# Precision kaon phenomenology on the lattice

Lattice QCD meets experiment worshop 2007
FermiLab

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# Outline

- precision kaon phenomenology:
  - $\left.\begin{array}{l} f_K/f_\pi \\ f_+^{K\pi}(0) \end{array}\right\}|V_{us}|$
  - $\hat{B}_K$
- status and outlook (what will be in 5 years time?)
- discussion of systematic errors
- some questions

for discussion of technical details and status and pecularities of simulations of the various fermion formulations cf. Lattice 2006 and Lattice 2007 plenary talks

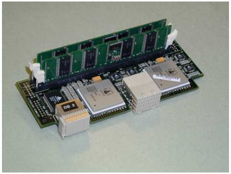
# Matrix elements in lattice QCD

- Correlation functions in terms of **Euclidean** path integral

  $\langle \mathcal{O}[\bar\psi,\psi,A]\rangle = \frac{1}{Z}\int D\bar\psi D\psi DA\, \mathcal{O}(\bar\psi,\psi,A)\, e^{-S_G(U)-S_q(\bar\psi,\psi,U)}$

  Ground state matrix elements for large Euclidean times
- discretisation – space time lattice as regulator $\sim \pi/a$

  Statistical sampling of path int. with e.g. QCDOC-computer by UKQCD/RBC



- from first principles:
  2+1 flavor (lattice) QCD has only 3 parameters $m_u = m_d$, $m_s$, $g(a)$
  tuning to physical point using hadronic input
  - lattice spacing: $a^{-1} = \frac{m_\rho^{\rm exp}}{am_V}$
  - quark masses: $\frac{am_H}{am_V} = \frac{m_H^{\rm exp}}{m_V^{\rm exp}}$ $(H = \pi, K, D, \ldots)$

# Estimating and controlling errors

After S.Sharpe, Lattice QCD: Present and Future, Orsay, 2004
assume staggered or DWF

## statistical error

O(100) measurements for 1% stat. precision but dependent on simulation and observable

## discretisation errors

$O^{\rm latt} = O^{\rm cont}(1 + (a\Lambda)^2 + (a\Lambda)^4 + \dots)$
two lattices with $a_{\rm min}$ and $\sqrt{2}a_{\rm min}$
for target precision $\epsilon$:

$$a_{\rm min} \approx \left(\frac{\epsilon}{2}\right)^{1/4} \frac{1}{0.5{\rm GeV}}$$

## finite volume errors (FVE)

- exponentially suppressed $\propto e^{m_\pi L}$,
- correction by analytical calculations
- or scaling study:
  for target precision $\epsilon$:

$$m_\pi L \approx -log(\epsilon)$$

## light dynamical quarks

- $N_f = 2, 2+1, 2+1+1$
- quark mass is a free parameter
- strange quark physical
- light $u$ & $d$ quarks are expensive:
  currently only $m_{u,d}/m_s \gtrsim 1/10$
  physical $m_{u,d}/m_s \approx 1/24$
- extrapolate using $\chi$PT
- naively

$$O^{\rm latt} = O^{\rm phys}\left(1 + c_1\left(\frac{m_\pi}{m_\rho}\right)^2 + c_2\left(\frac{m_\pi}{m_\rho}\right)^4 + \dots\right) -$$

for two lattices with
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## renormalization

perturbabtive or non-perturbabtive

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# Example

Current simulations: e.g. UKQCD+RBC domain wall fermions:

| | | | $\epsilon$ |
|---|---|---|---|
| cut-off | $a$ | 0.11fm | 1.4% |
| unphysical light quark mass | $m_\pi$ | 330MeV | 2.7% |
| finite volume | $L$ | 2.7fm | 1.0% |

# $|V_{us}|$ from $K_{l2}$ decay

- In 2004 Marciano first used the lattice determination of $f_K/f_\pi$ to determine $|V_{us}|$: *(Marciano, hep-ph/0402299)*

$$\frac{\Gamma(K\to\mu\bar\nu_\mu(\gamma))}{\Gamma(\pi\to\mu\bar\nu_\mu(\gamma))}=\frac{|V_{us}|^2}{|V_{ud}|^2}\left(\frac{f_K}{f_\pi}\right)^2\frac{m_K(1-m_\mu^2/m_K^2)}{m_\pi(1-m_\mu^2/m_\pi^2)}\times 0.9930(35)$$

