What would YOU do with 1 billion Z bosons ?

S. Heinemeyer, Brookhaven Natl. Lab.

BNL, 03/01

1. Introduction:
   - Where to get 1 billion Z bosons
   - What I would do with them

2. Precision Tests of the SM

3. Precision Tests of the MSSM

4. Conclusions

1. Introduction

Where to get 1 billion Z bosons

You need:

– a high luminosity e^+e^- linear collider (LC)
with e^+ and e^- polarization
(TESLA or NLC or . . .)

– 1 - 2 years of time
(depending on actual luminosity)

Then:

Let the collider run on the Z resonance

Example:

Luminosity at the Z resonance:

$$\mathcal{L} = 7 \times 10^{33}\ \text{cm}^{-2}\ \text{s}^{-1}$$

Cross section: $\sigma \approx 30$ nb

One year of operational time:

10^7 seconds

$\Rightarrow$ 2×10^9 Z bosons per year

Likewise at the WW threshold:

$\Rightarrow$ 10^6 W pairs per year

What I would do with 1 billion Z bosons

Definition:

LC at low energies: high luminosity
$\sim 10^9$ Z bosons/year
$\sim 10^6$ W pairs/year

$\left.\right\}$ **GigaZ**

$\to$ use the Z bosons to measure the left-right asymmetry: $A_{LR}\ (++, +-, -+, --)$

use A_{LR} to determine the

effective leptonic mixing angle: $\delta \sin^2\theta_{\text{eff}} \approx 1 \times 10^{-5}$

$\to$ use the W bosons (at threshold) to measure the

W boson mass: $\delta M_W \approx 6$ MeV

Expected accuracies:

	LEP2/Tev.	LHC	LC	GigaZ
M_W	30 MeV	15 MeV	15 MeV	6 MeV
$\sin^2\theta_{\text{eff}}$	0.00017	0.00017	0.00017	0.00001
m_t	3 GeV	2 GeV	0.2 GeV	0.2 GeV
m_h	?	0.2 GeV	0.05 GeV	0.05 GeV

What would YOU do with the improved accuracy in M_W and $\sin^2\theta_{\text{eff}}$?

Precision Observables (POs):

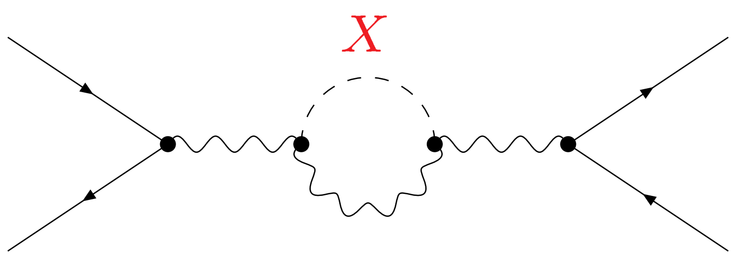
Comparison of electro-weak precision observables with theory:

EW Precision data:		Theory:
$\Delta\rho, \Delta r, M_W, \sin^2\theta_{\text{eff}}$	$\leftrightarrow$	SM, MSSM , ...

$\Downarrow$

Test of theory at quantum level:

Sensitivity to loop corrections



$\Downarrow$

Improve indirect constraints on unknown parameters: $m_h, m_{\tilde{t}}, \ldots$

Effects of "new physics"?

Theory: Precision observables M_W, $\sin^2\theta_{\mathrm{eff}}$:

1.) Theoretical prediction for M_W in terms of $M_Z, \alpha, G_\mu, \Delta r$:

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right) = \frac{\pi\,\alpha}{\sqrt{2}\,G_\mu}\left(\frac{1}{1-\Delta r}\right)$$

$\Updownarrow$

loop corrections

SM prediction for Δr, one-loop:

[*A. Sirlin '80*] [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\mathrm{loop}} = \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{\mathrm{rem}}(m_h)$$

$$\sim \log\frac{M_Z}{m_f} \qquad \sim m_t^2$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

2.) Effective mixing angle:

$$\sin^2\theta_{\mathrm{eff}} = \frac{1}{4\,|Q_f|}\left(1-\frac{\mathrm{Re}\,g_V^f}{\mathrm{Re}\,g_A^f}\right)$$

Higher order contributions:

$$g_V^f \to g_V^f + \Delta g_V^f, \quad g_A^f \to g_A^f + \Delta g_A^f$$

Further included corrections:

- SM: leading electroweak two-loop terms [*J. van der Bij, F. Hoogeveen '87*]
  [*R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere '92*]
  [*J. Fleischer, O.V. Tarasov, F. Jegerlehner '93*]
  [*A. Freitas, S.H., W. Hollik, W. Walter, G. Weiglein '00*]
- SM: QCD corrections up to $\mathcal{O}(\alpha\alpha_s^2)$
  [*A. Djouadi, C. Verzegnassi '87*]
  [*F. Halzen, B. Kniehl '91*]
  [*K. Chetyrkin, J. Kühn, M. Steinhauser '95*]
  [*L. Avdeev, J. Fleischer, S. Mikhailov, O. Tarasov '95*]
- MSSM: full one-loop corrections to Δr
  [*P. Chankowski, A. Dabelstein, W. Hollik, W. Mösle, S. Pokorski, J. Rosiek '94*]
  [*D. Garcia, J. Solà '94*]
- MSSM: Z-boson observables, one-loop
  [*D. Garcia, R. Jiménez, J. Solà '95*]
  [*D. Garcia, J. Solà '95*]
  [*A. Dabelstein, W. Hollik, W. Mösle '95*]
  [*P. Chankowski, S. Pokorski '96*]
- MSSM: leading $\mathcal{O}(\alpha\alpha_s)$ corrections
  [*A. Djouadi, P. Gambino, S. H., W. Hollik, C. Jünger, G. Weiglein '97*] [*S. H, W. Hollik, G. Weiglein '98*]

Interpretation in the SM:

Indirect determination of m_t from precision data:

$$m_t = 172^{+14}_{-11} \text{ GeV}$$

direct measurement:

$$m_t = 174.3 \pm 5.1 \text{ GeV}$$

One-loop corrections to precision observables:

$$\sim m_t^2$$
$$\sim \log m_h$$

$\Rightarrow$ Very high accuracy of measurements and theoretical predictions needed

Theoretical uncertainties:

– unknown higher-order corrections

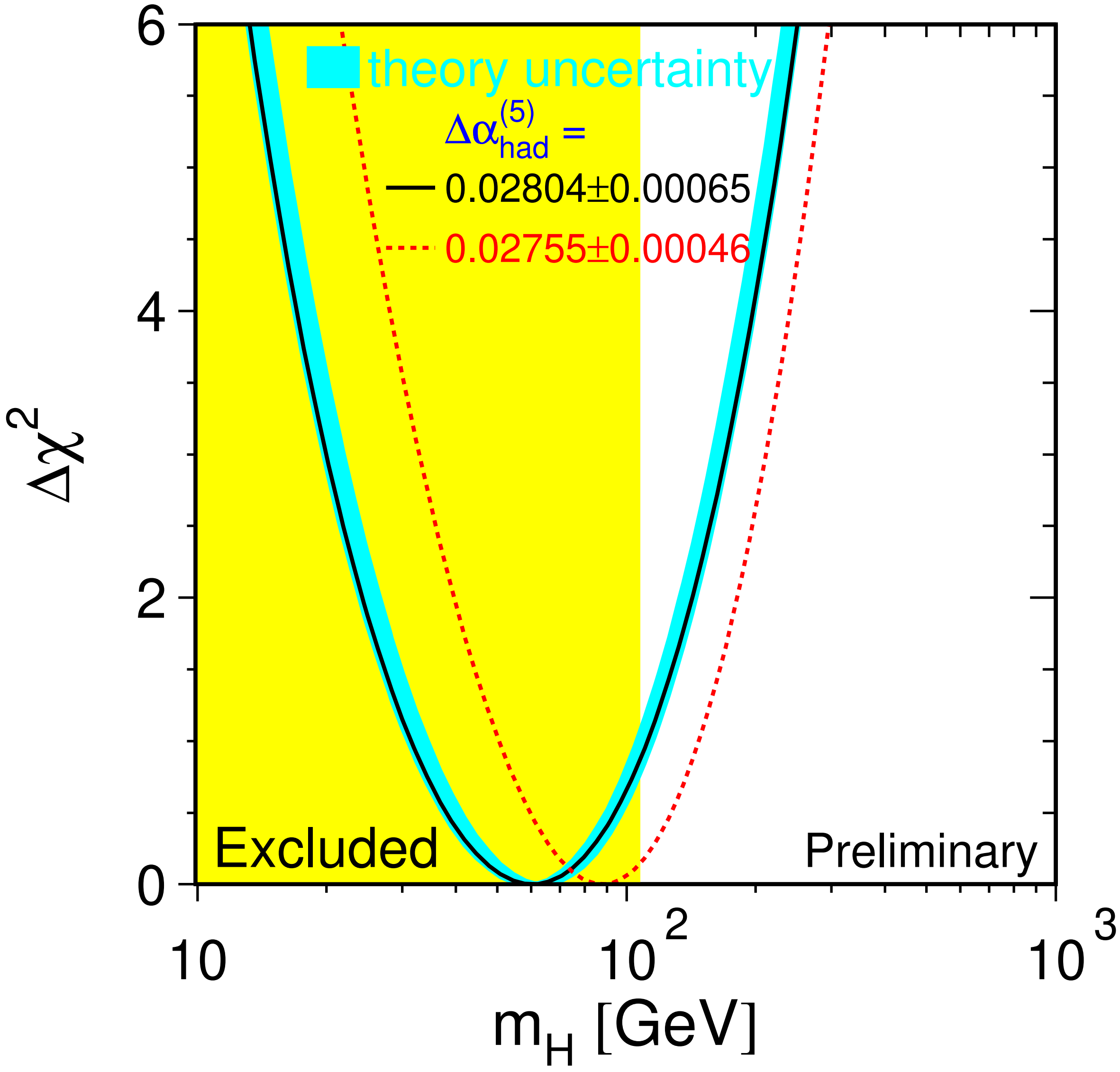
– exp. error of input parameters: m_t, $\Delta\alpha_{\text{had}}$, ...

