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Model of CSR induced bursts in slicing experiments

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Abstract

We suggest a model describing the CSR bursts observed in recent experiments at the Advanced Light Source at the LBL. The model is based on the linear theory of the CSR instability in electron rings. We describe how an initial perturbation of the beam generated by the laser pulse evolves in time when the beam is unstable due to the CSR wakefield.

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1 Introduction

In recent experiments at the Advanced Light Source at the LBL it was observed that beam slicing can induce bursts of coherent synchrotron radiation (CSR) which are correlated with the time of the slicing [1]. The beam current in these experiments exceeded the threshold current for the onset of the CSR instability which was determined in the previous experiments without slicing. Above the threshold, a pulse of a burst CSR followed the moment of slicing with a delay of about 25-30 μs. Such correlated with slicing bursts were observed in a lattice with a relatively large momentum compaction factor of $\alpha = 2.7 \cdot 10^{-3}$. The total power in bursts in the case of a large α grew exponentially with the bunch current. Similar results were obtained in the slicing experiments at BESSY [2]. The parameters of the slicing experiment at the ALS are summarized in Table 1.

Table 1: **Parameters of slicing experiment at the ALS.**

energy, GeV	1.5
revolution frequency, MHz	1.52
beam current, mA	2–20
momentum compaction α,	$(1.37/2.7) \times 10^{-3}$
relative energy spread δ_0,	1.0×10^{-3}
nominal bunch length σ_z, mm	4.2/5.9

In this paper we propose a model that give a qualitative explanation of some characteristic of the observed phenomenon. In this model we first calculate the energy and density perturbation induced by the interaction of the beam with the laser pulse in the undulator. We then track the evolution of this perturbation taking into account the CSR wake. Since the initial length of the density perturbation is much smaller than the bunch length (the duration of the laser pulse is of the order of 200 fs), it can be considered as a localized perturbation. During the evolution of this initial perturbation its amplitude grows with time. The perturbation is also moving with a group velocity and spread due to dispersion effects. Starting at the center of the bunch with the maximal peak current, after some time the perturbation moves to the slope of the distribution function of the beam, where the growth rate slows down.

In this paper we assume that the size of the slice is much smaller than the bunch length through the evolution of the slice.

The paper is organized as follows. In Section 2 we derive the initial perturbation of the beam density generated by the laser. In Section 3 we review some the theory of the CSR instability with the emphasis on the group velocity of the unstable perturbation. In Section 4 we compute the evolution of an initial unstable perturbation in the beam in a coasting beam, and in Section 5 we discuss modification of our result for a Gaussian bunch. The results of the paper are summarized in Section 6. In Appendix we show how to derive the mode amplitudes for a given initial profile of a perturbation.

2 Initial evolution of a slice

In this section, we consider evolution of a density perturbation generated by interaction of a laser pulse with a beam at an initial stage, when the wake effect can be neglected.

Interaction of an electron bunch with a short laser pulse in an undulator changes particle's energy by δ_{mod}. This energy change depends on the amplitude and phase of the laser field at the location of the particle and can be written as $\delta_{\mathrm{mod}}(z) = A(z)\sigma_\delta \cos k_L z$, where σ_δ is the rms energy spread in the beam, k_L is the wavenumber of the laser light, and $A(z)$ is the dimensionless amplitude of the modulation. The latter is determined by the laser pulse profile. For a Gaussian profile we assume that

$$A(z) = A_0 e^{-z^2/2\sigma_L^2} , \tag{1}$$

where σ_L is the rms laser pulse length, and A_0 is the modulation amplitude.

Assuming that the bunch length is much longer than the laser pulse, we will neglect below the variation of the beam density over the length of the slice. The initial energy distribution in the beam is characterized by a Gaussian distribution with an rms energy spread σ_δ, and the beam distribution function before slicing is given by

$$f_0(\delta) = \frac{1}{\sqrt{2\pi}} e^{-\delta^2/2\sigma_\delta^2} . \tag{2}$$

The distribution function after the interaction with the laser is

$$f(z,\delta) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma_\delta^2}\left(\delta + A(z)\sigma_\delta \cos k_L z\right)^2\right] . \tag{3}$$

Since the laser wavelength $\lambda_L = 2\pi/k_L$ is very small, we will average this distribution function over the laser wavelength

$$\bar{f}(z,\delta) = \frac{1}{\lambda_L}\int_{z-\lambda_L/2}^{z+\lambda_L/2} f(z,\delta)dz$$
$$= \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma_\delta^2}\delta^2 - \frac{1}{4}A(z)^2\right] R\left[A(z)\frac{\delta}{\sigma_\delta}, -\frac{1}{4}A(z)^2\right], \qquad (4)$$

where the function R is defined by the following formula

$$R(x,y) = I_0(x)I_0(y) + 2\sum_{n=1}^{\infty} I_{2n}(x)I_n(y)\,, \qquad (5)$$

with I_n the modified Bessel function of the nth order.

