sin^2\theta}_{\frac{1}{4}-\frac{1}{4}\cos^2\theta} - \frac{1}{2}\cos\theta\right] GM_W^2/\sqrt{2}$$

$$= 2M_W^2\left[1 - 2\cos\theta + \cos^2\theta\right]\frac{GM_W^2}{\sqrt{2}}$$

$$= GM_W^4\sqrt{2}\,(1-\cos\theta)^2$$

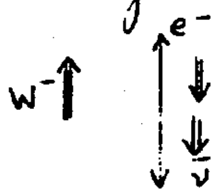
$$\frac{dW}{d\Omega} = \frac{|m|^2}{64\pi^2 M_W} = \frac{GM_W^3}{32\pi^2\sqrt{2}}(1-\cos\theta)^2$$

$$W = \frac{GM_W^3}{16\pi\sqrt{2}}\int_{-1}^{1} dz\,(1-z)^2 = \frac{GM_W^3}{6\pi\sqrt{2}}$$

For helicity -1, $\varepsilon \leftrightarrow \varepsilon^*$, and so the determinant changes sign. Thus for $W^- \to e^- \bar{\nu}$

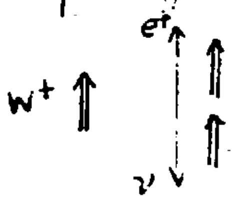
Helicity	$dW/d\Omega$
$+1$	$\dfrac{GM_W^3}{32\pi^2\sqrt{2}}(1-\cos\theta)^2$
0	$\dfrac{GM_W^3}{16\pi^2\sqrt{2}}\sin^2\theta$
-1	$\dfrac{GM_W^3}{32\pi^2\sqrt{2}}(1+\cos\theta)^2$

The angular dependences are easy to understand.



is forbidden ($\theta=0$), but $\theta=\pi$ is OK

Notice that for $W^+ \to e^+ \nu$, the situation is reversed: the e^+ is emitted preferentially along the polarization direction.

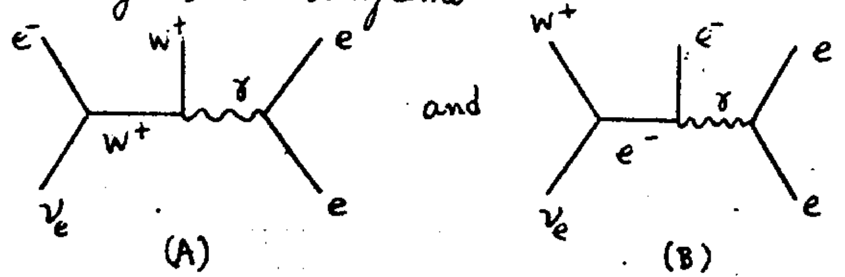


$\theta = 0$ allowed.

This is another example of C violation in the weak interactions.

Remark on W Production

In purely leptonic collisions, W production proceeds by the diagrams



It is plausible (and also true) that graph (B) provides the dominant contribution since $m_\mu^2 \ll M_W^2$. In addition, forward W production is favored. This means we can guess the polarization of the produced W from our knowledge of its decay: the W^+ will be produced in a left-handed state because the incident neutrino is left-handed. This in turn means that the positron from the decay $W^+ \to e^+ \nu$ will be emitted backward in the W^+ rest frame. Detailed computations for the case

$$\nu_\mu \, p \to \mu^- W^+ p$$

have been performed by many authors, including

T. D. Lee, P. Markstein, and C. N. Yang, Phys. Rev. Lett. 7, 429 (1961).

J. S. Bell and M. Veltman, Phys. Lett. 5, 94, 151 (1963).

R. W. Brown and J. Smith, Phys. Rev. D3, 207 (1971).

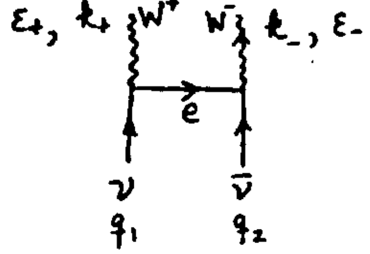
R. W. Brown, R. H. Hobbs, and J. Smith, Phys. Rev. D4, 794 (1971).

Unitarity Problems with $W^\pm$.

We have already mentioned that whereas the introduction of the intermediate boson softens the divergence of the S-wave amplitude for νe scattering, it introduces new divergences in other processes. The most celebrated example occurs in the reaction

$$\nu_e \bar{\nu}_e \longrightarrow W^+ W^-,$$

as first(?) explained by M. Gell-Mann, M. L. Goldberger, N. Kroll, and F. E. Low, Phys. Rev. 179, 1518 (1969). We compute the diagram



which, according to the theory we have built up so far, is the only contribution to the reaction.

We define
$$P = q_1 - k_+ = k_- - q_2.$$

$$-i\mathcal{M} = \frac{GM_W^2}{\sqrt{2}}\, \bar{v}(\nu, q_2)\gamma\cdot\varepsilon_-^*(1+\gamma_5)\frac{(m - i\gamma\cdot P)}{i(P^2+m^2)}$$

$$\gamma\cdot\varepsilon_+^*(1+\gamma_5)\,u(\nu, q_1).$$

CM Kinematics:

$$q_1 = (0, 0, Q, iQ) \qquad q_2 = (0, 0, -Q, iQ)$$
$$k_+ = (K\sin\theta, 0, K\cos\theta, iQ) \quad k_- = (-K\sin\theta, 0, -K\cos\theta, iQ)$$
$$K^2 = Q^2 - M_W^2$$
$$P^2 = -M_W^2 - 2q_1\cdot k_+$$
$$= -M_W^2 + Q^2 - QK\cos\theta$$

If $\hat{\varepsilon}_W$ is the W polarization in its rest frame, then in the CM

$$\varepsilon_\pm = \left(\hat{\varepsilon}_\pm + \frac{\vec{k}_\pm\cdot\hat{\varepsilon}_\pm\,\vec{k}_\pm}{M_W(Q+M_W)},\, i\frac{\vec{k}_\pm\cdot\hat{\varepsilon}_\pm}{M_W}\right)$$

For longitudinal polarization,
$$\varepsilon_\pm = \left(\frac{Q\hat{k}_\pm}{M_W}, \frac{iK}{M_W}\right) \xrightarrow[Q\to\infty]{} \frac{k_\pm}{M_W}.$$

At high energies, we therefore have
$$-i\mathcal{M} = \frac{G}{\sqrt{2}}\,\bar{v}(\nu, q_2)\gamma\cdot k_-(1+\gamma_5)\frac{(m-i\gamma\cdot P)}{i(P^2+m^2)}\gamma\cdot k_+(1+\gamma_5)\,u(\nu, q_1)$$

Using the Dirac eqn, $\gamma\cdot q_1\, u(\nu, q_1) = 0$ and $\bar{v}(\nu, q_2)\,\gamma\cdot q_2 = 0$, we replace

$$\gamma\cdot k_+ \longrightarrow \gamma\cdot(k_+ - q_1) = -\gamma\cdot P$$
$$\gamma\cdot k_- \longrightarrow \gamma\cdot(k_- - q_2) = \gamma\cdot P$$

to obtain

$$-i\mathcal{M} = \frac{G}{\sqrt{2}}\,\bar{v}(\nu, q_2)\,\gamma\cdot P\,\frac{(1+\gamma_5)}{P^2}\,\gamma\cdot P\;\gamma\cdot P\,(1+\gamma_5)\,u(\nu, q_1),$$

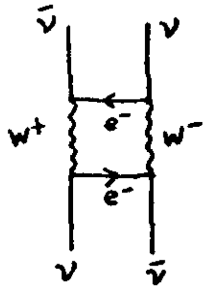
where we have everywhere discarded m. Now, $\gamma\cdot P\,\gamma\cdot P = P^2$ and $(1+\gamma_5)^2 = 2(1+\gamma_5)$, so we have

$$-i\mathcal{M} = G\sqrt{2}\;\bar{v}(\nu, q_2)\,\gamma\cdot P\,(1+\gamma_5)\,u(\nu, q_1).$$

Consequently,

$$|\mathcal{M}|^2 = 2G^2\,\mathrm{tr}\left[\gamma\cdot P(1+\gamma_5)(-i\gamma\cdot q_1)(1-\gamma_5)\,\gamma\cdot P(-i\gamma\cdot q_2)\right]$$
$$= 4G^2\,\mathrm{tr}\left[(1-\gamma_5)\,\gamma\cdot P\,\gamma\cdot q_1\,\gamma\cdot P\,\gamma\cdot q_2\right]$$
$$= 16G^2\left[2P\cdot q_1\,P\cdot q_2 - P^2\,q_1\cdot q_2\right]$$
$$= 16G^2\left[2\left[Q^2 - KQ\cos\theta\right]^2 + 2Q^2\left[Q^2 - KQ\cos\theta - M_W^2\right]\right]$$
$$= 32G^2Q^2\left[Q^2 - 2QK\cos\theta + K^2\cos^2\theta + Q^2 - KQ\cos\theta - M_W^2\right]$$
$$= 64G^2Q^4\left[1 - \frac{3K}{2Q}\cos\theta + \frac{K^2}{2Q^2}\cos^2\theta - \frac{M_W^2}{2Q^2}\right]$$

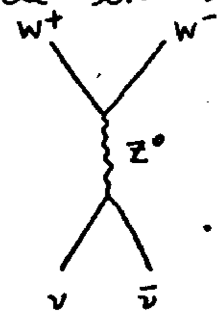
$\propto s^2$! Equivalently, the second-order contribution to the $\nu\bar{\nu}$ elastic scattering amplitude,



is proportional to s^2

Since the $k_\mu k_\nu/M_W^2$ piece of the W propagator gives rise to more severe divergences in each higher order, we must find some means of eradicating the divergence order by order. ~~[illegible]~~ To do so we shall have to invent new diagrams and, with them, new particles. [Discussions which parallel the present one may be found in S. Joglekar, Ann. Phys. 83, 427 (1974); A. de Rújula, in Proceedings of the First Formigal Winter School on Fundamental Physics 1973, p. 341 and C. H. Llewellyn-Smith, in Phenomenology of Particles at High Energies, ed. R.L. Crawford and R. Jennings (New York: Academic, 1974), p. 459.]

One possibility is to introduce a new neutral vector particle in the s-channel, and adjust its couplings to cancel the undesirable growth of the electron exchange graph.



What form can the couplings take? For the coupling to neutrinos, it is natural to choose

$$N\,\bar{v}(\nu, q_2)\,\gamma_\mu(1+\gamma_5)\,u(\nu, q_1).$$

The $W^+W^-Z^0$ coupling is more involved. To satisfy the requirements of T invariance and current conservation, it must be of the general form

$$A\{\lambda\,\delta_{\alpha\beta}(k_+ - k_-)_\nu + k_{+\beta}\,\delta_{\nu\alpha} - k_{-\alpha}\,\delta_{\nu\beta} + \kappa(\delta_{\nu\alpha}(k_+ + k_-)_\beta - \delta_{\nu\beta}(k_+ + k_-)_\alpha\}\,\varepsilon^*_{+\alpha}\,\varepsilon^*_{-\beta}$$

Hence

$$-i\mathcal{M} \approx AN\,\bar{v}(\nu, q_2)\,\gamma_\mu(1+\gamma_5)\,u(\nu, q_1)\,\frac{\delta_{\mu\nu} + \frac{R_\mu R_\nu}{M_Z^2}}{i(R^2 + M_Z^2)}$$

$$\{\lambda\,\delta_{\alpha\beta}(k_+ - k_-)_\nu + k_{+\beta}\,\delta_{\nu\alpha} - k_{-\alpha}\,\delta_{\nu\beta} + \kappa(\delta_{\nu\alpha} R_\beta - \delta_{\nu\beta} R_\alpha)\}\,\frac{k_{+\alpha}\,k_{-\beta}}{M_W^2},$$

where $R = k_+ + k_- = q_1 + q_2$.

$$R^2 \approx -s$$

We may neglect the $k_\mu k_\nu$ piece of the Z^0 propagator, as it gives a contribution to this diagram which is proportional to m_ν.

