Object Dependency of Resolution and Convergence Rate in OSEM with Filtering

S. Mustafovic, K.Thielemans, D. Hogg and P. Bloomfield

***Abstract*--. Convergence properties of the Maximum Likelihood Expectation Maximization (MLEM) algorithm depending on the activity distribution in the field of view is extended to MLEM/Ordered Subsets EM (OSEM) where different types of regularization are applied.**

It will be shown that although different parts of the image converge at different rates, pure and post filtered MLEM/OSEM achieves reasonably uniform resolution.

By contrast, inter iteration filtering (IF OSEM) with smoothing filters, such as Gaussian, renders images with varying spatial resolution that is dependent on the surrounding activity. Furthermore, a similar effect is noticed on images reconstructed with MAP using a Gaussian root prior.

We conclude that resolution non-uniformity is entirely due to the filtering.

***Index Terms*—iterative reconstruction, OSEM, convergence, resolution**

I. INTRODUCTION

The major advantage of iterative over analytical algorithms is the option of emission and detection process to be accurately modelled [11]. Furthermore, iterative algorithms allow statistical noise models to be included as well as incorporation of prior knowledge. Also, provided that some kind of regularisation is used, images obtained with iterative algorithms are more acceptable.

On the other hand, filter-back projection (FBP) as a linear algorithm produces images which have nearly spatially invariant, object independent resolution.

Pure MLEM/OSEM produces images which possess unacceptable noise properties as the iteration number increases. That is why regularisation is needed. Different types of regularisation can be used such as early stopping, based on different criteria [9], inter-iteration filtering (IF) where filtering is used in between the iterations [6,10], post filtering where one keeps on iterating up to the convergence after which post filtering is applied [12] as well as incorporating penalty term in the objective function leading to MAP algorithms [1,7,8].

Defining criteria for stopping the iteration before the image becomes too noisy is very difficult since different parts of the image converge at different rates leading to non-uniform and object dependent resolution [1]. To avoid this problem post filtering can be used after the convergence is achieved, but this leads to long reconstruction times.

Previously it was shown that standard regularisation penalties produce non-uniform resolution even for space-invariant tomographs [2] and a modified penalty was proposed that improved resolution properties.

This abstract concentrates on the non-uniform convergence properties and the influence of the activity distribution present in the image.

Even though some people desire and expect non-uniform resolution based on the idea that high-counts regions provide higher resolution as mentioned in [2], there are applications where uniform resolution or at least object independent resolution is of crucial importance. One obvious application would be to dynamic PET studies where there are different activity distributions in different frames and hence different resolution properties. Also, for cross-patient studies or single patient studies taken over a period of time, the same resolution properties across the image are desirable [3].

Post and inter–filtering as methods of regularising an image were compared previously [6] and it was concluded that the coefficient of variation (CV) and contrast of the inter-filtering was slightly better. However, that analysis did not account for the distribution of the radioactivity in the image itself. Our experiments show that smoothing filters, such as Metz and Gaussian, incorporated directly into reconstruction either as the part of the image updating process (inter-filtering) or as a prior (MAP with Gaussian root prior) influence the resolution properties of the object surrounded with a large activity. Images obtained in this way have spatially varying resolution.

Section I reviews the algorithms used and the methods are presented in Section II. The experimental results are

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presented in Section III. Finally, we conclude with a discussion.

I.ALGORITHM DESCRIPTION

Images for this study were reconstructed with OSEM through various schemes. OSEM requires projections to be organised into a number of subsets. The subsets were chosen as shown in [6]. It is worth mentioning that one pass through all the subsets is considered one full iteration. The number of subsets used was fixed to 10 subsets for all reconstructions.

A uniform image as a starting point was used in all experiments. In the case of post-filtering plain OSEM needed to be run to the convergence after which images were filtered. The convergence point was determined by the stability of resolution/coefficient of variation.

Inter-iteration filtering incorporated filtering directly in the reconstruction process where the filtering process itself was applied at different intervals i.e. filtered every 5 subsets, every 10 and every 20.