The last step in the calculation of the evolution of the density profile is to take into account the slippage due to the momentum compaction factor α when the beam travels down the ring after the interaction with the laser. After time t the slippage Δz is equal to $-\alpha ct\delta$. The (averaged) beam distribution function $f(t,z,\delta)$ at time t is related to the initial function $\bar{f}$ through $f(t,z,\delta) = \bar{f}(z+t\alpha c\delta,\delta)$. The linear density (or beam current I) distribution is

$$I(z,t) = I_0\int_{-\infty}^{\infty} d\delta \bar{f}(z+t\alpha c\delta,\delta)\,, \qquad (6)$$

where I_0 is the value of the unperturbed current at the location of the slice. Using Eqs. (4) and (5) one can integrate Eq. (6) numerically. The result of such integration is shown in Fig. 1 for $A_0 = 6$ (that is when the maximal energy modulation is 6 times larger than the initial rms energy spread of the beam). As one can see from this plot, the an initially localized perturbation widens with time and its amplitude goes down. Eventually it smears out and disappears.

In the next two sections we will consider the dynamics of the initial perturbation taking into account the CSR wake.

3 Review of the theory of CSR instability

Before proceeding to the problem of slice dynamics with the CSR wake, we present here the main elements of the theory of CSR instability [3] necessary for our subsequent calculation. This theory is developed for a coasting beam model which is valid for perturbation with a characteristic length much smaller than the bunch length σ_z. In the model, the equilibrium beam current I_0 does not depend on z and is equal to the current of the bunch at the location of the perturbation. The theory also ignores the shielding effect of conducting walls and assumes a vacuum value for the CSR impedance Z.

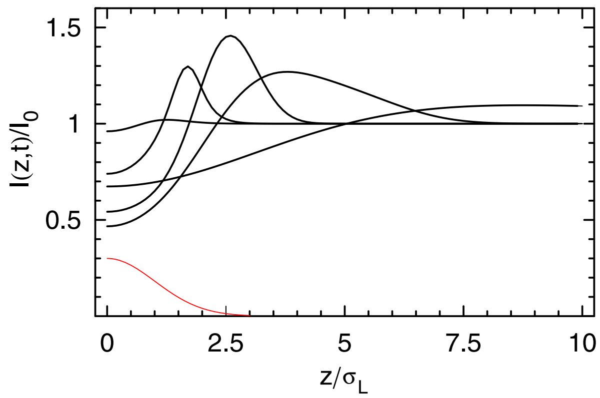

Figure 1: Density distributions at times $ct\alpha/\sigma_L = 0.05,\ 0.2,\ 0.5,\ 1,\ 3$ (broader distributions correspond to later times) for $A_0 = 6$. The red curve shows the laser profile Eq. (1). The plot shows only positive values of z—the curves are symmetric about the value $z = 0$.

In the linear approximation, the stability is considered for perturbations in the form $a_k\, e^{i(kz-\omega(k)t)}$, where a_k is the amplitude of the perturbation and $\omega(k)$ is the frequency which depends on the wavenumber k. This dependence is found from the dispersion equation. The growth rate of the instability $\Gamma(k)$ is given by the imaginary part of the frequency $\Gamma(k) = \mathrm{Im}\,(\omega(k))$. The important parameter in the theory [3] is

$$\Lambda = \frac{n_b r_e}{\alpha\gamma\sigma_\delta^2}\,, \tag{7}$$

where γ is the relativistic factor, r_e is the classical electron radius, n_b is the number of particles in the beam per unit length (equal to I_0/ec), and σ_δ is the relative energy spread of the unperturbed bunch. In a ring with a constant bending radius R the mode with the wave number k is unstable if