Now, $\quad k_+ \cdot k_- = -K^2 - Q^2 = -2Q^2 + M_W^2$

$$= -\tfrac{1}{2}s + M_W^2 .$$

$$\therefore -i\mathcal{M} = \frac{AN}{M_W^2(-s + M_Z^2)} \bar{v}(\nu, q_2)\Big\{\lambda(M_W^2 - \tfrac{1}{2}s)\,\gamma\cdot(k_+ - k_-)$$
$$+ (M_W^2 - \tfrac{1}{2}s)\,\gamma\cdot(k_+ - k_-) - x\tfrac{1}{2}s\;\gamma\cdot(k_+ - k_-)\Big\}$$
$$\cdot(1+\gamma_5)\,u(\nu, q_1)$$

$$\lim_{E\to\infty} (-i\mathcal{M}) \longrightarrow \frac{AN}{2M_W^2}\,\bar{v}(\nu, q_2)\Big\{\lambda + 1 + x\Big\}\,\gamma\cdot(k_+ - k_-)\,\cdot$$
$$(1+\gamma_5)\,u(\nu, q_1)$$

In terms of the variable $P = q_1 - k_+ = k_- - q_2$ introduced in the electron-exchange graph, we may replace $\gamma\cdot(k_+ - k_-) = -2\gamma\cdot P$ between spinors. [The $\gamma\cdot q_1$, $\gamma\cdot q_2$ terms vanish by the Dirac eqn.] Then in the high-energy limit, the sum of our two diagrams, is

$$(-i\mathcal{M}) \longrightarrow \left[G\sqrt{2} - \frac{AN}{M_W^2}(\lambda + 1 + x)\right]\bar{v}(\nu, q_2)\,\gamma\cdot P(1+\gamma_5)\,u(\nu, q_1).$$

This result shows that the objectionable high-energy behavior of the electron-exchange graph can indeed be cancelled by an s-channel pole term, if the couplings of the latter are chosen to make

$$G\sqrt{z} - AN(\lambda + 1 + x) = 0, \qquad (*)$$

which evidently can be done. Having introduced a neutral vector boson with finite couplings to $\nu\bar{\nu}$, we are forced to predict neutral current interactions, i.e. Z^0 exchange in the t-channel.

Remarks.

(1) Clearly the condition (*) does not fix all the Z^0 couplings relative to the $We\nu$ vertex. We may, however, go on to consider and control all conceivable graphs in the theory which potentially blow up at high energies. By so doing, we would fix all the couplings and arrive at a theory with the structure of a Gauge Field Theory; see Joglekar's article.

(2) The introduction of a neutral vector boson is not the only possible cure for the divergence of the electron exchange diagram. The introduction of a heavy lepton will serve as well, as we shall see later.

The prediction of neutral current effects in unitary theories of the weak interactions (most specifically in the Weinberg model to be discussed below) and the availability of high energy neutrino beams spurred the search for neutral current events. In the Summer of 1973, muonless events in deep inelastic $\nu_\mu N$ and $\bar{\nu}_\mu N$ collisions were reported by the Gargamelle collaboration working at the CERN PS and by Fermilab experiment 1A. Subsequently these observations have been confirmed and extended. Of particular interest for our discussion of leptons is the report by the Gargamelle group of three events of the type

$$\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e.$$

The space-time structure of the neutral current interaction has not yet been established. The most popular prejudice is that it will be some combination of $V \pm A$. However, considerable effort has gone into preparations for a general phenomenological analysis. These efforts are summarized in Summer School lectures by

S.L. Adler, "Neutrino Interaction Phenomenology and Neutral Currents," IAS preprint

J.J. Sakurai, "Neutral Currents without Gauge Theory Prejudices," CERN-TH. 2099

and in L. Wolfenstein's talk at the 1975 SLAC Conference.

Neutral Current Interactions - General Space-time Structure.

We repeat our earlier exercise for the neutral current case. We now consider the reactions

$$\nu_\mu e \to \nu_\mu e$$

$$\bar{\nu}_\mu e \to \bar{\nu}_\mu e$$

It is clear from writing down the matrix elements that the cross sections for these ~~elements~~ processes may be obtained from our charged-current results by interchanging

$$p_2 \longleftrightarrow q_2 \quad \text{for } \nu_\mu e$$

$$p_1 \longleftrightarrow q_2 \quad \text{for } \bar{\nu}_\mu e$$

(i) Scalar interaction, $\nu_\mu e$

$$|M|^2 = 32\,(p_1\cdot p_2)(q_1\cdot q_2) = 32 m^2 E^2 y^2$$

$$\frac{d\sigma}{dy} = \frac{1}{2}\,\frac{|M|^2}{128\pi^2 m E}\cdot 4\pi = \frac{|M|^2}{64\pi m E}$$

$$= \frac{mE}{2\pi}\,y^2$$

(ii) Scalar interaction, $\bar{\nu}_\mu e$

$$|M|^2 = 32\,(p_1\cdot p_2)(q_1\cdot q_2)$$

$$\frac{d\sigma}{dy} = \frac{mE\,y^2}{2\pi}$$

(iii) Pseudoscalar interaction, $\nu_\mu e$
Same as scalar

(iv) Pseudoscalar interaction, $\bar{\nu}_\mu e$
Same as scalar

(v) Tensor coupling, $\nu_\mu e$

$$|m|^2 = 64\left[2q_1\cdot p_2\, p_1\cdot q_2 + 2q_1\cdot q_1\, q_2\cdot p_2 - q_1\cdot q_2\, p_1\cdot p_2\right]$$
$$= 64m^2E^2\left[2(1-y)^2 + 2 - y^2\right]$$
$$= 64m^2E^2\left[4 - 4y + y^2\right]$$
$$= 256m^2E^2\left[1-\tfrac{1}{2}y\right]^2$$

$$\frac{d\sigma}{dy} = \frac{4mE}{\pi}\left(1-\tfrac{1}{2}y\right)^2$$

(vi) Tensor coupling; $\bar{\nu}_\mu e$

$$|m|^2 = 64\left[2q_1\cdot p_2\, p_2\cdot q_2 + 2p_1\cdot q_2\, p_2\cdot q_1 - p_1\cdot p_2\, q_1\cdot q_2\right]$$

$$\frac{d\sigma}{dy} = \frac{4mE}{\pi}\left(1-\tfrac{1}{2}y\right)^2$$

(vii) ST Interference, $\nu_\mu e$

$$|m|^2 = 128\left(p_1\cdot q_1\, p_2\cdot q_2 - p_1\cdot q_2\, q_1\cdot p_2\right)$$
$$= 128m^2E^2\left(1 - (1-y)^2\right)$$
$$= 128m^2E^2\left(2y - y^2\right)$$

$$\frac{d\sigma}{dy} = \frac{4mE\left(1-\tfrac{1}{2}y\right)y}{\pi}$$

(viii) ST Interference, $\bar{\nu}_\mu e$

$$|m|^2 = 128\left(p_1\cdot q_2\, p_2\cdot q_1 - q_2\cdot p_2\, p_1\cdot q_1\right)$$

$$\frac{d\sigma}{dy} = -\frac{4mE\left(1-\tfrac{1}{2}y\right)y}{\pi}$$

(ix) V−A, $\nu_\mu e$

$$|m|^2 = 256\,(p_1\cdot q_1)(p_2\cdot q_2)\,.$$

$$\frac{d\sigma}{dy} = \frac{4mE}{\pi}$$

(x) V−A, $\bar{\nu}_\mu e$

$$|m|^2 = 256\,(p_1\cdot q_2)(p_2\cdot q_1)$$

$$\frac{d\sigma}{dy} = \frac{4mE}{\pi}(1-y)^2$$

(xi) V+A, $\nu_\mu e$

$$|m|^2 = 256\; p_1\cdot q_2\;\; p_2\cdot q_1$$

$$\frac{d\sigma}{dy} = \frac{4mE}{\pi}(1-y)^2$$

(x) V+A, $\bar{\nu}_\mu e$

$$|m|^2 = 256\; q_2\cdot p_2\;\; q_1\cdot p_1$$

$$\frac{d\sigma}{dy} = \frac{4mE}{\pi}$$

Again we can compose a Table of the various angular dependences.

Coupling	$\nu_\mu e$	$\bar{\nu}_\mu e$
S, P	$\frac{mEy^2}{2\pi}$	$\frac{mEy^2}{2\pi}$
T	$\frac{4mE}{\pi}(1-\frac{1}{2}y)^2$	$\frac{4mE}{\pi}(1-\frac{1}{2}y)^2$
(S,P)-T	$\pm\frac{4mE}{\pi}y(1-\frac{1}{2}y)$	$\pm\frac{4mE}{\pi}y(1-\frac{1}{2}y)$
V−A	$\frac{4mE}{\pi}$	$\frac{4mE}{\pi}(1-y)^2$
V+A	$\frac{4mE}{\pi}(1-y)^2$	$\frac{4mE}{\pi}$

Again we may write

$$\frac{d\sigma}{dy} = \frac{d\sigma}{dy}(SPT) + \frac{d\sigma}{dy}(V-A) + \frac{d\sigma}{dy}(V+A).$$

Just as in the charged-current case, we have a "confusion theorem:" any V±A angular distribution can be imitated by a suitable choice of SPT couplings.

An interesting parameter of the neutral current interaction is the mean value of y,

$$\langle y\rangle \equiv \int_0^1 dy\, y\frac{d\sigma}{dy} \Big/ \int_0^1 dy\, \frac{d\sigma}{dy}.$$

General neutral current case:

	$\langle y \rangle_{\nu_\mu e}$	$\langle y \rangle_{\bar{\nu}_\mu e}$
S, P	0.75	0.75
T	0.3929	0.3929
(S,P) T	0.625	0.625
V−A	0.5	0.25
V+A	0.25	0.5

$$\int_0^1 y^3\, dy \Big/ \int_0^1 y^2\, dy = \tfrac{1}{4} \Big/ \tfrac{1}{3}$$

$$\int_0^1 \left(y - y^2 + y^3/4\right) dy \Big/ \int_0^1 \left(1 - y + y^2/4\right) dy$$

$$= \left(\tfrac{1}{2} - \tfrac{1}{3} + \tfrac{1}{16}\right) \Big/ \left(1 - \tfrac{1}{2} + \tfrac{1}{12}\right)$$

$$= \left(\frac{24-16+3}{48}\right) \Big/ \left(\frac{12-6+1}{12}\right) = \frac{11}{48} \cdot \frac{12}{7} = \frac{11}{28} = .3929$$

$$\int_0^1 \left(y^2 - \frac{y^3}{2}\right) dy \Big/ \int_0^1 \left(y - \frac{y^2}{2}\right) dy =$$

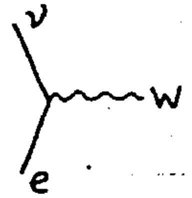
$$= \left(\tfrac{1}{3} - \tfrac{1}{8}\right) \Big/ \left(\tfrac{1}{2} - \tfrac{1}{6}\right) = \frac{5}{24} \cdot \frac{3}{1} = \frac{5}{8} = .625$$

Weinberg Model Predictions for Lepton Scattering

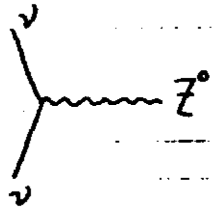
The Weinberg model [S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).] is a simple example of a renormalizable theory of weak and electromagnetic interactions which contains neutral currents but no additional leptons. It furthermore is not in conflict with any experimental information, so it is reasonable for us to study its consequences both as an example of gauge field theories in general and as a possibly correct candidate theory.

It is beyond the scope of these lectures to present a theoretical motivation of the model. It suffices to remark that a consistent pursuit of the divergence-cancelling program we embarked upon last time leads naturally to the coupling structure of the Weinberg model, although it was first derived from a more visionary perspective.

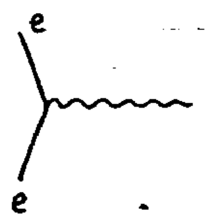
For our purposes we need only note the Feynman rules for leptonic processes. Diagrammatically, these are



$$= \left(\frac{GM_W^2}{\sqrt{2}}\right)^{1/2} \bar{\nu}\gamma_\mu(1+\gamma_5)e$$



$$= \frac{1}{\sqrt{2}}\left(\frac{GM_Z^2}{\sqrt{2}}\right)^{1/2} \bar{\nu}\gamma_\mu(1+\gamma_5)\nu$$



$$= \frac{1}{\sqrt{2}}\left(\frac{GM_Z^2}{\sqrt{2}}\right)^{1/2} \bar{e}\gamma_\mu\left\{R(1-\gamma_5)+L(1+\gamma_5)\right\}e \,,$$

where $R = 2\sin^2\theta_W$, $L = 2\sin^2\theta_W - 1$,

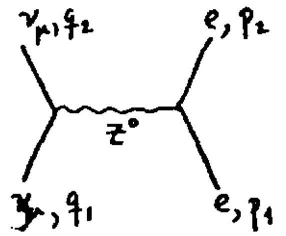
and $M_W^2 = M_Z^2(1-\sin^2\theta_W) = (37.3\ \mathrm{GeV}/c^2)^2/\sin^2\theta_W$,

The [illegible] because [illegible] the Theory [illegible]

The important coupling parameter θ_W is known as the Weinberg angle. It will sometimes be convenient to write $x_W \equiv \sin^2\theta_W$.

We now treat reactions which occur only through the neutral current interaction. A useful reference is De Rújula, Georgi, Glashow, and Quinn, Rev. Mod. Phys. **46**, 391 (1974).