Todays theoretical uncertainty:

$$\delta M_W^{\text{theo}} \approx 6 \text{ MeV}$$
$$\delta \sin^2\theta_{\text{eff}}^{\text{theo}} \approx 0.00004$$

Global fit to all data in the SM:

[*LEPEWWG '00*]



($\Rightarrow M_H < 188$ GeV, 95% C.L.)

Recently:

Shift by ~ 30 GeV due to BES II analysis

$\Rightarrow M_H < 210$ GeV, 95% C.L.

What if SUSY will be discovered?

Assume: SUSY particles accessible at next generation of colliders

Exact SUSY: $m_f = m_{\tilde{f}}, \ldots$

$\Rightarrow$ SUSY must be broken: $M_{\mathrm{SUSY}} \lesssim 1$ TeV

SUSY breaking mechanism not well understood; different models for SUSY breaking:

"Hidden sector": SUSY breaking $\longrightarrow$ Visible sector: MSSM

"Gravity-mediated": mSUGRA
"Gauge-mediated": GMSB
"Anomaly-mediated": AMSB
...

Unconstrained MSSM:

no particular SUSY breaking mechanism assumed, parameterization of possible SUSY-breaking terms

$\Rightarrow$ 105 new parameters: masses, mixing angles, phases

Current situation:

– Good phenomenological description for universal breaking terms

– no preference for certain SUSY-breaking scenario

Possible future situation:

Measurements of (SUSY) observables:

$\Rightarrow$ Determination of SUSY parameters

$\Rightarrow$ Patterns of SUSY breaking (new high scales)

Role of PO's:

Indirect information from precision observables

$\rightarrow$ constraints to unaccessible scales

$\rightarrow$ complementary to direct production processes

Stringent direct test of SUSY:

Light Higgs boson h required

Tree level: $m_h < M_Z$

Yukawa couplings: $\frac{e\, m_t}{2 M_W s_W}, \frac{e\, m_t^2}{M_W s_W}, \ldots$

Dominant corrections to m_h from $t - \tilde{t}$-sector :

$$\begin{aligned} \Delta m_h^2 &\sim \frac{m_t^4}{M_W^2} \log\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right) \\ &\approx \frac{m_t^4}{M_W^2} \log\left(\frac{(M_{\mathrm{SUSY}}^2 + m_t^2)^2 - m_t^2 X_t^2}{m_t^4}\right) \end{aligned}$$

$\tilde{t}$ -mass-matrix:

$$M_{\tilde{t}}^2 = \begin{pmatrix} M_{\mathrm{SUSY}}^2 + m_t^2 + DT_1 & m_t X_t \\ m_t X_t & M_{\mathrm{SUSY}}^2 + m_t^2 + DT_2 \end{pmatrix}$$

$\Rightarrow m_h \lesssim 135$ GeV

[S.H., W. Hollik, G. Weiglein '99]

High-precision measurement of m_h:

LHC :	$\delta m_h \approx 0.2$ GeV
LC :	$\delta m_h \approx 0.05$ GeV
Muon Collider :	$\delta m_h \approx 0.1$ MeV

$\Rightarrow m_h$ will be precision observable!

SM:

free parameters: m_t, m_h

$\to$ direct contribution via loop effects

$\Rightarrow$ Prediction for precision observables
M_W, $\sin^2\theta_{\mathrm{eff}}$

MSSM:

free parameters: $m_t, M_{\mathrm{SUSY}}, X_t$

$\to$ direct contribution via loop effects

$\to$ indirect effect via contribution to m_h

$\Rightarrow$ Prediction for precision observables
M_W, $\sin^2\theta_{\mathrm{eff}}$

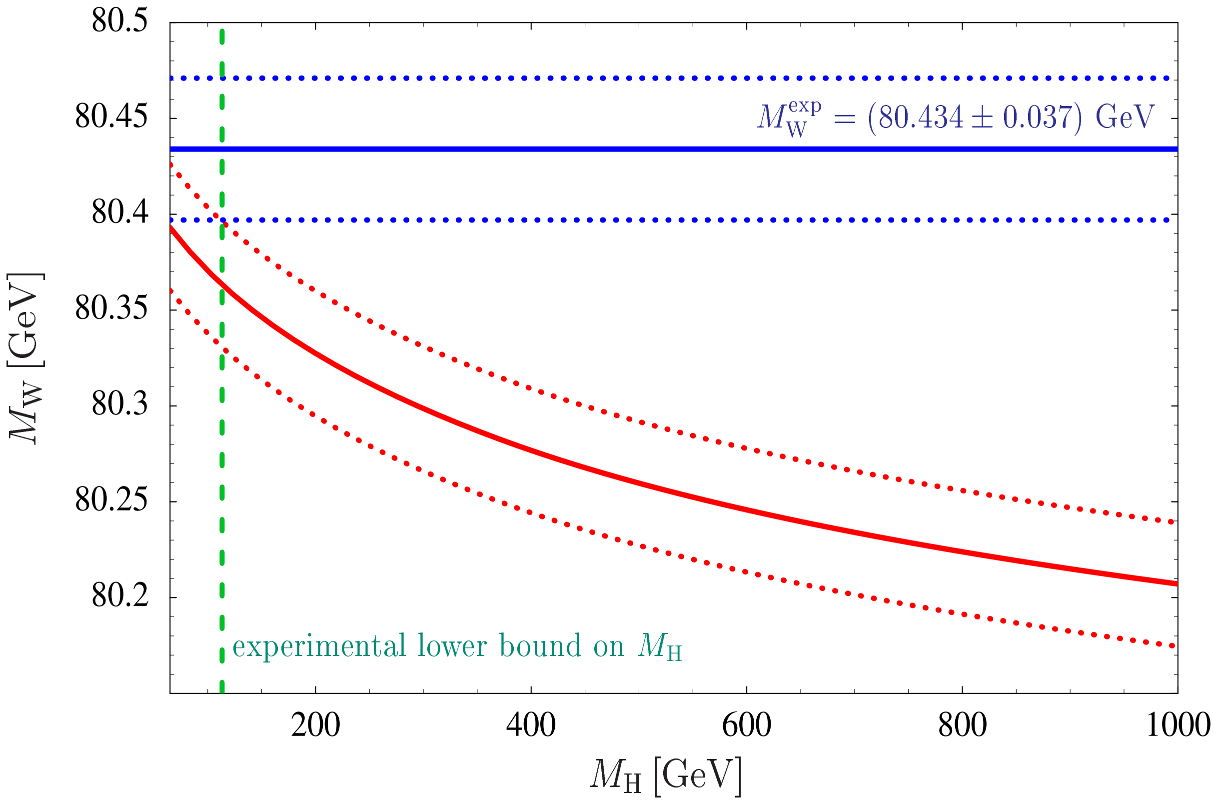
Prediction for precision observable m_h

2. Precision tests of the SM:

Present situation:

SM prediction for M_W vs. experimental result:

[A. Freitas, S.H., W. Hollik, W. Walter, G.Weiglein '00]



$\Rightarrow$ no overlap at the 1 σ level

$\Rightarrow$ light Higgs preferred

Theoretical uncertainty dominated by uncertainty in input parameters: m_t, M_H, ...

