Self-Consistent Relativistic Hydrodynamics

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It is an illusion that Hydro Equations are of the form:

$$\partial_\mu T_f^{\mu\nu}(x,t) = 0 \,, \qquad T_f^{\mu\nu}(x,t) = (\epsilon_f + p_f)\, u_f^\mu u_f^\nu - p_f g^{\mu\nu} \,,$$

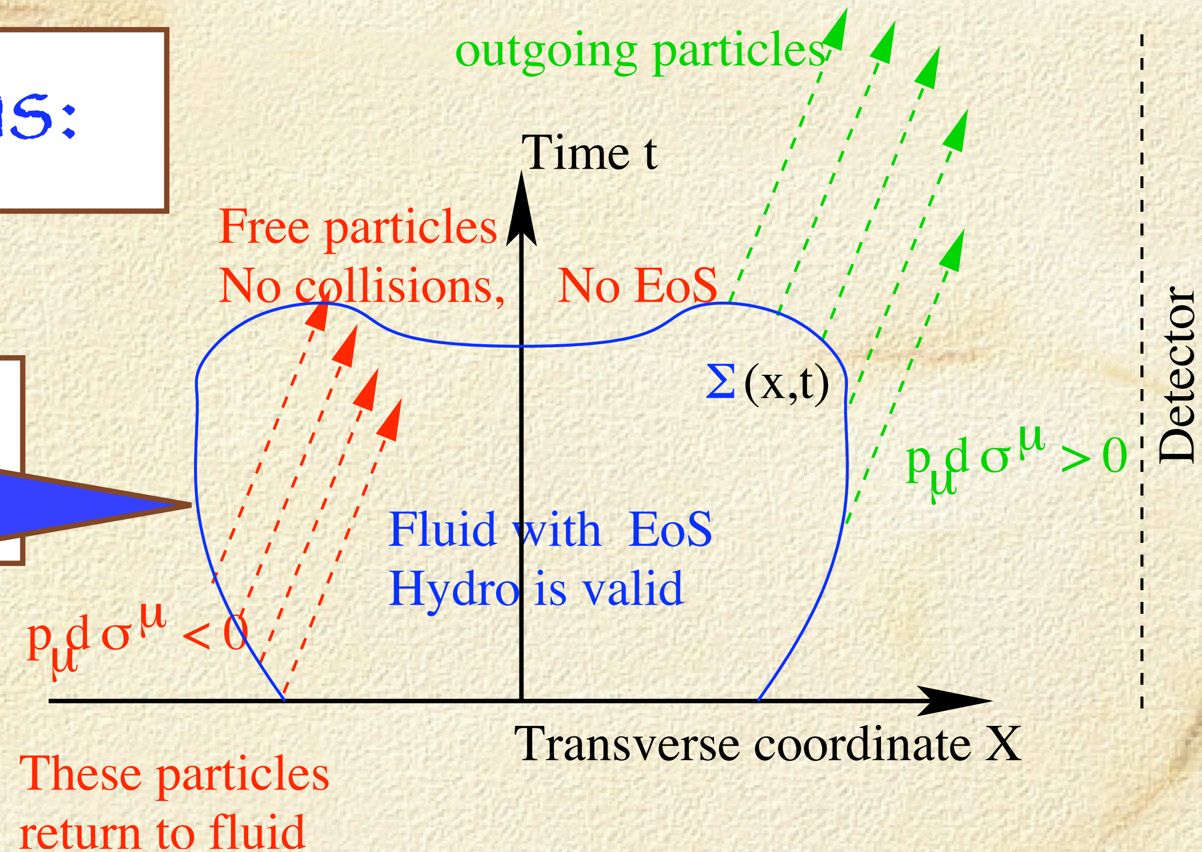
$$\partial_\mu N_f^{\mu}(x,t) = 0 \,, \qquad N_f^{\nu}(x,t) = n_f u_f^\nu \,,$$

Without boundary conditions (=**Freeze-out Procedure**)

Hydro Equations **do not make any sense** at all!

