# Appendix C

# Details of Acceptance Measurements

Details of the acceptance measurements used in the calculations of the single event sensitivity are described. The numbers after corrections are printed in blue (2005.May.20).

## C.1 Monte Carlo Based Acceptance

The trigger acceptance ($A_{trigCT}$), the acceptance of the $\pi^+$ fiducial cuts ($A_{fidCT}$) and the $\pi^+$ stop efficiency ($A_{nidif}$) for the $K^+ \to \pi^+\gamma\gamma$ are measured with the Monte Carlo simulation. In addition, the acceptance of the fiducial cuts on the photon clusters ($A_{fidBV}$), except for the timing cut, and the photon cuts ($A_{PV-MC}$) are measured. The $K^+ \to \pi^+\gamma\gamma$ events are generate by assuming the ChPT $O(p^6)$ with $\hat{c} = 1.8$, ChPT $O(p^4)$ with $\hat{c} = 1.6$, and the phase space distribution for the $\pi^+$ with $P_{\pi^+}$ >200 MeV/c. Events are generated for both the $\pi^+$ nuclear interaction and decay-in-flight (NIDIF) turned on and off. The calculations of the trigger acceptance are summarized in Tables C.1, C.2 and C.3; the acceptance factor is obtained as

$$\begin{aligned} A_{trigCT} &= 0.03455 \pm 0.00008 \quad (\text{for ChPT } O(p^6)) \\ &= 0.00609 \pm 0.00003 \quad (\text{for ChPT } O(p^4)) \\ &= 0.03490 \pm 0.00010 \quad (\text{for Phase Space}). \end{aligned}$$

The "UFATE", "USTMED" and "USTOP_HEX" are prepared in order to estimate the acceptance loss by the $\pi^+ \to \mu^+ \to e^+$ decay chain requirements due to an incorrect assignment of the stopping counter. The UFATE requires that a track does not decay-in-flight nor interact with the Range Stack materials before it comes to rest in the simulation. The USTMED requires that a charged track does not come to rest in the dead materials in the Range Stack in the simulation. The USTOP_HEX requires that the stopping counter in the offline analysis agrees with the true stopping position in the simulation.

The calculations of the acceptance of $\pi^+$ fiducial cuts are summarized in Tables C.4, C.5 and C.6; the acceptance factor is obtained as

$$\begin{aligned} A_{fidCT} &= 0.89802 \pm 0.00073 \quad (\text{for ChPT } O(p^6)) \\ &= 0.62606 \pm 0.00279 \quad (\text{for ChPT } O(p^4)) \\ &= 0.89701 \pm 0.00092 \quad (\text{for Phase Space}). \end{aligned}$$

The $\pi^+$ stop efficiency is obtained by taking the ratio of the factors with NIDIF turned on and off.

$$\begin{aligned} A_{nidif} &= \frac{A_{trigCT}(\text{NIDIFon})}{A_{trigCT}(\text{NIDIFoff})} \times \frac{A_{fidCT}(\text{NIDIFon})}{A_{fidCT}(\text{NIDIFoff})} \\ &= 0.49226 \pm 0.00215 \quad (\text{for ChPT } O(p^6)) \\ &= 0.52350 \pm 0.00647 \quad (\text{for ChPT } O(p^4)) \\ &= 0.49943 \pm 0.00274 \quad (\text{for Phase Space}). \end{aligned}$$

The calculation of the acceptance of fiducial cuts on the photon cluster are summarized in Tables C.7, C.8 and C.9; the acceptance factor is obtained as

$$\begin{aligned} A_{fidBV} &= 0.55209 \pm 0.00180 \quad (\text{for ChPT } O(p^6)) \\ &= 0.51235 \pm 0.00503 \quad (\text{for ChPT } O(p^4)) \\ &= 0.56083 \pm 0.00225 \quad (\text{for Phase Space}). \end{aligned}$$

