Low harmonic distortion in a Josephson arbitrary waveform synthesizer

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The use of broadband integrated filters has enabled practical operating margins for ac waveforms synthesized from the perfectly quantized voltage pulses of Josephson junction arrays. This improvement enabled the digital synthesis of arbitrary waveforms with low harmonic distortion; the second and higher harmonics are all at least 94 dB below the fundamental [−94 dBc (carrier)]. This is a 47 dB lower distortion compared to that of the same sine wave synthesized by the semiconductor-based code generator which drives the array. We present operating margins for synthesized dc voltages and demonstrate synthesized waveforms with multiple tones at kilohertz and megahertz frequencies with harmonic distortion and intermodulation products below −80 dBc. [S0003-6951(00)00133-9]

We have achieved practical operating margins for arbitrary ac waveform synthesis using Josephson junction arrays. We demonstrate a Josephson arbitrary waveform synthesizer with output voltage three times higher and harmonic distortion 14 dB lower than that of previous circuits.[1] This improved performance is due to the use of nondissipative low-pass filters on the output voltage taps and a four times larger array of 4096 Josephson junctions. With this circuit it is now possible to synthesize arbitrary waveforms with low harmonic distortion and stable, reproducible voltage amplitude and phase. This precision arbitrary waveform synthesizer will be useful in high-performance audio- and radio-frequency (rf) applications, including ac voltage standards, low-noise radar, and electronic instrument calibration.

We have been developing an ac voltage standard source based on controlling the perfectly quantized voltage pulses of Josephson junctions since 1996.[2] The time-integrated area of every Josephson pulse is precisely equal to $h/2e$, the ratio of Planck's constant and twice the electron charge. Digital synthesis using perfectly quantized pulses allows the generation of voltage waveforms with unprecedented accuracy and stability. In previous simulations and experiments we have shown how voltage noise and timing jitter, intrinsic to all semiconductor-switched digital code generators, create harmonic distortion in a synthesized waveform.[3] We have also shown how the perfect area quantization of Josephson pulses can recover the low harmonic distortion of the original digital code that assumed perfect voltage levels and perfect timing.

One of the main challenges in creating a practical and useful Josephson arbitrary waveform synthesizer is generating large voltages. The maximum output voltage of a series array of junctions is $V=nNf(h/2e)$, where n is the number of quantized output pulses per input pulse, N is the number of series junctions, and f is the input pulse frequency. The area $h/2e$ of each quantized pulse is very small, approximately 2 μV/GHz. In order to generate a waveform with a 1 V peak amplitude, many pulses (5×10^{14} pulses/s) must be synthesized and controlled. Thus, high-voltage output requires both many junctions and a high pulse-repetition frequency. High pulse rates are particularly difficult to achieve using semiconductor code generators; typical high-speed digital code generators clocked at 12 GHz generate pulses at a maximum bit rate of 6 GHz. Since this method generates pulses of only one polarity, it can synthesize only unipolar ac waveforms with a dc offset.

We recently described a method for synthesizing bipolar waveforms with much higher output voltage in which the array is driven with a combined input waveform consisting of the two-level digital code and a sine wave.[3] The circuit schematic for this method is shown in Fig. 1. The modulator algorithm converts the desired analog signal $V(t)$ into a ''perfect'' digital code; it reduces in-band distortion from the quantization process, called quantization noise, and pushes it out of band, usually to higher frequencies.[4] We use a second-order two-level delta–sigma modulator algorithm that has been modified for use with this bipolar bias technique. The periodic code of length N_s is loaded into the circulating memory of the semiconductor code generator, which attempts to reproduce the ideal digital code with its two-level output. The array is biased by the combined signal of the code generator and a sine wave through a directional coupler ''C.'' The low-pass filter removes the out-of-band quantization noise and produces the desired synthesized signal $V'(t)$. For a clock (and sampling) frequency f_s, the minimum signal frequency is $f_1=f_s/N_s$.

The optimum operating margins occur for this method when the sine frequency f is an odd half-integer multiple of

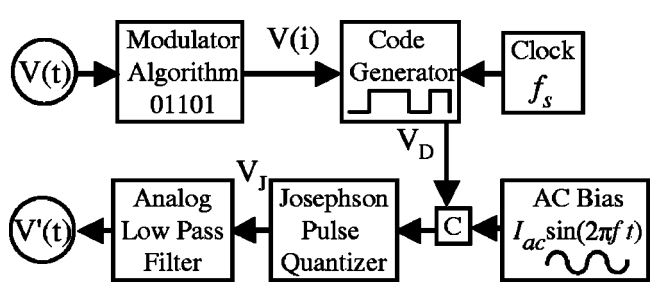

FIG. 1. Bipolar circuit schematic of the Josephson arbitrary waveform synthesizer based on a Josephson junction pulse-quantizing array.