Two Basic Problems:

Here points are causally connected!!!



- Hydro is valid in some domain. There is no Hydro & EoS outside this domain.
  How to stop solving Hydro and do not affect its solution on time-like parts of FO hypersurface?

$d\sigma^\mu$ is external normal 4-vector

- How to convert fluid into free streaming particles?
  How to connect domains with EoS and without EoS?

Hydro with Particle Emission

- Hydro does not describe transition from the matter with EoS to the gas of free streaming particles. It is kinetic process.
  But at time-like parts the transition region is not wide: 0.5 - 1.5 fm only!
  L. Bravina et al, PRC 60 (1999)

- System has two subsystems (domains): Fluid and Gas.
  Conservations laws MUST be written for the whole system!

- Gas of free particles has no EoS, but its hydro variables can be found from the cut-off distribution function which accounts for outgoing particles only: K.A.B., Nucl. Phys. A 606 (1996)

$$p_\mu d\sigma^\mu > 0$$

$$\phi_g = \phi_{eq}(x, t^*, p)\,\Theta(p_\rho d\sigma^\rho)\ ,$$

$$T_g^{\mu\nu}(x, t^*) = \int \frac{d^3p}{p_0} p^\mu p^\nu\, \phi_{eq}(x, t^*, p)\,\Theta(p^\mu d\sigma_\mu)\ .$$

Correct Hydro Equations

- Due to causality the FO criterion F(x,t) = 0 MUST be formulated for the Gas of free particles. Otherwise there is a problem with conservation laws: if particle spectra in the Fluid are frozen (=no collisions!), then Gas cannot have any other temperature! = No solution!

- Same temperature of Fluid and Gas cannot be at time-like parts of the FO hypersurface.

- Energy-momentum and charge conservation laws are valid for Fluid and Gas together (K.A.B., Nucl. Phys. A 606 (1996)):

$$\partial_\mu T^{\mu\nu}_{tot} = 0$$

$$T^{\mu\nu}_{tot}(x,t) = \Theta^*_f \, T^{\mu\nu}_f(x,t) + \Theta^*_g \, T^{\mu\nu}_g(x,t)\,,$$

$$\Theta^*_f = 1 - \Theta^*_g\,, \quad \Theta^*_g = \Theta(F(x,t)) : \quad \Theta^*_g = 1 \;\; for \;\; gas \;\; only!$$

