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Time Scales in the JPL and CfA Ephemerides

E. M. Standish

Caltech / Jet Propulsion Laboratory
JPL 301-150; Pasadena, CA 91109; USA
email: ems@smyles.jpl.nasa.gov

Abstract. Over the past decades, the IAU has repeatedly attempted to correct its definition of the basic fundamental argument used in the ephemerides. Finally, they have defined a time system which is physically possible, according to the accepted standard theory of gravitation: TCB ("Barycentric Coordinate Time"). Ironically, this time scale is mathematically and physically equivalent to T_{eph}, the time scale that has been used by JPL and by MIT (the group later went to CfA) in their ephemeris creation processes since the 1960's. TCB differs from T_{eph} by only a constant offset and a constant rate. As such, TCB provides an equivalent alternative to T_{eph}, but it does *not* allow increased accuracy as others have implied.

Key words: time – ephemeris time – ET – TDB – TCB

1. Introduction

Throughout the time since the 1960's, the basic ephemeris creation processes, at JPL and MIT/CfA, have remained similar to each other and have remained basically unchanged: the basic equations of motion are unaltered, and the implied definitions of the relevant parameters remain the same. (See, e.g., Ash et al. 1967; Devine 1967; Standish et al. 1992.) In particular, the independent variable of the ephemerides has remained unchanged. At MIT/CfA, this time has been properly referred to as "CT" (Coordinate Time); at JPL it has been (somewhat erroneously) referred to at first as "ET" (Ephemeris Time) and then later as "TDB" (Barycentric Dynamical Time). Nevertheless, the usage of the basic time-scale in both ephemeris programs has been essentially the same; the independent variable has been treated as a coordinate time.

This paper shows that the independent variable in the JPL and CfA ephemerides (hereafter referred to in this paper as "T_{eph}") is physically and mathematically equivalent to the IAU 1994 definition of TCB (Barycentric Coordinate Time), differing by only an offset and a constant rate.

The JPL and MIT/CfA ephemeris improvement process, used since the 1960's, is briefly discussed in Section 2. In Section 3 it is shown, incidentally, why T_{eph} is not equal to ET or to TDB. Section 4 describes the time transformation between two different times in the solar system, and the computation of the transformation is described in Section 5. The usage of TCB and of T_{eph} is described in Section 6; the scaling of the ephemerides in order to fit the chosen coordinate time is described in Section 7; the conclusions are given in Section 8.

2. The History of Modern Ephemerides

In the 1960's, the program of ephemeris development began at JPL, because existing ephemerides were not accurate enough for spacecraft navigation. A group at MIT, now at the Harvard-Smithsonian Center for Astrophysics (CfA), had also initiated such an ephemeris program for support of solar system observations and resulting scientific analyses. By the early 1970's, the ephemerides from JPL and MIT had become the world's standards; since 1984, JPL's DE200 has been the basis of the published ephemerides from all of the major almanac offices. The ephemerides have also served for spacecraft navigation, mission planning, reduction and analysis of the most precise of astronomical observations, and for the testing of the various proposed alternative theories of the laws of gravitation.

The basic ephemeris creation processes at JPL and at MIT/CfA have not been altered since the 1960's. Since that time, both programs have been characterized by the following features:

– the equations of motion represent the currently accepted laws of gravitation, including relativistic terms expressed through order $1/c^2$ (see, e.g., Newhall et al. 1983),

Send offprint requests to: E. M. Standish

- the equations include all forces known to produce observable effects in the motions,
- the ephemerides are produced by a numerical integration of the equations of the motion governing the major bodies of the solar system (The choice of using numerical integration was inevitable, for analytical theories cannot meet the accuracies demanded by modern spacecraft navigation or by modern-day precision solar system studies.), and
- the accuracy of the actual integration program has been validated to an order of magnitude smaller than that of any observable effect (a necessity for proper ephemeris adjustment).

With these features, the ephemerides approximate a system which is physically possible somewhere in the universe, according to the accepted standard theory of gravitation. Inaccuracies due to integration error, truncation of the relativistic part of the equations of motion, and intentional omission of small perturbing forces have been kept to a level below that of observational significance. How closely the ephemerides resemble our own solar system depends more upon the accuracy of the initial conditions of the integration. In turn, the accuracy of the initial conditions depends upon how well they are adjusted by the least squares process of fitting the ephemerides to precise astrometrical observations of the sun, moon and planets.