The calculation of the acceptance of photon cuts are summarized in Tables C.10, C.11 and C.12; the acceptance factor is obtained as

$$\begin{aligned} A_{PV-MC} &= 0.28075 \pm 0.00220 \quad (\text{for ChPT } O(p^6)) \\ &= 0.22925 \pm 0.00591 \quad (\text{for ChPT } O(p^4)) \\ &= 0.26813 \pm 0.00268 \quad (\text{for Phase Space}). \end{aligned}$$

| | NIDIF on | | NIDIF off | |
|---|---|---|---|---|
| Cut | Events | Acceptance | Events | Acceptance |
| $K_B$ | 4999684 | – | 4999707 | – |
| $T \bullet 2$ | 2246678 | 0.44936 | 2332350 | 0.46650 |
| $6_{CT} + 7_{CT}$ | 1873487 | 0.83389 | 2298723 | 0.98558 |
| $16_{CT} + 17_{CT}$ | 112603 | 0.06010 | 184823 | 0.08040 |
| UFATE | 96775 | 0.85944 | 184823 | 1.00000 |
| USTMED | 96406 | 0.99619 | 184504 | 0.99827 |
| USTOP_HEX | 88512 | 0.91812 | 172730 | 0.93619 |
| $A_{trigCT}$ | 0.01770 ± 0.00006 | | 0.03455±0.00008 | |

Table C.1: Trigger acceptance for the $K^+ \to \pi^+\gamma\gamma$ (ChPT $O(p^6)$ with $\hat{c} = 1.8$).

| | NIDIF on | | NIDIF off | |
|---|---|---|---|---|
| Cut | Events | Acceptance | Events | Acceptance |
| $K_B$ | 4999673 | – | 4999719 | – |
| $T \bullet 2$ | 2242190 | 0.44847 | 2336778 | 0.46738 |
| $6_{CT} + 7_{CT}$ | 1880363 | 0.83863 | 2301050 | 0.98471 |
| $16_{CT} + 17_{CT}$ | 32794 | 0.01744 | 37075 | 0.01611 |
| UFATE | 22236 | 0.67805 | 37075 | 1.00000 |
| USTMED | 21867 | 0.98341 | 36839 | 0.99364 |
| USTOP_HEX | 17081 | 0.78113 | 30460 | 0.82684 |
| $A_{trigCT}$ | 0.00342 ± 0.0000261 | | 0.00609 ± 0.00003 | |

Table C.2: Trigger acceptance for the $K^+ \to \pi^+\gamma\gamma$ (ChPT $O(p^4)$ with $\hat{c} = 1.6$).

| | NIDIF on | | NIDIF off | |
|---|---|---|---|---|
| Cut | Events | Acceptance | Events | Acceptance |
| $K_B$ | 3124797 | – | 3124818 | – |
| $T \bullet 2$ | 1403625 | 0.44919 | 1456344 | 0.46606 |
| $6_{CT} + 7_{CT}$ | 1173809 | 0.83626 | 1437534 | 0.98708 |
| $16_{CT} + 17_{CT}$ | 72813 | 0.02330 | 117799 | 0.08195 |
| UFATE | 62817 | 0.86272 | 117799 | 1.00000 |
| USTMED | 62555 | 0.99583 | 117558 | 0.99795 |
| USTOP_HEX | 56667 | 0.90588 | 109048 | 0.92761 |
| $A_{trigCT}$ | 0.01813 ± 0.00008 | | 0.03490 ± 0.00010 | |

Table C.3: Trigger acceptance for the $K^+ \to \pi^+\gamma\gamma$ (Phase Space).

| | NIDIF on | | NIDIF off | |
|---|---|---|---|---|
| Cut | Events | Acceptance | Events | Acceptance |
| SETUP | 87994 | – | 171761 | – |
| LAY1617 | 86046 | 0.97786 | 169441 | 0.98649 |
| COS3D | 85739 | 0.99643 | 169110 | 0.99805 |
| ZFRF | 85739 | 1.00000 | 169110 | 1.00000 |
| ZUTOUT | 85717 | 0.99974 | 169080 | 0.99982 |
| PGGBOX | 75910 | 0.88559 | 154244 | 0.91226 |
| $A_{fidCT}$ | $0.86267 \pm 0.00116$ | | $0.89802 \pm 0.00073$ | |

Table C.4: $\pi^+$ fiducial acceptance for the $K^+ \to \pi^+\gamma\gamma$ (ChPT $O(p^6)$ with $\hat{c} = 1.8$).

| | NIDIF on | | NIDIF off | |
|---|---|---|---|---|
| Cut | Events | Acceptance | Events | Acceptance |
| SETUP | 16898 | – | 30176 | – |
| LAY1617 | 15481 | 0.91614 | 28315 | 0.93833 |
| COS3D | 15456 | 0.99839 | 28307 | 0.99972 |
| ZFRF | 15456 | 1.00000 | 28307 | 1.00000 |
| ZUTOUT | 15452 | 0.99974 | 28306 | 0.99997 |
| PGGBOX | 9876 | 0.63914 | 18892 | 0.66742 |
| $A_{fidCT}$ | $0.58445 \pm 0.00379$ | | $0.62606 \pm 0.00279$ | |