Equations for FO Hypersurface

- Free streaming particles move along straight lines, therefore

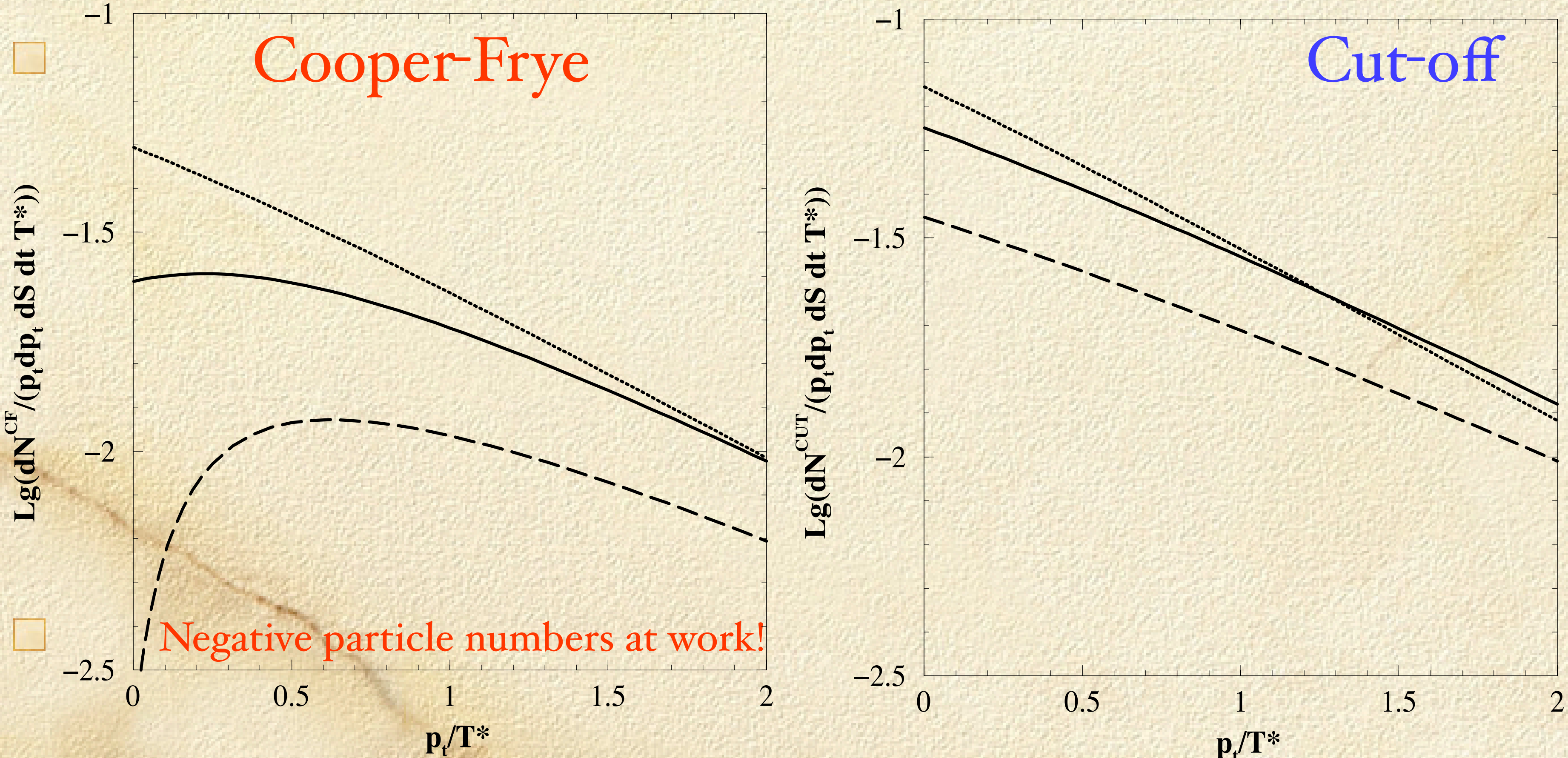
$$
\begin{aligned}
Gas: \quad & \partial_\mu T_g^{\mu\nu}(x,t) &\equiv& \; 0 \quad \Rightarrow \\
Fluid: \quad & \Theta_f^* \, \partial_\mu T_f^{\mu\nu}(x,t) &=& \; 0 \; , \\
Boundary \;\; conditions \;\; at \;\; \Sigma(x,t^*): \quad & d\sigma_\mu T_f^{\mu\nu}(x,t^*) &=& \; d\sigma_\mu T_g^{\mu\nu}(x,t^*) \; .
\end{aligned}
$$

- Boundary conditions define Equations for the FO hypersurface!
- Equations for Fluid vanish Everywhere outside the Fluid domain!
- It was proven that this system does not have a causal paradox due to recoil.