$$\frac{\Lambda}{R^{2/3}} > 0.63k^{2/3}\,. \tag{8}$$

The ALS ring has magnets with various bending radii. In this case, the parameter $\Lambda/R^{2/3}$ has to be properly averaged over the ring (see [4]). Note that for a given RF voltage in the ring, the linear density n_b scales as $n_b \propto \sigma_z^{-1} \propto \alpha^{-1/2}$. With this scaling, using Eqs. (7) and (8), we find that the threshold current for the instability $I_{\rm th}$ can be written as

$$I_{\rm th} = Dk^{2/3}\alpha^{3/2}\,, \tag{9}$$

where D is a constant that depends on the bending radii of the magnets in the ring. In the ALS experiments it was found that $D = 1.22\times10^4$ mA·cm$^{2/3}$

at $E = 1.5$ GeV [4]. An example of the growth rate $\Gamma(k)$ calculated for the ALS ring using this value of D is shown in Fig. 2. The maximum of this function is reached for $k = k_{\max} = 8.7$ cm^{-1} corresponding to the wavelength 7.2 mm, and is equal to 0.45 μs^{-1}.

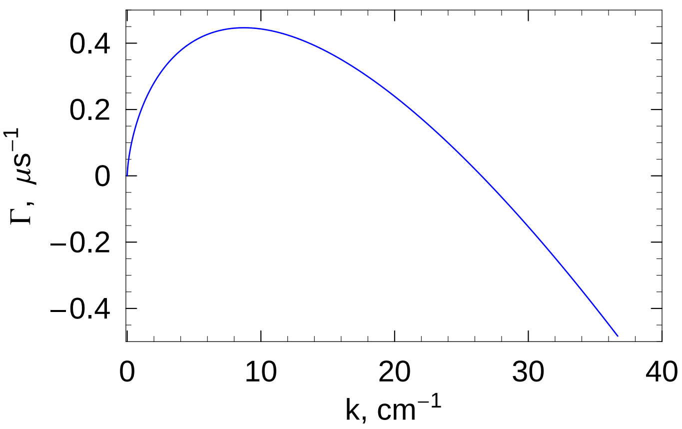

Figure 2: The growth rate of the CSR instability versus the wave number k for the ALS ring with $\alpha = 2.7 \times 10^{-3}$ and the average single bunch current of 15 mA.

Another important characteristic of the dispersion function $\omega(k)$ is the group velocity of the perturbation

$$v_g(k) = \frac{d\mathrm{Re}\,\omega(k)}{dk}\,. \tag{10}$$

This group velocity is calculated for the ALS (for the same parameters as in Fig. 2) and is shown in Fig. 3. The value of the group velocity at the maximum of the growth rate, which we denote by V_g, is equal to 0.67 mm/μs. Due to this velocity an initial perturbation will propagate along the z-axis toward the head of the bunch. It will also spread out due to dispersion effects.

4 Slice evolution in unstable beam

To calculate the time evolution of a localized initial perturbation (slice) induced by the interaction with the laser beam we will use the model of the coasting beam described in the previous section. This approach is valid while

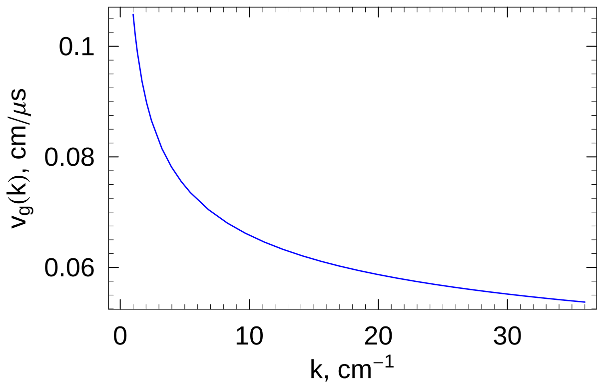

Figure 3: Group velocity as a function of the wavenumber k.

the slice remains localized in the vicinity of its initial position and its width is much shorter than the bunch length σ_z. We will also use the linear theory which assumes that the density perturbation δn is much smaller than the equilibrium beam density n_0. Numerically, for the parameters of the ALS experiment, those approximations turn out to be not very good, however, this simplified theory gives an insight into the mechanism of the CSR bursts induced by the laser slicing and can be used for qualitative analysis of the phenomenon.

To find the time evolution of an initial perturbation, in the linear theory of a coasting beam instability, we need to integrate $a_k\, e^{i(kz-\omega(k)t)}$ over the spectrum of wavenumbers all the modes (see Section 3):

$$\delta n(z,t) = \mathrm{Re} \int_0^\infty a(k) e^{i[kz-\omega(k)t]} dk\,. \tag{11}$$

The amplitudes a_k are determined by the initial perturbation of the distribution function and can be calculated following the standard technique described in Appendix 1. For our purposes, the exact expression for this function is not important.