Table C.5: $\pi^+$ fiducial acceptance for the $K^+ \to \pi^+\gamma\gamma$ (ChPT $O(p^4)$ with $\hat{c} = 1.6$).

| | NIDIF on | | NIDIF off | |
|---|---|---|---|---|
| Cut | Events | Acceptance | Events | Acceptance |
| SETUP | 56349 | – | 108462 | 0.99463 |
| LAY1617 | 55111 | 0.97803 | 106930 | 0.98588 |
| COS3D | 54933 | 0.99677 | 106728 | 0.99811 |
| ZFRF | 54933 | 1.00000 | 106728 | 1.00000 |
| ZUTOUT | 54913 | 0.99964 | 106712 | 0.99985 |
| PGGBOX | 48578 | 0.88464 | 97291 | 0.91172 |
| $A_{fidCT}$ | $0.86209 \pm 0.00145$ | | $0.89701 \pm 0.00092$ | |

Table C.6: $\pi^+$ fiducial acceptance for the $K^+ \to \pi^+\gamma\gamma$ (Phase Space)

| Cut | Events | Acceptance |
|---|---|---|
| SETUP | 75910 | – |
| NREG | 58008 | 0.76417 |
| EG1 | 53044 | 0.91443 |
| COSOPXY | 51965 | 0.97966 |
| COSOPZ | 51616 | 0.99328 |
| DIPANG | 51592 | 0.99953 |
| EG2 | 41909 | 0.81231 |
| $A_{fidBV}$ | $0.55209 \pm 0.00180$ | |

Table C.7: $\gamma$ fiducial acceptance for the $K^+ \to \pi^+\gamma\gamma$ (ChPT $O(p^6)$ with $\hat{c} = 1.8$).

| Cut | Events | Acceptance |
|---|---|---|
| SETUP | 9876 | – |
| NREG | 7296 | 0.73876 |
| EG1 | 6622 | 0.90762 |
| COSOPXY | 6338 | 0.95711 |
| COSOPZ | 6238 | 0.98422 |
| DIPANG | 6235 | 0.99952 |
| EG2 | 5060 | 0.81155 |
| $A_{fidBV}$ | $0.51235 \pm 0.00503$ | |

Table C.8: $\gamma$ fiducial acceptance for the $K^+ \to \pi^+\gamma\gamma$ (ChPT $O(p^4)$ with $\hat{c} = 1.6$).

| Cut | Events | Acceptance |
|---|---|---|
| SETUP | 48578 | – |
| NREG | 37705 | 0.77617 |
| EG1 | 34727 | 0.92102 |
| COSOPXY | 34028 | 0.97987 |
| COSOPZ | 33872 | 0.99542 |
| DIPANG | 33857 | 0.99956 |
| EG2 | 27244 | 0.80468 |
| $A_{fidBV}$ | $0.56083 \pm 0.00225$ | |

Table C.9: $\gamma$ fiducial acceptance for the $K^+ \to \pi^+\gamma\gamma$ (Phase Space).

| Cut | Events | Acceptance |
|---|---|---|
| SETUP | 41909 | – |
| HEX | 19334 | 0.46133 |
| RSPVCUT | 16761 | 0.86692 |
| BV·BVL | 14132 | 0.84483 |
| COSGG | 13886 | 0.98259 |
| TZMAX | 13865 | 0.99849 |
| EC | 13053 | 0.94144 |
| PVCUT | 12107 | 0.92753 |
| TGPVCUT | 11766 | 0.97183 |
| $A_{PV-MC}$ | $0.28075 \pm 0.00220$ | |

Table C.10: Photon-cut acceptance for the $K^+ \to \pi^+\gamma\gamma$ (ChPT $O(p^6)$ with $\hat{c} = 1.8$).

| Cut | Events | Acceptance |
|---|---|---|
| SETUP | 5060 | – |
| HEX | 2682 | 0.53004 |
| RSPVCUT | 2297 | 0.85645 |
| BV·BVL | 1813 | 0.78929 |
| COSGG | 1655 | 0.91285 |
| TZMAX | 1648 | 0.99577 |
| EC | 1399 | 0.84891 |
| PVCUT | 1244 | 0.88921 |
| TGPVCUT | 1160 | 0.93248 |
| $A_{PV-MC}$ | $0.22925 \pm 0.00591$ | |