$\underline{\nu_\mu e \longrightarrow \nu_\mu e}$



$$K = (q_2 - q_1) = (p_1 - p_2)$$
$$K^2 = -2 q_1 \cdot q_2 = 2mEy$$

$$-i\mathcal{M} = \frac{G M_Z^2}{2\sqrt{2}} \; \bar{u}(\nu, q_2)\gamma_\mu (1+\gamma_5) u(\nu, q_1) \; \frac{\delta_{\mu\nu} + \frac{K_\mu K_\nu}{M_Z^2}}{i(K^2 + M_Z^2)}$$

$$\cdot \; \bar{u}(e, p_2)\gamma_\nu \{R(1-\gamma_5) + L(1+\gamma_5)\} u(e, p_1)$$

In this diagram, the $K_\mu K_\nu$ term is again impotent. We therefore have

$$|\mathcal{M}|^2 = \frac{G^2 M_Z^4}{8[K^2 + M_Z^2]^2} \; tr\,[\gamma_\mu (1+\gamma_5)(-i\gamma\cdot q_1)(1-\gamma_5)\gamma_\lambda (-i\gamma\cdot q_2)]$$

$$\times \; tr\,[\gamma_\mu \{R(1-\gamma_5) + L(1+\gamma_5)\}(m - i\gamma\cdot p_1)\{R(1+\gamma_5) + L(1-\gamma_5)\}$$
$$\gamma_\lambda (m - i\gamma\cdot p_2)]$$

1^{st} trace: $\; -2\, tr[(1-\gamma_5)\gamma_\mu \, \gamma\cdot q_1 \, \gamma_\lambda \, \gamma\cdot q_2]$

$$= -8\left[q_{1\mu} q_{2\lambda} - \delta_{\mu\lambda} q_1\cdot q_2 + q_{2\mu} q_{1\lambda} - \varepsilon_{\chi\rho\sigma\tau}\delta_{\chi\mu} q_{1\rho}\delta_{\lambda\sigma} q_{2\tau}\right]$$

2^{d} trace: $= - \; tr\,[\{R^2(1+\gamma_5) + L^2(1-\gamma_5)\}\gamma_\mu \, \gamma\cdot p_1 \, \gamma_\lambda \, \gamma\cdot p_2]$

$$= -8\Big[(R^2 + L^2)\{p_{1\mu} p_{2\lambda} - \delta_{\lambda\mu} p_1\cdot p_2 + p_{2\mu} p_{1\lambda}\}$$
$$+ (R^2 - L^2)\, \varepsilon_{\alpha\beta\gamma\delta}\, \delta_{\alpha\mu}\, p_{1\beta}\, \delta_{\gamma\lambda}\, p_{2\delta}\Big]$$

$$|m|^2 = \frac{G^2 M_Z^4 \cdot 64}{8\,[M_Z^2+K^2]^2}\Big\{(R^2+L^2)(2\,p_1\cdot q_1\; p_2\cdot q_2 + 2\,p_1\cdot q_2\; p_2\cdot q_1)$$

$$-(R^2-L^2)\;\varepsilon_{\alpha\beta\gamma\delta}\,\varepsilon_{\kappa\rho\sigma\tau}\,\delta_{\alpha\mu}\,p_{1\beta}\,\delta_{\gamma\lambda}\,p_{2\delta}\,\delta_{\kappa\mu}\,q_{1\rho}\,\delta_{\lambda\sigma}\,q_{2\tau}\Big\}$$

$$\varepsilon_{\mu\beta\lambda\delta}\,\varepsilon_{\mu\rho\lambda\tau}\;p_{1\beta}\,p_{2\delta}\,q_{1\rho}\,q_{2\tau}$$

$$= 2\left[\delta_{\beta\rho}\,\delta_{\delta\tau} - \delta_{\beta\tau}\,\delta_{\delta\rho}\right] p_{1\beta}\,p_{2\delta}\,q_{1\rho}\,q_{2\tau}$$

$$= 2\,p_1\cdot q_1\; p_2\cdot q_2 - 2\,p_1\cdot q_2\; p_2\cdot q_1$$

$$|m|^2 = \frac{32\,G^2 M_Z^4}{[M_Z^2+K^2]^2}\left[p_1\cdot q_1\; p_2\cdot q_2\; L^2 + p_1\cdot q_2\; p_2\cdot q_1\; R^2\right]$$

$$\frac{d\sigma}{dy} = \frac{|m|^2}{64\pi m E} = \frac{G^2}{\left[1+\frac{2mEy}{M_Z^2}\right]^2}\,\frac{\left[L^2 m^2E^2 + R^2 m^2E^2(1-y)^2\right]}{2\pi m E}$$

$$= \frac{G^2 m E}{2\pi}\,\frac{1}{(1+2mEy/M_Z^2)^2}\left[L^2 + R^2(1-y)^2\right]$$

$$\frac{d\sigma}{dy} = \frac{G^2 m E}{2\pi}\,\frac{\left[(2x_w-1)^2 + 4x_w^2(1-y)^2\right]}{\left[1+2mEy/M_Z^2\right]^2}$$

For $M_Z^2 \gg mE$,

$$\sigma = \frac{G^2 m E}{2\pi}\left[(2x_w-1)^2 + 4x_w^2/3\right]$$

$$= \frac{\sigma_{\nu_\mu e}(V-A)}{4}\left[(2x_w-1)^2 + 4x_w^2/3\right]$$

$$= 4.30\times 10^{-42}\,\mathrm{cm}^2\left(\frac{E}{1\,\mathrm{GeV}}\right)\left[(2x_w-1)^2 + 4x_w^2/3\right]$$

Normalized Angular Distributions.

$$\frac{1}{\sigma}\frac{d\sigma}{dy} = \frac{(2x_W-1)^2 + 4x_W^2(1-y)^2}{(2x_W-1)^2 + 4x_W^2/3}$$

$\bar{\nu}_\mu e \longrightarrow \bar{\nu}_\mu e$

We have only to make the replacement $g_1 \leftrightarrow g_2$:

$$\frac{d\sigma}{dy} = \frac{G^2 mE}{2\pi}\frac{[(2x_W-1)^2(1-y)^2 + 4x_W^2]}{[1+\frac{2mEy}{M_Z^2}]^2}.$$

For $M_Z^2 \gg mE$,

$$\sigma = \frac{G^2 mE}{2\pi}[(2x_W-1)^2/3 + 4x_W^2]$$

$$= 4.30 \times 10^{-42}[(2x_W-1)^2/3 + 4x_W^2]\ \mathrm{cm}^2\left(\frac{E}{1\,\mathrm{GeV}}\right)$$

$$\frac{1}{\sigma}\frac{d\sigma}{dy} = \frac{(2x_W-1)^2(1-y)^2 + 4x_W^2}{(2x_W-1)^2/3 + 4x_W^2}$$

The Gargamelle Collaboration quotes

$$\sigma_{\nu_\mu e} < 0.26 \times 10^{-41}\ \mathrm{cm}^2 \qquad (90\%\ \mathrm{C.L.})$$

$$\sigma_{\bar{\nu}_\mu e} = 1.3 \pm 0.8 \times 10^{-41}\ \mathrm{cm}^2$$

Both lie in the range allowed by the Weinberg model (see Figure p. 54). Together they imply $0.1 \leq \sin^2\theta_W \leq 0.4$.

More generally, we may express

$$R = -g_A + g_V \qquad g_A = \tfrac{1}{2}(L-R)$$
$$L = g_A + g_V \qquad g_V = \tfrac{1}{2}(L+R)$$

so that

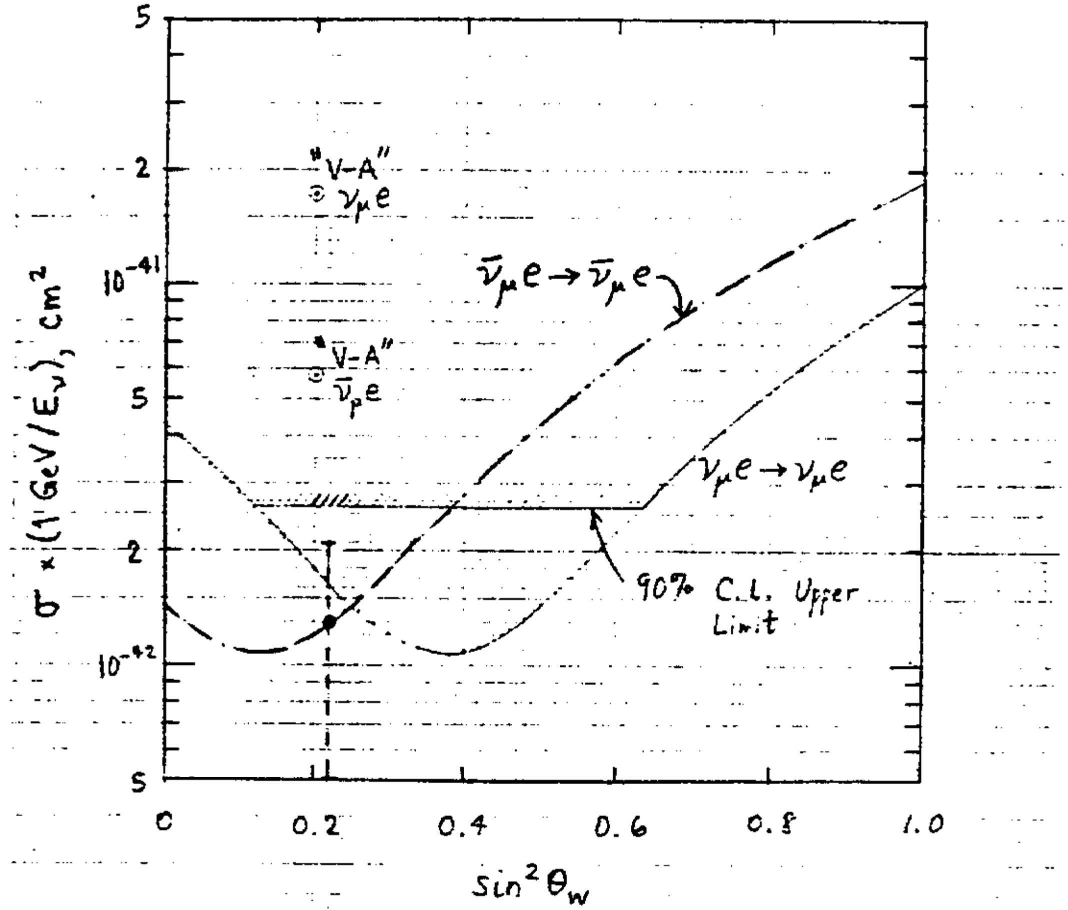
$$\sigma(\nu_\mu e) = \frac{G^2 m E}{\pi}\left[(g_A+g_V)^2 + \frac{1}{3}(g_A-g_V)^2\right]$$

$$\sigma(\bar{\nu}_\mu e) = \frac{G^2 m E}{\pi}\left[\frac{1}{3}(g_A+g_V)^2 + (g_A-g_V)^2\right]$$

Evidently in the Weinberg model,

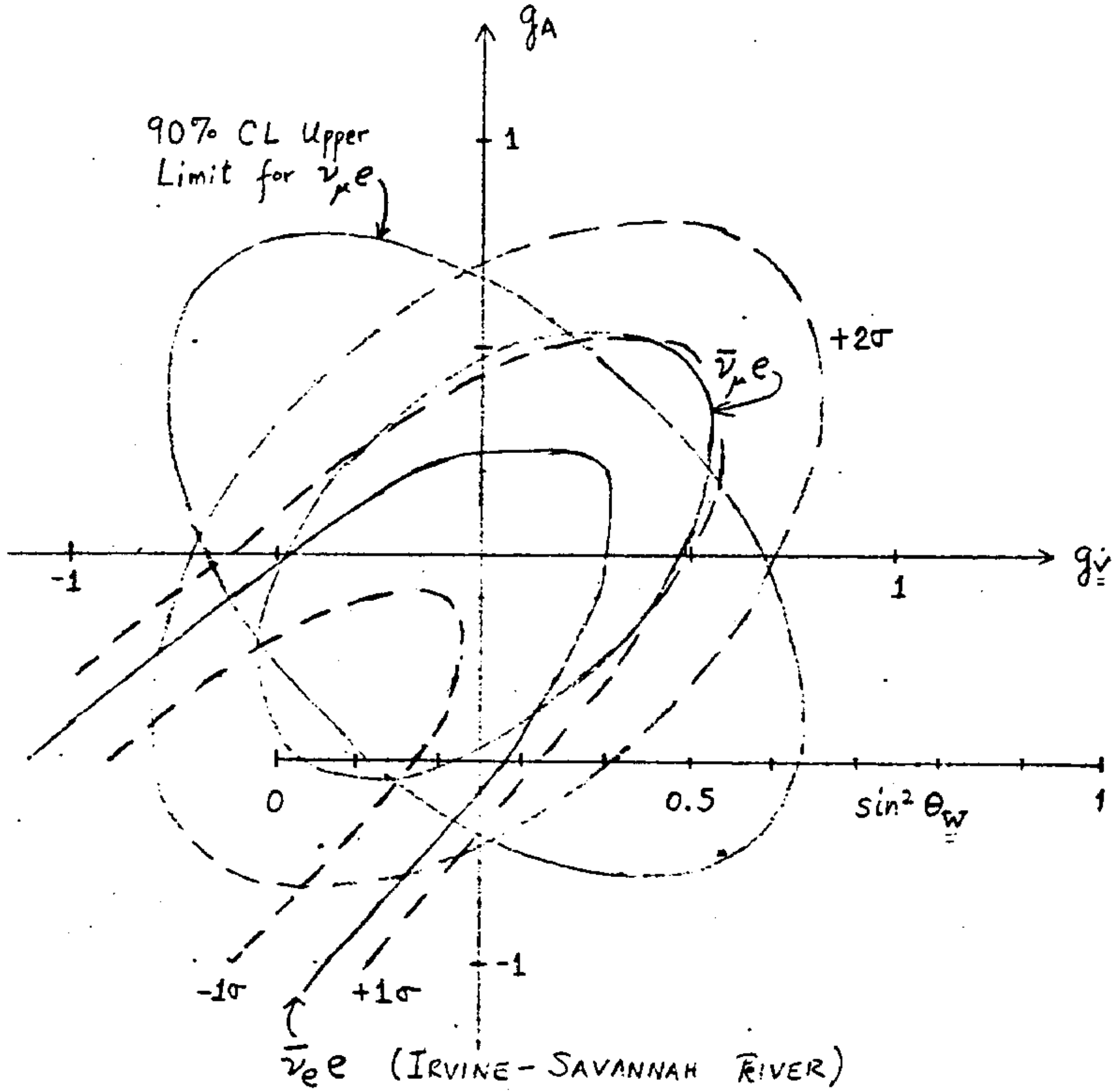
$$g_A = -\tfrac{1}{2},$$
$$g_V = -\tfrac{1}{2} + 2\sin^2\theta_W ;$$

and in the conventional charged-current theory, $g_A = g_V = 0$. The available data then allow values of g_A and g_V within the shaded region of the Figure on p.55