Additionally, maximum a posteriori (MAP) with a Gaussian root prior was implemented where it was assumed that images are locally smooth and as such pass unaltered by Gaussian filter. Gaussian outputs a “weighted average” of each pixel's neighbourhood, with the average weighted more towards the value of the central pixels and therefore providing gentler smoothing than the one obtained with simple averaging. The prior will only be applied if the pixel values possess huge variations. The approach is similar to median root prior but differs in that instead of smoothness, privilege is given to those solutions, which are roots of the median [8].

The reconstructions were implemented using the object-oriented software library PARAPET [15,16].

II. METHODS

In the experimental study we have used simulated and real data. The real data were obtained from a HiDAC, 3D small animal PET scanner, which has about 1mm resolution.

The camera consists of four planar, rectangular detector banks each consisting of 8 HiDAC modules, rotating backwards and forwards every 6sec over 180°.

The data acquired in list mode was rebinned into 0.5mm bins where the axial field of view was set to 100mm and diameter to 60mm. The maximum acceptance angle was 59.03° discretized in 15 steps. We have used 160 views for the rebinning, resulting in the projection data of size $15 \times 160 \times 161$. Data were reconstructed on a grid $161 \times 161 \times 244$ cubic voxels of side 0.5mm.

The simulated data was forward projected and the resulting sinograms were used for reconstruction purposes. The sizes and angles were the same as for the experimental data. The effects of attenuation, scatter or noise were not simulated so that only resolution effects could be examined.

Furthermore, we have simulated two cylinders and two line sources placed such that one lied in between the cylinders and the other one is placed a bit further apart. The line sources were longer than the cylinders.

The real data consisted of a 1h scan of a germanium cylinder (external length 7.2cm and source length 6.2 cm with external diameter 3.6cm and source diameter 3cm) with a 10cm long aluminum oxide (Al_2O_3) filament line source aligned to it.

These configurations are similar to the 2D case in [1], to illustrate the effect of convergence of FWHM by surrounding activity (Fig. 1a).

Resolution was measured by FWHM of the line source where two values were recorded, one obtained from the part of the line source surrounded with the activity and the other one from the opposing end where there was no surrounding activity present.

A Gaussian smoothing filter with FWHM = 2mm was used in all experiments. This filter was incorporated directly into the reconstruction either after the normal OSEM image update (inter filtering) or as a prior (MAP with Gaussian root prior) or applied to the converged images reconstructed with pure OSEM (post filtering).

III. EXPERIMENTAL RESULTS

The analysis of the inter filtered case showed that the part of the image sandwiched between the two cylinders failed to achieve the same resolution as the one obtained in the post filtered case (Fig.1b and 1c). This indicates that application of inter filtering with smoothing filters renders images with spatially varying resolution. The more frequent the filter is applied the bigger the difference in the resolution properties between the two parts of the line sources (Fig. 2a). As the frequency of filtering is decreased the effect is less pronounced and the resolution properties approximate to the nonregularized case (Fig. 2b).

Furthermore, images reconstructed with MAP with Gaussian root prior showed that the same effect, i.e. non-uniform resolution, is present (Fig. 1d)

We conclude that these resolution non–uniformities are due exclusively to the filtering with smoothing filters (IF OSEM) or the interaction between likelihood function and the prior for MAP. Moreover, resolution properties depend on the object in the case of inter-iteration filtering whereas this effect is not present in the case of post-filtering. .

Similarly, simulations of the line source and the two cylinders placed this time further apart did not show this effect. Hence showing the object dependency once again.

Furthermore, the real data was reconstructed using 3D pure and post-filtered OSEM, IF OSEM where filtering was performed as explained in section II. Once again it was confirmed that smoothing filters incorporated in the reconstruction produce images with spatially varying resolution (Fig. 3).

IV. DISCUSSION

We have investigated object dependency and the convergence rate in the pure and regularized OSEM (post/inter filtering) as well as MAP with the Gaussian root prior. It was found that once a smoothing filter is applied either as a part of image updating process (IF OSEM) or in MAP the obtained images have spatially varying resolution depending on the activity distribution in the image.