$\delta m_t = 5.1$ GeV $\Rightarrow \delta M_W \approx 31$ MeV

Future theoretical uncertainties:

Parametric uncertainties from experimental errors of input parameters:

$$\delta m_t = \pm 130 \text{ MeV} \quad \Rightarrow \delta M_W \approx \pm 1 \text{ MeV}$$
$$\delta \sin^2\theta_{\text{eff}} \approx \pm 0.4 \times 10^{-5}$$

$$\delta m_h = \pm 50 \text{ MeV}$$

$$\delta \alpha_s = \pm 1 \times 10^{-3}$$

$$\delta M_Z = \pm 2.1 \text{ MeV} \quad \Rightarrow \delta M_W \approx \pm 2.5 \text{ MeV}$$
$$\delta \sin^2\theta_{\text{eff}} \approx \pm 1.4 \times 10^{-5}$$

$$\delta(\Delta\alpha) = \pm 5 \times 10^{-5} \quad \Rightarrow \delta M_W \approx \pm 1 \text{ MeV}$$
$$\delta \sin^2\theta_{\text{eff}} \approx \pm 1.8 \times 10^{-5}$$

$\Rightarrow$ Error of M_Z non-negligible!

Estimate for future theoretical uncertainties:

– unknown higher-order corrections

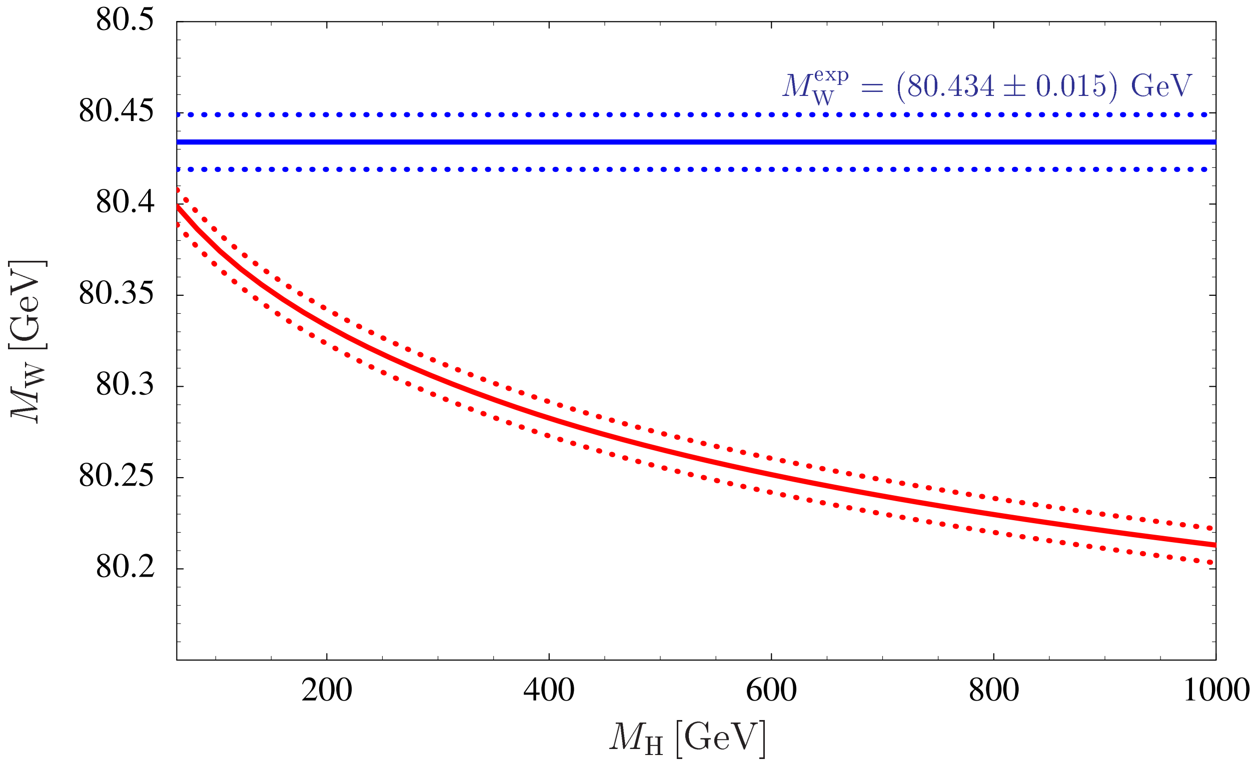
– $\delta(\Delta\alpha)$ uncertainty

$$\delta M_W^{\text{theo}} \approx \pm 3 \text{ MeV}, \quad \delta \sin^2\theta_{\text{eff}}^{\text{theo}} \approx \pm 3 \times 10^{-5}$$

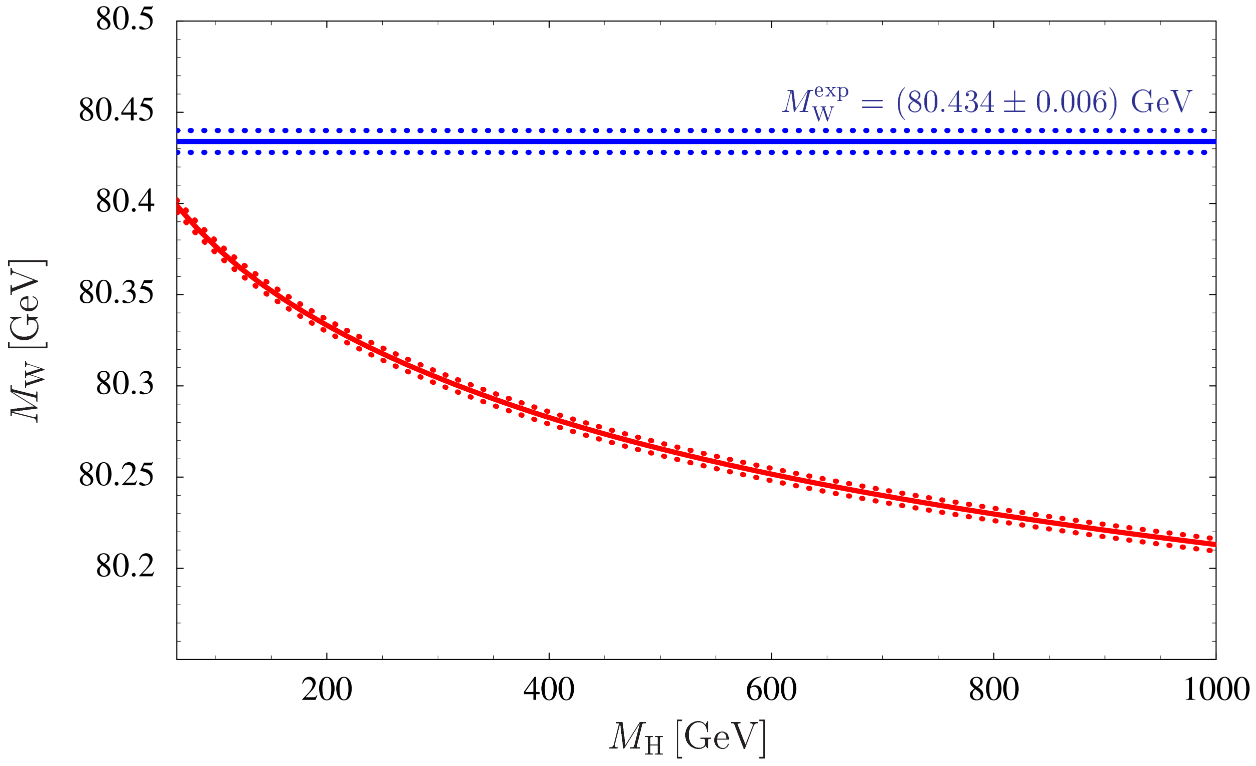
$M_W(m_h)$ in the SM:

prospective future accuracies:

LHC precision:



GigaZ precision:

