Overcoming Depolarizing Resonances with Dual Helical Partial Siberian Snakes

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Acceleration of polarized protons in the energy range of 5 to 25 GeV is challenging. In a medium energy accelerator, the depolarizing spin resonances are strong enough to cause significant polarization loss but full Siberian snakes cause intolerably large orbit excursions and are also not feasible since straight sections usually are too short. Recently, two helical partial Siberian snakes with double pitch design have been installed in the Brookhaven Alternating Gradient Synchrotron (AGS). With a careful setup of optics at injection and along the energy ramp, this combination can eliminate the intrinsic and imperfection depolarizing resonances otherwise encountered during acceleration to maintain a high intensity polarized beam in medium energy synchrotrons. The observation of partial snake resonances of higher than second order will also be described.

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Introduction.—The acceleration of polarized beams in circular accelerators is complicated by the presence of numerous depolarizing spin resonances. During acceleration, a spin resonance is crossed whenever the number of spin precessions per turn ν_{sp} (also known as spin tune) equals the frequency with which spin-perturbing magnetic fields are encountered. There are mainly two kinds of resonances: imperfection resonances due to magnet field errors and misalignments and intrinsic resonances due to vertical betatron motion in quadrupoles. An imperfection resonance happens when $\nu_{sp} = G\gamma = n$, where n is an integer, γ is the Lorentz factor, and $G = 1.7928$ is the anomalous magnetic moment of the proton. Strong intrinsic resonances occur when $\nu_{sp} = G\gamma = kP \pm \nu_y$, where k is an integer, P is the superperiodicity of the synchrotron, and ν_y is the vertical betatron tune. In the AGS, $P = 12$. There are seven intrinsic resonances in the AGS and most of them cause significant depolarization if uncorrected.

All imperfection resonances can be overcome by introducing a local spin rotator with a rotating axis in the horizontal plane, called a partial Siberian snake [1]. Such a device can be built as a helically twisted dipole magnet, which distorts the orbit in a snakelike manner. In tribute to the Novosibirsk inventors of this scheme, the name Siberian snake is used. For a ring with a partial snake of strength s, the spin tune ν_{sp} satisfies [1]

$$\cos\pi\nu_{sp} = \cos\frac{s\pi}{2}\cos G\gamma\pi, \tag{1}$$

where $s = 1$ corresponds to a full Siberian snake, which rotates the spin by 180°. For $s < 1$, the device is called partial Siberian snake. The partial snake is often referred as percentage of a full snake. When s is small, the spin tune is nearly equal to $G\gamma$ except when $G\gamma$ is close to an integer n, where the spin tune ν_{sp} is shifted away from the integer by $\pm s/2$. Thus the partial snake can overcome all imperfection resonances, provided that the resonance strengths are much smaller than the spin tune gap created by the partial snake.

A partial snake is particularly interesting for medium energy synchrotrons such as the AGS, as a full snake is not practical due to the large integrated field strength required and lack of long straight sections. Over the past ten years, a 5% ($s = 0.05$) solenoidal partial snake has been used successfully to overcome imperfection resonances in the AGS [2]. In addition, coherent spin resonances excited by an ac dipole were used to overcome the four strong intrinsic spin resonances [3]. However, there are limitations of this combination. First, the solenoid partial snake causes coupling between the horizontal and vertical betatron motions resulting in the so-called coupling spin resonances, which cause modest polarization loss in the AGS [4]. A 5.9% normal conducting (warm) helical partial snake [5] with much less coupling has been installed to replace the solenoidal partial snake. Second, ac dipole operation requires large beam coherence amplitude at low energies for the given emittance. Combined with beam physical size, the required aperture is large comparing to the beam pipe, and it limits the beam intensity. In addition, the ac dipole does not work for weak intrinsic resonances, which are strong enough to generate polarization loss but too weak to be overcome by the coherent spin resonance excited by an ac dipole.