Figures 2a, 2b give a clear intuitive explanation for this behavior in the interfiltering case. There are two competing effects on the FWHM. The normal OSEM update decreases the FWHM, while the filtering step increases it. At the convergence, the balance between these two updates is influenced by two factors: the frequency of filtering (higher the frequency, higher FWHM) and the convergence rate of pure OSEM in that point (slower convergence, higher FWHM). This latter factor gives an interesting connection between the object dependency of the local convergence rate and the resolution obtained in an inter filtering case. It is clear that this connection will also exists when filtering occurs after every subitertaion, which in the case of 1 subset is the original EMS algorithm. It is also clear that exactly the same behavior will happen in any algorithm, i.e. with different update steps than (OS) EM, that uses interfilterinng. This is because any algorithm will have different local convergence rates depending on surrounding activity. So we generally conclude: for any iterative algorithm, interfiltering with spatially invariant smoothing filtering will lead to object dependent resolution.

This situation is very similar to the case analyzed in [4]. Instead of intefiltering, there, a (smoothing) filtering term is added to the likelihood. It was found that having a spatially invariant penalty term leads to object dependent (and hence non-uniform) resolution. In this case, no obvious connection with the convergence rate is present. Indeed, the analysis in [4] is independent of the algorithm used to find the MAP maximum. In contrast, for interfiltering, the object dependency will vary if a different algorithm is used, as the local convergence rate would be different.

It is likely that, similar to the MAP case [3,4,5], uniform resolution could be obtained in an interfiltering case by adapting the filter locally. We plan to investigate this in the future. To do this, the fixed-point equations for EMS (and its variations) will have to be analyzed. This is worthwhile as we found that when no surrounding activity is present, interfiltering gives a better resolution vs. noise (measured as CV in a uniform region) trade-off compared to postfiltering (data not shown but see also [6]).

At the moment, it is unclear if a filtering approach is better than using a penalty term. To investigate this, we included some results on MAP with a Gaussian root prior. Although this algorithm does not correspond to maximizing an objective function (there is generally no penalty term associated with a root prior), we used this algorithm because a clear connection exists with the filter. We were surprised to see (Fig. 1d) that the FWHM obtained by this algorithm is a lot smaller than in the case of postfiltering (and hence interfiltering). Obviously, this will depend on the choice of beta, and one would have to look at the corresponding noise properties. We leave this for future work.

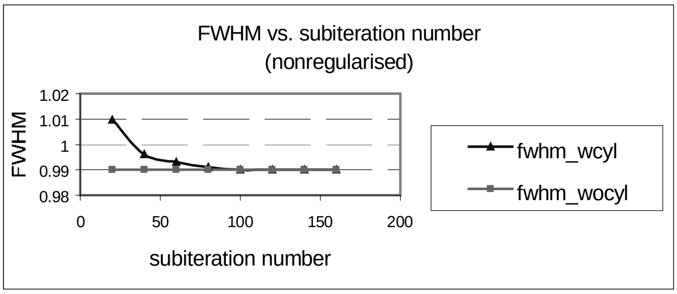

Figure 1a. Resolution vs. subiteration number (nonregularised OSEM) for two parts of the line source (part of the line source aligned with a cylinder – triangles, and part of the line source on its own – squares)

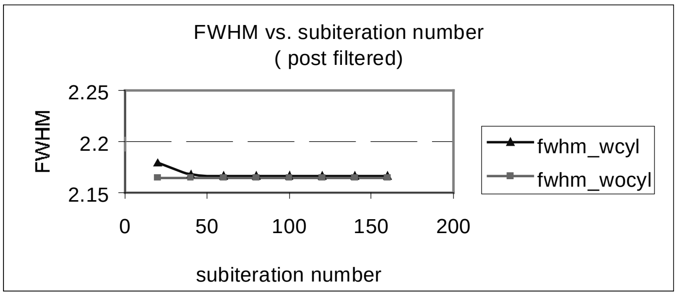

Figure 1b. Resolution vs. subiteration number (post filtered OSEM) for two parts of the line source (part of the line source aligned with a cylinder –triangles, and part of the line source on its own – squares)