- experimental values for decay rates from KLOE 2006

$$\begin{aligned}\Gamma(K\to\mu\bar\nu_\mu(\gamma)) &= 2.528(2)\times 10^{-14}\mathrm{MeV}\\ \Gamma(\pi\to\mu\bar\nu_\mu(\gamma)) &= 3.372(9)\times 10^{-14}\mathrm{MeV}\end{aligned}$$

- yields

$$\frac{|V_{us}|^2}{|V_{ud}|^2}\frac{f_K^2}{f_\pi^2}=0.07602(23)_{\rm exp}(27)_{\rm rad}\qquad \delta=2\times 0.23\%$$

- use

$$|V_{ud}|=0.97372(10)_{\rm exp}(15)_{\rm nucl}(19)_{\rm RC}\qquad \delta<1‰$$

from nuclear Beta decay *(Marciano, Kaon 2007)*

and $f_K/f_\pi$ from the lattice

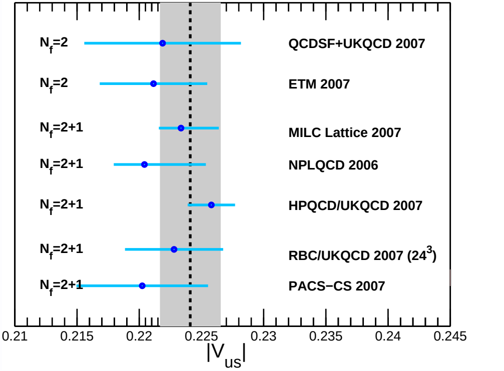
# $|V_{us}|$ from $K_{l2}$ decay

Recent (2006/2007) dynamical lattice computations of $f_K/f_\pi$

- many different groups (QCDSF+UKQCD, ETM, MILC, HPQCD+UKQCD, NPLQCD, RBC+UKQCD, PACS-CS)
- many different formulations (impr. Wilson, tmQCD, staggered, DWF, overlap)
- 2 or 2+1 flavour
- lattice spacing $a = 0.06 - 0.13\text{fm}$, sometimes scaling study otherwise crude estimate of cut-off effects
- lightest meson masses $m_\pi^{\text{min}} = 210 - 330\text{MeV}$
- lattice volume $m_\pi L = 3 - 5$

# $|V_{us}|$ from $K_{l2}$ decay

Summary $f_K/f_\pi \rightarrow |V_{us}|$



# $|V_{us}|$ from $K_{l2}$ decay

Weighted average, error as for MILC-result:

$$f_K/f_\pi = 1.198(10) \qquad \delta = (0.3\%)^{\text{stat}} \underbrace{(0.8\%)^{\text{syst}}}_{\text{FVE}, a, \chi}$$

$$|V_{us}| = 0.2241 \begin{array}{c}(05)^{\Gamma} \\ (0.2\%)^{\Gamma}\end{array} \begin{array}{c}(19)^{f_K/f_\pi} \\ (0.8\%)^{f_K/f_\pi}\end{array} \begin{array}{c}(01)^{V_{ud}} \\ (0.03\%)^{V_{ud}}\end{array}$$

- reliability of analytical finite volume corrections is crucial
- $\chi$PT is now being tested
  - up to which quark mass does it work and at which order?
  - is $\chi$PT improvable order by order?
  - compare $SU(2) \times SU(2)$ with $SU(3) \times SU(3)$
- most collaborations are still generating configurations and will reduce their errors in the near future

# $|V_{us}|$ from $K_{l3}$ decay

$$\Gamma(K \to \pi l \nu) = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{\rm EW} I \, [1 + 2\Delta_{SU(2)} + 2\Delta_{\rm EM}] \times |V_{us} f_+^{K^0\pi^-}(0)|^2$$

- $S_{\rm EW}$ short distance EW corrections
- $I$ phase space integral (via FF shape from experiment)
- $\Delta_{SU(2)}$ iso-spin breaking corrections
- $\Delta_{\rm EM}$ long distance EM corrections