Table C.11: Photon-cut acceptance for the $K^+ \to \pi^+\gamma\gamma$ (ChPT $O(p^4)$ with $\hat{c} = 1.6$).

| Cut | Events | Acceptance |
|---|---|---|
| SETUP | 27244 | – |
| HEX | 13551 | 0.49739 |
| RSPVCUT | 11582 | 0.85470 |
| BV·BVL | 9292 | 0.80228 |
| COSGG | 9121 | 0.98160 |
| TZMAX | 9109 | 0.99868 |
| EC | 8486 | 0.93161 |
| PVCUT | 7738 | 0.91186 |
| TGPVCUT | 7305 | 0.94404 |
| $A_{PV-MC}$ | $0.26813 \pm 0.00268$ | |

Table C.12: Photon-cut acceptance for the $K^+ \to \pi^+\gamma\gamma$ (Phase Space).

## C.2 $K_{\mu 2}$-Based Acceptance

A number of acceptance factors are measured with the $K_{\mu 2}$ monitor data. The acceptance factors of the basic reconstruction cuts ($A_{RD}$ and $A_{recon}$) are measured at first because these cuts are used as the setup cuts in the other acceptance measurements. The measurements of $A_{RD}$ and $A_{recon}$ are summarized in Tables C.13 and Table C.14, respectively; the acceptance factors are

$$\begin{aligned} A_{RD} &= 0.99995 \pm 0.00001 \\ A_{recon} &= 0.99561 \pm 0.00012. \end{aligned}$$

The acceptance factor of the beam and Target analysis ($A_{rest}$) is measured by selecting the events within the $\pm 2\sigma$ of the $K_{\mu 2}$ momentum peak. The measurement of $A_{rest}$ is summarized in Table C.15; the acceptance factor is obtained as

$$A_{rest} = 0.50697 \pm 0.00077.$$

The accidental loss due to the online and offline photon veto cuts ($A_{PV-K_{\mu 2}}$) is measured after requiring the offline stopping layer to be 18 or less to avoid muons that reach the Barrel photon detectors. The measurement of $A_{PV-K_{\mu 2}}$ is summarized in Table C.16; the acceptance factor is obtained as

$$A_{PV-K_{\mu 2}} = 0.50697 \pm 0.00077.$$

In addition, the accidental loss due to the online requirement of $18_{ct}$+$19_{ct}$ ($A_{\bar{\mu}}$) is measured with the $K_{\mu 2}$ monitor data. $A_{\bar{\mu}}$ is obtained as

$$A_{\bar{\mu}} = 0.99625 \pm 0.00065.$$

| Cut | Events | Acceptance |
|---|---|---|
| SETUP | 381644 | – |
| RDTRK | 381644 | 1.00000 |
| TRKTIM | 381625 | 0.99995 |
| $A_{RD}$ | 0.99995 ± 0.00001 | |

Table C.13: $K_{\mu 2}$-based acceptance factor $A_{RD}$.

| Cut | Events | Acceptance |
|---|---|---|
| SETUP | 291543 | – |
| UTC/RANGE | 291543 | 1.00000 |
| ITGQUAL | 290264 | 0.99561 |
| $A_{recon}$ | $0.99561 \pm 0.00012$ | |

Table C.14: $K_{\mu 2}$-based acceptance factor $A_{recon}$.

| Cut | Events | Acceptance |
|---|---|---|
| SETUP | 422384 | 0.00000 |
| TIC | 421583 | 0.99810 |
| TIMCON | 420153 | 0.99661 |
| DCBIT | 368523 | 0.87712 |
| DELC2 | 354726 | 0.96256 |
| TGTCON | 352068 | 0.99251 |
| DELCO | 318250 | 0.90394 |
| CKTRS | 313104 | 0.98383 |
| CKTAIL | 305714 | 0.97640 |
| B4DEDX | 300786 | 0.98388 |
| CPITRS | 299408 | 0.99542 |
| CPITAIL | 299202 | 0.99931 |
| TARGF | 286837 | 0.95867 |
| DTGTTP | 286779 | 0.99980 |
| RTDIF | 283525 | 0.98865 |
| PIGAP | 281302 | 0.99216 |
| TGB4 | 265541 | 0.94397 |
| KIC | 261507 | 0.98481 |
| TGGEO | 258124 | 0.98706 |
| EIC | 248578 | 0.99171 |
| B4EKZ | 240027 | 0.99487 |
| TGZFOOL | 240027 | 1.00000 |
| BWTRS | 231548 | 0.96467 |
| RVUPV | 227839 | 0.98398 |
| B4TRS | 216661 | 0.95094 |
| B4CCD | 214135 | 0.98834 |
| $A_{rest}$ | $0.50697 \pm 0.00077$ | |