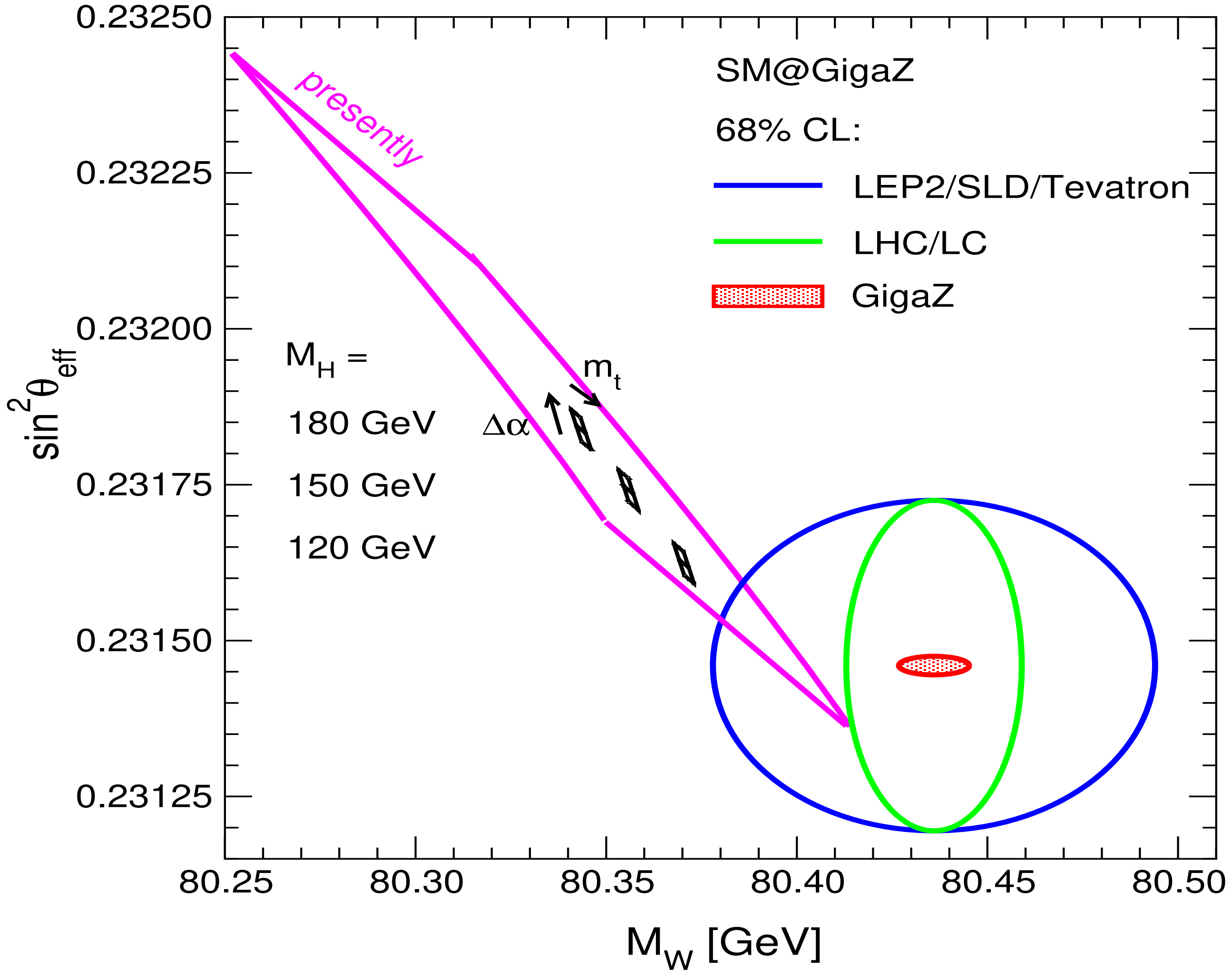


SM prediction for M_W and $\sin^2\theta_{\text{eff}}$

vs. prospective accuracies at

LEP2/SLD/Tevatron , LHC/LC , GigaZ:

[*J. Erler, S.H., W. Hollik, G. Weiglein and P. Zerwas '00*]

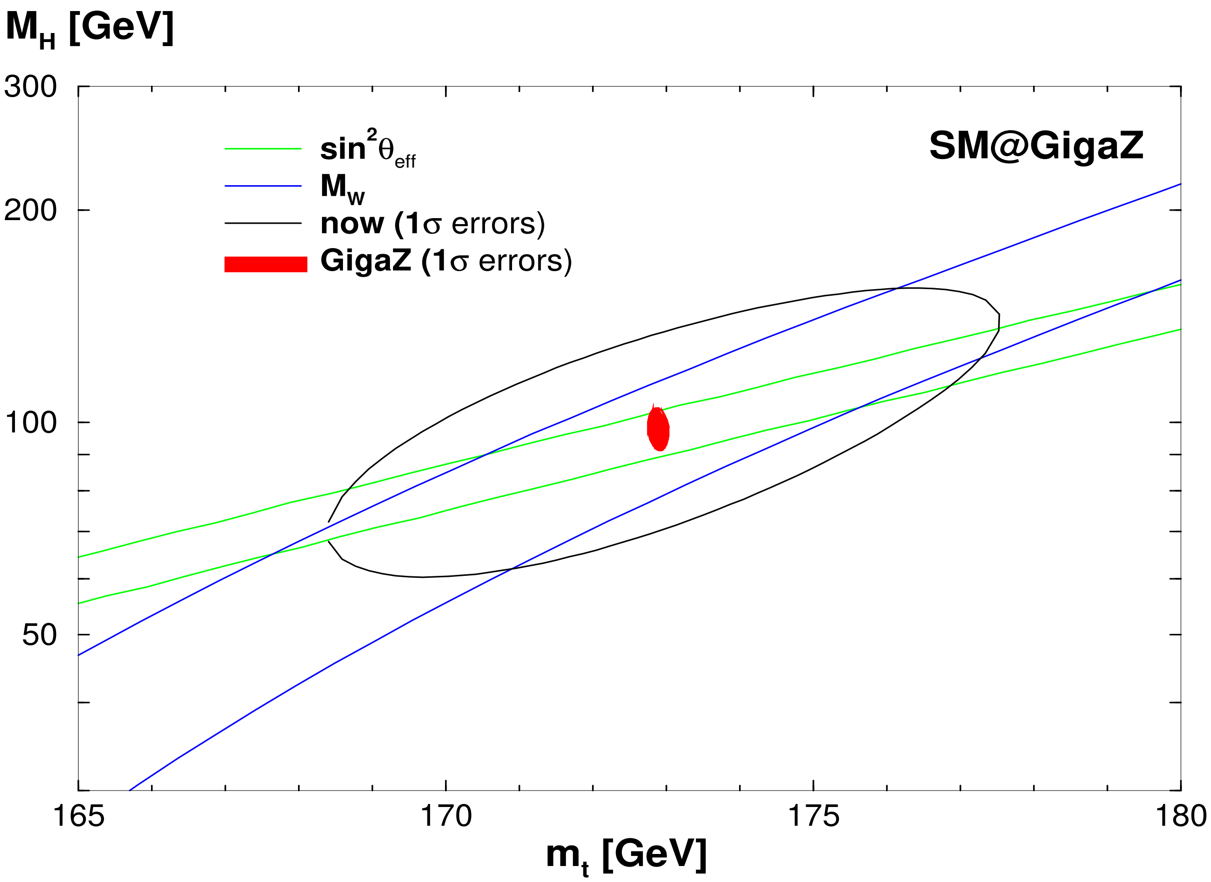


$\Rightarrow$ ultimate precision test of the SM

extreme high sensitivity to new physics scales

Indirect determination of m_t, M_H at GigaZ:

[*J. Erler, S.H., W. Hollik, G. Weiglein and P. Zerwas '00*]



Expected precisions of indirect M_H-determinations:

[*J. Erler, S.H., W. Hollik, G. Weiglein and P. Zerwas '00*]

	M_W	$\sin^2\theta_{\mathrm{eff}}$	all
now	200 %	62 %	60 %
Tevatron Run IIA	77 %	46 %	41 %
Tevatron Run IIB	39 %	28 %	26 %
LHC	28 %	24 %	21 %
LC	18 %	20 %	15 %
GigaZ	12 %	7 %	7 %