Table by *Moulson hep-ex/0703013* (KLOE, KTeV, ISTRA+, NA48)

| | | | Approx contrib to % err | | | |
|---|---|---|---|---|---|---|
| Mode | $\lvert V_{us} f_+^{K\pi}(0)\rvert$ | % err | BR | $\tau$ | $\Delta$ | $I$ |
| $K_L$ $e3$ | 0.21639(55) | 0.25 | 0.09 | 0.19 | 0.10 | 0.09 |
| $K_L$ $\mu3$ | 0.21649(68) | 0.31 | 0.10 | 0.18 | 0.15 | 0.17 |
| $K_S$ $e3$ | 0.21555(142) | 0.66 | 0.65 | 0.03 | 0.10 | 0.09 |
| $K^\pm$ $e3$ | 0.21844(101) | 0.46 | 0.38 | 0.11 | 0.24 | 0.09 |
| $K^\pm$ $\mu3$ | 0.21809(125) | 0.57 | 0.31 | 0.10 | 0.45 | 0.17 |
| average | 0.21673(46) | 0.21 | | | | |

$\to$ sub-1%-precision for $f_+^{K\pi}(0)$ required

# $|V_{us}|$ from $K_{l3}$: Simulations

Recent computations of $f_+^{K\pi}(0)$

| Ref. | | $f_+(0)$ | % err | $m_\pi$ [GeV] | $a$ [fm] | $N_f$ |
|---|---|---|---|---|---|---|
| *Leutwyler & Roos* | *Z.Phys.C25:91,1984* | 0.961(8) | 0.8 | | | |
| *Bijnens & Talvera* | *hep-ph/0303103* | 0.978(10) | 1.0 | | | |
| *Cirigliano et al.* | *hep-ph/0503108* | 0.984(12) | 1.2 | | | |
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| *Becirevic et al.* | *hep-ph/0403217* | 0.960(5)(7) | 0.9 | $\gtrsim 0.5$ | 0.07 | 0 |
| *HPQCD,MILC* | *hep-lat/0412044* | 0.962(6)(9) | 1.1 | ‡ | ‡ | 2+1 |
| *JLQCD* | *hep-lat/0510068* | 0.967(6) | 0.6 | $\gtrsim 0.55$ | 0.09 | 2 |
| *RBC* | *hep-ph/0607162* | 0.968(9)(6) | 1.1 | $\gtrsim 0.49$ | 0.12 | 2 |
| *RBC+UKQCD* | *arXiv:0710.5136* | 0.964(5) | 0.5 | $\gtrsim 0.33$ | 0.114 | 2+1 |
| *UKQCD/QCDSF* | *arXiv:0710.2100* | 0.965(?) | | $\gtrsim 0.5$ | 0.08 | 2 |
| *ETM ???* | | hopefully soon... | | | | |
| *MILC ???* | | hopefully soon... | | | | |

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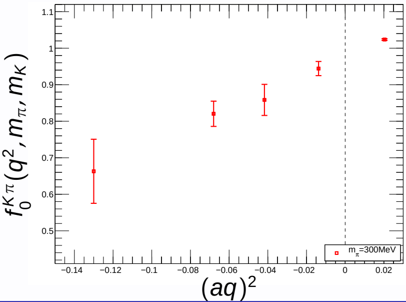
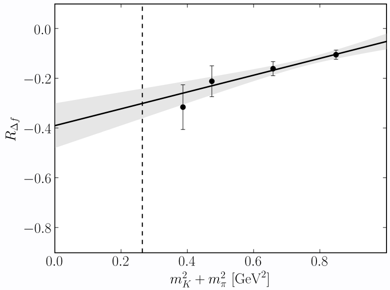
# $K_{l3}$-decay - 4 steps on the lattice

$$\langle\pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu, \quad q_\mu=(p_K-p_\pi)_\mu$$