Note that asymptotically, for large values of t, the dominant contribution to the integral (11) comes from the harmonics which have the fastest growth rate Γ_{max}. In this limit, we can expand the function $\omega(k)$ about the value of k_{max} corresponding to Γ_{max},

$$\omega(k) \approx \omega(k_{\mathrm{max}}) + \omega'(k_{\mathrm{max}})(k-k_{\mathrm{max}}) + \frac{1}{2}\omega''(k_{\mathrm{max}})(k-k_{\mathrm{max}})^2\,, \tag{12}$$

and replace $a(k)$ by its value $a(k_{\mathrm{max}})$ at the point of the maximal increment. The prime in this equation denotes the derivative with respect to k. Then the integral (11) can be calculated analytically

$$\delta n(z,t) \propto \frac{1}{\sqrt{t}} e^{i(z-tV_g)^2/2t\omega''(k_{\max})} e^{ik_{\max}z - i\omega(k_{\max})t} . \tag{13}$$

In this equation, we took into account that $\Gamma'(k_{\max}) = 0$ and used notation $V_g = \operatorname{Re}\omega'(k_{\max})$. Note that $\omega''(k_{\max})$ has both real and (negative) imaginary parts.

The plot of the function given by Eq. (13) for the ALS parameters is shown in Fig. 4. Each line gives the profile of the perturbation at a given time. The line are drawn for the first 5 μs with the time step of 1 μs. The result shows that an initial perturbation exponentially grows with time, becomes wider due to the dispersion effects and is moving away from the center of the bunch. The dashed line in the plot indicates a Gaussian beam

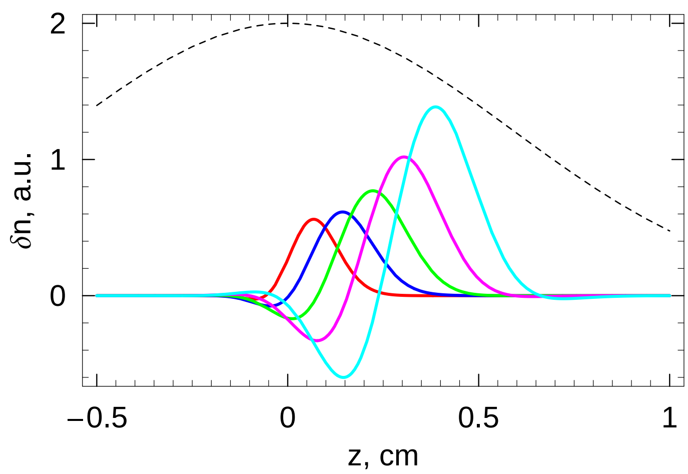

Figure 4: The time evolution of the initial perturbation.

profile with the rms length of 5.9 mm. We assumed that the initial location of the slice corresponded to the center of the beam $z = 0$.

5 Slice evolution in a Gaussian bunch

As we emphasized above, the analysis in the previous section was based on the coasting beam approximation and is valid in the approximation that the slice does not move far from its original position.

We can, however, draw some qualitative conclusions about the slice evolution at later times. As we see from Fig. 4 the slice is moving with the group velocity V_g (see Eq. (13)). The amplitude of the slice grows exponentially with the growth rate that is determined by the local value of the beam current I. As the slice moves away from the center, the value of the current at the location of the slice decreases and the growth rate goes down. Eventually, the slice arrives at the region where the imaginary part of the frequency corresponding to the dominant wavenumber in the slice becomes negative and it starts to decay. The time scale involved into this process can be estimated as the time needed for traversing of the bunch length with the group velocity V_g, and for the ALS experiment it is of the order of

$$t \sim \frac{\sigma_z}{V_g} \sim 10\ \mu\text{sec}\,. \tag{14}$$

In our consideration above we neglected nonlinear effects in the slice dynamics. They become important when the density perturbation is comparable to the beam density. Due to the large initial density perturbation (see Fig. 2) they may be of importance in the ALS experiments.

