Muon polarization effects in the Front End of the Neutrino Factory

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Abstract

We summarize the methods used for simulation of polarization effects in the front end of a possible neutrino factory. We first discuss the helicity of muons in the pion decay process. We find that, neglecting acceptance considerations, the average helicity asymptotically approaches a magnitude of 0.185 at large pion momenta. Next we describe the methods used for tracking the spin through the complicated electromagnetic field configurations in the front end of the neutrino factory, including rf phase rotation and ionization cooling channels. Various depolarizing effects in matter are then considered, including multiple Coulomb scattering and elastic scattering from atomic electrons. Finally, we include all these effects in a simulation of a 480 m long, double phase rotation front end scenario.

1 Introduction

A polarized muon beam should provide a valuable handle for understanding the physics at a neutrino factory since the polarization has a strong effect on the angular distributions of the neutrinos coming from the muon decay [1]. In a similar manner polarization information is also thought to be valuable for understanding the physics in a muon collider [2].

Muon beam polarization is normally described in terms of the helicity of each muon, which is defined to be the projection of the spin vector **s** in the muon rest frame along its momentum **p**. The spin vector is normalized to 1 in the muon rest frame. It is well known [3] that the positive muon has helicity = -1 in the positive pion rest frame. However, in the transformation of the muon into the LAB frame an angle develops between the transformed muon's momentum

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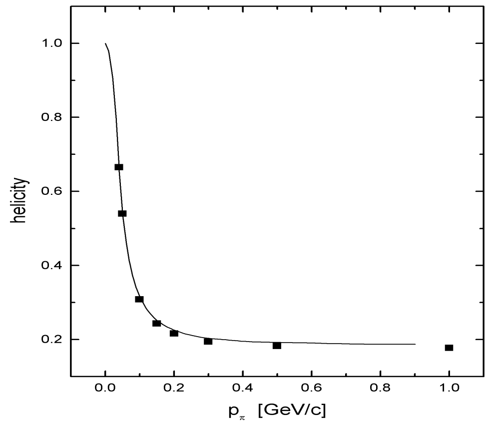

Fig. 1. Magnitude of the average muon helicity in the LAB frame as a function of pion momentum.

and its spin in the rest frame. As a result the magnitude of the helicity in the LAB frame is generally less than 1.

After the muon has been created it must propagate through a maze of complicated electric and magnetic fields in the front end of a neutrino factory or muon collider. Since the spin evolution follows a different set of equations than the momentum, small depolarization effects can be introduced that reduce the average polarization of the muon beam. In addition the cooling channels contain substantial amounts of material in the absorbers, absorber windows, and *rf* cavity windows if pillbox cavities are used. Large angle Coulomb scattering and elastic scattering from atomic electrons have a finite probability of flipping the muon spin direction and can thus lead to further depolarization of the muon beam. Many additional details concerning these calcualtions can be found in a previous note [13].

2 Muon polarization in pion decay

Expressions for the muon helicity in the LAB frame can be found in the literature [4–6] as a function of the production angle in the pion rest frame. Fig. 1 shows the variation of the average muon helicity in the LAB frame as a function of the pion momentum. The solid curve is from a calculation using the Wigner formalism, while the points are from the simulation code Icool [8], which uses the Hayakawa expression [4].

The Icool simulation used a monochromatic pion beam with 0 transverse emittance. We averaged the helicity for 10000 random pion decays. We see that both simulations agree and that the helicity has the value -1 as it should at very low pion momentum. When the average helicity function shown in Fig. 1 is convoluted with the accepted muon momentum spectrum for typical muon collider scenarios [9], the average polarization of the accepted muon beam is typically $\sim$ 20%. As the pion momentum increases, the helicity falls rapidly and approaches an asymptotic value given by the expression

$$\langle h \rangle = \tfrac{1}{2} \left(1 - \frac{E^{*2}}{p^{*2}}\right) \ln \left[\frac{E^* + p^*}{E^* - p^*}\right] + \frac{E^*}{p^*} \qquad (1)$$

where $p^* = 30$ MeV/c and $E^* = 110$ MeV refer to the muon momentum and energy in the pion rest frame. This expression evaluates to -0.185.