$\nu_\mu e$ upper limit F.J. Hasert, *et al.* PL **46B**, 121 (1973): $0.1 < \sin^2\theta_W < 0.65$ (and ...)

$\bar{\nu}_\mu e$ point. J. Morfin's talk at 1975 SLAC Conference. $2\sigma \Rightarrow \sin^2\theta_W < 0.4$

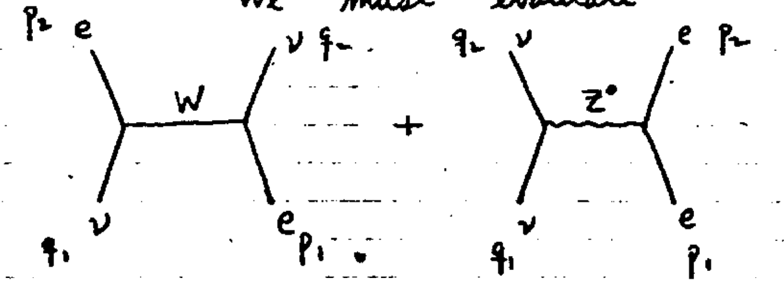

g_A
1
90% CL Upper
Limit for $\nu_\mu e$
$+2\sigma$
$\bar{\nu}_\mu e$
-1
1
g_V
0
0.5
$\sin^2\theta_W$
1
-1
-1σ
$+1\sigma$
$\bar{\nu}_e e$ (Irvine - Savannah River)

Modifications to charged current processes.

We now study the changes to the "allowed" processes $\nu_e e$ and $\bar{\nu}_e e$ scattering which result from the introduction of neutral currents.

$\nu_e e \to \nu_e e$.

We must evaluate



$$K = q_2 - q_1 = p_1 - p_2$$
$$P = p_2 - q_1 = q_2 - p_1$$

$$-i\mathcal{M} = \frac{G M_W^2}{i\sqrt{2}\,(P^2+M_W^2)}\, \bar{u}(\nu,q_2)\gamma_\mu(1+\gamma_5)u(\nu,q_1)\,\bar{u}(e,p_2)\gamma_\mu(1+\gamma_5)u(e,p_1)$$
$$+ \frac{G M_Z^2}{2i\sqrt{2}\,(K^2+M_Z^2)}\, \bar{u}(\nu,q_2)\gamma_\mu(1+\gamma_5)u(\nu,q_1)\,\cdot$$
$$\bar{u}(e,p_2)\gamma_\mu\{L(1+\gamma_5)+R(1-\gamma_5)\}u(e,p_1),$$

where for later convenience I have Fierz re-ordered the W-exchange term. In the limit of large IVB masses, we have

$$|\mathcal{M}|^2 = \frac{G^2}{2}\, \mathrm{tr}[\gamma_\mu(1+\gamma_5)(-i\gamma\cdot q_1)(1-\gamma_5)\gamma_\nu(-i\gamma\cdot q_2)] \times$$
$$\Big\{ \mathrm{tr}[\gamma_\mu(1+\gamma_5)(m-i\gamma\cdot p_1)(1-\gamma_5)\gamma_\nu(m-i\gamma\cdot p_2)] +$$
$$\tfrac{1}{4}\,\mathrm{tr}\big[\gamma_\mu(L(1+\gamma_5)+R(1-\gamma_5))(m-i\gamma\cdot p_1)(L(1-\gamma_5)+R(1+\gamma_5))(m-i\gamma\cdot p_2)\big]$$
$$+ \mathrm{tr}\big[\gamma_\mu(1+\gamma_5)(m-i\gamma\cdot p_1)(L(1-\gamma_5)+R(1+\gamma_5))(m-i\gamma\cdot p_2)\big]\Big\}$$

1st trace: $-2\,\mathrm{tr}\left[(1-\gamma_5)\gamma_\mu\, \gamma\cdot q_1\, \gamma_\nu\, \gamma\cdot q_2\right]$

$$= -8\left[q_{1\mu} q_{2\nu} - \delta_{\mu\nu}\, q_1\cdot q_2 + q_{1\nu} q_{2\mu} - \varepsilon_{\alpha\beta\gamma\delta}\, \delta_{\alpha\mu}\, q_{1\beta}\, \delta_{\gamma\nu}\, q_{2\delta}\right]$$

2d term, first trace:

$$-2\,\mathrm{tr}\left[(1-\gamma_5)\gamma_\mu\, \gamma\cdot p_1\, \gamma_\nu\, \gamma\cdot p_2\right]$$

$$= -8\left[p_{1\mu} p_{2\nu} - \delta_{\mu\nu}\, p_1\cdot p_2 + p_{1\nu} p_{2\mu} - \varepsilon_{\kappa\lambda\rho\sigma}\, \delta_{\kappa\mu}\, p_{1\lambda}\, \delta_{\rho\nu}\, p_{2\sigma}\right]$$

2d term, 2d trace:

$$-\tfrac{1}{2}\,\mathrm{tr}\left[\{R^2(1+\gamma_5) + L^2(1-\gamma_5)\}\gamma_\mu\, \gamma\cdot p_1\, \gamma_\nu\, \gamma\cdot p_2\right]$$

$$= -2\left[(R^2+L^2)(p_{1\mu} p_{2\nu} - \delta_{\mu\nu}\, p_1\cdot p_2 + p_{1\nu} p_{2\mu}) + (R^2-L^2)\,\varepsilon_{\kappa\lambda\rho\sigma}\cdots\right]$$

2d term, 3d trace:

$$= -2L\,\mathrm{tr}\left[(1-\gamma_5)\gamma_\mu\, \gamma\cdot p_1\, \gamma_\nu\, \gamma\cdot p_2\right]$$

$$2^d \text{ term} = -\left(8+8L+2(R^2+L^2)\right)\left(p_{1\mu} p_{2\nu} - \delta_{\mu\nu}\, p_1\cdot p_2 + p_{1\nu} p_{2\mu}\right)$$

$$+\left[8+8L+2(L^2-R^2)\right]\varepsilon_{\kappa\lambda\rho\sigma}\, \delta_{\kappa\mu}\, p_{1\lambda}\, \delta_{\rho\nu}\, p_{2\sigma}$$

This is identical to the calculation of $\nu_\mu e$ scattering, with the replacement $L \to L+2$.

$$\therefore\quad \frac{d\sigma}{dy} \approx \frac{G^2 mE}{2\pi}\left[(2x_w+1)^2 + 4x_w^2(1-y)^2\right]$$

$$\sigma \approx \frac{G^2 mE}{2\pi}\left[(2x_w+1)^2 + 4x_w^2/3\right]$$

By the same token, for $\bar{\nu}_e e \to \bar{\nu}_e e$ we have

$$\frac{d\sigma}{dy} = \frac{G^2 m E}{2\pi}\left[(2x_w+1)^2(1-y)^2 + 4x_w^2\right]$$

$$\sigma = \frac{G^2 m E}{2\pi}\left[(2x_w+1)^2/3 + 4x_w^2\right].$$

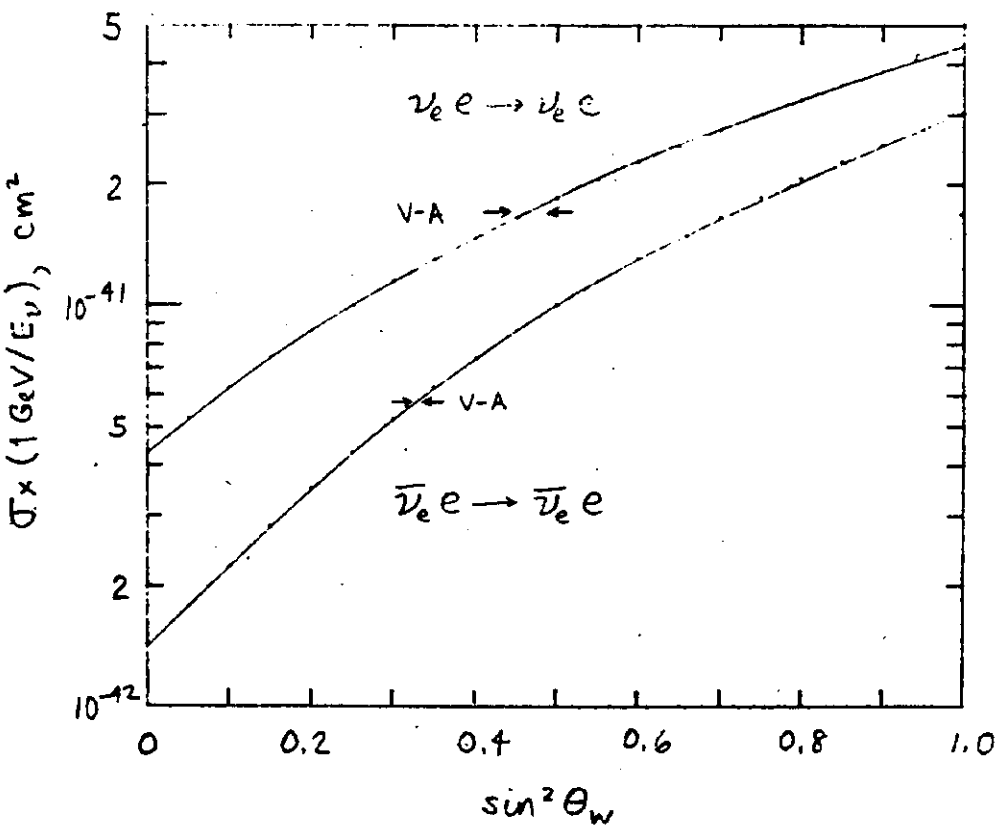
The predicted cross sections, as functions of the Weinberg angle, are shown in the following Figure (p. 59).

There are data on $\bar{\nu}_e e$ scattering from the heroic reactor experiments of Reines and collaborators. These have been interpreted by Chen and Lee, PRD 5, 1874 (1972)
[Figure is based on a later report: F. Reines, H.W. Sobel, H.S. Gurr, Irvine ppt. UCI-10P-19-90.]
to give bounds on g_A and g_V, here defined as

$$g_A = \tfrac{1}{2}(L-R) - 1$$

$$g_V = \tfrac{1}{2}(L+R) - 1$$

so that for the conventional 4-fermion theory, $g_A = g_V = 0$. (This definition is consistent with our discussion of $\nu_\mu e$ scattering).
The allowed region is shown on p. 55.
It includes the Weinberg model, with $\sin^2\theta_W \lesssim 0.3$.


$\nu_e e \to \nu_e e$
V-A
V-A
$\bar{\nu}_e e \to \bar{\nu}_e e$
$\sigma \times (1\ \mathrm{GeV}/E_\nu)$, cm^2
5
2
10^{-41}
5
2
10^{-42}
0
0.2
0.4
0.6
0.8
1.0
$\sin^2\theta_W$

$\langle y \rangle$ in νe scattering.