Table C.15: $K_{\mu 2}$-based acceptance factor $A_{rest}$.

| Cut | Events | Acceptance |
|---|---|---|
| SETUP | 25831 | – |
| Online PV | 23615 | 0.98106 |
| STLAY | 23340 | 0.98835 |
| RSHEX | 22522 | 0.96495 |
| PVCUT | 21410 | 0.95063 |
| NREG | 19979 | 0.93316 |
| $A_{PV}^{K_{\mu 2}}$ | $0.77345 \pm 0.00260$ | |

Table C.16: $K_{\mu 2}$-based acceptance factor $A_{PV}^{K_{\mu 2}}$.

## C.3 $K_{\pi 2}$-Based Acceptance

The acceptance factor of the OPSVETO cut and the acceptance factors related to the track energy in the Target are measured with the $K_{\pi 2}$ monitor data ($A_{opt}$ and $A_{tgkin}$). In order to select the $K_{\pi 2}$ events, the reconstruction cuts (PASS1CUT), beam and kinematic cuts except for those whose acceptances are being measured here, Transient Digitizer cuts (TDCUT), and $\pm 2\sigma$ cuts on the $K_{\pi 2}$ momentum, range and kinetic energy around the peaks are imposed as the setup cuts. In addition, the photon clustering in the Barrel photon detectors is performed and the $\pi^0$ in the $K_{\pi 2}$ events is tagged in order to confirm that the two photons reach the Barrel region, because the acceptance measurements might be screwed up due to the photon shower in the Target. The measurement of $A_{opt}$ is summarized in Table C.17; the acceptance factor is obtained as

$$A_{opt} = 0.98317 \pm 0.00100.$$

The measurement of $A_{tgkin}$ is summarized in Table C.18; the acceptance factor is obtained as

$$A_{tgkin} = 0.89142 \pm 0.00246.$$

In addition, the acceptance factor of the timing cut on the photon clusters is measured with the $K_{\pi 2}$ monitor data ($A_{BVTIME}$). The acceptance for a single photon is measured to be $0.98023 \pm 0.00140$. In the analysis, however, events with two photon-clusters are accepted; the number is squared, conservatively, for the $A_{BVTIME}$ and obtained as

$$A_{BVTIME} = 0.96085 \pm 0.00194.$$

## C.4 $\pi_{scat}$-Based Acceptance

The acceptance factors of the Kinematic cuts ($A_{kin}$) and the Transient Digitizer Cuts ($A_{TD}$) are measured with the $\pi_{scat}$ monitor data. To measure $A_{kin}$, the reconstruction cuts, small energy deposit in the B4 counter, a prompt timing in the Target, Range Stack photon-veto cut and the Transient Digitizer cuts are imposed in the setup cuts. The measurement of $A_{kin}$ is summarized in Table C.19; the acceptance factor is obtained as

$$A_{kin} = 0.61280 \pm 0.00675.$$

To measure $A_{TD}$, the same setup cuts used in the $A_{kin}$ acceptance measurements, with $\pi^+\gamma\gamma$ box cuts, the cut on the range-momentum relation, z-position cuts in the Range Stack and Drift Chamber and the cut on the I-counter energy are imposed, but the Transient Digitizer cuts are unused. The measurement of $A_{TD}$ is summarized in Table C.20 ; the acceptance factor is obtained as

$$A_{TD} = 0.34872 \pm 0.00548.$$

## C.5 Categorization of the Acceptances

The acceptance factors obtained in the previous sections are categorized and summarized in Tables C.21, C.22 and C.23.

| Cut | Events | Acceptance |
|---|---|---|
| SETUP | 16397 | – |
| OPSVETO | 16121 | 0.98317 |
| $A_{ops}$ | $0.98317 \pm 0.00100$ | |

Table C.17: $K_{\pi 2}$-based acceptance factor $A_{ops}$.

| Cut | Events | Acceptance |
|---|---|---|
| SETUP | 15997 | – |
| TGDEDX | 15424 | 0.96418 |
| TGER | 15418 | 0.99961 |
| TGLIKE | 15107 | 0.97983 |
| EPITG | 15100 | 0.99954 |
| EPIMAXK | 14918 | 0.98795 |
| TGEDGE | 14850 | 0.99544 |
| PHIVTX | 14381 | 0.96842 |
| PHIVTX2 | 14290 | 0.99367 |
| DRP | 14260 | 0.99790 |
| $A_{tgkin}$ | $0.89142 \pm 0.00246$ | |