3 Spin tracking

When spin tracking is required each muon needs an associated 3-component, normalized spin vector in the muon rest frame. We start with the known spin vector in the pion rest frame. The spin vector is then boosted to the LAB frame using the pion properties and the appropriate Lorentz transformation. A second transformation can then be made to the muon rest frame [4,7].

The muon spin vector in the rest frame evolves in electric and magnetic fields according to the Thomas-BMT equation [7,10]. This differs from the equation of motion for the momentum vector because of additional terms proportional to the g-factor anomaly for the muon, which has the value 1.16510^{-3}.

As a test of likely depolarization effects in the neutrino factory due to the differing evolution equations, we examined the spin tracking in an early example of the double phase rotation front end scenario [11]. This example uses a pion input beam file made from the interactions of 24 GeV/c protons on a mercury target. There is a solenoidal field of 20 T around the target that tapers down to 1.25 T over a distance of about 4 m and then continues at that value over the 45 m long rf phase rotation channel. The actual phase rotation is done with a sequence of rf cavities that vary in frequency from 30 to 45 MHz and that have electric field gradients up to 6 MV/m. There were no windows or other materials in the muon path in the phase rotation channel. The phase rotation is followed by a 1.8 m long liquid hydrogen minicooler and then a 10 m long field reversal region where the solenoid field changes from +1.25 T to -1.25 T. No significant changes in helicity were found in the regions of rapidly varying magnetic field or in the *rf* cavities.

When the muons were propagated through the phase rotation system with spin tracking turned off we found that the average helicity of all muons accepted in the channel at the entrance to the minicooler is -0.189 $\pm$ 0.010. There is no material in the beam up to this point. With spin tracking enabled the average helicity of the accepted muons drops slightly to -0.181 $\pm$ 0.010. A special run with the g-factor anomaly set to 0 gave -0.188 $\pm$ 0.010, so most of the deviation can be attributed to the extra terms in the spin equation of motion.

4 Depolarization effects in matter

Each scattering of a muon in matter has a finite probability of changing the muon's spin direction. This in turn can lead to a depolarization of the muon beam. We have considered two possible depolarizing processes, elastic and multiple Coulomb scattering. Both of these processes change the momentum vector of the muon. Our approach is to first change the spin vector in each interaction in such a way that the helicity is conserved. Then, using the appropriate kinematics for the event, we determine the quantum mechanical probability for the helicity to flip in the interaction. We then randomly flip muon spin directions with that probability.

Continuous energy loss (Bethe-Bloch) is another important process that is simulated for particles in matter. However, in the continuous energy loss process the code scales the magnitude of the momentum vector to correspond to the reduced kinetic energy available after a step. The direction of the momentum is unchanged in this process, and thus the helicity is not changed by it.

The helicity is forced to be conserved in scattering events by first computing the helicity h before each scattering. Then, if primes refer to quantities after the scattering, the spin vector after the scattering is taken as

$$\hat{s\prime} = h\hat{p\prime} + \sqrt{1-h^2}\frac{(\hat{p\prime}\times\hat{s})\times\hat{p\prime}}{|(\hat{p\prime}\times\hat{s})\times\hat{p\prime}|} \tag{2}$$

As a test of the effectiveness of this algorithm, we show in Fig. 2 the behavior of the helicity in the 1.8 m long liquid hydrogen minicooler. The helicity remains approximately constant, even though numerous scattering events occur and the kinetic energy drops by a large amount.

