## **Global-local Analysis of Laminated Composite Plates**

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Laminated composites are widely used structural materials, especially in Laerospace, automotive, and defense applications. Their popularity can be attributed mainly to their high load-carrying capacity-to-weight ratio. Another characteristic of composite laminates is that their properties can be tailored—by changing, for example, the thickness, orientation, or stacking sequence of individual layers—with the goal of obtaining a desired response under given loading conditions.

To be able to optimize the properties of laminated composites in this manner, however, analytical and computational tools that can accurately and efficiently predict their behavior are needed. Conventional (equivalent single-layer) plate/ shell theories do not resolve the layered setup and are therefore inadequate for modeling important damage mechanisms at the ply level, such as delamination. On the other hand, due to the small thickness of individual layers compared with the overall structural dimensions, general analysis techniques like 3D finite element methods are prohibitively expensive. Therefore, a new specialized analysis technique was developed.

The proposed technique [1] is a finite element formulation, based on the multiscale plate theory developed by Williams [2], and represents the first step toward the development of an efficient computational framework that can be used to study the response and damage distribution within composite laminates subjected to dynamic impact loading.

The formulation is built around the idea of expressing the displacement field as the summation of a) a global field, varying continuously over the thickness of the entire laminate, and b) a local field, whose definition varies from layer to layer. This layerwise definition of the local field allows discontinuities between layers (delamination) to be accounted for in a straightforward manner, and affords much improved accuracy, but incurs higher computational cost. However, the presence of a global component makes it possible to use the full global-local representation in regions where high resolution is required (e.g., in the immediate vicinity of the collision site as in a large structure impacted by a small projectile), and a more economical global-only representation elsewhere, thereby maintaining overall computational efficiency.

Cohesive-zone models (CZMs) are used to predict the initiation and evolution of delamination. This enhances the predictive capability of the formulation, since CZMs do not require any assumptions about the pre-existence of delaminations in the structure. A hybrid finite element approach is adopted in which Lagrange multiplier fields, defined on the interface between adjacent layers, are used to enforce the appropriate interfacial constraint (the CZM or, in the perfectly bonded case, the persistent-contact condition).

Four-node (Q4T1) and nine-node (Q9T3) plate elements, based on this approach, were implemented and a set of standard benchmark problems were solved for verification purposes. One problem (Figs. 1 and 2) involves delamination in a simply supported cross-ply plate subject to pressure loading on its top surface. Another problem (Fig. 3), involving a cantilever plate subject to a transverse force, illustrates the robustness of the Q9T3 element in bending-dominated cases.

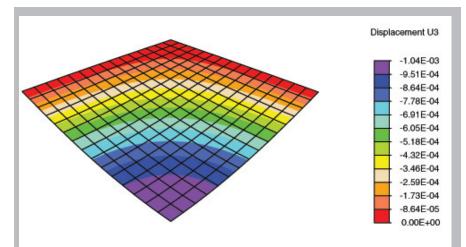


Fig. 1. Transverse displacement in a simply supported cross-ply laminate subject to pressure loading on its top surface. This square plate is relatively thick (with an aspect ratio of 10), and consists of three layers. Due to symmetry, only one quarter of the plate is modeled. The exact mid-point deflection is  $1.041 \times 10^{-3}$  in.

## **Materials**

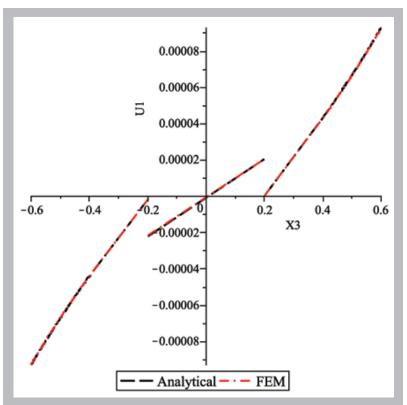


Fig. 2. Variation of the in-plane displacement,  $u_1$ , through the thickness of the laminate shown in Fig. 1 (at the mid-point of its edge coinciding with  $x_1$ =0). The finite-element and analytical solutions are in excellent agreement. Interfacial displacement discontinuities due to delamination are clearly visible.

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 H.M. Mourad, T.O. Williams, and F.L. Addessio, "Finite element analysis of inelastic laminated plates: A global-local formulation with delamination," in preparation.
T.O. Williams, *Int. J. Solids Struct.*, **36**, 3015–3050 (1999).

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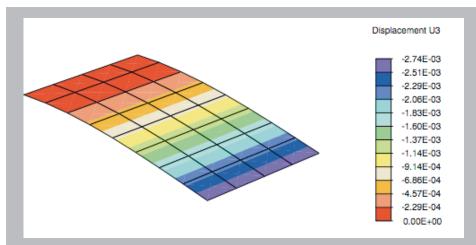


Fig. 3. Transverse displacement in a cantilever plate subject to transverse loading at its tip. First-order elements (like Q4T1) are known to exhibit overly stiff behavior (locking) in such bending-dominated problems. The Q9T3 element was therefore used in this analysis and predicted the correct behavior. The exact tip deflection is 2.743 mm.