Example: FO of Simple Wave

$$EoS \quad p = \frac{1}{3}\varepsilon \quad \text{(massless pions)}$$

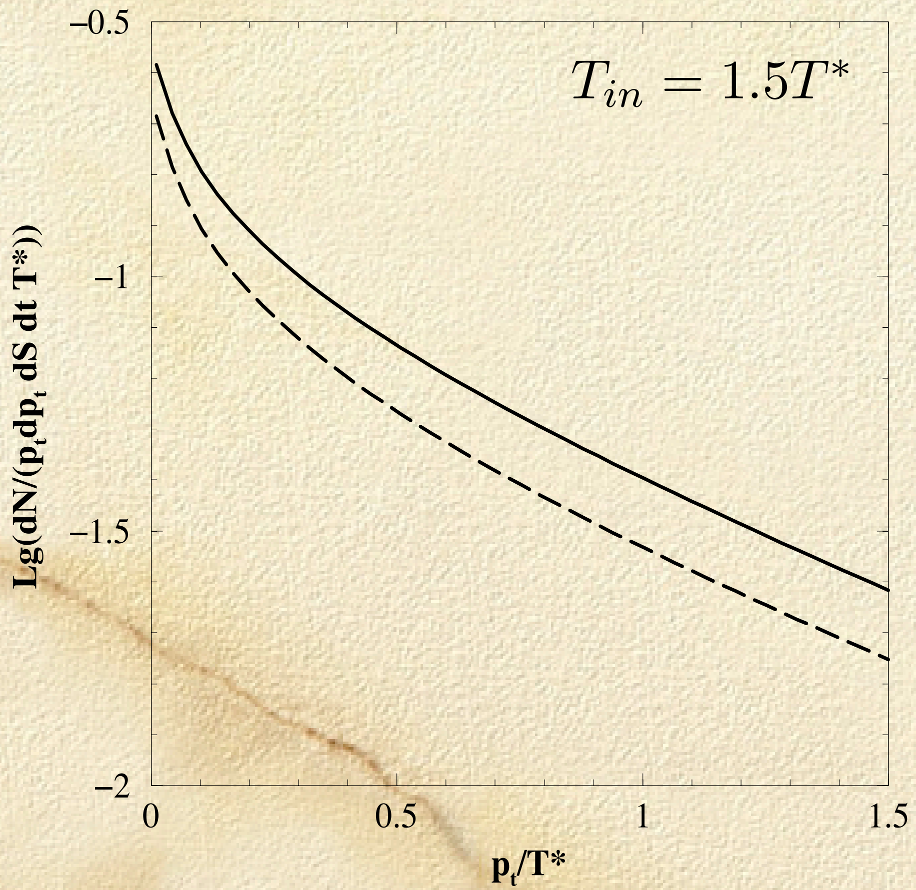
FO temperature is T^*, initial temperature in the wave is T_{in}.
Center of mass rapidity interval is $y_{C.M.} \in [-2; 2]$;
$T_{in} = 1.1T^*$ (dotted line), $T_{in} = 1.5T^*$ (solid line) and $T_{in} = 1.9T^*$ (dashed)
Transverse momentum spectra supposed to be exponential