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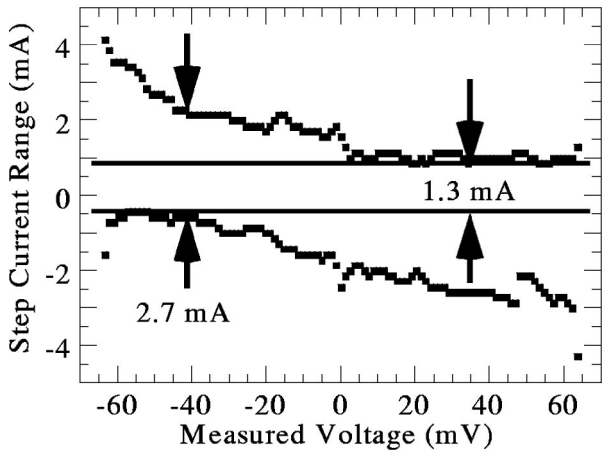

FIG. 2. Bias current range for 101 patterns of a 100 bit code synthesizing dc voltages between -63.5 and $+63.5$ mV.

the clock frequency f_s (eg., 3/2, 5/2).[5] The relative phase between the clock and the sine wave is maintained with a 10 MHz reference signal. In this bipolar method,[1] the maximum output frequency is determined by the sine frequency, and can result in at least a sixfold increase in peak-to-peak voltage [for $f=(3/2)f_s$] compared to the unipolar method using the same code generator.

The circuit improvements for the results presented in this letter include increasing the number of junctions from 1000 to 4096, and replacing the previous 50 Ω resistor output voltage taps with nondissipative superconducting low-pass filters. The junctions are 2 μm diameter with a critical current of 8.2 mA and a normal state resistance of 2.7 mΩ. They are spaced at 7 μm along a 50 Ω coplanar waveguide that is terminated with a 50 Ω resistor. The voltage across the array is measured through two low-pass filters each having a 3 GHz cutoff frequency. The filters contain compact square superconducting inductor coils that are 4.25 turns with 5 μm Nb wire spaced at 1.5 μm. The superconducting integrated circuit is cooled to 4 K in a 1 m long cryoprobe. For these measurements the clock frequency was 3 GHz and the sine frequency was 7.5 GHz.

We measured the bias range of dc voltage steps synthesized with a periodic 100 bit pattern. Figure 2 shows the current range of the 101 different voltage steps. The current range for all steps is greater than 2.7 mA. The 1.3 mA current range common to all steps defines the operating range for this circuit. This range is two times larger than the previously measured operating range for the smaller 1000-junction arrays with resistive filters.[6] Furthermore, for the resistive-filtered array, two specific bit patterns were found to be extremely sensitive to the amplitude of the two-level code and its phase relative to the sine frequency, so there were essentially no operating margins for code amplitude and relative phase. The operating range of the 4096-junction array with the low-pass filters is not limited by any single bit pattern. An overlapping current range can be found for a finite range of every bias parameter, including phase and amplitude, resulting in the nonzero operating margins for the waveform synthesizer.

These improved margins are a result of the improved broadband response of the low-pass filters. The filters provide a high impedance for signals at frequencies above 3 GHz. These high-frequency signals remain in the broadband transmission line, preventing standing waves in the distributed array that occurred in previous circuits with resistive bias taps. Controlling the filter characteristics, especially the stop-band transfer function, is critical to improving the operating margins of the array.