*Becirevic et al.:*

1) compute $f_0^{K\pi}(q^2_{\rm max})$ - very precise
2) compute $f_0^{K\pi}(q^2)$ for Kaon and pion with lattice Fourier momenta
3) interpolate $f_0^{K\pi}(q^2)$ to $q^2=0$
4) extrapolate in the quark mass to the physical point
   $f_+^{K\pi}(0) = 1 + f_2 + f_4 + \dots$ we compute
   *Gasser&Leutwyler, 1984*
   $\Delta f = f_+^K(0,m_\pi,m_K) - (1+f_2(m_\pi,m_K))$
   $\longrightarrow$ need $<20\%$ precision for $\Delta f$



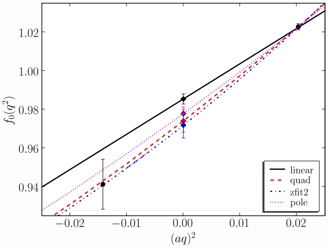
# $|V_{us}|$ from $K_{l3}$ decay - uncertainties and improvements

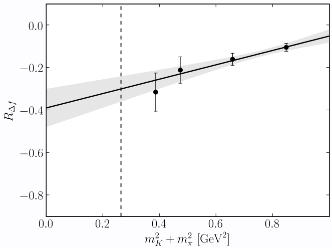
**Uncertainties (e.g. *RBC+UKQCD hep-lat/0702026*)**

$$f_+^{K\pi}(0) = 0.9644(33)^{\rm stat}(34)^{q^2,\chi}(14)^{a}$$

- **stat.:** all-to-all propagators, larger stats.
- $q^2$ **interpol.:** due to Fourier momenta on the lattice → twisted boundary conditions (*Boyle et al. hep-lat/0703005)*
- $\chi$ **extrapol.:** simulate at lighter quark masses and new results in $\chi$PT -

  Bijnens has NNLO-expressions but tedious to evaluate
- **cut-off:** scaling study



# $V_{us}$ from $K_{l3}$ decay - comparison

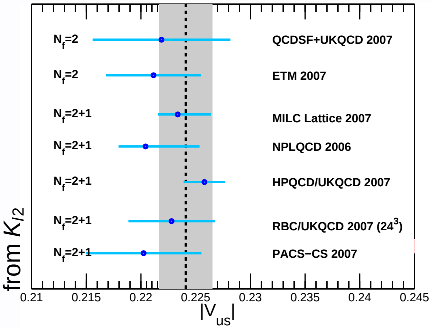
Best value currently *RBC+UKQCD*:

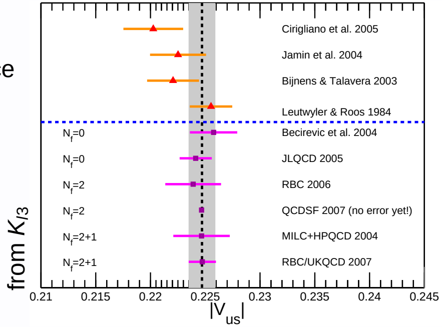
$$f_+^{K\pi}(0) = 0.9644(33)^{\text{stat}}(34)^{q^2,\chi}(14)^{a}$$

$$|V_{us}^{K_{l3}}| = 0.2247 \quad \begin{matrix} (5)^{\Gamma} & (8)^{\text{stat}} & (8)^{q^2,\chi} & (3)^{a} \\ (0.2\%)^{\Gamma} & (0.3\%)^{\text{stat}} & (0.4\%)^{q^2,\chi} & (0.1)^{a}\% \end{matrix}$$

# Summary $|V_{us}|$

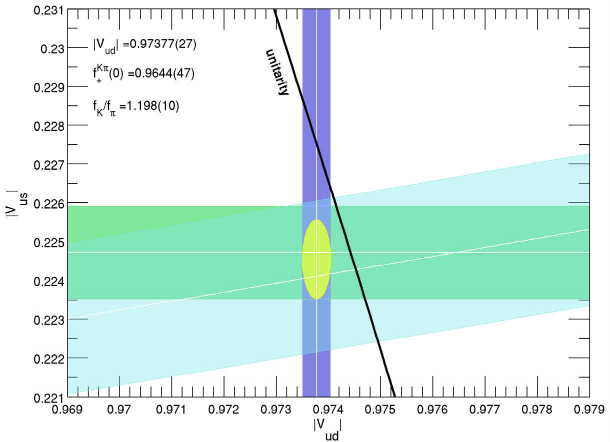
- nice agreement between two independent lattice methods
- in many cases Leutwyler & Roos (1984!) is still used to determine $|V_{us}|$ (cf. PDG) this will hopefully change as more independent lattice result build up confidence