Table C.18: $K_{\pi 2}$-based acceptance factor $A_{tgkin}$.

| Cut | Events | Acceptance |
|---|---|---|
| SETUP | 5204 | 1.00000 |
| UTCQUAL | 4856 | 0.93313 |
| RNGMOM | 4807 | 0.98991 |
| RSDEDX | 4097 | 0.85230 |
| PRRF | 3189 | 0.88781 |
| $A_{kin}$ | $0.61280 \pm 0.00675$ | |

Table C.19: $\pi_{scat}$-based acceptance factor $A_{kin}$.

| Cut | Events | Acceptance |
|---|---|---|
| SETUP | 7562 | – |
| FITPI | 6195 | 0.81999 |
| RSHEX2 | 6065 | 0.97902 |
| Online PID | 4081 | 0.67288 |
| EV5 | 3425 | 0.83926 |
| ELVETO | 3180 | 0.92847 |
| TDFOOL | 3179 | 0.99969 |
| TDVAR | 2637 | 0.82951 |
| $A_{TD}$ | $0.34872 \pm 0.00548$ | |

Table C.20: $\pi_{scat}$ based acceptance factor $A_{TD}$.

| Definition | Symbol | Acceptances | Method |
|---|---|---|---|
| Trigger | $A_{trigCT}$ | $0.03455 \pm 0.00008$ | Monte Carlo |
| $A^{\pi^+\gamma\gamma}_{\text{Trigger}} = 0.03455 \pm 0.00008$ | | | |
| $\pi^+$ Reconstruction Efficiency | $A_{RD}$<br>$A_{Recon}$ | $0.99995 \pm 0.00001$<br>$0.99561 \pm 0.00012$ | $K_{\mu 2}$<br>$K_{\mu 2}$ |
| $A^{\pi^+\gamma\gamma}_{\text{Reconstruction}} = 0.99556 \pm 0.00012$ | | | |
| $\pi^+$ Fiducial Cuts | $A_{fidCT}$ | $0.89802 \pm 0.00073$ | Monte Carlo |
| $A^{\pi^+\gamma\gamma}_{\text{Fiducial}} = 0.89802 \pm 0.00073$ | | | |
| $\pi^+$ Stop Efficiency | $A_{nidif}$ | $0.49226 \pm 0.00215$ | Monte Carlo |
| $A^{\pi^+\gamma\gamma}_{\pi^+\text{StopEff.}} = 0.49226 \pm 0.00215$ | | | |
| $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ Decay Chain | $A_{TD}$ | $0.34872 \pm 0.00548$ | $\pi_{scat}$ |
| $A^{\pi^+\gamma\gamma}_{\pi^+\rightarrow\mu^+\rightarrow e^+} = 0.34872 \pm 0.00548$ | | | |
| $\gamma$ Reconstruction and Fiducial | $A_{fidBV}$<br>$A_{BVTIME}$ | $0.55209 \pm 0.00180$<br>$0.96085 \pm 0.00194$ | Monte Carlo<br>$K_{\pi 2}$ |
| $A^{\pi^+\gamma\gamma}_{\gamma\text{Recon.\&.Fid.}} = 0.53047 \pm 0.00204$ | | | |
| Extra Photon Veto Cuts | $A_{PV-MC}$<br>$A_{PV-K_{\mu 2}}$<br>$A_{\bar{\mu}}$ | $0.28075 \pm 0.00220$<br>$0.77345 \pm 0.00260$<br>$0.99625 \pm 0.00065$ | Monte Carlo<br>$K_{\mu 2}$<br>$K_{\mu 2}$ |
| $A^{\pi^+\gamma\gamma}_{\text{ExtraPV}} = 0.21633 \pm 0.00185$ | | | |
| Kinematic Cuts | $A_{ops}$<br>$A_{tgkin}$<br>$A_{kin}$ | $0.98317 \pm 0.00100$<br>$0.89142 \pm 0.00246$<br>$0.61280 \pm 0.00675$ | $K_{\pi 2}$<br>$K_{\pi 2}$<br>$\pi_{scat}$ |
| $A^{\pi^+\gamma\gamma}_{\text{Kinematics}} = 0.53706 \pm 0.00613$ | | | |
| Beam and Target | $A_{rest}$ | $0.50697 \pm 0.00077$ | $K_{\mu 2}$ |
| $A^{\pi^+\gamma\gamma}_{\text{Beam\&Target}} = 0.50697 \pm 0.00077$ | | | |
| $T \bullet 2$ Efficiency($\pi^+\gamma\gamma$) | $A_{T\bullet 2}(\pi^+\gamma\gamma)$ | $0.9357 \pm 0.0011$ | $K_B$ |
| $A^{\pi^+\gamma\gamma}_{T\bullet 2\text{Eff.}} = 0.93570 \pm 0.00110$ | | | |
| $A^{\pi^+\gamma\gamma}(\text{ChPT } O(p^6)) = 1.5502 \pm 0.0344 \times 10^{-4}$ | | | |