<u>V-A Theory</u>

$$d\sigma(\nu e \rightarrow \nu e)/dy \propto 1$$

$$\langle y \rangle = \int_0^1 dy \; y \, d\sigma/dy \Big/ \sigma = \tfrac{1}{2}$$

$$d\sigma(\bar{\nu} e \rightarrow \bar{\nu} e)/dy \propto (1-y)^2$$

$$\langle y \rangle = \int_0^1 dy \; y(1-y)^2 \Big/ \sigma$$

$$= 3\int_0^1 dy \left[y - 2y^2 + y^3\right] = 3\left[\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right]$$

$$= \frac{3}{12}\left[6 - 8 + 3\right] = \underline{\underline{\frac{1}{4}}}$$

<u>Weinberg Theory</u>

$\bar{\nu}_\mu e$:

$$\langle y \rangle = \frac{\int \left[(1-2x)^2 (1-y)^2 + 4x^2\right] y \, dy}{\int \left[\qquad \right] dy}$$

$$= \frac{\dfrac{(1-2x)^2}{12} + \dfrac{4x^2}{2}}{\dfrac{(1-2x)^2}{3} + 4x^2}$$

$$= \frac{1 - 4x + 4x^2 + 24x^2}{4 - 16x + 16x^2 + 48x^2}$$

$$= \frac{28x^2 - 4x + 1}{64x^2 - 16x + 4}$$

x	$\langle y \rangle_{\bar{\nu}_\mu e}$	$\langle y \rangle_{\nu_\mu e}$	$\langle y \rangle_{\nu_e e}$	$\langle y \rangle_{\bar{\nu}_e e}$
0	.2500	0.5000	0.5000	0.2500
0.1	.2895	0.4949	0.4977	0.2692
0.2	.3929	0.4677		0.2992
0.3	.4677	0.3929	0.4888	0.3242
0.4	.4949	0.2895		0.3430
0.5	.5000	0.2500	0.4808	0.3571
0.6	.4977	0.2692	0.4774	0.3679
0.7		0.2992	0.4745	0.3763
0.8	.4888	0.3242	0.4720	0.3830
0.9		0.3430		0.3884
1.0	.4808	0.3571	0.4677	0.3929

For $\nu_\mu e$ scattering,

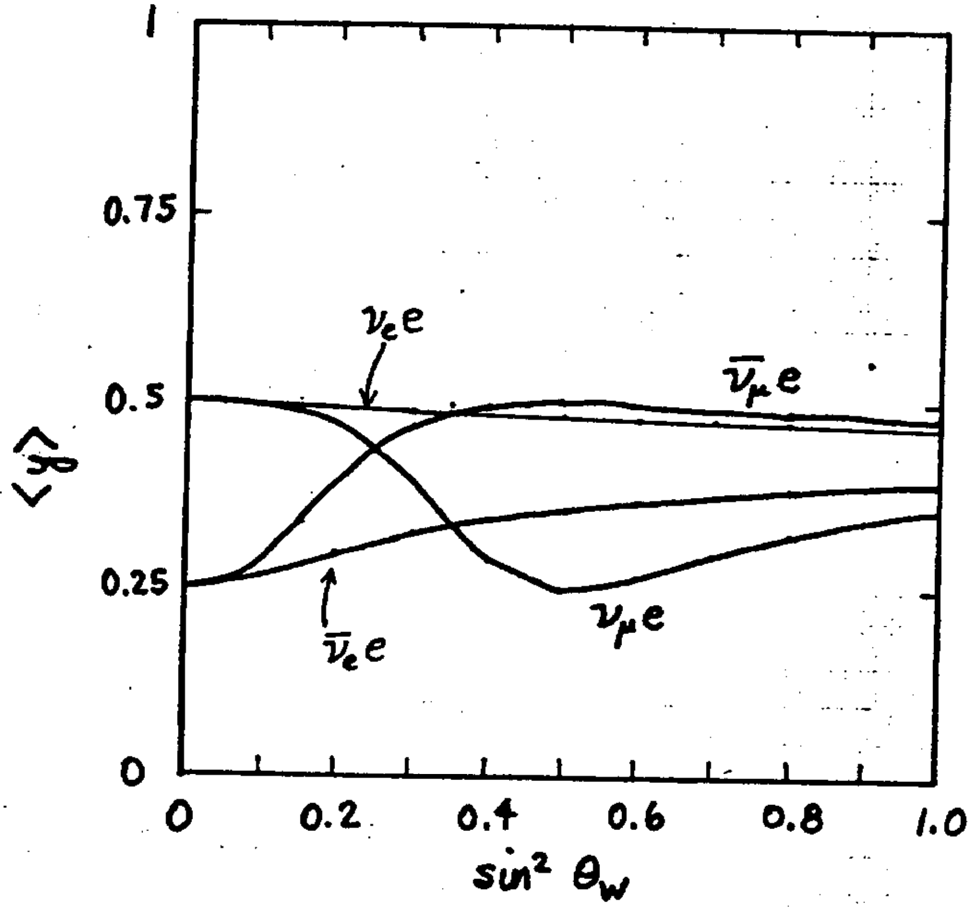
$$\langle y \rangle = \frac{\frac{(1-2x)^2}{2} + \frac{4x^2}{12}}{(1-2x)^2 + \frac{4x^2}{3}} = \frac{\frac{3}{2}(1-2x)^2 + x^2}{3(1-2x)^2 + 4x^2}$$

For $\nu_e e$ scattering,

$$\langle y \rangle = \frac{\frac{(1+2x)^2}{2} + \frac{4x^2}{12}}{(1+2x)^2 + \frac{4x^2}{3}} = \frac{\frac{3}{2}(1+2x)^2 + x^2}{3(1+2x)^2 + 4x^2}$$

For $\bar{\nu}_e e$ scattering,

$$\langle y \rangle = \frac{\frac{(1+2x)^2}{12} + \frac{4x^2}{2}}{\frac{(1+2x)^2}{3} + 4x^2} = \frac{(1+2x)^2 + 24x^2}{4(1+2x)^2 + 48x^2}$$


1
0.75
0.5
0.25
0
0
0.2
0.4
0.6
0.8
1.0
$\langle y \rangle$
$\sin^2 \theta_W$
$\nu_e e$
$\bar{\nu}_\mu e$
$\bar{\nu}_e e$
$\nu_\mu e$

Leptonic decays of Z°.

We recall that the rate for $W^\pm$ decay into leptons is

$$W(W^- \to e^- \bar{\nu}) = \frac{G^2 M_W^3}{6\pi\sqrt{2}}.$$

The computation of $Z \to l\bar{l}$ is completly analogous, so that we may simply read off the answer.

$\underline{Z^\circ \longrightarrow \nu\bar{\nu}}$

$$|m(Z^\circ \to \nu\bar{\nu})|^2 = \frac{1}{2}\frac{M_Z^2}{M_W^2}\,|m(W \to e\nu)|^2$$

$$\therefore\ W(Z^\circ \to \nu\bar{\nu}) = \frac{G^2 M_Z^3}{12\pi\sqrt{2}} = 3.32 \times 10^{17}\ \mathrm{sec}^{-1}\left(\frac{M_Z}{1\ \mathrm{GeV}/c^2}\right)^3$$

$Z^\circ \longrightarrow e^+e^-$

$$-i\,m = \left(\frac{G M_Z^2}{\sqrt{2}}\right)^{1/2}\frac{1}{\sqrt{2}}\ \bar{e}\,\gamma_\mu\left[R(1-\gamma_5) + L(1+\gamma_5)\right]e$$

For the case of longitudinal polarization, we have

$$W(Z^\circ \to e^+e^-) = \frac{(R^2+L^2)}{2}\,\frac{G M_Z^3}{6\pi\sqrt{2}}$$

$$= W(Z^\circ \to \nu\bar{\nu}) \cdot (R^2+L^2).$$

In the Weinberg model, we have explicitly

$$W(W \to e\nu) = \frac{G^2}{6\pi\sqrt{2}} \frac{(37.3 \text{ GeV})^3}{\sin^3\theta_W}$$

$$= 3.446 \times 10^{22} \text{ sec}^{-1} / \sin^3\theta_W$$

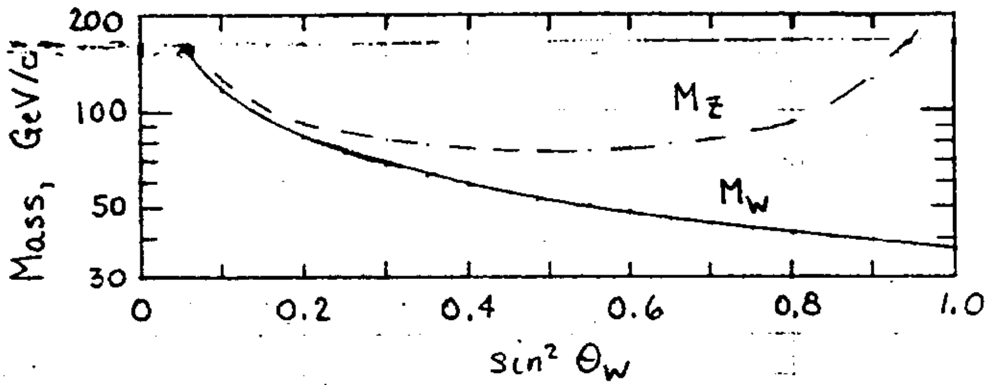
$$W(Z^0 \to \nu\bar{\nu}) = \frac{G^2}{12\pi\sqrt{2}} \frac{(37.3 \text{ GeV})^3}{\sin^3\theta_W \, [1-\sin^2\theta_W]^{3/2}}$$

$$= \tfrac{1}{2} W(W \to e\nu) / [1-\sin^2\theta_W]^{3/2}$$

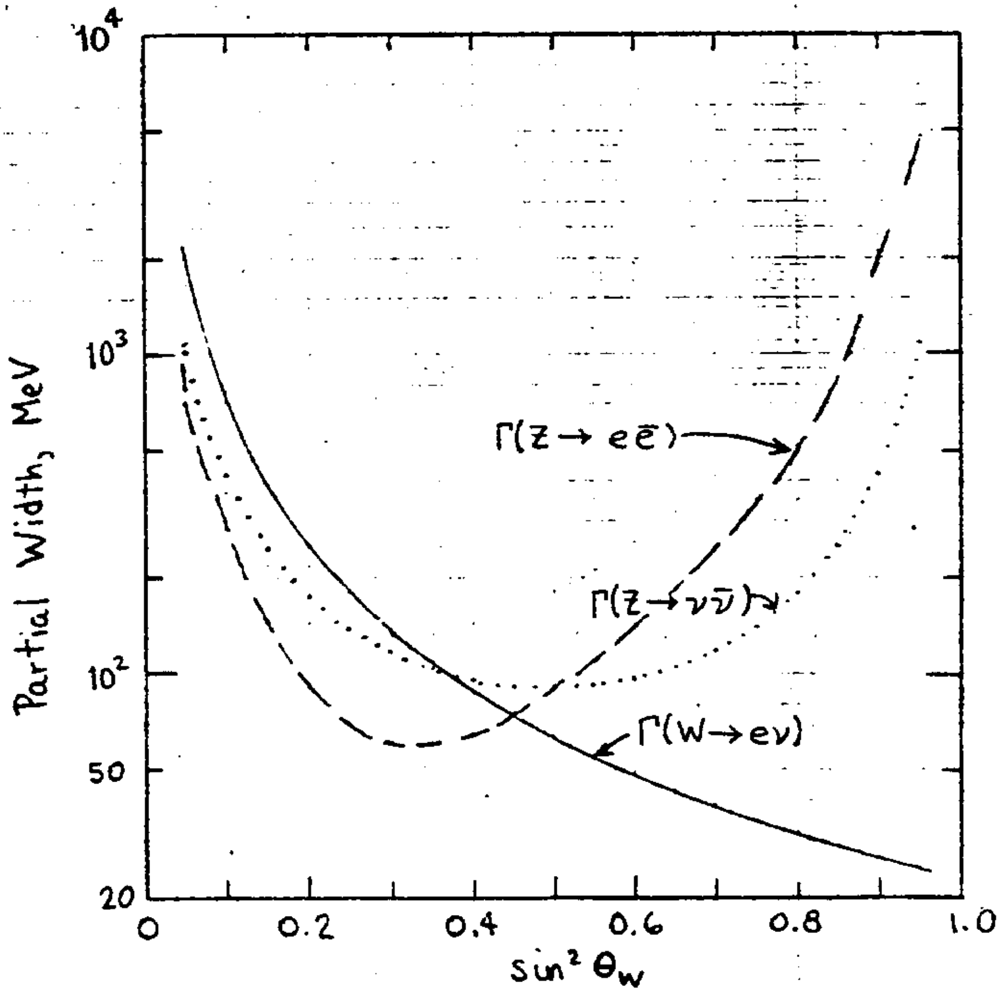
$$W(Z^0 \to e^+e^-) = W(Z^0 \to \nu\bar{\nu})\left([2x-1]^2 + 4x^2\right)$$

The intermediate vector boson masses and the partial widths implied by these expressions are plotted on the next page (65).