# First row unitarity

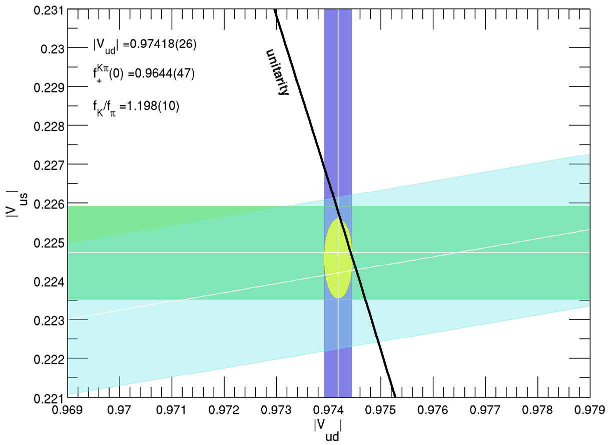
slight tension with unitarity with Marciano's $|V_{ud}|$ *hep-ph/0510099*




Plot inspired by FlaviaNet Kaon working group (cf. *Moulson hep-ex/0703013*)

# First row unitarity

less tension with Unitarity with Towner and Hardy's $|V_{ud}|$ *arXiv:0710.3181*

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## *CP*-violation in kaon decays:
## neutral kaon mixing ($\Delta S = 2$)

# $B_K$ - $\Delta S = 2$ neutral Kaon mixing

$$\frac{\langle \overline{K^0} | Q^{\Delta S=2}(\mu) | K^0 \rangle}{\frac{8}{3} |\langle 0 | A_4 | K^0 \rangle|^2}; \qquad O^{\Delta S=2} = O_{\mathrm{VV+AA}} - O_{\mathrm{VA+AV}}$$

- currently precision for $\hat{B}_K \leftrightarrow$ lattice chiral symmetry (domain wall, overlap fermions)

  $$O^{\Delta S=2} = Z_{\mathrm{VV+AA}}(g_0, a\mu) O_{\mathrm{VV+AA}}(g_0)$$

  otherwise operator mixing, e.g. for Wilson fermions

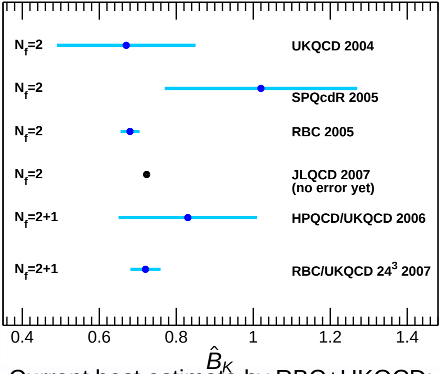
  $$O^{\Delta S=2} = Z_{\mathrm{VV+AA}}(g_0, a\mu) \left( O_{\mathrm{VV+AA}}(g_0) + \sum_{i=1}^{4} \Delta_i(g_0) O_i(g_0) \right)$$

  and similarly for staggered fermions
  which blows up error
- improvement through non-perturbative renormalization
- new development mixed action: valence and sea with domain wall and staggered fermions, respectively (*Aubin, Laiho, van der Water*) analysis under way

# $B_K$ - $\Delta S = 2$ neutral Kaon mixing

- recent activities by (HPQCD+UKQCD, Bae-Kim-Lee-Sharpe, RBC+UKQCD JLQCD, Aubin-Laiho-van de Water)
- staggered, DWF and overlap or Domain Wall on staggered sea with 2+1 or 2 dynamical quark flavors
- $a \approx 0.1$fm - help from quenched studies
- perturbative or non-perturbative matching (preferred)
- NLO $\chi$PT ($SU(2) \times SU(2)$ or $SU(3) \times SU(3)$)

# $B_K$ - $\Delta S = 2$ neutral Kaon mixing




Current best estimate by RBC+UKQCD:

$$\hat{B}_K = 0.720(13) \underbrace{(37)}_{(1\%)^{\mathrm{FVE}}(4\%)^{a}(2\%)^{\chi PT}}$$