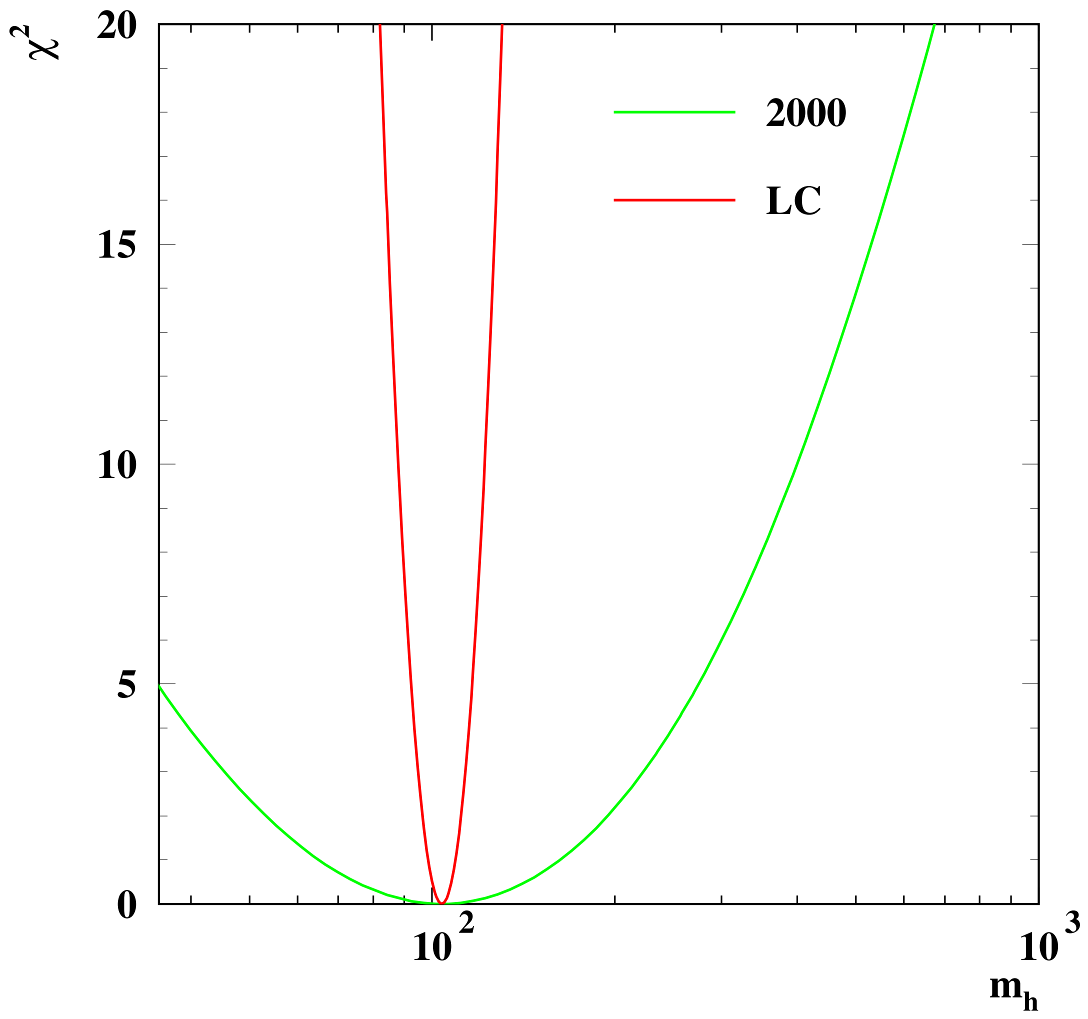
New "Blue band" plot:

Fit for the SM Higgs boson mass

Today → GigaZ

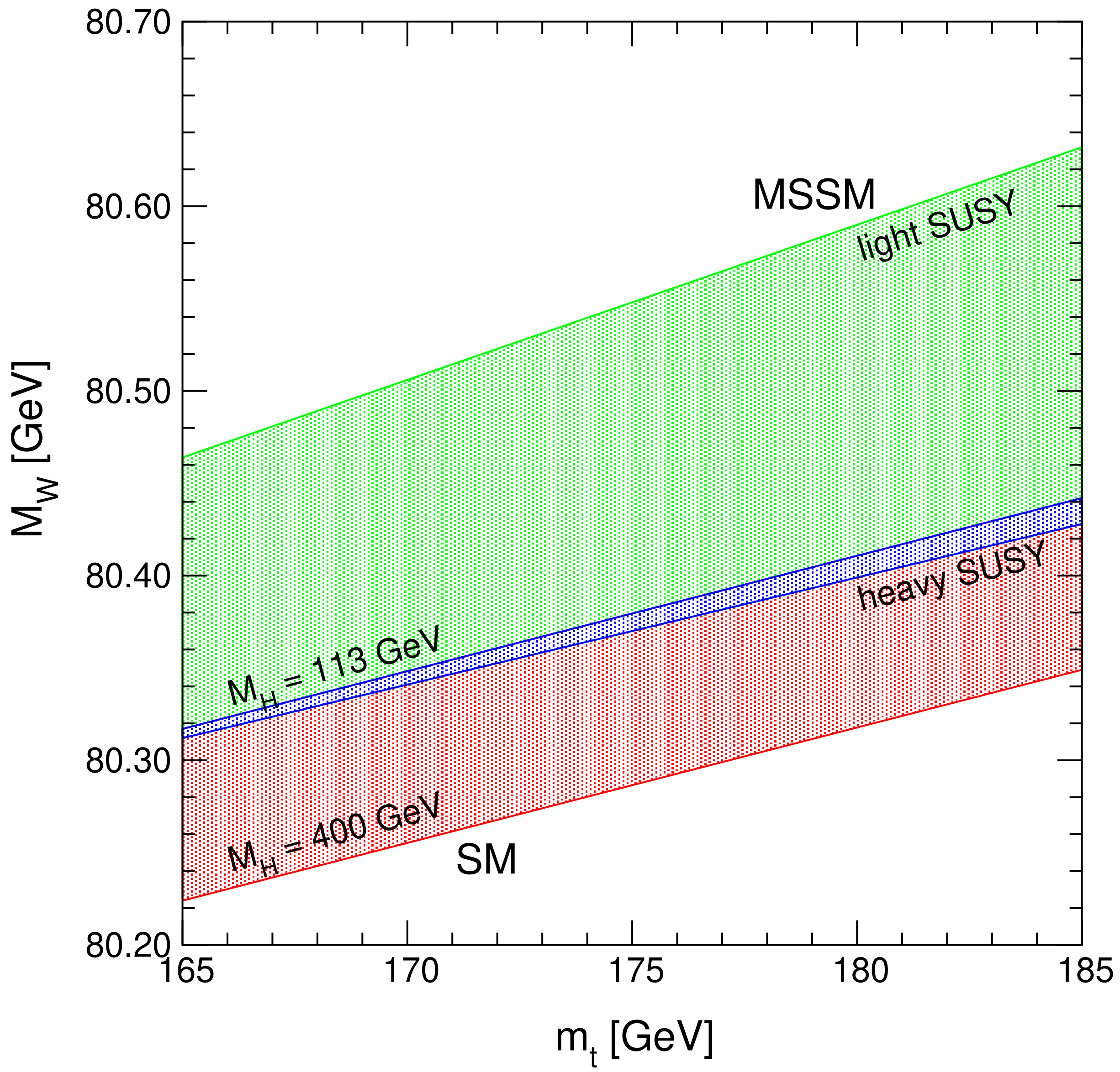
(neglected theoretical uncertainty)

[R. Hawkings and K. Mönig '00]