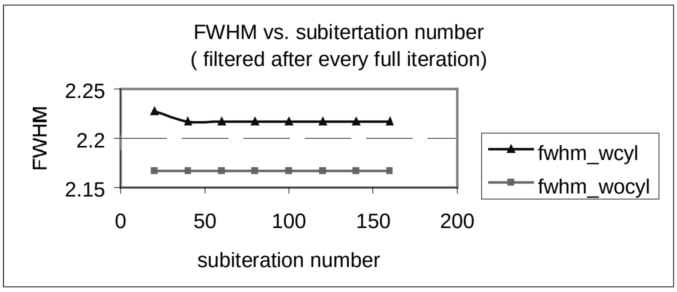

Figure 1c Resolution vs. subiteration number (inter filtered OSEM) for two parts of the line source (part of the line source aligned with a cylinder –triangles, and part of the line source on its own – squares)

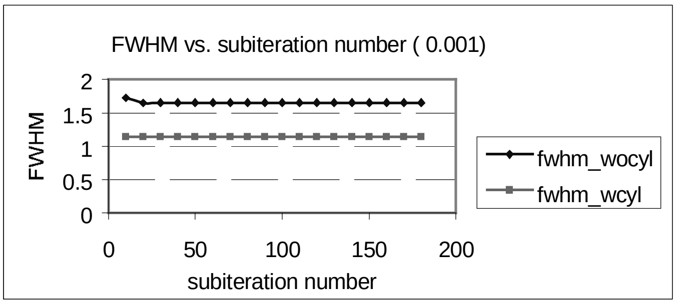

Figure 1d. Resolution vs. subiteration number (MAP with Gaussian root prior) for two parts of the line source (part of the line source aligned with a cylinder – triangles, and part of the line source on its own – squares)

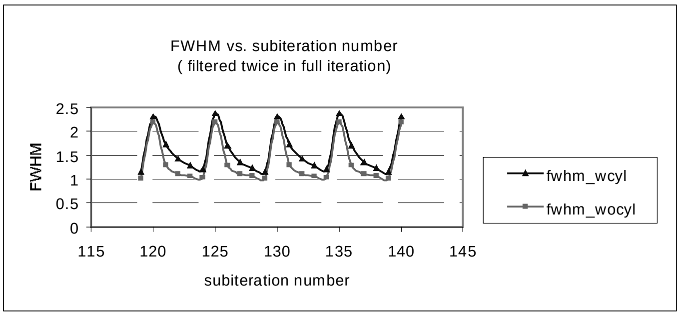

Figure 2a. Resolution vs. subiteration number for inter-filtered OSEM where filtering was applied twice in every full iteration

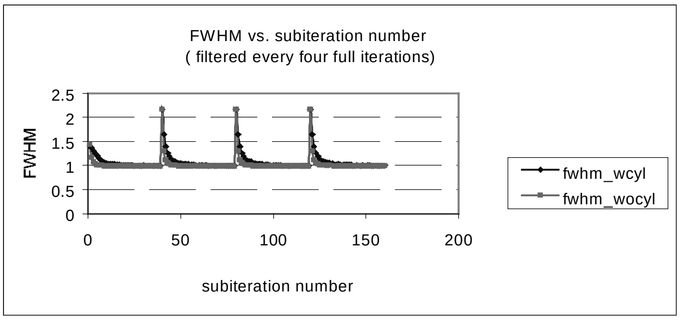

Figure 2b. Resolution vs. subiteration number for inter-filtered OSEM where filtering was applied every four full iteration

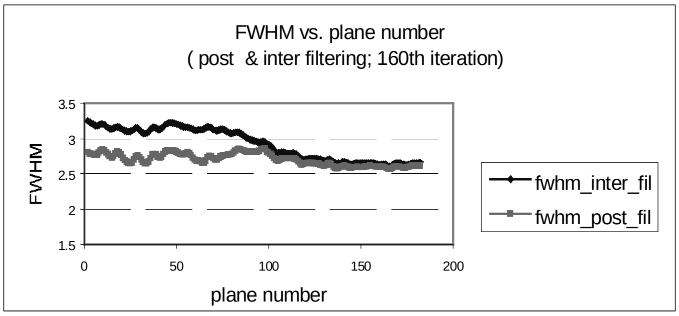

Figure 3 Comparison of resolution vs. plane number for real data of inter and post filtering

