Wall/roof: 2x4 steel stud wall with R-11 insulation

Description

- ½-in. gypsum board
  - Thermal conductivity – 1.11 Btu-in/h-ft^2-F
  - Density – 50 lb/ft^3
  - Specific Heat – 0.26 Btu/lb-F
- 3.5-in. R-11 fiberglass batts
  - Thermal conductivity – 0.32 Btu-in/h-ft^2-F
  - Density – 5.3 lb/ft^3
  - Specific Heat – 0.23 Btu/lb-F
- 2x4 steel studs
  - Thermal conductivity – 314 Btu-in/h-ft^2-F
  - Density – 490 lb/ft^3
  - Specific Heat – 0.12 Btu/lb-F
- ½-in. plywood
  - Thermal conductivity – 0.8 Btu-in/h-ft^2-F
  - Density – 34 lb/ft^3
  - Specific Heat – 0.29 Btu/lb-F
- ½-in wood siding
  - Thermal conductivity – 0.5 Btu-in/h-ft^2-F
  - Density – 34 lb/ft^3
  - Specific Heat – 0.30 Btu/lb-F

Comment:
Total response of the thermal bridge is to be calculated by multiplying response factors or z-transfer function coefficients by the interior surface area.

COMPUTATION RESULTS

Three-dimensional model

Table 14.1
Resistance, transmittance and capacitance of the wall

	IP		*SI*	
R-value	3.56525	ft^2 °F h/Btu	0.62748	m^2 K/W
R^{-1}	0.28049	Btu/h ft^2 °F	1.59367	W/m^2 K
Capacitance	2.11033	Btu/ft^2 °F	43.12196	kJ/m^2 K

Table 14.2
Dimensionless 3D z-transfer function coefficients

n	b_n	c_n	d_n
0	0.32878	2.67848	1.00000
1	0.33525	-2.71613	-0.48144
2	-0.08624	0.65392	0.05880
3	-0.00082	-0.03925	-0.00035

$\Sigma\, c_n = 0.57701$, $E_1 = -0.00007$

Table 14.3
3D response factors calculated with the help of the finite difference computer code HEATING 7.2 [Btu/h ft^2 °F]

n	X_n	Y_n
0	7.5127402E-01	9.2218698E-02
1	-4.0014305E-01	1.3844753E-01
2	-5.3407216E-02	3.7042459E-02
3	-1.2928685E-02	9.4937698E-03
4	-3.2237717E-03	2.4408562E-03
5	-8.1208086E-04	6.2750822E-04
6	-2.0583342E-04	1.6127638E-04
7	-5.2384202E-05	4.1436561E-05
8	-1.3367560E-05	1.0642903E-05
9	-3.4170764E-06	2.7327212E-06
10	-8.7443869E-07	7.0144334E-07
11	-2.2392920E-07	1.8000275E-07

Table 14.4
3D response factors ratio, dimensionless 3D response factors and transfer functions of the first order

n	X_n/X_{n-1}	Y_n/Y_{n-1}	$R*X_n$	$R*Y_n$	$R*X'_n$	$R*Y'_n$
0			2.67848	0.32878	2.67848	0.32878
1	-0.53262	1.50130	-1.42661	0.49360	-2.11324	0.40932
2	0.13347	0.26756	-0.19041	0.13207	0.17530	0.00553
3	0.24208	0.25629	-0.04609	0.03385	0.00272	-0.00001
4	0.24935	0.25710	-0.01149	0.00870	0.00032	0.00003
5	0.25190	0.25709	-0.00290	0.00224	0.00005	0.00001
6	0.25346	0.25701	-0.00073	0.00057	0.00001	
7	0.25450	0.25693	-0.00019	0.00015		
8	0.25518	0.25685	-0.00005	0.00004		
9	0.25562	0.25676	-0.00001	0.00001		
10	0.25590	0.25668	-0.00000	0.00000		
11	0.25608	0.25662	-0.00000	0.00000		

$\alpha = 0.25635$, $\tau_1 = 0.73464$

Equivalent wall model: 3 layers plane wall

Table 14.4
Structure factors and time constants

Structure factors		*Time constants [h]*	
φ_{ii}	0.26747	$R \cdot C \cdot \varphi_{ii}$	0.267
φ_{ie}	0.12097	$R \cdot C \cdot \varphi_{ie}$	0.121
φ_{ee}	0.49058	$R \cdot C \cdot \varphi_{ee}$	0.491
		$R \cdot C$	7.524

Table 14.6a
Thermophysical properties of the equivalent wall - IP units

Layer n	R_n ft^2-°F-h/Btu	C_n Btu/ft^2-°F	l_n in	k_n Btu-in/h-ft^2-°F	ρ_n lb/ft^3	c_{pn} Btu/lb-°F
1	0.69116	0.41667	0.5	0.723	40	0.25
2	2.54035	0.90329	3.5	1.378	12.39	0.25
3	0.33374	0.79037	1	2.996	37.94	0.25

Table 14.6b
Thermophysical properties of the equivalent wall - SI units

Layer	R_n	C_n	l_n	k_n	ρ_n	c_{pn}
n	m^2 K/W	kJ/m^2 K	m	W/m K	kg/m^3	kJ/kg K
1	0.12165	8.51408	0.013	0.104	640	1.048
2	0.44710	18.45750	0.089	0.199	198.21	1.048
3	0.05874	16.15035	0.025	0.432	607	1.048

Table 14.7
Dimensionless z-transfer function coefficients and first time constants for the equivalent wall

n	b_n	c_n	d_n	τ_n
0	0.25171	2.86965	1.00000	
1	0.57492	-2.11946	-0.13039	0.489
2	0.04311	0.11967	0.00015	0.148
3	0.00003	-0.00009		0.081

$\Sigma\, c_n = 0.86976$, $\alpha = .12920$

Table 14.8
Response factors for the equivalent wall [Btu/h ft^2 °F]

n	X_n	Y_n
0	8.048947E-01	7.059990E-02
1	-4.895279E-01	1.704613E-01
2	-3.038878E-02	3.430645E-02
3	-3.912231E-03	4.456582E-03
4	-5.054356E-04	5.758099E-04
5	-6.530120E-05	7.439348E-05
6	-8.436780E-06	9.611483E-06
7	-1.090014E-06	1.241784E-06
8	-1.408276E-07	1.604359E-07
9	-1.819463E-08	2.072798E-08
10	-2.350709E-09	2.678012E-09
11	-3.037066E-10	3.459935E-10

Frequency response for the three-dimensional model and equivalent wall; dimensionless amplitude and phase angle

Table 14.9a
3-D model

	Transmittance		*Admittance*	
period	amplitude	phase angle	amplitude	phase angle
48	0.99	-7°	1.06	14°
24	0.98	-14°	1.21	25°
12	0.91	-26°	1.65	35°
6	0.71	-48°	2.53	33°

Table 14.9b
Equivalent wall

	Transmittance		*Admittance*	
period	amplitude	phase angle	amplitude	phase angle
48	1.00	-7°	1.04	15°
24	0.99	-14°	1.16	27°
12	0.96	-27°	1.54	41°
6	0.87	-52°	2.42	50°