3. T_{eph}, the Time Used in Ephemeris Creation

The independent variable of the equations of motion, used by JPL and by CfA, is referred to in this paper as "T_{eph}". In the past, T_{eph} has been referred to at JPL, first, as "ET", and then, as "TDB". However, the use of either of these terms for "T_{eph}" is incorrect. Even though the *intention* was to make ET and TDB the independent variable in the equations of motion (i.e., a smooth-flowing, coordinate time), the unfortunate definitions of ET and TDB confused this concept with their practical realizations:

- ET : As initially defined, ET was the independent variable of the *existing planetary and lunar theories* (see Expl. Suppl. 1961, pp. 66-71); this is a fundamentally different definition from the independent variable in the *fundamental equations of motion*. ET, then, is subject to the inadequacies of those (analytical) theories.
- TDB : In 1976, the basic time scale was chosen to be of terrestrial origin (TAI or TDT), and TDB was then defined to differ from TDT by *only periodic terms* (see Guinot and Seidelmann 1988). Such a relation yields a time scale which is physically impossible. The intention was to remove a secular drift from the difference, TDB-TDT; however, the remaining difference cannot be characterized by merely periodic terms.
- T_{eph} : It is this quantity, as used by JPL and MIT/CfA in their ephemeris programs, that has always been what ET and TDB were intended to be.

Therefore, $T_{eph} \neq$ ET and $T_{eph} \neq$ TDB.

T_{eph} is *approximately* equal to them in value, but not exactly. On the other hand, as will be shown, T_{eph} is mathematically and physically equivalent to the newly-defined TCB (see Seidelmann and Fukushima 1992), differing from it by only an offset and a constant rate.

4. Time Transformations

Let T be a coordinate time; i.e., the independent variable of the equations of motion governing the bodies of the solar system. Arbitrary clocks in the solar system will run at rates different from T, including, of course, clocks on or near the surface of the earth. The difference in the rate of T_i, the time kept by an arbitrary clock, and the rate of T is given by the expression,

$$\frac{dT_i}{dT} \;=\; 1 \;-\; \frac{1}{c^2}(U_i + \frac{v_i^2}{2}) \qquad [1]$$

where U_i is the gravitational potential at clock "i" due to the masses of the bodies in the solar system (including the earth), and v_i is the clock's solar-system barycentric velocity.

Equation 1 assumes that T_i and T are measured in the same inherent units (e.g., SI seconds). However, one may also consider T_i and T to be measured in different units; this could be expressed by multiplying the right side of Equation 1 by a constant factor, K. Integration of Equation 1, with K included, yields

$$T_i - T = \;(K-1)(T-T_0) \;-\; K\int_{T_0}^{T} \frac{1}{c^2}(U_i + \frac{v_i^2}{2})\; dt \quad [2]$$

where it is assumed that the two times are synchronized at T_0 so that $T_i = T$ at T_0.

The difference between T representing TCB and T representing T_{eph} is entirely determined by merely the value of K. Mathematically and physically, any choice for the constant, K, is valid.

If it is assumed for the moment that T_i represents "Terrestrial Time" (TT, or, formerly, TDT), then

- for $K = 1$, T in Equation 2 becomes TCB. For a clock on or near the geoid, the difference, TT-TCB, contains a large linear drift (0.5 seconds/year) as well as a relatively small, quasi-periodic signature — mainly an annual term with amplitude of 1.6 milliseconds of time.
- On the other hand, as shown below, one can allow K to be set automatically by having $T \approx$ TT. In this case, $K-1$ becomes some sort of mean value of the integral, and T in Equation 2 becomes T_{eph}, the independent variable used by JPL and CfA for their ephemerides throughout the past many years.

The second choice has been implicitly adopted by much of the astronomical community: the difference, $T_{eph}-$ TT, is often ignored, since the two time scales differ by no more than 2 milliseconds of time. Such an approximation is not valid, however, when considering TCB - TT: the large linear drift amounts to 0.5 seconds/year.

5. Computing $T-$TT

The integral of Equation 2 can be evaluated numerically, using the trapezoidal rule or something similar of higher order, with the integrand computed straight from the planetary and lunar ephemerides. However, since the integrand is observer–dependent, it would be necessary to evaluate the equation for each different clock. Clearly, this is undesirable; however, it is necessary for clocks far from the geoid: GPS satellites, interplanetary spacecraft, rotations of planets, etc.