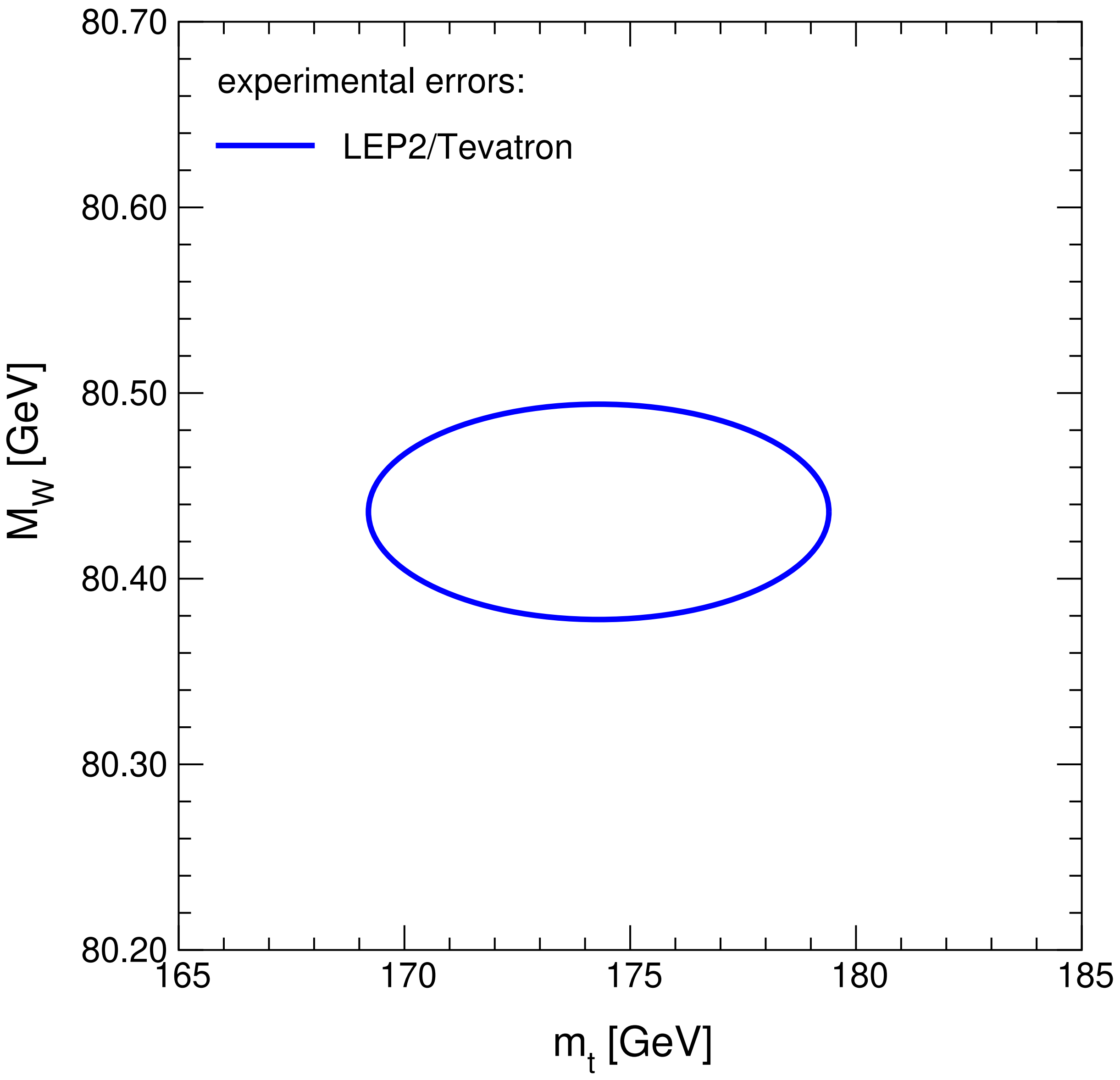
3. Precision tests of the MSSM

Prediction for M_W in the SM and the MSSM :

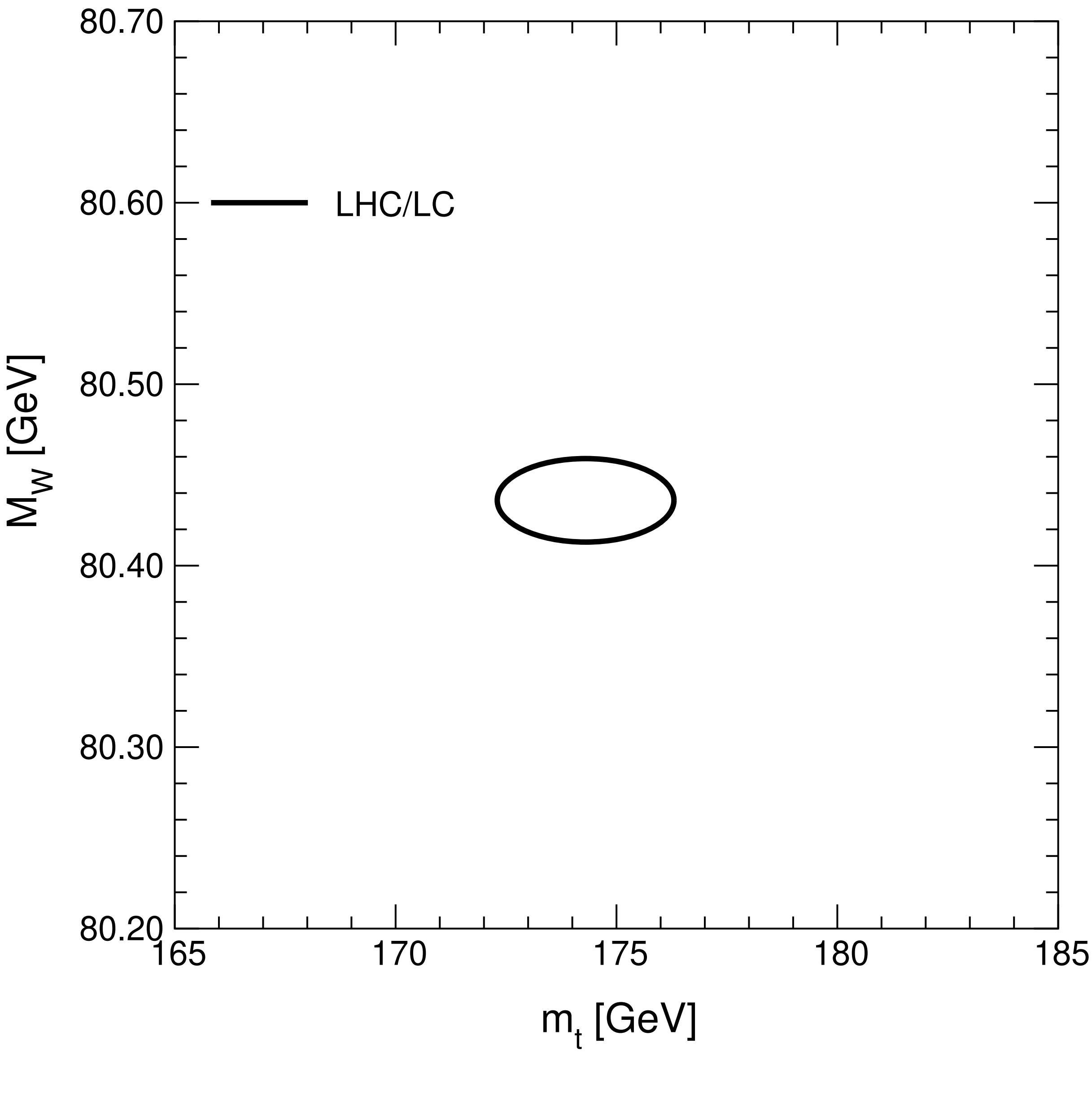


SM uncertainty: unknown Higgs mass

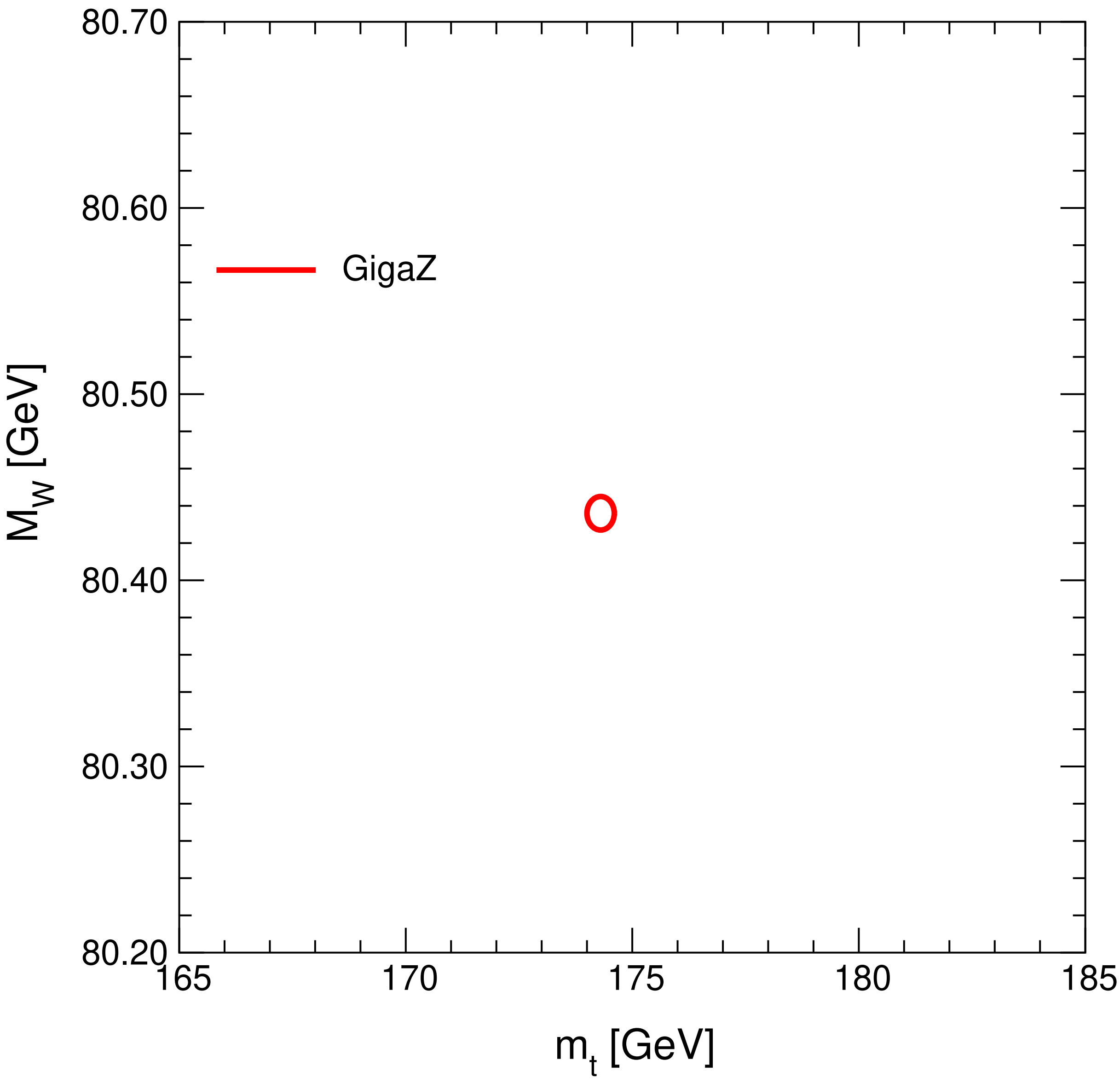
MSSM uncertainty: unknown masses
of SUSY particles



experimental errors:
LEP2/Tevatron
80.70
80.60
80.50
80.40
80.30
80.20
165
170
175
180
185
M_W [GeV]
m_t [GeV]