On the other hand, the depolarizing resonance strength in the medium energy synchrotrons is not very strong. A strong enough partial snake can generate a large enough spin tune gap to overcome both intrinsic and imperfection resonances, as long as the vertical tune is placed inside the gap. The challenge is then to operate the synchrotron with a betatron tune close to an integer. An experiment with an 11.4% partial snake to overcome a low energy intrinsic resonance in the AGS showed that this scheme works [6].

In general, the partial snake strength required for a medium energy synchrotron is 0.15 to 0.3. Although the idea is simple, there are two issues. First, it is hard to build one single magnet to fit into the short available straight sections in the existing synchrotrons. If it were possible to build such a strong partial snake, this single device would eliminate depolarization from all spin resonances. For the AGS the challenge amounts to building a 36° spin rotator with a maximum length of 2.6 m and internal orbit excursion of less than about 4 cm. A solenoid spin rotator would require a field of at least 7 T, which would lead to an unacceptable level of orbit coupling and also to strong coupling spin resonances. The most compact solution consists of a 3 T helical dipole with variable pitch. The two ends have a helical pitch that is twice the helical pitch at the center. This field profile allows for a compact matching of the outside orbit to the helical orbit inside the magnet. Such a superconducting (cold) partial snake has been installed in the AGS. It is capable of 22% strength at the AGS extraction energy. The second issue is the control of the machine lattice and orbit with the betatron tune so close to an integer. This will be discussed later.

Two partial snakes.—With a weak (~5%) partial snake the stable spin direction reverses direction at all imperfection resonances but is very close to the vertical direction at half-integer values of $G\gamma$. It is therefore possible to inject and extract vertically polarized beam at these energies without much polarization loss.

For a strong partial snake, however, polarization loss due to spin mismatch at injection and extraction is no longer negligible. A 20% snake will lead to a 10% polarization loss due to this spin direction mismatch. This could be solved with appropriate spin rotators in the injection and extraction beam lines. However, a single additional partial snake located in the synchrotron can provide the spin direction matching at injection and extraction and also increase the effective partial snake strength if its position is chosen properly.

The location and the precession axis direction of multiple partial snakes has to be chosen very carefully to maintain control of the spin tune in a similar way as is necessary for multiple full snakes. For practical partial snakes the precession axis direction is always very close to longitudinal, which leaves only the location and strength of the partial snakes as free parameters.

The spin tune for two partial snakes separated by the fraction $1/m$ of the ring circumference is given by [7]:

$$\cos\pi\nu_{\mathrm{sp}} = \cos\frac{s_1\pi}{2}\cos\frac{s_2\pi}{2}\cos G\gamma\pi - \sin\frac{s_1\pi}{2}\sin\frac{s_2\pi}{2}\cos\frac{G\gamma\pi(m-2)}{m}, \quad (2)$$

where $s_1\pi$ and $s_2\pi$ are the rotation angles of the two partial snakes.

Separating the two partial snakes by one third of the ring is of particular interest since it will introduce a periodicity of three units in the spin tune dependence on $G\gamma$. Since both the superperiodicity of the AGS (12) and the vertical betatron tune (~9) are divisible by three, the spin tune will be the same at all strong intrinsic resonances, namely $\nu_{\mathrm{sp}} = (s_1 + s_2)/2$ for $G\gamma = 3n$. With both snakes at equal strength s, $\nu_{\mathrm{sp}} = s$ effectively doubling the strength of one partial snake. At the injection and extraction energies, for which $G\gamma = 3n + 1.5$, the two partial snakes cancel. The polarization direction in the AGS is therefore exactly vertical and no polarization is lost due to spin direction mismatch.