For clocks on or near the geoid, where T_i represents TT, there is an alternative to integrating Equation 2 for each different clock. One may separate the integrand into an observer–dependent part and an observer–independent part: an approximation whose error is below 1 nanosecond for observers on or near the earth's surface. Then, the observer–dependent part can be integrated analytically (Thomas 1975) so that Equation 2 becomes

$$T - \mathrm{TT} = K\ [\ \frac{1}{c^2}(\mathbf{r_i}-\mathbf{r_E})\cdot\mathbf{v_E} \quad + \quad L_G(T-T_0) + \frac{1}{c^2}\int_{T_0}^{T}(U_E+\frac{v_E^2}{2})\ dt\ ] \quad + \quad (1-K)\ (T-T_0) \qquad [3]$$

where the subscript i again denotes the barycentric observer, the subscript E denotes the geocenter, and the potential U_E includes the potential from all the bodies except the earth.

- The first term is small and quasi-periodic, but valid only for observers reasonably near the surface of the earth; i.e., near the geoid. Since its amplitude is less than about 2 microseconds of time, the error introduced by the observer's not being exactly on the geoid is negligible.
- The second term provides the linear difference between a clock on the geoid and a clock at the geocenter: $L_G = W_0/c^2$, where W_0 is the potential of the geoid. In practice, since the observing clocks are not located exactly on the geoid, the rate of each clock is adjusted so that, on the average, it maintains the TAI or TT time-scale.
- The third term is *in*-dependent of the observer; it *may* be computed once over the time-span of a given ephemeris and then stored for subsequent interpolation. However, this is not necessary, since analytical expressions for $T-T_E$ are now available, accurate to the nanosecond level (Fukushima 1995).
- The fourth term allows the rate to be adjusted automatically in order to have $T \approx$ TT, as described below.

At JPL, the third term of Equation 3 has been approximated using, in turn, the expressions from Moyer (1971), Moyer (1981), Hirayama et al. (1987), and presently from Fairhead and Bretagnon (1990), the latter two as improved by Fukushima (1995). These analytical formulae express the integral of only the *difference* between $U_E+v_E^2$ and its mean value, L_C. As such, the analytical formulae express the last two terms of Equation 3 with $1-K$ being equal to $-L_C$ and T being T_{eph}.

6. The Usage of TCB or T_{eph}

When evaluating Equation 3 with T_{eph}, one may either
- ignore the second and fourth terms of Equation 3 and use one of the analytical expressions for the quasi-periodic part of the third term, or
- evaluate all four terms, using the numerical quadrature of the integral and setting $1-K = -(L_G+L_C)$ in order to remove all secular drift.

On the other hand, if operating in TCB, one may either
- use the numerical quadrature of the integral and set $K = 1$, or
- use one of the analytical expressions for the quasi-periodic part of the third term and set $1-K = L_C$ in order to complete the full integral.

Determination of the rate (L_C) can be done by fitting the analytical formulae to a numerical quadrature of the integral in Equation 3, as was done by Fukushima (1995).

7. Automatic Ephemeris Scaling

In the ephemeris creation process, the system automatically adjusts itself to fit the choice of units, including the units of T_{eph} as determined by the choice of K. This is similar to physics, but not identical.
- In physics, one chooses the units of distance, mass, and time; the numerical value of G, in those units, is then determined by observation; its numerical value is automatically adjusted in order to give a consistent system: one which agrees with the laws of motion and with the observations of measurement. Further, now that the meter is defined in terms of the speed of light ($c \doteq 299792458 m/sec$), the unit of distance is effectively expressed in time units.
- In astronomy, one adopts the gaussian gravitational constant, the mass of the sun, and the unit of time determined by the choice of K, (or equivalently, by the computation of $T-$TT in Equation 3). The unit of length, the au, is related to the units of time by the relation, $au[sec] = S[\frac{km}{au}]/c[\frac{km}{sec}]$, where S is the scale factor. The adjustment of the unit of length by use of the parameter S is analogous to the adjustment of G in physics.

In the ephemeris programs at JPL and CfA, Equation 3 is computed such that T-TT remains small, and consistency is maintained via the scale factor, S; in this manner, the value of K is automatically determined such that there is no drift between T and TT.

8. Conclusions

JPL and CfA (formerly MIT) ephemeris creation programs have been producing the world's standards for three decades. Both of these programs have used for their independent variable, a coordinate time which is physically and mathematically equivalent to the IAU's recently introduced TCB, differing only by a constant offset and a constant rate. Operating in either of these time systems produces identical results, except for a scaling factor which may be used to easily convert from one system to the other.

The IAU introduction of TCB does *not* allow greater accuracy, as has been implied by Seidelmann and Fukushima (1992) and by Soffel and Brumberg (1991). It presents an equivalent alternative to the system that has been used for decades at CfA and at JPL.

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