# Conclusion

Hand-wavy, crude, hopefully conservative estimate for simulation in five years time:
E.g. domain wall fermions assuming 100-1000TFlop/s computer:

| | | | $\epsilon$ |
|---|---|---|---|
| cut-off | $a$ | 0.06fm | 0.1% |
| unphysical light quark mass | $m_\pi$ | 200MeV | 1.0% |
| finite volume | $L$ | 5fm | 0.7% |

$f_K/f_\pi$
- closer to the chiral limit
- detailed study of finite volume effects
- currently dominated by staggered fermions which is already very far
- precision now 0.8%; then $\lesssim$ 0.5%

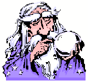
$f_+^{K\pi}(0)$
- precision can easily be increased by larger statistics/smarter methods (which are available)
- systematic due to $q^2$ interpolation can be removed
- $\chi$PT at NNLO is lacking
- precision now 0.5%; then $\lesssim$ 0.2%

$\hat{B}_K$
- NPR mandatory
- precision now 5.4% then $\lesssim$ 3.5%

# Not the only one...

V. Lubicz at SuperB IV, November 2006

**Estimates of error for 2015**



| Hadronic matrix element | Current lattice error | 6 TFlop Year | 60 TFlop Year | 1-10 PFlop Year |
|---|---|---|---|---|
| $f_+^{K\pi}(0)$ | 0.9% (22% on 1-$f_+$) | 0.7% (17% on 1-$f_+$) | 0.4% (10% on 1-$f_+$) | **< 0.1% (2.4% on 1-$f_+$)** |
| $\hat{B}_K$ | 11% | 5% | 3% | **1%** |

# Outlook

- status: predictions for $N_f = 2, 2+1$ QCD for

| quantity | | | % err |
|---|---|---|---|
| $f_+^{K\pi}(0)$ | = | 0.964(5) | 0.5% |
| $f_K/f_\pi$ | = | 1.198(10) | 0.8% |
| $\hat{B}_K$ | = | 0.720(39) | 5.4% |

- What will improve these results in the next five years?:
  - 100-1000TFlop/s computer
  - larger volume
  - smaller lattice spacing
  - lighter dynamical quarks $\rightarrow m_\pi \lesssim 200$MeV
  - tests of chiral perturbation theory
- other:
  - $\epsilon'/\epsilon$
  - Matrix elements for $K \rightarrow \pi l^+ l^-$ and $K \rightarrow \pi \nu \bar{\nu}$?
  - lepton universality - $K \rightarrow \mu \nu_\mu$ vs. $K \rightarrow e \nu_e$?

# BACKUP

$$\Delta S = 1 \; (K \to \pi\pi|_{I=0,2})$$

*The Holy Grail*

# $\Delta S = 1$

$$\langle \pi\pi(I)| - i\mathcal{H}|K^0\rangle = A_I e^{i\delta_I}$$

$$u,d,c,s \quad \mathcal{H}_c^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{\sigma=\pm} \left\{ k_1^\sigma(\mu) \mathcal{O}_1^\sigma(\mu) + k_2^\sigma(\mu) \mathcal{O}_2^\sigma(\mu) \right\}$$

$$u,d,s \quad \mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)$$

- $\mathcal{O}_\pm^\sigma$, $O_{1,2}$ current-current

  $O_{3,4,5,6}$ QCD penguins

  $O_{7,8,9,10}$ EW Penguins

- no penguins in $\mathcal{H}_c^{\Delta S=1}$
- challenge is to compute $\langle \pi\pi(I)| - i\mathcal{H}|K^0\rangle = A_I e^{i\delta_I}$

  *(Maiani, Testa Phys.Lett.B245:585-590,1990)*

- both Hamiltonians are being investigated on the lattice:

  - influence of charm quark on $\Delta I = \frac{1}{2}$ $\rightarrow$ study $\mathcal{H}_c^{\Delta S=1}$ *(Giusti et al. hep-lat/0407007) Hernandez plenary Lattice 2006) and talk Lattice 2007)*

  - phenomenological calculations $\rightarrow$ study $\mathcal{H}^{\Delta S=1}$