Table C.21: Categorization of acceptance factors for the $K^+ \rightarrow \pi^+\gamma\gamma$ (ChPT $O(p^6)$).

| Definition | Symbol | Acceptances | Method |
|---|---|---|---|
| Trigger | $A_{trigCT}$ | $0.00609 \pm 0.00003$ | Monte Carlo |
| $A^{\pi^+\gamma\gamma}_{\text{Trigger}} = 0.00609 \pm 0.00003$ | | | |
| $\pi^+$ Reconstruction Efficiency | $A_{RD}$<br>$A_{Recon}$ | $0.99995 \pm 0.00001$<br>$0.99561 \pm 0.00012$ | $K_{\mu 2}$<br>$K_{\mu 2}$ |
| $A^{\pi^+\gamma\gamma}_{\text{Reconstruction}} = 0.99556 \pm 0.00012$ | | | |
| $\pi^+$ Fiducial Cuts | $A_{fidCT}$ | $0.62606 \pm 0.00279$ | Monte Carlo |
| $A^{\pi^+\gamma\gamma}_{\text{Fiducial}} = 0.62606 \pm 0.00279$ | | | |
| $\pi^+$ Stop Efficiency | $A_{nidif}$ | $0.52350 \pm 0.00647$ | Monte Carlo |
| $A^{\pi^+\gamma\gamma}_{\pi^+\text{StopEff.}} = 0.52350 \pm 0.00647$ | | | |
| $\pi^+ \to \mu^+ \to e^+$ Decay Chain | $A_{TD}$ | $0.34872 \pm 0.00548$ | $\pi_{scat}$ |
| $A^{\pi^+\gamma\gamma}_{\pi^+\to\mu^+\to e^+} = 0.34872 \pm 0.00548$ | | | |
| $\gamma$ Reconstruction and Fiducial | $A_{fidBV}$<br>$A_{BVTIME}$ | $0.51235 \pm 0.00503$<br>$0.96085 \pm 0.00194$ | Monte Carlo<br>$K_{\pi 2}$ |
| $A^{\pi^+\gamma\gamma}_{\gamma\text{Recon.\&.Fid.}} = 0.49229 \pm 0.00493$ | | | |
| Extra Photon Veto Cuts | $A_{PV-MC}$<br>$A_{PV-K_{\mu 2}}$<br>$A_{\bar{\mu}}$ | $0.22925 \pm 0.00591$<br>$0.77345 \pm 0.00260$<br>$0.99625 \pm 0.00065$ | Monte Carlo<br>$K_{\mu 2}$<br>$K_{\mu 2}$ |
| $A^{\pi^+\gamma\gamma}_{\text{ExtraPV}} = 0.17665 \pm 0.00459$ | | | |
| Kinematic Cuts | $A_{ops}$<br>$A_{tgkin}$<br>$A_{kin}$ | $0.98317 \pm 0.00100$<br>$0.89142 \pm 0.00246$<br>$0.61280 \pm 0.00675$ | $K_{\pi 2}$<br>$K_{\pi 2}$<br>$\pi_{scat}$ |
| $A^{\pi^+\gamma\gamma}_{\text{Kinematics}} = 0.53706 \pm 0.00613$ | | | |
| Beam and Target | $A_{rest}$ | $0.50697 \pm 0.00077$ | $K_{\mu 2}$ |
| $A^{\pi^+\gamma\gamma}_{\text{Beam\&Target}} = 0.50697 \pm 0.00077$ | | | |
| $T \bullet 2$ Efficiency$(\pi^+\gamma\gamma)$ | $A_{T\bullet 2}(\pi^+\gamma\gamma)$ | $0.9357 \pm 0.0011$ | $K_B$ |
| $A^{\pi^+\gamma\gamma}_{T\bullet 2\text{Eff.}} = 0.93570 \pm 0.00110$ | | | |
| $A^{\pi^+\gamma\gamma}(\text{ChPT } O(p^4)) = 1.5358 \pm 0.0554 \times 10^{-5}$ | | | |