6 Conclusion

We developed a simple model of evolution of the initial perturbation in the slicing experiments. In the model, the beam dynamics is considered in the linear approximation. In contrast to the case when the CSR wake is neglected, the model predict that, under certain conditions, the initial perturbation does not decay, but, on contrary, gets amplified. It is moving away from the center of the bunch, and spreads out. This should be reflected in the spectrum of coherent synchrotron radiation observed in the experiment. Although our model gives only a qualitative explanation of the phenomenon it might be used as a guide for a more detailed studies based on numerical simulation of the effect.

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APPENDIX

Following the derivation from Ref. [3] we consider the beam distribution function $\rho(\delta, z, s)$ as a sum of the equilibrium distribution function ρ_0 and a perturbation ρ_1

$$\rho = \rho_0(\delta) + \rho_1(\delta, z, t)\,, \tag{A1}$$

with $\rho_1 \ll \rho_0$. Note that the equilibrium beam density (number of particles per unit length) n_b is equal to $n_b = \int \rho_0(\delta) d\delta$, and the density perturbation n_1 is given by $n_1(z,t) = \int \rho_1(\delta, z, t) d\delta$. Linearizing the Vlasov equation and assuming that ρ_1 has a z dependence $\propto e^{ikz}$ where k is the wavenumber of the perturbation, we find

$$\frac{\partial \rho_1}{\partial t} - ick\eta\delta\rho_1 = -\frac{r_0 c}{\gamma}\frac{d\rho_0}{d\delta} Z(k) \int_{-\infty}^{-\infty} d\delta\, \rho_1(\delta, t)\,, \tag{A2}$$

where $Z(k)$ is the CSR impedance equal to

$$Z(k) = iA\frac{k^{1/3}}{R^{2/3}}\,. \tag{A3}$$

with $A = 3^{-1/3}\Gamma\left(\frac{2}{3}\right)\left(\sqrt{3}i - 1\right) = 1.63i - 0.94$, where Γ is the gamma-function.

To solve the problem with initial condition $\rho_1(\delta, t)|_{t=0} = \rho_1^{(0)}(\delta)$ we define the Laplace image $\hat{\rho}_1(\delta, p)$,

$$\hat{\rho}_1(\delta, p) = \int_0^{\infty} dt \rho_1(\delta, t) e^{-pt}\,, \tag{A4}$$

and make the Laplace transform of Eq. (A2):

$$p\hat{\rho}_1 - ick\eta\delta\hat{\rho}_1 = -\frac{r_0 c}{\gamma}\frac{d\rho_0}{d\delta} Z(k) \int_{-\infty}^{-\infty} d\delta\, \hat{\rho}_1(\delta, p) + \rho_1^{(0)}(\delta)\,, \tag{A5}$$

The solution of this equation is

$$\hat{\rho}_1 = \frac{\rho_1^{(0)}(\delta)}{p - ick\eta\delta} - \frac{cr_0 Z(k)}{\gamma(p - ick\eta\delta)} \frac{d\rho_0}{d\delta} \hat{n}_1 \tag{A6}$$

where

$$\hat{n}_1 = \frac{1}{D(p,k)} \int \frac{d\delta\, \rho_1^{(0)}(\delta)}{p - ick\eta\delta}, \tag{A7}$$

and

$$D(p,k) = 1 + \frac{r_0 c Z(k)}{\gamma} \int \frac{d\delta\, (d\rho_0/d\delta)}{p - ick\eta\delta}\,. \tag{A8}$$

With this solution, one can find the time-dependent function $\rho_1(\delta, t)$ by making the inverse Laplace transformation

$$\rho_1(\delta, t) = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} dp \hat{\rho}_1(\delta, p) e^{pt}\,. \tag{A9}$$

From Eq. (A6) we see that there are two contributions to this solution. The first term, which does not depend on the impedance $Z(k)$ is the ballistic motion of particles is responsible for the distortion of the slice studied in Section 2. The second one is due to the collective interaction of the particles via the impedance $Z(k)$—it describes the effect of the CSR instability on the slice.

The standard analysis of the initial value problem (see, e.g., [5], p. 138) leads to the conclusion that after some transient period, the dominant contribution to the "collective" part of the perturbation comes from the poles of the function $D(p,k)$, and

$$\rho_1(\delta, t) = \tilde{\rho}_1(\delta) e^{-\omega(k)t}\,, \tag{A10}$$

where $\omega(k)$ satisfies the equation $D(-i\omega, k) = 0$. Eqs. (A6-A8) allow to explicitly relate the function $\tilde{\rho}_1(\delta)$ through the initial perturbation $\rho_1^{(0)}(\delta)$.