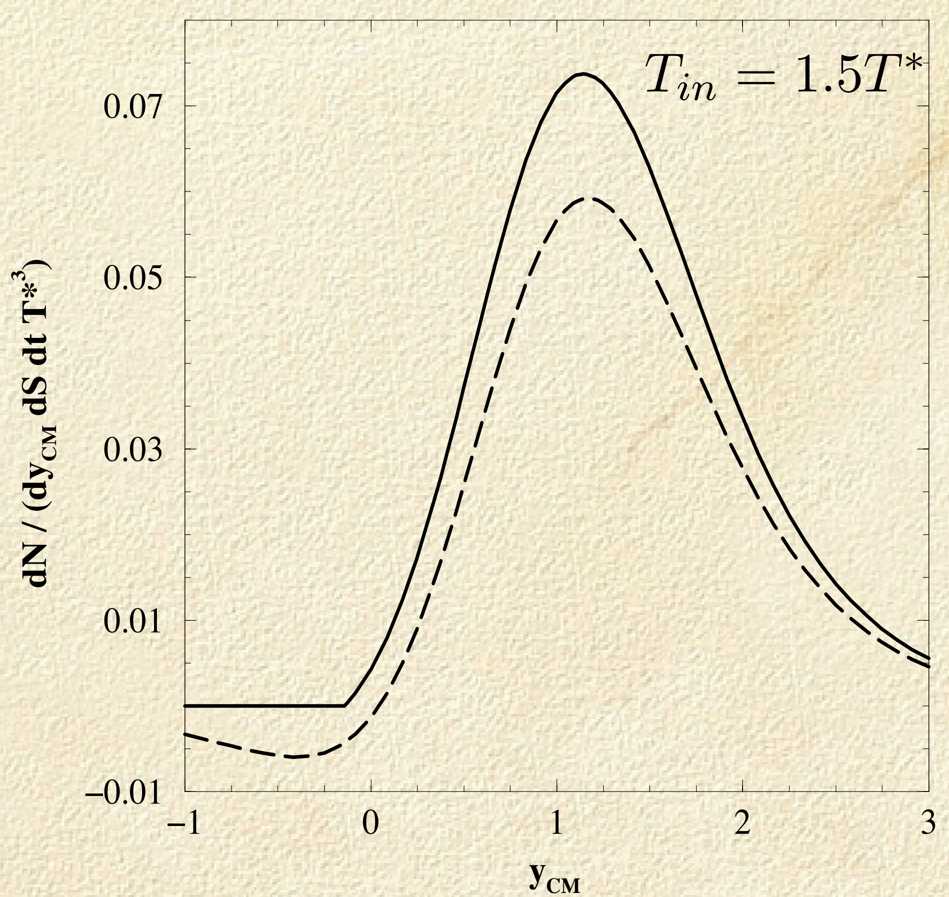


Example: FO of Simple Wave

- Evidence of negative particle number disappears when rapidity interval is wide.

- Effect is not small even for rapidity spectra: (Cooper-Frye results are dashed lines)





We need Other Kind of Hydro to Resolve RHIC puzzles!

Problems of Present Hydro-Cascades

- Best indication that we have no control of Hydro is indicated by HBT puzzles at RHIC!
- So far to reproduce nearly exponential spectra and angular dependence of flow is not a great deal!
  It is important ideologically, but not a proof!
- Therefore, let us check the Hydro-Cascade!

No term can be NEGLECTED!!!

For switch criterion $F(x.t) = 0$ between ideal fluid F and $non-ideal$ Hadronic Gas G described by cascade:

$$T^{\mu\nu}_{tot}(x,t) = \Theta^*_f\, T^{\mu\nu}_f(x,t) + \Theta^*_g\, [T^{\mu\nu}_g(x,t) + \tau^{\mu\nu}_g(x,t)]\,,$$

with

$$\Theta^*_f = 1 - \Theta^*_g\,, \quad \Theta^*_g = \Theta(F(x,t)) : \quad \Theta^*_g = 1 \;\; for \;\; gas \;\; only!$$

If at the switch hypersurface

$$d\sigma_\mu T^{\mu\nu}_f(x,t^*) \neq d\sigma_\mu [T^{\mu\nu}_g(x,t^*) + \tau^{\mu\nu}_g(x,t^*)]\,,$$

Then equations of motion are as follows:

$$\begin{aligned} \Theta^*_f\, \partial_\mu T^{\mu\nu}_f(x,t) &= -\Theta^*_g\, \partial_\mu [T^{\mu\nu}_f(x,t) + \tau^{\mu\nu}_g(x,t^*)] \\ &- \sigma_\mu [T^{\mu\nu}_f(x,t^*) - T^{\mu\nu}_g(x,t^*) - \tau^{\mu\nu}_g(x,t^*)]\, \underline{\delta(t - t^*(x))}\,, \end{aligned}$$

No matter how small is coefficient in front of δ-function
IT CANNOT BE NEGLECTED!!!

Hydro-Cascade problems are similar to old Hydro problems!

- Welcome To Hydro-Cascade!