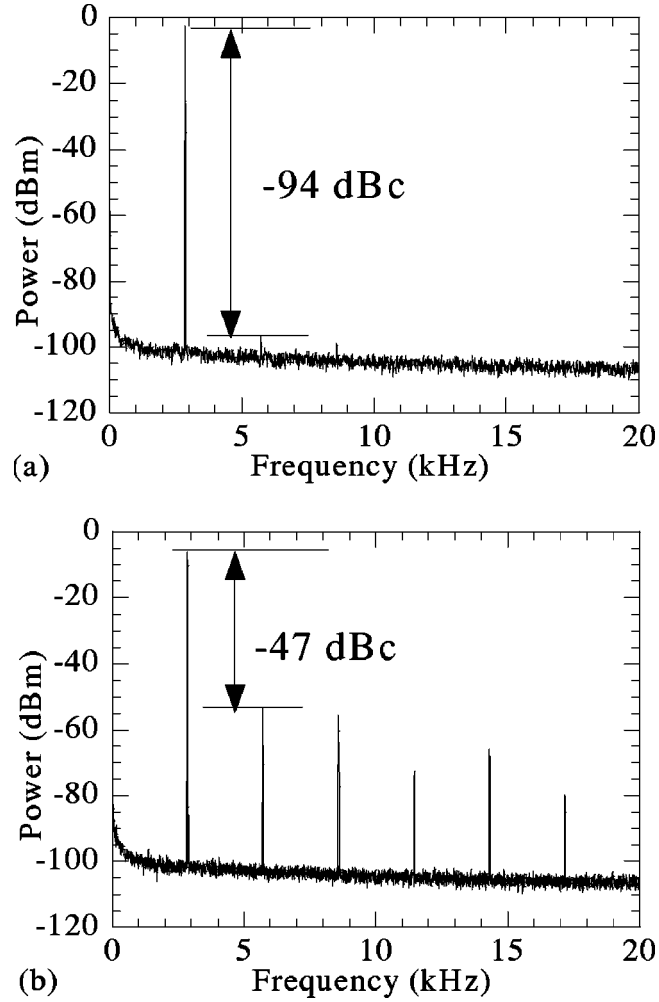

FIG. 3. (a) 2.86 kHz sine wave synthesized by the Josephson arbitrary waveform synthesizer. Peak amplitude is 60.3 mV. (b) Same tone synthesized by the semiconductor digital code generator and used to drive the Josephson junction array. Resolution bandwidth for the spectra is 13.8 Hz.

In addition to the measured operating margins for dc voltage synthesis, we have found similar operating margins for arbitrary synthesized waveforms. In previous demonstrations using resistive filters we observed increased harmonic distortion whenever the code amplitude or phase was altered from the minimum distortion values (thus no margins). With the low-pass filter circuits, we observe the same low distortion over a range for each bias parameter, indicating nonzero operating margins.

Figure 3(a) shows the power spectrum of a 2.86 kHz sine wave synthesized using the low-pass filter circuit and a 1048 576-bit-long digital code. The peak voltage amplitude is 0.95 times the maximum 63.5 mV voltage. The waveform has very low distortion; the second and higher harmonics are all at least 94 dB below the fundamental (-94 dBc). This waveform was amplified by a differential preamplifier with a 1 MHz bandwidth and a 50× gain. The preamp was necessary to reject the common-mode signal from the termina-

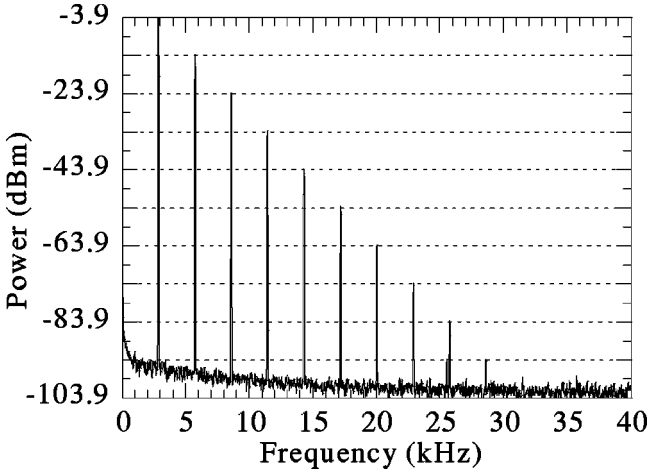

FIG. 4. Multiple-tone staircase waveforms synthesized by the Josephson arbitrary waveform synthesizer. 2.86 kHz sine wave with consecutive higher harmonics in decrements of 10 dB. Resolution bandwidth is 47.7 Hz.