Table C.22: Categorization of the acceptance factors for the $K^+ \to \pi^+\gamma\gamma$ (ChPT $O(p^4)$).

| Definition | Symbol | Acceptances | Method |
|---|---|---|---|
| Trigger | $A_{trigCT}$ | $0.03490 \pm 0.00010$ | Monte Carlo |
| $A^{\pi^+\gamma\gamma}_{\text{Trigger}} = 0.03490 \pm 0.00010$ | | | |
| $\pi^+$ Reconstruction Efficiency | $A_{RD}$<br>$A_{Recon}$ | $0.99995 \pm 0.00001$<br>$0.99561 \pm 0.00012$ | $K_{\mu 2}$<br>$K_{\mu 2}$ |
| $A^{\pi^+\gamma\gamma}_{\text{Reconstruction}} = 0.99556 \pm 0.00012$ | | | |
| $\pi^+$ Fiducial Cuts | $A_{fidCT}$ | $0.89701 \pm 0.00092$ | Monte Carlo |
| $A^{\pi^+\gamma\gamma}_{\text{Fiducial}} = 0.89701 \pm 0.00092$ | | | |
| $\pi^+$ Stop Efficiency | $A_{nidif}$ | $0.49943 \pm 0.00274$ | Monte Carlo |
| $A^{\pi^+\gamma\gamma}_{\pi^+\text{StopEff.}} = 0.49943 \pm 0.00274$ | | | |
| $\pi^+ \to \mu^+ \to e^+$ Decay Chain | $A_{TD}$ | $0.34872 \pm 0.00548$ | $\pi_{scat}$ |
| $A^{\pi^+\gamma\gamma}_{\pi^+\to\mu^+\to e^+} = 0.34872 \pm 0.00548$ | | | |
| $\gamma$ Reconstruction and Fiducial | $A_{fidBV}$<br>$A_{BVTIME}$ | $0.56083 \pm 0.00225$<br>$0.96085 \pm 0.00194$ | Monte Carlo<br>$K_{\pi 2}$ |
| $A^{\pi^+\gamma\gamma}_{\gamma\text{Recon.\&.Fid.}} = 0.53887 \pm 0.00242$ | | | |
| Extra Photon Veto Cuts | $A_{PV-MC}$<br>$A_{PV-K_{\mu 2}}$<br>$A_{\bar{\mu}}$ | $0.26813 \pm 0.00268$<br>$0.77345 \pm 0.00260$<br>$0.99625 \pm 0.00065$ | Monte Carlo<br>$K_{\mu 2}$<br>$K_{\mu 2}$ |
| $A^{\pi^+\gamma\gamma}_{\text{ExtraPV}} = 0.20661 \pm 0.00218$ | | | |
| Kinematic Cuts | $A_{ops}$<br>$A_{tgkin}$<br>$A_{kin}$ | $0.98317 \pm 0.00100$<br>$0.89142 \pm 0.00246$<br>$0.61280 \pm 0.00675$ | $K_{\pi 2}$<br>$K_{\pi 2}$<br>$\pi_{scat}$ |
| $A^{\pi^+\gamma\gamma}_{\text{Kinematics}} = 0.53706 \pm 0.00613$ | | | |
| Beam and Target | $A_{rest}$ | $0.50697 \pm 0.00077$ | $K_{\mu 2}$ |
| $A^{\pi^+\gamma\gamma}_{\text{Beam\&Target}} = 0.50697 \pm 0.00077$ | | | |
| $T \bullet 2$ Efficiency$(\pi^+\gamma\gamma)$ | $A_{T\bullet 2}(\pi^+\gamma\gamma)$ | $0.9357 \pm 0.0011$ | $K_B$ |
| $A^{\pi^+\gamma\gamma}_{T\bullet 2\text{Eff.}} = 0.93570 \pm 0.00110$ | | | |
| $A^{\pi^+\gamma\gamma}(\text{Phase Space}) = 1.5395 \pm 0.0363 \times 10^{-4}$ | | | |

Table C.23: Categorization of the acceptance factors for the $K^+ \to \pi^+\gamma\gamma$ (Phase Space).