(65)



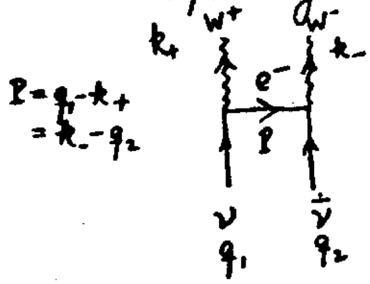
Mass, GeV/c²
200
100
50
30
M_Z
M_W
0
0.2
0.4
0.6
0.8
1.0
$\sin^2 \theta_W$



Partial Width, MeV
10^4
10^3
10^2
50
20
$\Gamma(Z \to e\bar{e})$
$\Gamma(Z \to \nu\bar{\nu})$
$\Gamma(W \to e\nu)$
0
0.2
0.4
0.6
0.8
1.0
$\sin^2 \theta_W$

The Heavy Lepton Alternative

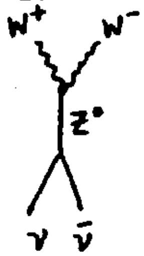
We recall that the matrix element corresponding to the diagram



for $\nu\bar{\nu} \to W^+W^-$ is (at high energies, for longitudinally polarized W's)

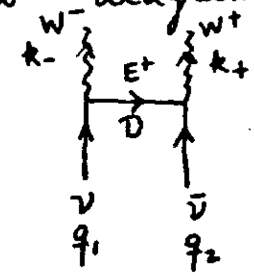
$$-iM = \frac{2G}{\sqrt{2}}\,\bar{v}(\nu, q_2)\,\gamma\cdot P(1+\gamma_5)\,u(\nu, q_1),$$

which has an objectionable HE behavior. Our discussion of neutral current phenomena and the Weinberg model was inspired by the observation that the undesirable growth of the amplitude could be cancelled by the diagram



Here we explore another alternative, the introduction of a new u-channel exchange diagram. We invent for this purpose a new heavy lepton, E^+, which carries the same lepton number as the electron but opposite charge. The new diagram to be evaluated is

$D = q_1 - k_-$
$= k_+ - q_2$



Notice that because we are now exchanging a positively charged lepton, the $W^\pm$ appear at the other vertices, compared with the electron exchange diagram. If the $\nu E W$ coupling is called g, the matrix element is

$$-iM = g^2\, \bar{v}(\nu, q_2)\, \gamma\cdot\varepsilon_+^*\, (1+\gamma_5)\frac{(M_E - i\gamma\cdot D)}{i(D^2+M_E^2)}\, \gamma\cdot\varepsilon_-^*$$

$$\cdot\ (1+\gamma_5)\, u(\nu, q_1).$$

Neglecting M_E everywhere, and replacing

$$\varepsilon_\pm^* = k_\pm / M_W,$$

we find

$$-iM = -\frac{2g^2}{M_W^2}\cdot\frac{1}{D^2}\cdot \bar{v}(\nu, q_2)\, \gamma\cdot k_+\, \gamma\cdot D\, \gamma\cdot k_-\, (1+\gamma_5)\, u(\nu, q_1)$$

Between spinors we can substitute

$$\gamma\cdot k_+ = \gamma\cdot D; \quad \gamma\cdot k_- = -\gamma\cdot D,$$

so that

$$-iM = \frac{2g^2}{M_W^2}\cdot\frac{1}{D^2}\ \bar{v}(\nu, q_2)\, \gamma\cdot D\, \gamma\cdot D\, \gamma\cdot D\, (1+\gamma_5)\, u(\nu, q_1)$$

$$= \frac{2g^2}{M_W^2}\ \bar{v}(\nu, q_2)\, \gamma\cdot D\, (1+\gamma_5)\, u(\nu, q_1).$$

To compare with the electron exchange diagram, we note that between spinors $\gamma \cdot D = -\gamma \cdot P$. The sum of the two lepton exchange diagrams is then

$$-i\mathcal{M} = 2\left[\frac{G}{\sqrt{2}} - \frac{g^2}{M_W^2}\right] \bar{v}(\nu, q_2)\, \gamma \cdot P\, (1+\gamma_5)\, u(\nu, q_1)$$

which can be made to vanish if the $\nu E W$ coupling is chosen as

$$g^2 = \frac{G M_W^2}{\sqrt{2}},$$

which is equal to the $\nu e W$ coupling. An example of a complete theory with a family of "wrong-sign" heavy leptons and no neutral intermediate vector bosons is the Georgi-Glashow

Since neutral currents seem well-established, this model can no longer be taken as realistic.

model, Phys. Rev. Lett. 28, 1494 (1972). Apparently there is nothing to prevent our contemplating a model with both heavy leptons and neutral currents. Indeed several such schemes have been discussed in the literature.

Production of Wrong-Sign Heavy Leptons.

Since ν_μ and $\bar{\nu}_\mu$ beams are more intense than ν_e, $\bar{\nu}_e$ beams, let us consider the production of the μ-like heavy lepton M. [Substitute $\nu_e \to \nu_\mu$, $e \to \mu$, $E \to M$ in our previous discussion.] We must compute

$q_1, \bar{\nu}_\mu$ $\quad$ e^-, p_1 $\quad$ In the point-coupling limit,

$$-i\mathcal{M} = \frac{G}{\sqrt{2}}\, \bar{v}(\nu, q_1)\gamma_\mu(1+\gamma_5)v(M, p_2)\,\bar{u}(\nu, q_2)\gamma_\mu(1+\gamma_5)u(e, p_1)$$

The matrix-element-squared is identical to that for $\nu_e e \to e \nu_e$ (see p. 20). It is

$$|\mathcal{M}|^2 = 256\; p_1\cdot q_1\; p_2\cdot q_2\; \frac{G^2}{2}$$

The kinematics can be read off our results for $\nu_\mu e \to \mu^- \nu_e$:

$$p_1\cdot q_1 = -mE; \quad p_2\cdot q_2 = -mE + (M^2 - m^2)/2,$$

where M is the heavy lepton mass. Thus,

$$\frac{d\sigma}{dy} = \frac{|\mathcal{M}|^2}{64\pi m E}\left[1 - \frac{(M^2-m^2)}{2mE}\right]$$

$$= \frac{2G^2 mE}{\pi}\left[1 - \frac{(M^2-m^2)}{2mE}\right]^2, \quad \text{which is the}$$

same as our result for $\nu_\mu e^- \to \mu^- \nu_e$, with $\mu \to M$.

<u>Heavy lepton decay</u>: $M^- \to \bar{\nu}_\mu\, e^-\, \bar{\nu}_e$.

This is virtually identical to the familiar calculation of $\mu^- \to \nu_\mu e^- \bar{\nu}_e$. In the point coupling limit the matrix element is

$$-i\mathcal{M} = \frac{G}{\sqrt{2}}\bar{u}(e,p)\gamma_\mu(1+\gamma_5)v(\nu_e,q)\,\bar{v}(M,k)\gamma_\mu(1+\gamma_5)v(\nu_\mu,\ell),$$

where we have labelled the momenta as

$$\begin{array}{llcll} M^- : & k & \qquad & e^- : & p \\ \bar{\nu}_\mu : & \ell & & \bar{\nu}_e : & q \end{array}$$

$$|\mathcal{M}|^2 = \frac{G^2}{2}\,\mathrm{tr}\left[\gamma_\mu(1+\gamma_5)(-i\gamma\cdot q)(1-\gamma_5)\gamma_\nu(m-i\gamma\cdot p)\tfrac{1}{2}(1+i\gamma_5\gamma\cdot s)\right]$$
$$\times\,\mathrm{tr}\left[\gamma_\mu(1+\gamma_5)(-i\gamma\cdot\ell)(1-\gamma_5)\gamma_\nu(-M-i\gamma\cdot k)\tfrac{1}{2}(1+i\gamma_5\gamma\cdot \mathcal{S})\right]$$

where s is the electron spin 4-vector, and $\mathcal{S}$ is the heavy lepton spin 4-vector.

1st trace: $-i\,\mathrm{tr}\left[(1-\gamma_5)\gamma_\mu\gamma\cdot q\,\gamma_\nu(m-i\gamma\cdot p)(1+i\gamma_5\gamma\cdot s)\right]$

$$= -i\,\mathrm{tr}\left[(1-\gamma_5)\gamma_\mu\gamma\cdot q\,\gamma_\nu\left\{m - i\gamma\cdot p + im\gamma_5\gamma\cdot s + \gamma\cdot p\,\gamma_5\gamma\cdot s\right\}\right]$$
$$= -\mathrm{tr}\left[(1-\gamma_5)\gamma_\mu\gamma\cdot q\,\gamma_\nu\gamma\cdot p\right] + m\,\mathrm{tr}\left[(1-\gamma_5)\gamma_\mu\gamma\cdot q\,\gamma_\nu\gamma\cdot s\right]$$
$$= \mathrm{tr}\left[(1-\gamma_5)\gamma_\mu\gamma\cdot q\,\gamma_\nu\,\gamma\cdot(ms-p)\right]$$
$$= 4\left[q_\mu(ms_\nu - p_\nu) - \delta_{\mu\nu}(ms\cdot q - p\cdot q) + q_\nu(ms_\mu - p_\mu)\right.$$
$$\left. - \epsilon_{\alpha\beta\gamma\delta}\,\delta_{\alpha\mu}\,q_\beta\,\delta_{\gamma\nu}(ms_\delta - p_\delta)\right]$$

2nd trace is the same form, with the substitution
$q \to \ell,\quad p \to k,\quad m \to -M,\quad s \to \mathcal{S}$.

$\therefore$ 2$^{\text{nd}}$ trace $= 4\left[l_\mu(-Ms_\nu - k_\nu) - \delta_{\mu\nu}(-Ms\cdot l - k\cdot l) + l_\nu(-Ms_\mu - k_\mu)\right]$
$- \varepsilon_{\kappa\lambda\rho\sigma}\,\delta_{\kappa\mu}\,l_\lambda\,\delta_{\rho\nu}(-Ms_\sigma - k_\sigma)$

$$\therefore |m|^2 = 8G^2\Big[2q\cdot l\,(ms-p)\cdot(-Ms-k) \cancel{-2(ms\cdot q - p\cdot q)(-Ms\cdot l - k\cdot l)}$$
$$+2(ms\cdot l - p\cdot l)(-Ms\cdot q - k\cdot q)$$
$$+2\,q\cdot l\,(ms-p)\cdot(-Ms-k) - 2q\cdot(-Ms\cdot q - k\cdot q)(ms-p)\cdot l\Big]$$

$$= 32\,G^2\,q\cdot l\,(ms-p)\cdot(-Ms-k)$$

Neutrinos go unobserved, so we integrate over their momenta:

$$dW = \frac{(2\pi)^4}{2E_M}\,\frac{d^3p}{(2\pi)^3 2E_e}\,\frac{d^3q}{(2\pi)^3 2\omega_e}\,\frac{d^3l}{(2\pi)^3 2\omega_\mu}\,\delta^{(4)}(p+q+l-k)\,|M|^2$$

$$= \frac{2G^2}{(2\pi)^5}\,\frac{d^3p}{E_M E_e}\,(ms-p)\cdot(-Ms-k)\underbrace{\int\frac{d^3q}{\omega_e}\,\frac{d^3l}{\omega_\mu}\,l\cdot q\,\delta^{(4)}(l+q-Q)}_{\equiv\, \mathcal{I}}$$

$Q = k-p = q+l$

Evaluate $\mathcal{I}$ in the frame for which $\vec{q} = -\vec{l}$, $Q = (\vec{0}, i\Omega)$.

$$\int d^3l\;\delta^{(3)}(q+l-Q) = 1;$$

$$\therefore\ \mathcal{I} = \int\frac{d^3q}{\omega_e\,\omega_\mu}\,(q\cdot l)\,\delta(\omega_e+\omega_\mu-\Omega)$$

In our special frame, $\omega_e = \omega_\mu$, $q\cdot l = -2\omega^2$

$$\mathcal{I} = -8\pi\int\frac{d\omega\cdot\omega^2}{\omega^2}\cdot\omega^2\,\delta(2\omega-\Omega)$$
$$= -\underline{\pi\,\Omega^2} = \pi Q^2$$

$$dW = \frac{2\pi G^2}{(2\pi)^4} Q^2 (ms-p)\cdot(M\mathcal{s}-k) \frac{d^3p}{E_e E_M}$$