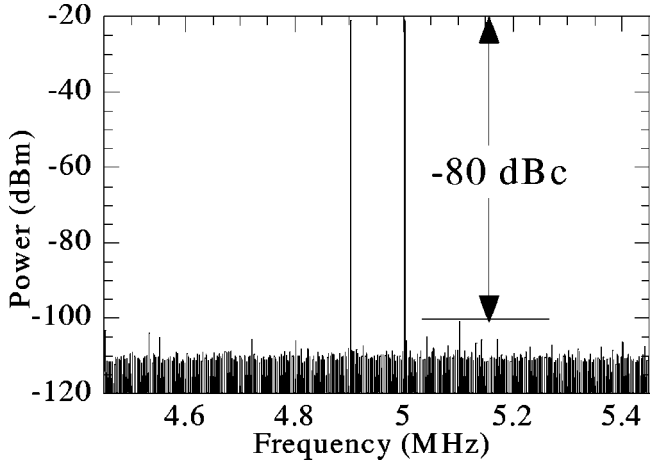

FIG. 5. 4.90 and 5.00 MHz tones synthesized by the Josephson arbitrary waveform synthesizer. Resolution bandwidth is 10 Hz.

tion resistor. The common-mode rejection specification of the preamp is only 95 dB and distortion is specified at only 0.01% (-80 dBc). The amplified waveform was measured with a 10 MHz fast Fourier transform (FFT) spectrum analyzer with a harmonic distortion specification of only -75 dBc. The width of each tone and the noise floor is limited by the resolution bandwidth. Thus, the measured distortion of our synthesized waveform is at or below the specifications of our measurement equipment.

Figure 3(b) shows the power spectrum of the digital code generator that drives the Josephson array. The harmonic distortion is only -47 dBc. This distortion is within the code amplitude operating margins of the Josephson array, so the array can still perfectly quantize this waveform and reduce the distortion by at least 47 dB.

Next, we demonstrate the arbitrary synthesis capability of this source by synthesizing multiple-tone waveforms. Figure 4 shows the power spectrum of a 2.86 kHz tone combined with higher harmonics whose amplitudes are intentionally decreased in 10 dB increments. No distortion or intermodulation products are observed within the resolution of the spectrum analyzer. Immediately across the on-chip array the voltage amplitude is perfectly defined through the digital code and the perfectly quantized pulses. However, the bandwidth of the output filters, cryoprobe wiring, and preamp all limit the absolute accuracy of the voltage amplitudes. Nevertheless, this spectrum demonstrates the relative voltage accuracy, stability, and low distortion of the Josephson synthesizer.

Figure 5 shows the power spectrum of two rf tones with harmonic distortion products that are at least 80 dB below the synthesized tones. The code length is 300 000 bits corresponding to a 10 kHz minimum frequency. Imperfect synthesis of the digital code would appear as distortions at harmonics of 10 kHz, which is one tenth of the spacing between the tones. The array output is buffered by a 100 MHz differential preamp with unity gain to reduce the common-mode signal to the spectrum analyzer. The output voltage leads from the array to the top of the cryoprobe are designed for audio frequencies may result in significant rf coupling between bias lines. Nevertheless, even with this poor rf design, -80 dBc is a significant improvement over the -53 dBc harmonic distortion from the semiconductor digital code generator.

In conclusion, we demonstrated a Josephson arbitrary waveform synthesizer with operating margins. We have synthesized audio-frequency waveforms of single and multiple tones with harmonic distortion on the order of our measurement capabilities. By improving common-mode and rf circuit design, we hope to demonstrate improved performance at higher frequencies. These improvements should enable this system to be used as an ac voltage standard and for characterizing the frequency response of high-performance electronic instrumentation.

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[1] S. P. Benz, C. A. Hamilton, C. J. Burroughs, and T. E. Harvey, IEEE Trans. Instrum. Meas. **48**, 266 (1999).
[2] S. P. Benz and C. A. Hamilton, Appl. Phys. Lett. **68**, 3171 (1996).
[3] S. P. Benz, C. A. Hamilton, C. J. Burroughs, T. E. Harvey, L. A. Christian, and J. X. Przybysz, IEEE Trans. Appl. Supercond. **8**, 42 (1998).
[4] J. C. Candy, in *Delta–Sigma Data Converters: Theory, Design, and Simulation*, edited by S. R. Norsworthy, R. Schreier, and G. C. Temes (IEEE, Piscataway, NJ, 1997), p. 5.
[5] S. P. Benz, C. A. Hamilton, C. J. Burroughs, and T. E. Harvey, IEEE Trans. Appl. Supercond. **9**, 3306 (1999).
[6] S. P. Benz, C. A. Hamilton, and C. J. Burroughs, in *Extended Abstracts of the 7th International Superconductive Electronics Conference* (ISEC'99, Berkeley, CA, 1999), p. 115.