Even using the presently installed warm helical partial snake with a rotation angle of 10.6° (5.9%) at extraction energy, a very substantial reduction of the injection and extraction spin mismatch can be achieved. At the same time, the effective strength of the partial snakes at the strong intrinsic resonances is significantly increased. Since it is not practical to ramp the two partial snake magnets, their fields are kept constant. This results in the snake strength reduction with the energy ramping up as shown in Fig. 1. The snake strength quoted in this Letter is the strength at extraction energy. Figure 2 shows the spin tune and the vertical betatron tune in the AGS with two partial snakes of 2.11 T (10% partial snake) and 1.53 T (5.9% partial snake), respectively. The partial snakes have to be located as shown in Fig. 3, spaced one third of the ring apart. In this case the polarization loss due to injection and extraction mismatch is about 1%. For a single partial snake with strength of 15.9%, the loss would be 6%.

In general, the intrinsic resonance is only associated with the vertical betatron tune ν_y for vertical polarization, as the vertical spin can only be affected by the horizontal magnetic field. However, in the presence of a partial snake, the stable spin direction is not purely vertical. Therefore, the perturbing fields that rotate the spin away from the stable spin direction have vertical as well as horizontal components. Particles undergoing horizontal betatron os-

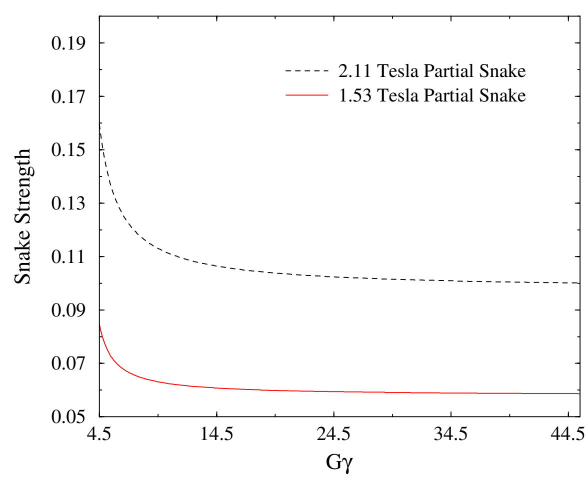

FIG. 1 (color online). The partial snake strength as a function of $G\gamma$. Note the snake strength drops quickly at low energies but is almost constant at high energies.

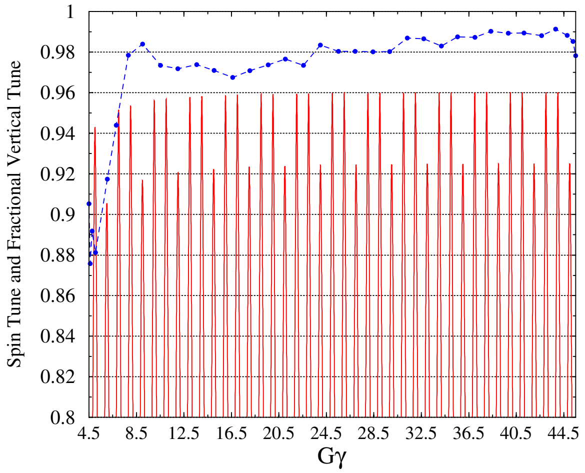

FIG. 2 (color online). Fractional part of measured vertical tune (dots connected with dashed line) along the energy ramp and spin tune for the combination of 2.11 T and 1.53 T partial snakes.

cillations encounter vertical field deviations at the horizontal oscillation frequency. As a result, resonances with the spin tune are driven by the horizontal betatron oscillations and will occur whenever the spin tune satisfies $G\gamma = k \pm \nu_x$, where k is an integer [8]. To avoid these horizontal spin intrinsic resonances, the horizontal betatron tune should also be put into the spin tune gap generated by the partial snakes. However, at present such optics is difficult to achieve in the AGS without upgrading power supplies of betatron tune quadrupoles. Since this depolarizing effect would be stronger for a stronger partial snake, the total snake strength is then a compromise between overcoming vertical intrinsic resonances and minimizing the effect of horizontal resonances. Several cold snake strengths were tested and a 10% cold snake strength gives the best polarization at AGS extraction.