In the heavy lepton rest frame, $\mathcal{s} = (\hat{\mathcal{s}}, 0)$, $k = (\vec{0}, iM)$. Then $Q = (-\vec{p}, i(M-E_e))$

$$p^2 + m^2 = E^2 \qquad (M\mathcal{s}-k) = (M\hat{\mathcal{s}}, -iM)$$

$$2p\,dp = 2E\,dE \qquad (ms-p) = (m\hat{s} + E_e(\hat{p}\cdot\hat{s})\hat{p} - E_e\hat{p}, iE_e\hat{p}\cdot\hat{s} - iE_e)$$

$$\therefore\ p^2 dp = pE\,dE$$

$$dW = \frac{G^2}{(2\pi)^4 M} Q^2 (ms-p)\cdot(M\mathcal{s}-k)\, p\,dE\cdot d\Omega$$

$$Q^2 = p^2 - (M-E_e)^2 = p^2 - E^2 - M^2 + 2ME_e$$

$$= 2ME_e - M^2 - m_e^2 \approx 2ME_e - M^2 = M(2E_e - M).$$

$$dW = \frac{G^2}{(2\pi)^4}(2E_e - M)[Mm\,\hat{s}\cdot\hat{\mathcal{s}} + ME_e(\hat{p}\cdot\hat{s})\hat{p}\cdot\hat{\mathcal{s}} + ME_e\,\hat{p}\cdot\hat{\mathcal{s}} + ME_e\,\hat{p}\cdot\hat{s} - ME_e]\, E_e\,dE_e\,d\Omega,$$

where I have now set $E_e = p_e$ everywhere

$$dW = \frac{G^2}{(2\pi)^4} M\left(1 - \frac{2E}{M}\right)(1 - \hat{p}\cdot\hat{s})[ME_e(1 - \hat{p}\cdot\hat{\mathcal{s}})]\,E_e\,dE_e\,d\Omega.$$

Let $\varepsilon \equiv 2E_e/M$: $E = M\varepsilon/2$, $dE = M\,d\varepsilon/2$

$$dW = \frac{G^2}{(2\pi)^4} M^3(1-\varepsilon)\frac{\varepsilon}{2}\frac{M^2}{4}\varepsilon\,d\varepsilon\,(1-\hat{p}\cdot\hat{s})(1-\hat{p}\cdot\hat{\mathcal{s}})$$

$$= \frac{G^2 M^5}{4(2\pi)^4} \underbrace{\frac{(1-\hat{p}\cdot\hat{s})}{2}}_{\text{picks out helicity } -1 \text{ in the } m_e=0 \text{ approximation.}} (1-\hat{p}\cdot\hat{\mathcal{s}})(\varepsilon^2 - \varepsilon^3)\,d\varepsilon\,d\Omega$$

Define $\hat{s}\cdot\hat{p} = \cos\theta$

$$dW = \frac{G^2M^5}{4(2\pi)^3}(1-\cos\theta)(\varepsilon^2-\varepsilon^3)\,d\varepsilon\, d(\cos\theta)$$

$$\therefore\ dW = \frac{G^2M^5}{4(2\pi)^3}(\varepsilon^2 d\ \varepsilon^3)\,d\varepsilon\int_{-1}^{1}dz(1+z)$$

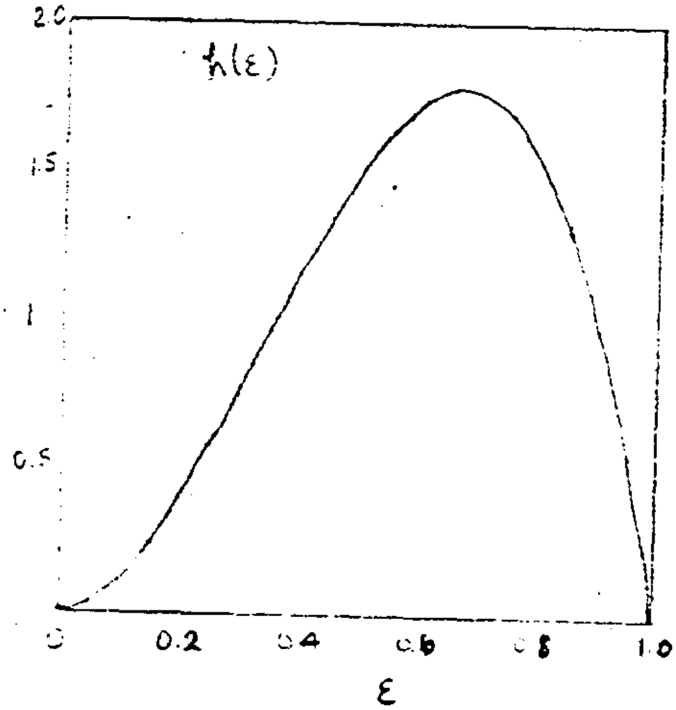
$$W = \frac{G^2M^5}{2(2\pi)^3}\int_0^1 d\varepsilon(\varepsilon^2-\varepsilon^3)$$

$$= \frac{G^2M^5}{192\pi^3} = 3.47\times 10^{10}\left(\frac{M}{1\,\text{GeV}}\right)^5 \sec^{-1}$$

the same $\overbrace{\text{expression}}^{\text{for the total rate}}$ as for μ-decay.

Normalized e^- spectrum:

$$h(\varepsilon) = 12(\varepsilon^2-\varepsilon^3)$$



A search for heavy muons (M^+) of the Georgi-Glashow variety was reported by B. C. Barish, et al., Phys. Rev. Lett. 32, 1387 (1974), who looked for evidence of the reaction

$$\nu_\mu N \to M^+ + \text{hadrons}$$
$$\hookrightarrow \mu^+ \nu_\mu \nu_\mu$$

[For the decays $E^+ \to e^+ \nu_e \nu_e$ and $M^+ \to \mu^+ \nu_\mu \nu_\mu$, the expected rate is

$$W = \frac{G^2 M^5}{96\pi^3} = 7.94 \times 10^{10} \left(\frac{M}{1\,\text{GeV}}\right)^5 \text{sec}^{-1};$$

twice the rate for the decays without identical neutrinos in the final state.]

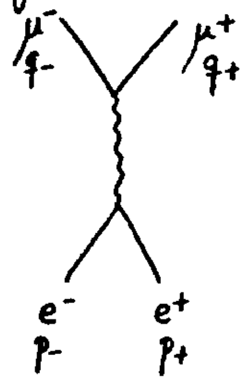
Assuming a branching ratio to the $\mu\nu\nu$ mode of 30%, they can place a 90% CL lower limit on the M^+ mass:

$$M_{M^+} \gtrsim 8.4 \text{ GeV}/c^2.$$

This is particular interest because the measured muon $(g-2)$ requires the Georgi-Glashow M^+ with full Fermi coupling strength $G/\sqrt{2}$ to be less massive than 7 GeV/c^2. See, e.g. J. Primack and H. R. Quinn, Phys. Rev. D6, 3171 (1972); K. Fujikawa, et al., PR D6, 2923 (1972).

Parity Violation in $e^+e^- \to \mu^+\mu^-$

The most important mechanism for the reaction $e^+e^- \to \mu^+\mu^-$ is of course the one-photon annihilation diagram. In a theory with neutral currents, the s-channel Z^0 pole can contribute as well. Interference between the vector-coupling photon diagram and the axial vector coupling portion of the Z^0 diagram will in general give rise to parity-violating effects in the angular distribution of μ-pairs. Let us investigate these in the specific context of the Weinberg model. Kinematics (in HE limit):



$$p_+ \cdot p_- \approx q_+ \cdot q_- \approx -\tfrac{1}{2} s$$
$$p_- \cdot q_- \approx p_+ \cdot q_+ = -\tfrac{1}{4} s(1-z)$$
$$p_- \cdot q_+ \approx p_+ \cdot q_- \approx -\tfrac{1}{4} s(1+z)$$

$$-iM = e^2\, \bar{u}(\mu,q_-)\gamma_\mu v(\mu,q_+)\frac{1}{i(-s)}\bar{v}(e,p_+)\gamma_\mu u(e,p_-)$$

$$+\frac{GM_Z^2}{2\sqrt{2}}\bar{u}(\mu,q_-)\gamma_\mu[R(1-\gamma_5)+L(1+\gamma_5)]v(\mu,q_+)\frac{1}{i(M_Z^2-s)}$$

$$\cdot\,\bar{v}(e,p_+)\gamma_\mu[R(1-\gamma_5)+L(1+\gamma_5)]u(e,p_-)$$

$$|M|^2 = \frac{e^4}{s^2}\,\mathrm{tr}[\gamma_\mu(-i\gamma\cdot q_+)\gamma_\nu(-i\gamma\cdot q_-)]\,\mathrm{tr}[\gamma_\mu(-i\gamma\cdot p_-)\gamma_\nu(-i\gamma\cdot p_+)]$$

$$+\frac{G^2M_Z^4}{8[s-M_Z^2]^2}\,\mathrm{tr}[\gamma_\mu[R(1-\gamma_5)+L(1+\gamma_5)](-i\gamma\cdot q_+)[R(1+\gamma_5)+L(1-\gamma_5)]\gamma_\nu(-i\gamma\cdot q_-)]$$

$$\times\,\mathrm{tr}[\gamma_\mu[R(1-\gamma_5)+L(1+\gamma_5)](-i\gamma\cdot p_-)[R(1+\gamma_5)+L(1-\gamma_5)]\gamma_\nu(-i\gamma\cdot p_+)]$$

$$+\frac{e^2GM_Z^2}{\sqrt{2}\,s[s-M_Z^2]}\,\mathrm{tr}[\gamma_\mu(-i\gamma\cdot q_+)[R(1+\gamma_5)+L(1-\gamma_5)]\gamma_\nu(-i\gamma\cdot q_-)]$$

$$\times\,\mathrm{tr}[\gamma_\mu(-i\gamma\cdot p_-)[R(1+\gamma_5)+L(1-\gamma_5)]\gamma_\nu(-i\gamma\cdot p_+)]$$

1st term: $\frac{32e^4}{s^2}\left[q_+\cdot p_-\, q_-\cdot p_+ + q_+\cdot p_+\, q_-\cdot p_-\right]$

$$= \frac{256\pi^2\alpha^2}{s^2}\left[\frac{s^2}{16}\right]\left[(1+z)^2+(1-z)^2\right] = 64\pi^2\alpha^2(1+z^2)$$

Note that the purely EM cross section is therefore

$$\frac{d\sigma_{EM}}{d\Omega} = \frac{1}{4}\cdot\frac{|M|^2}{64\pi^2 s} = \frac{\alpha^2(1+z^2)}{4s}$$

$$\frac{d\sigma}{dz} = \frac{\pi\alpha^2(1+z^2)}{2s} \qquad \sigma = \frac{4\pi\alpha^2}{3s}$$

2nd term: $\dfrac{G^2}{8\left[1-\frac{s}{M_Z^2}\right]^2} \, 4 \times \mathrm{tr}\left[\left\{R^2(1+\gamma_5)+L^2(1-\gamma_5)\right\}\gamma_\mu\,\gamma\cdot q_+\,\gamma_\nu\,\gamma\cdot q_-\right]$

$$\times\ \mathrm{tr}\left[\left\{R^2(1+\gamma_5)+L^2(1-\gamma_5)\right\}\gamma_\mu\,\gamma\cdot p_-\,\gamma_\nu\,\gamma\cdot p_+\right]$$

$$= \frac{G^2}{2\left[1-\frac{s}{M_Z^2}\right]^2}\Big\{(R^2+L^2)^2(32\, q_+\cdot p_-\, q_-\cdot p_+ + 32\, q_+\cdot p_+\, q_-\cdot p_-)$$
$$+(R^2-L^2)^2(32\, q_+\cdot p_-\, q_-\cdot p_+ - 32\, q_+\cdot p_+\, q_-\cdot p_-)\Big\}$$

$$= \frac{16G^2}{\left[1-\frac{s}{M_Z^2}\right]^2}\left\{\frac{(R^2+L^2)^2}{8}(1+z^2)s^2 + \frac{(R^2-L^2)^2}{4}z\,s^2\right\}$$

$$= \frac{2G^2}{\left[\frac{1}{s}-\frac{1}{M_Z^2}\right]^2}\left\{(R^2+L^2)^2(1+z^2)+2(R^2-L^2)^2 z\right\}$$