REFERENCES

1. J.Nuyts, "ML and MAP Reconstruction for Emission and Transmission Tomography", Nuclear Science Symposium and Medical Imaging Conference, Lyon, 2000
2. J. A. Fessler and W.L. Rogers, "Spatial resolution properties of the penalized-likelihood image reconstruction methods: Space invariant tomographs", IEEE Tr. Im. Proc., 5(9) ,Sep. 1996, pp. 1346-58,
3. J.W. Stayman and J. Fessler, " Regularization for uniform spatial resolution properties in penalized-likelihood image reconstruction", IEEE Trans. on Med. Imaging, March 8, 2000
4. J.A. Fessler and W.L.Rogers. " Uniform quadratic penalties cause nonuniform image resolution (and sometimes vice versa)", in Conf. Rec. of the IEEE Nuc. Sci.Symp. Med. Im.Conf., Vol.4., pp. 1915-1919, 1994
5. Qi J., R. Leahy, " Resolution and noise properties of MAP reconstruction for fully 3-D PET", Vol. 19, No5, May 2000, 493-506
6. M. Jacobson, R. Levkovitz, A. Ben-Tal, K.Thielemans, T.Spinks, D.Belluzzo, E.Pagani, V. Bettinardi, M. Gilardi, A.Zverovich and G.Mitra, "Enhanced 3D PET OSEM reconstruction using inter-update Metz filtering", Phys. Med.Biol. 45 (2000) 2417-2439
7. Green P.J "Bayesian reconstruction from emission tomography data using a modified EM algorithm", IEEE Tr. Med. Im., Vol.9, No 1, March 1990, pp. 84-93.
8. S. Alenius, " On noise reduction in iterative image reconstruction algorithms for emission tomography : median root prior", Ph.D. Thesis, 1999
9. E. Veklerov and J. Llacer "Stopping rule for the MLE algorithm based on statistical hypothesis testing," IEEE Tr. Med. Im. Vol.6, No. 4, pp.313-319, December 1987
10. S.Silverman, M.C. Jones, J.D. Wilson, D.W. Nychka " A smooth EM approach to indirect estimation problems, with particular reference to stereology and emission tomography" , Journal Royal Statistical Society, 1990, 271-324.
11. B.F. Hutton, H.M. Hudson, F.J. Beekman " A clinical perspective of accelerated statistical reconstruction", European Journal of Nuclear Medicine Vol. 24.No. 7, July 1997
12. Beekman F.J., Slijpen T.P.E., Niessen J.W., "Selection of task-dependent diffusion filters for the post-processing of SPECT images", Phys. Med. Biol. 43, Feb. 1998, pp. 1713-1730
13. A.P. Jeavons, R.A. Chandler, C.A. Dettmar, " A 3D HiDAC PET camera with sub-millimeter resolution for imaging small animals ", IEEE NSS and MIC Conference, Toronto Canada, 1998
14. Y. Vardi, L.A.Shepp, L. Kaufman " A statistical model for positron emission tomography", Journal of the American Statistical Association, Vol. 80, No. 389, March 1985
15. Labbé C., Thielemans K., Belluzzo D., Bettinardi V., Giardi M.C., Hague D.S., Jacobson M., Kaiser S, Levkovitz R, Margalit T, Mitra G., Morel C., Spinks T.J., Valente P., Zaidi H., Zverovich A. " An object –oriented library for 3D PET reconstruction using parallel computing ", 1999
16. Labbé C., Thielemans K., Zaidi H., Morel C., " An object-oriented library incorporating efficient projector/back projection operators for volume reconstruction in 3D PET", Proc. 1999 Int. meeting on fully three-dimensional image reconstruction in radiology and nuclear medicine, pp. 137-140,1999