Helical snake operation.—The AGS injection and extraction energies are chosen as $G\gamma = 4.5$ and 45.5, respectively. The extraction energy is chosen such that the spin transmission between AGS and Relativistic Heavy Ion Collider (RHIC) is optimized [9]. At low energies, the helical magnets cause significant lattice distortion. Four

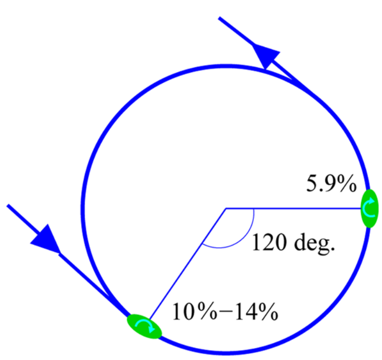

FIG. 3 (color online). Locations of the partial snakes and the injection and extraction regions that give minimum polarization loss due to spin direction mismatch.

compensation quads are added to each of the two helical snake magnets, since the quadrupole component of a helical magnet is focusing in both transverse planes. However, with the compensating quadrupoles, the vertical tune is moved into the spin tune gap only after $G\gamma = 5$. Although the two intrinsic resonances crossed at the beginning of the ramp are weak, the acceleration rate is also slow. This could cause some polarization loss, and simulation studies are underway to quantify the effect and find a proper vertical tune path for the ramp. The vertical orbit has to be corrected very carefully so that the vertical tune could be set to around 8.98 throughout the energy ramp. Polarization at AGS extraction reached 65% for an intensity of 1.5×10^{11} protons/bunch with a 10% cold partial snake and 5.9% warm partial snake. In comparison, the polarization was around 52% for an intensity of 1.0×10^{11} protons/bunch with an ac dipole and a 5.9% warm partial snake. The transverse emittances were similar for the two operation modes.

The lattice distortions near injection due to the helical magnets are significant. Even with compensation quadrupoles, the vertical tune has to be lower than 8.90 near injection (as shown in Fig. 2) so that the beam size due to beta function distortions does not exceed the aperture limit. A solenoid was added in the middle of the cold snake to compensate orbit coupling introduced by the helical field at lower energies. Since the solenoid field cannot be ramped during operation, the field was set to minimize coupling near the $0 + \nu_y$ resonance.

To maintain polarization in the AGS, we have to place the vertical tunes along the energy ramp into the spin tune gap generated by the two partial snakes. Moreover, due to the so-called partial snake resonances [10], the available tune space is reduced even further. The partial snake resonances occur when

$$\nu_{\rm sp} = k \pm l\nu_y, \tag{3}$$

where k and $l(>1)$ are integers. This is the same condition as for full snake resonances [11,12]. The polarization was measured as a function of the vertical betatron tune in the vicinity of several intrinsic resonances. Figure 4 depicts the effect of the partial snake resonances near the two intrinsic resonances for the 14% cold partial snake and 5.9% warm partial snake.

The high order snake resonance locations can be calculated by solving Eqs. (2) and (3), and they agree well with the measured values as marked in Fig. 4. The snake resonance strength is proportional to the strength of the nearby intrinsic resonance. The intrinsic resonance strength can be calculated from DEPOL [13] for a given lattice. For the weak intrinsic resonance $(12 + \nu_y)$, there is only a benign effect from the snake resonances and polarization reaches a plateau above 8.96. For the strong intrinsic resonance $(36 + \nu_y)$, the data show the effect from the second, third, and fourth order partial snake resonances. The vertical chromaticity was set close to zero along the energy ramp