80.70
80.60
80.50
80.40
80.30
80.20
LHC/LC
M_W [GeV]
165
170
175
180
185
m_t [GeV]

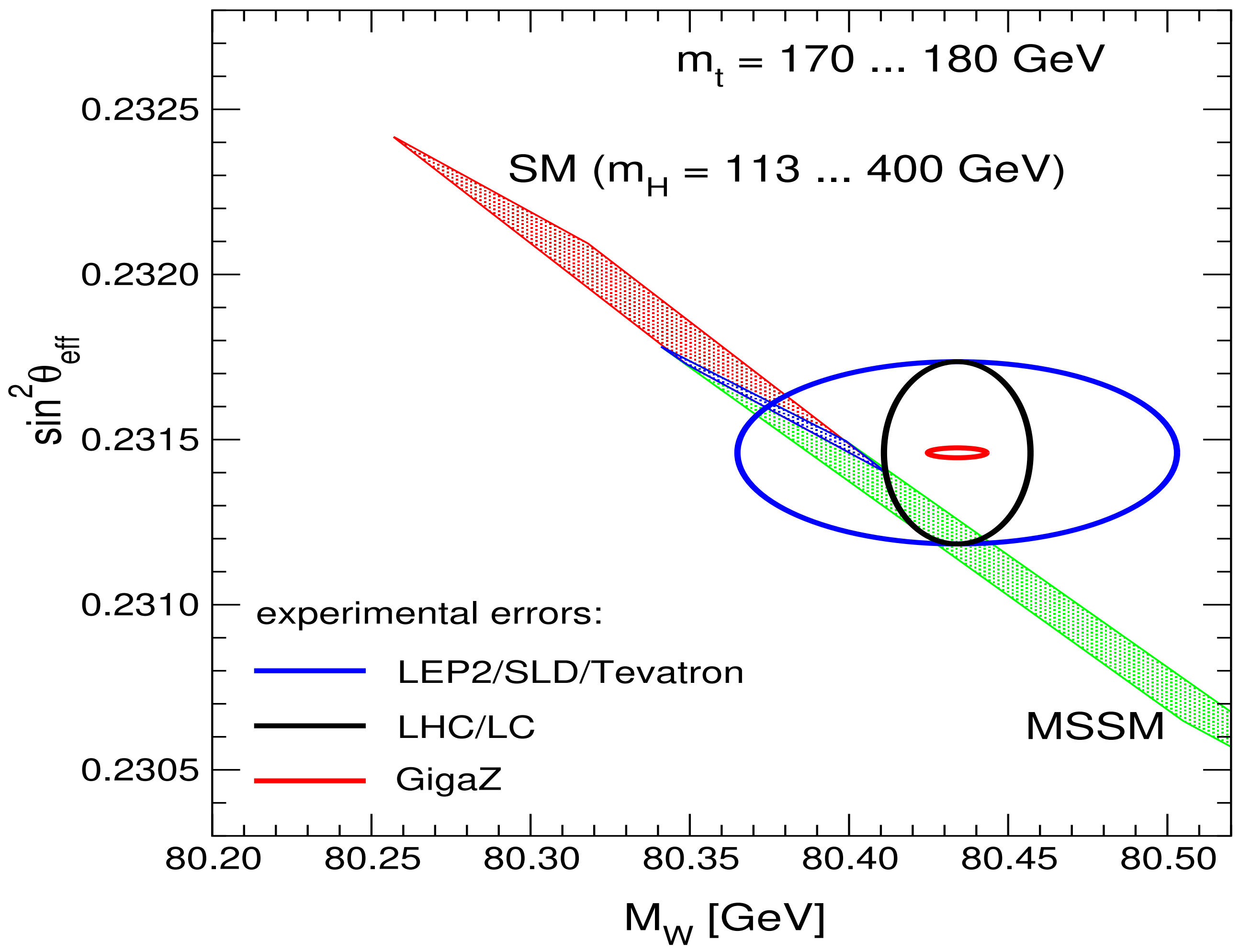


80.70
80.60
80.50
80.40
80.30
80.20
GigaZ
M_W [GeV]
165
170
175
180
185
m_t [GeV]

Prediction for M_W, $\sin^2\theta_{\text{eff}}$ in the SM and MSSM

vs. prospective accuracies at

LEP2/SLD/Tevatron , LHC/LC, GigaZ:



$\Rightarrow$ large improvement of experimental accuracy:

LEP2/SLD/Tevatron $\to$ LHC/LC $\to$ GigaZ

$\Rightarrow$ Very sensitive test of theory

The Higgs mass as a precision observable

(Scenario I:)

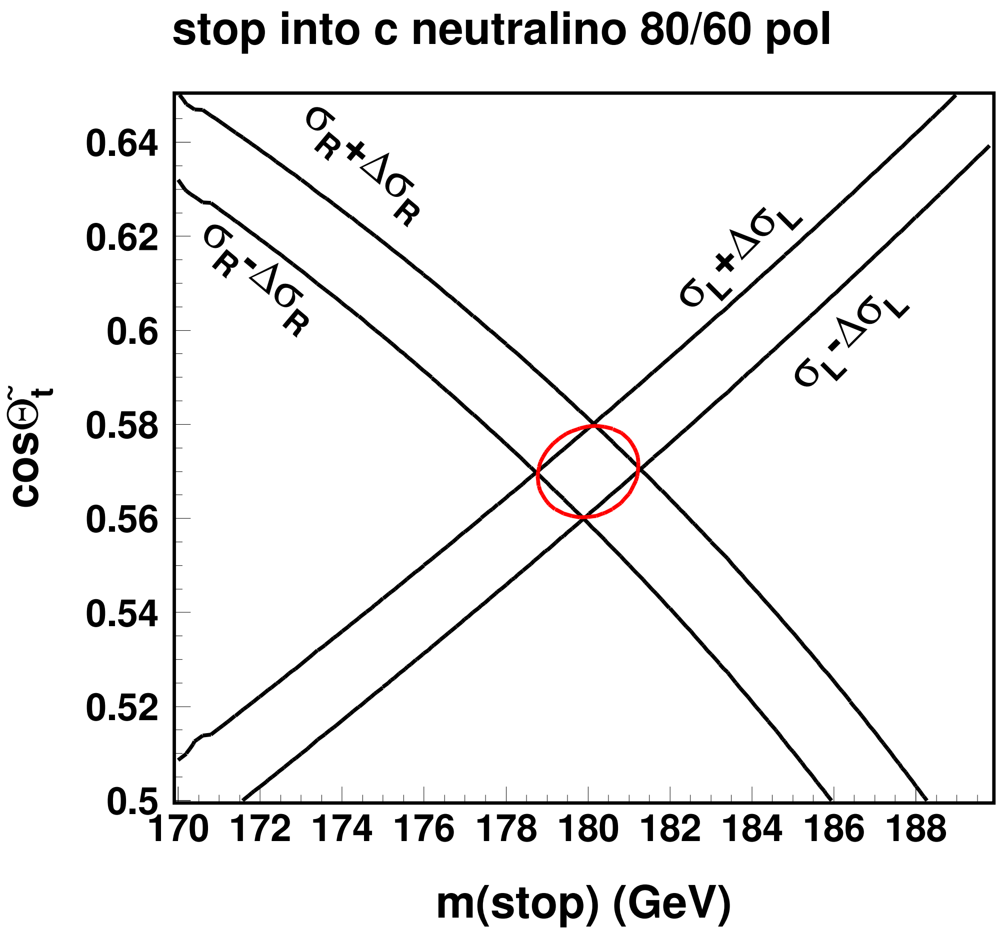
$\to$ Combination of direct and indirect information on $\tilde{t}$ sector parameters:

Direct information:

$e^+e^- \to \tilde{t}_1\tilde{t}_1$ at high luminosity LC

(80% pol. e^- beam, 60% pol. e^+ beam, $\sqrt{s} = 500$ GeV, $\mathcal{L} = 500$ fb^{-1})