Norum & Rossmanith [12] have given an earlier estimate of the spin flip probability based on the non-relativistic calculation of the spin flip probability by Ford & Mullin [17]. The calculated spin flip probability in multiple Coulomb scattering is known to be very small [14–17]. We have computed the spin flip probability by integrating the 1st order QED matrix element [16] over scattering angles. For typical step size $\sim$ 1 mm in liquid hydrogen we find this

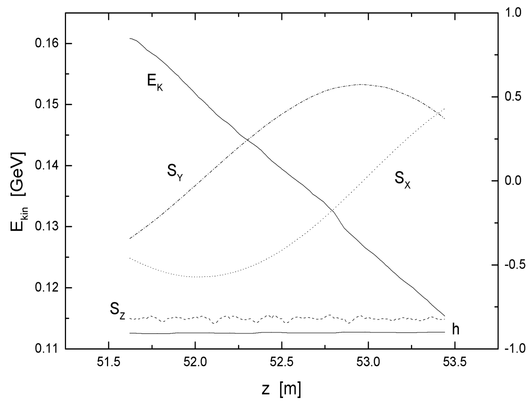

Fig. 2. Kinetic energy, spin vector components, and helicity for one particle in the liquid hydrogen minicooler.

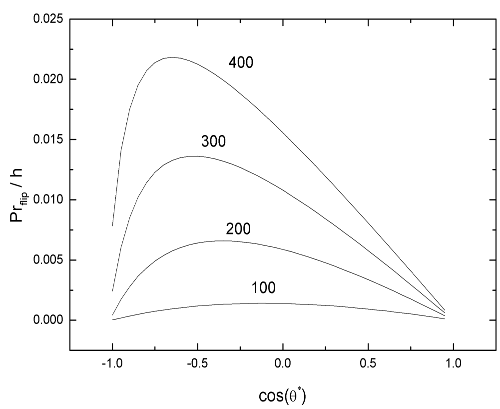

Fig. 3. First order QED calculation of the probability that a muon scattered elastically through an angle θ^* ends up with its spin in the opposite direction from the direction before the scattering. The probability is divided by the particle's helicity. The numbers on the curves indicate the muon LAB momentum in MeV/c.

probability is less than 10^{-7} and is thus indeed negligible.

We next consider the spin flip probability in elastic scattering events from the atomic electrons. We have computed [14]the lowest order QED matrix element for the helicity flip probability in elastic scattering. The results are shown in Fig. 3 for several values of the muon LAB momentum.

The spin flip probability vanishes in both the forward and backwards directions. For muons with a typical momentum of 200 MeV/c, the maximum spin flip probability in an elastic scattering event is $\sim 5 10^{-3}$. For low momentum muons the flip probability is near 0, in agreement with the general principle

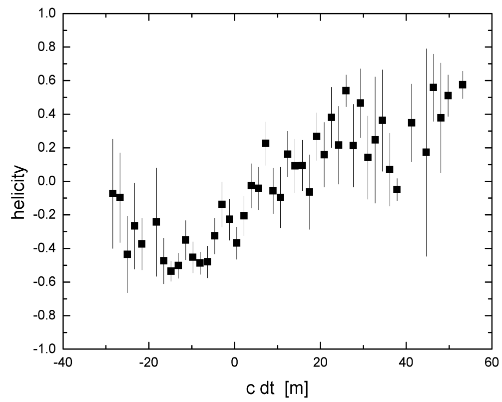

Fig. 4. Correlation of average helicity with bunch position at the end of the double phase rotation system

that in non-relativistic scattering the particle's spin tends to remain aligned with its initial direction.

5 Conclusions

We have simulated polarization effects in the 480 m long, double phase rotation front end for the Neutrino Factory [11]. Figure 4 shows the average helicity in each bunch at the end of the system.

We see that the average bunch helicity varies from about -60 to +60 %. There is a strong correlation of the strength of the helicity with its temporal location in the bunch train. However, the average helicity is not a meaningful measure for the distribution as a whole. We define instead the effective polarization

$$P_{eff} = \int h^2(t) I(t) dt \tag{3}$$

where I(t) is the normalized intensity at time t. The effective polarization was 0.355 ± 0.007 at 52 m into the channel (at the entrance to the minicooler, after the decay region). The effective polarization at the end of the channel was 0.350 ± 0.015. Thus the loss in effective polarization was only 1.4%.

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