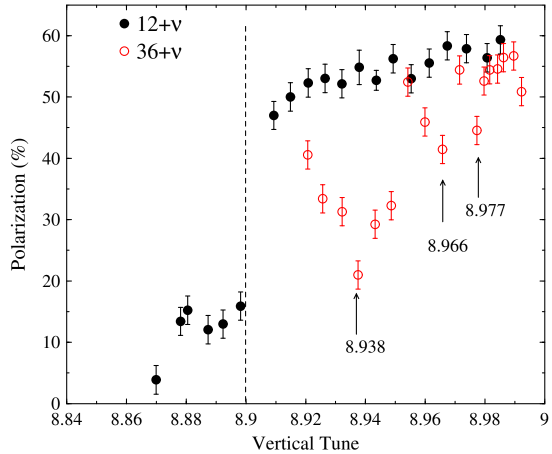

FIG. 4 (color online). Polarization as function of vertical tunes at two intrinsic resonances with different resonance strength. The dashed line shows the position of the spin tune gap for a combined 19.9% (14% + 5.9%) partial snake strength. The locations of high order ($l = 2$, 3, and 4) snake resonances near intrinsic resonance $36 + \nu_y$ are marked.

to reduce betatron tune spread due to momentum spread. This helps to reduce the depolarization from the snake resonances. As expected, the higher the resonance order, the weaker the resonance strength shown as less of a polarization dip. In addition, when the vertical tune is pushed beyond 8.99, the associated large orbit distortions [see discussion after Eq. (4)] are likely the cause of the polarization drop of the last data point.

In a synchrotron, the vertical rms closed orbit is given by

$$y_{\mathrm{co,rms}} \approx \frac{\beta_{\mathrm{av}}}{2\sqrt{2}|\sin \pi \nu_y|}\sqrt{N}\theta_{\mathrm{rms}}, \tag{4}$$

where β_{av}, N, ν_y, and θ_{rms} are, respectively, the average vertical β function, the number of dipoles with field errors, the vertical betatron tune, and the rms steering errors. As seen from Eq. (4), the closed orbit amplitude is greatly enhanced when the betatron tune is close to an integer for the same steering errors. As the imperfection resonance strength is proportional to the closed orbit amplitude and beam energy, the imperfection resonance can still be important at high energies even with two partial snakes installed. Since the betatron tune is close to 9, the 9th and multiple of 9th harmonics are strong. The strength of the imperfection resonance calculated for AGS lattice with large orbit distortion and vertical tune close to 9 could be comparable to the partial snake strength. If they have opposite phase, the imperfection resonance just cancels the effect of the two partial snakes. In fact, we observed polarization loss when the amplitudes of the 9th harmonic of the closed orbit are large. A measurement of polarization as a function of the 9th harmonic orbit amplitude is shown in Fig. 5. The depolarization occurs at the expected amplitude.

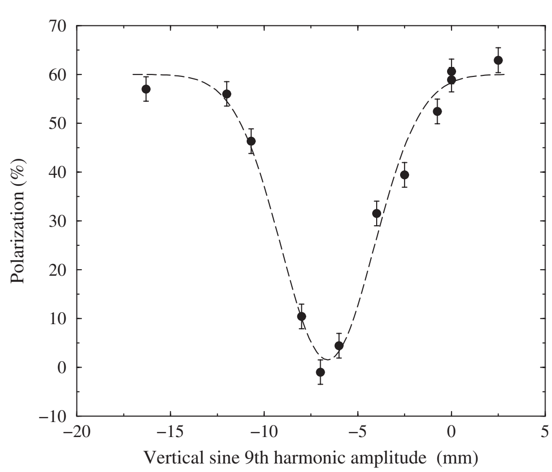

FIG. 5. Measured polarization as function of the sine 9th harmonic amplitude at $36 + \nu_y$. The dashed line is to guide the eyes. The location of the polarization dip agrees with calculation.

Conclusions.—Acceleration of 1.5×10^{11} protons/bunch to 24 GeV with 65% polarization was achieved in the AGS using 5.9% and 10% helical partial snakes. The new partial Siberian snake design using helical dipoles with varying pitch made it possible to build such compact partial snakes. The dual-snake scheme avoids depolarization from both imperfection and intrinsic spin resonances in medium energy accelerators and also maintains good matching to the vertical polarization in the injection and extraction beam lines. It provides a better approach to maintain polarization with high intensity in the medium energy synchrotrons.

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