Hence the purely weak cross section is

$$\frac{d\sigma_w}{d\Omega} = \frac{1}{4}\,\frac{|m|^2}{64\pi^2 s} = \frac{G^2 s}{128\pi^2\left[1-\frac{s}{M_Z^2}\right]^2}\{\ \} .$$

$$\frac{d\sigma}{dz} = \frac{G^2 s}{64\pi\left[1-\frac{s}{M_Z^2}\right]^2}\{\ \}$$

$$\sigma = \frac{G^2 s}{24\pi\left[1-\frac{s}{M_Z^2}\right]^2}(R^2+L^2)$$

3^{d} term: $\dfrac{e^2 G M_Z^2}{\sqrt{2}\, s^2 [1 - M_Z^2/s]}\ tr\left[[(R+L) + (R-L)\gamma_5]\gamma_\mu\, \gamma\cdot q_+\, \gamma_\nu\, \gamma\cdot q_-\right]$

$$tr\left[[(R+L) + (R-L)\gamma_5\, \gamma_\mu\, \gamma\cdot p_-\, \gamma_\nu\, \gamma\cdot p_+\right]$$

$$= \frac{128\,\pi\alpha\, G M_Z^2}{\sqrt{2}\, s^2 [1 - M_Z^2/s]} \left\{ (R+L)^2 (q_+\cdot p_- q_-\cdot p_+ + q_+\cdot p_+ q_-\cdot p_-) \right.$$

$$\left. + (R-L)^2 (q_+\cdot p_- q_-\cdot p_+ - q_+\cdot p_+ q_-\cdot p_-) \right\}$$

$$= \frac{8\pi\alpha\, G M_Z^2}{\sqrt{2}\,[1 - M_Z^2/s]} \left\{ (R+L)^2 \cdot 2(1+z^2) \quad + 4(R-L)^2 z \right\}$$

$$= \frac{16\pi\alpha\, G M_Z^2}{\sqrt{2}\,[1 - M_Z^2/s]} \left\{ \div 2 \right\}$$

$$\therefore \quad \frac{d\sigma}{d\Omega} = \frac{\alpha^2 (1+z^2)}{4s} + \frac{\alpha G M_Z^2}{16\pi\sqrt{2}\,[s - M_Z^2]} \left\{ (R+L)^2 (1+z^2) + 2(R-L)^2 z \right\}$$

$$+ \frac{G^2 s}{128\pi^2 \left[1 - \frac{s}{M_Z^2}\right]^2} \left\{ (R^2+L^2)^2 (1+z^2) + 2(R^2-L^2)^2 z \right\}$$

$$A \equiv \frac{\frac{d\sigma}{d\Omega}(z) - \frac{d\sigma}{d\Omega}(-z)}{+} \approx \frac{4z\,(R-L)^2 \alpha\, G M_Z^2}{16\pi\sqrt{2}\,[s - M_Z^2]} \cdot \frac{2s}{\alpha^2 (1+z^2)}$$

$$\approx \frac{G M_Z^2}{2\pi\alpha\sqrt{2}} \frac{(R-L)^2}{[1 - M_Z^2/s]} \cdot \frac{z}{1+z^2}$$

In the Weinberg model, $R - L = 1$

$$\therefore \quad A = \frac{G}{2\pi\alpha\sqrt{2}\left[\frac{1}{M_Z^2} - \frac{1}{s}\right]} \cdot \frac{z}{1+z^2}$$

$$= \frac{G}{2\pi\alpha\sqrt{2}\left[\frac{(1-x_W)\, x_W}{(37.3\ \mathrm{GeV}/c^2)^2} - \frac{1}{s}\right]} \quad \frac{z}{1+z^2}$$

Numerically, we have

$$A = 1.80\times 10^{-4} \Big/ \left[\frac{(1-x_w)\,x_w}{(37.3)^2} - \frac{(1\ \mathrm{GeV})^2}{s}\right] \quad \frac{z}{1+z^2}$$

$$= 0.25 \Big/ \left[(1-x_w)\,x_w - \frac{(37.3\ \mathrm{GeV})^2}{s}\right] \quad \frac{z}{1+z^2}$$

$$= a(x_w, s) \quad z/(1+z^2).$$

Front-back ratio.

$$\frac{F-B}{F+B} = 2\pi \frac{\alpha G M_Z^2}{16\pi\sqrt{2}\,[s-M_Z^2]} \; 4 \frac{(R-L)^2}{2} \cdot \frac{3s}{4\pi\alpha^2}$$

$$= \frac{3\,G\,M_Z^2\,s}{16\pi\,\alpha\sqrt{2}\ [s-M_Z^2]}$$

$$= \frac{3G}{16\pi\alpha\sqrt{2}} \Big/ \left[\frac{1}{M_Z^2} - \frac{1}{s}\right]$$

$$= \frac{3G\,(37.3\ \mathrm{GeV})^2}{16\pi\alpha\sqrt{2}} \Big/ \left[x(1-x) - \frac{(37.3)^2}{s}\right]$$

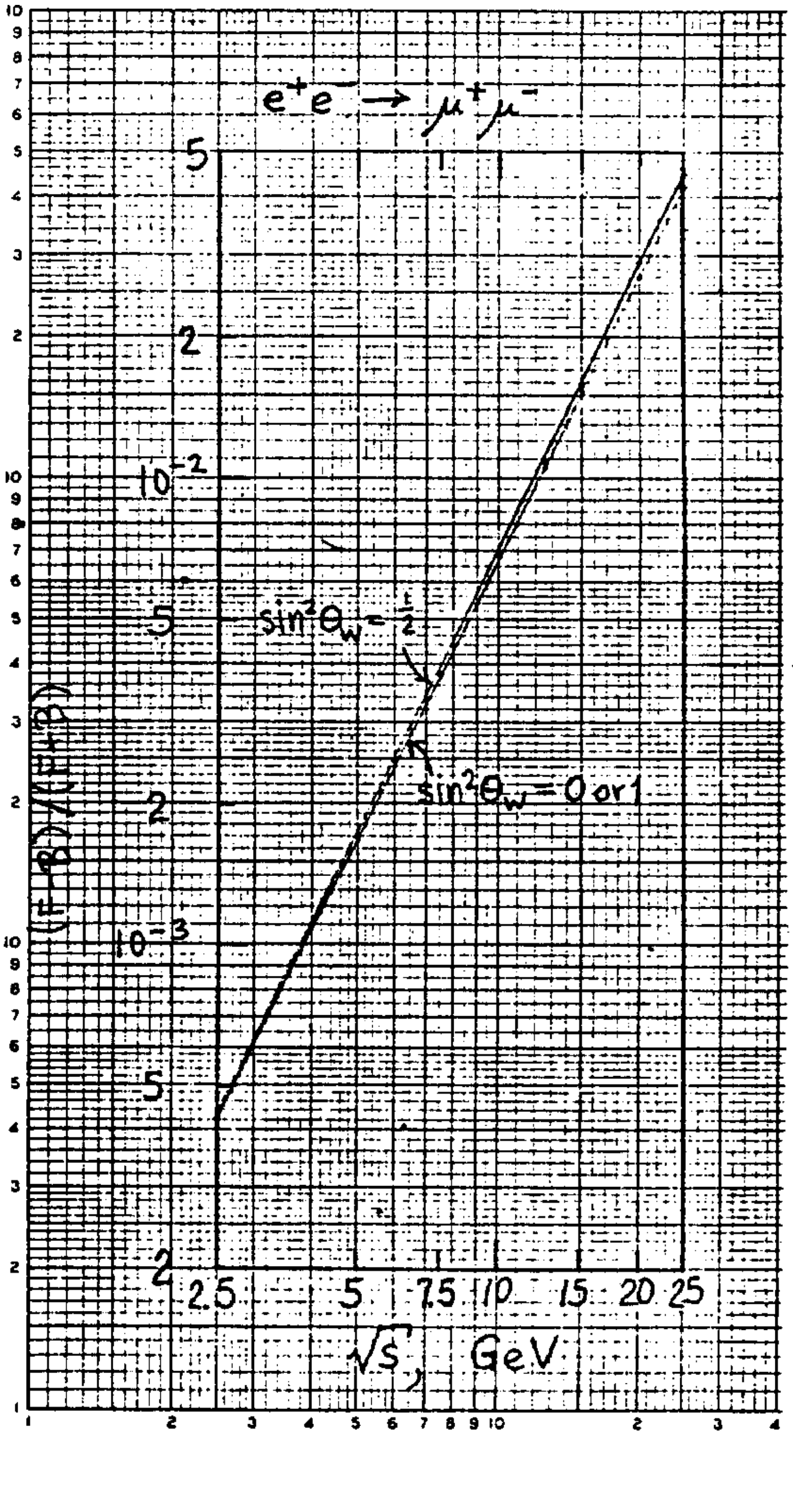
$$= 3.13\times 10^{-2} \Big/ \left[x_w(1-x_w) - \frac{(37.3)^2}{s}\right]$$

$$= \frac{3}{8}\, a(x_w, s)$$

The front-back ratio is plotted on p. 81. It is quite insensitive to the value of the Weinberg angle over a realistic range of CM energies. By exploiting the polarization of the stored electron and positron beams, and by restricting one's acceptance in $\cos\theta$ and φ, it is possible to enhance one's sensitivity to the parity-violating effect. This has been discussed, for example, by J. Godine and A. Hankey, PR D6, 3301 (1972), by A. Love, Lett. NC 5, 113 (19[illegible]) by V.K. Cung, *et al.*, Phys. Lett. 41B, 355 (1972). The polarization of stored beams has been measured by J.G. Learned, *et al.*, Phys. Rev. Lett. 35, 1688 (1975), and has been reviewed didactically by J.D. Jackson, LBL-4232. U. Camerini, *et al.* have mounted a SPEAR experiment to search for the parity violating asymmetry.

The asymmetry we have computed is so small as to be confused with 2γ EM effects.

(81)



$e^+e^- \rightarrow \mu^+\mu^-$
$(F-B)/(F+B)$
$\sin^2\theta_W = \frac{1}{2}$
$\sin^2\theta_W = 0$ or 1
5
2
10^{-2}
5
2
10^{-3}
5
2
2.5
5
7.5
10
15
20
25
$\sqrt{s}$, GeV

A Glimpse of Hadronic Weak Interactions - Charm

We conclude this series of lectures with a too brief mention of weak interactions of hadrons. The key element in the description of hadronic weak interactions is the hadronic weak current, J. Neglecting the space-time structure of the current, we may summarize all of our common experience by the quark model transcription of the Cabibbo theory:

$$J = \bar{u}(d\cos\theta_C + s\sin\theta_C)$$
$$J^\dagger = (\bar{d}\cos\theta_C + \bar{s}\sin\theta_C)\,u$$

In gauge field theories, the neutral current is not *ad hoc*. Rather, it is specified (at least in part) as

$$J^0 = [J, J^\dagger]$$

In the Cabibbo theory, we have

$$J_0 = \bar{u}d\bar{d}u \cos^2\theta + \bar{u}s\bar{d}u \sin\theta\cos\theta$$
$$+ \bar{u}d\bar{s}u \sin\theta\cos\theta + \bar{u}s\bar{s}u \sin^2\theta$$
$$- \bar{d}u\bar{u}d \cos^2\theta - \bar{d}u\bar{u}s \sin\theta\cos\theta$$
$$- \bar{s}u\bar{u}d \sin\theta\cos\theta - \bar{s}u\bar{u}s \sin^2\theta$$

To simplify this expression we use the Fermion anticommutation relations

$$\{a, b\} = 0 = \{\bar{a}, \bar{b}\}$$
$$\{a, \bar{b}\} = \delta_{ab}$$

$$(\bar{u}d\bar{d}u - \bar{d}u\bar{u}d)\cos^2\theta = (d\bar{d}\bar{u}u - \bar{d}du\bar{u})\cos^2\theta$$
$$= (-\bar{d}d\bar{u}u + \bar{u}u + \bar{d}d\bar{u}u - \bar{d}d)\cos^2\theta$$
$$= (\bar{u}u - \bar{d}d)\cos^2\theta$$

$$(\bar{u}s\bar{s}u - \bar{s}u\bar{u}s)\sin^2\theta = (\bar{u}u - \bar{s}s)\sin^2\theta$$

$$(\bar{u}d\bar{s}u - \bar{s}u\bar{u}d)\sin\theta\cos\theta = (\bar{u}ud\bar{s} - u\bar{u}\bar{s}d)\sin\theta\cos\theta$$
$$= (-\bar{u}u\bar{s}d + \bar{u}u\bar{s}d - \bar{s}d)\sin\theta\cos\theta = -\bar{s}d\sin\theta\cos\theta$$
$$(\bar{u}s\bar{d}u - \bar{d}u\bar{u}s)\sin\theta\cos\theta = -\bar{d}s\sin\theta\cos\theta$$

$$\therefore\ J_\mu^0 = \bar{u}u - \bar{d}d\cos^2\theta - \bar{d}s\sin\theta\cos\theta$$
$$- \bar{s}d\sin\theta\cos\theta - \bar{s}s\sin^2\theta,$$

which contains $|\Delta S| = 0, 1$ elements.

It is a familiar fact that $\Delta S \neq 0$

neutral currents have not been observed. Indeed, there are stringent limits from processes such as $K_L \to \mu^+\mu^-$, $K^\pm \to \pi^\pm \nu\bar{\nu}$, and from the $K_L - K_S$ mass difference. Thus, we are called upon to cancel the strangeness-changing neutral current. The cancellation is provided by the Glashow-Iliopoulos-Maiani mechanism, in which

$$J^{GIM} = \bar{u}(d\cos\theta_c + s\sin\theta_c) + \bar{c}(s\cos\theta_c - d\sin\theta_c).$$

The charmed quark c is an SU(3) singlet with charge $= +2/3$ and charm $= +1$. The ordinary u, d, s quarks are charmless. It is elementary to verify that.

$$J^0 = [J^{GIM}, J^{GIM+}] = \bar{u}u + \bar{c}c - \bar{s}s - \bar{d}d,$$

which is free of $\Delta S \neq 0$ terms.

It is now apparent that a renormalizable theory of the weak and electromagnetic interactions can be constructed along the lines of a gauge field theory. To do so, it is necessary to predict the existence of $W^\pm$, Z°, charmed hadrons, and perhaps wrong-sign heavy leptons.

An extended discussion of high-energy νN reactions is given in the lecture notes of B.C. Barish, CALT 68-535, Dec., 1975.