[R. Keränen, H. Nowak, A. Sopczak '00]



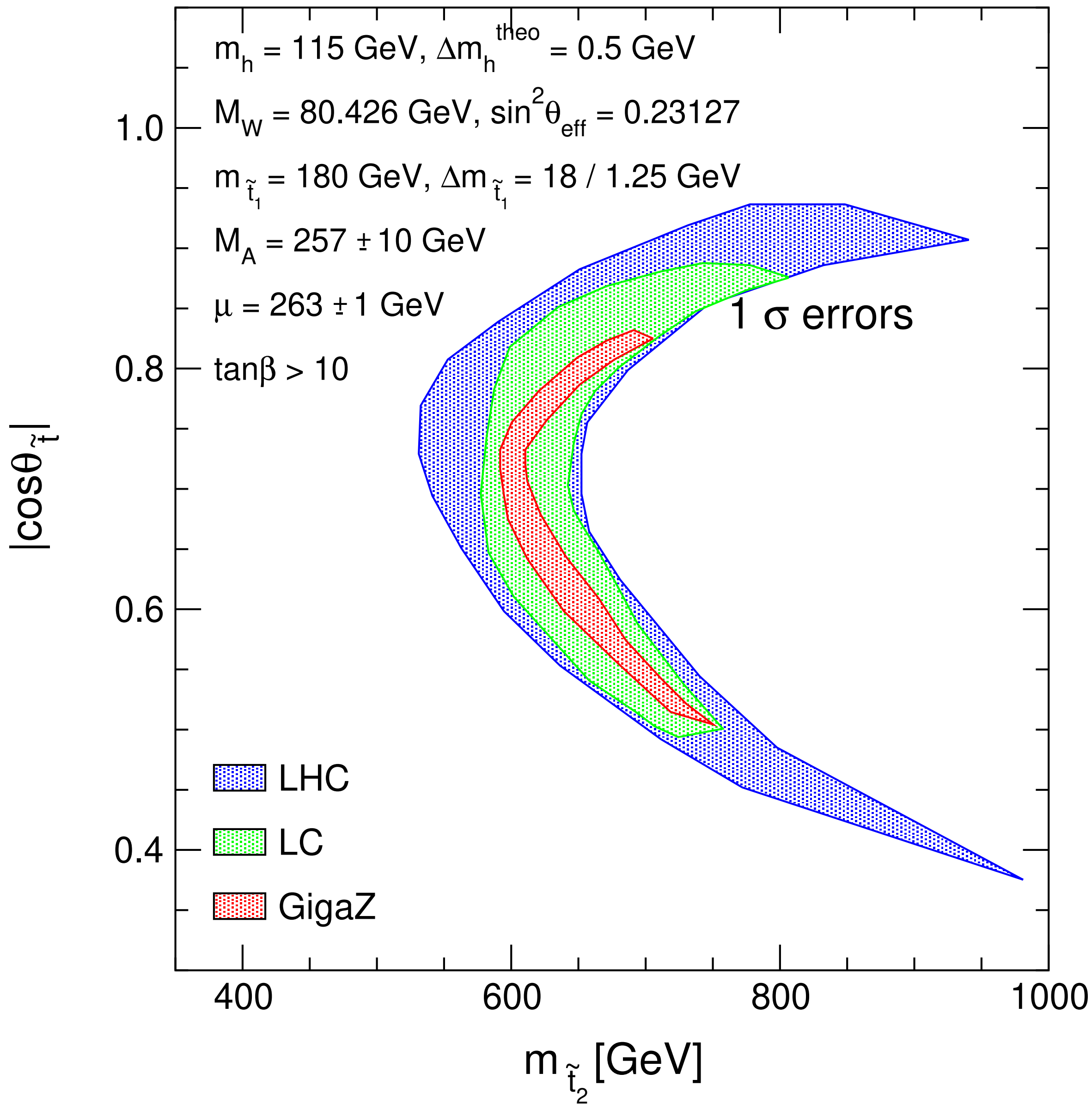
$\Rightarrow$ precise direct determination of $m_{\tilde{t}_1}$ and $\theta_{\tilde{t}}$

Indirect information:

Constraints on $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$ from precision observables $m_h, M_W, \sin^2\theta_{\text{eff}}$ at LHC , LC , GigaZ :

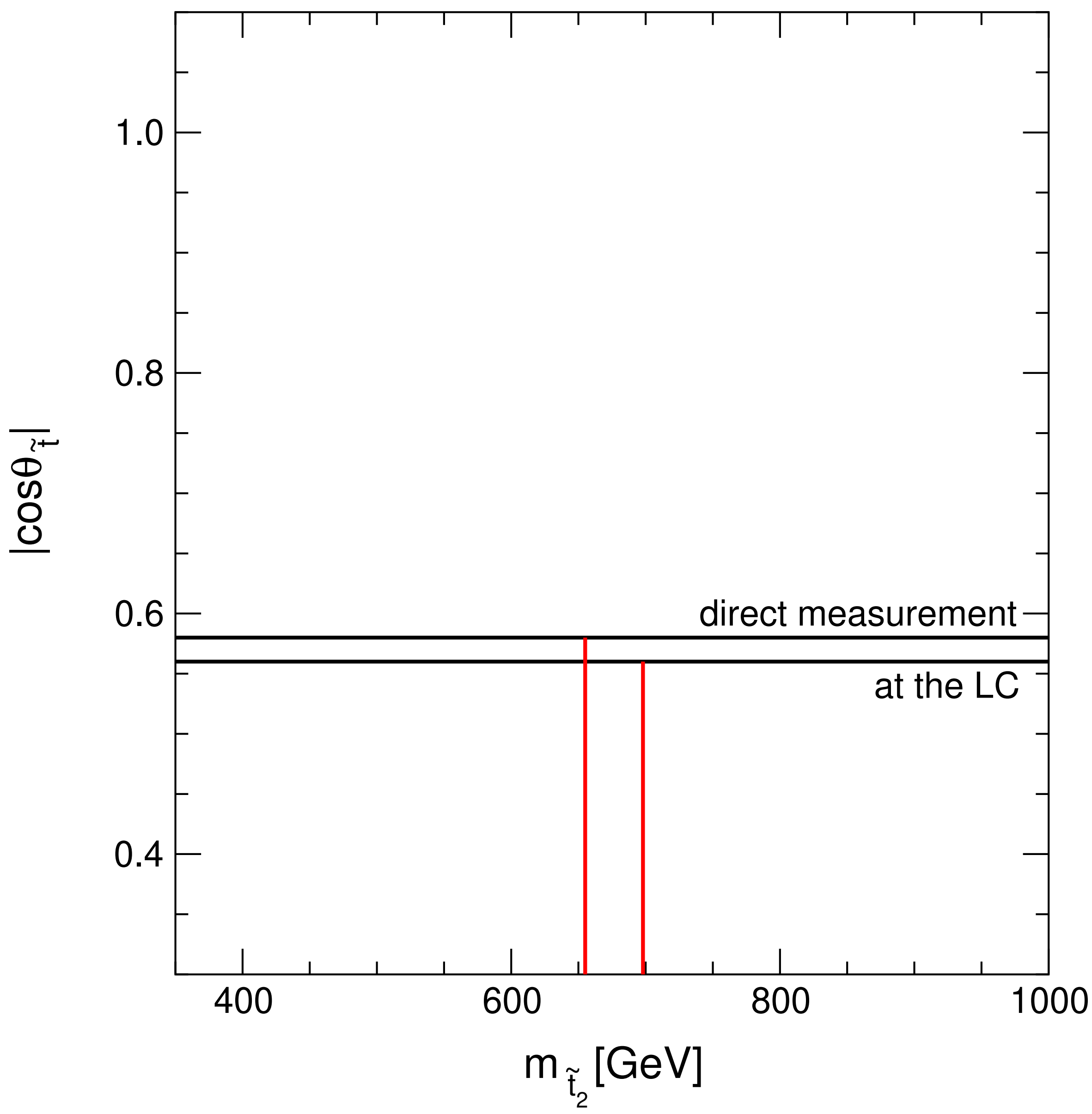
[S.H., G. Weiglein '00]

Allowed region in $m_{\tilde{t}_2}$-$\cos\theta_{\tilde{t}}$ plane:
$(M_A = 257 \pm 10$ GeV$, \tan\beta > 10)$:



Complementary of direct $\leftrightarrow$ indirect information

$\Rightarrow$ Indirect determ. of $m_{\tilde{t}_2}$ with high precision



$|\cos\theta_{\tilde{t}}|$
1.0
0.8
0.6
0.4
direct measurement
at the LC
400
600
800
1000
$m_{\tilde{t}_2}$ [GeV]

Scenario II:

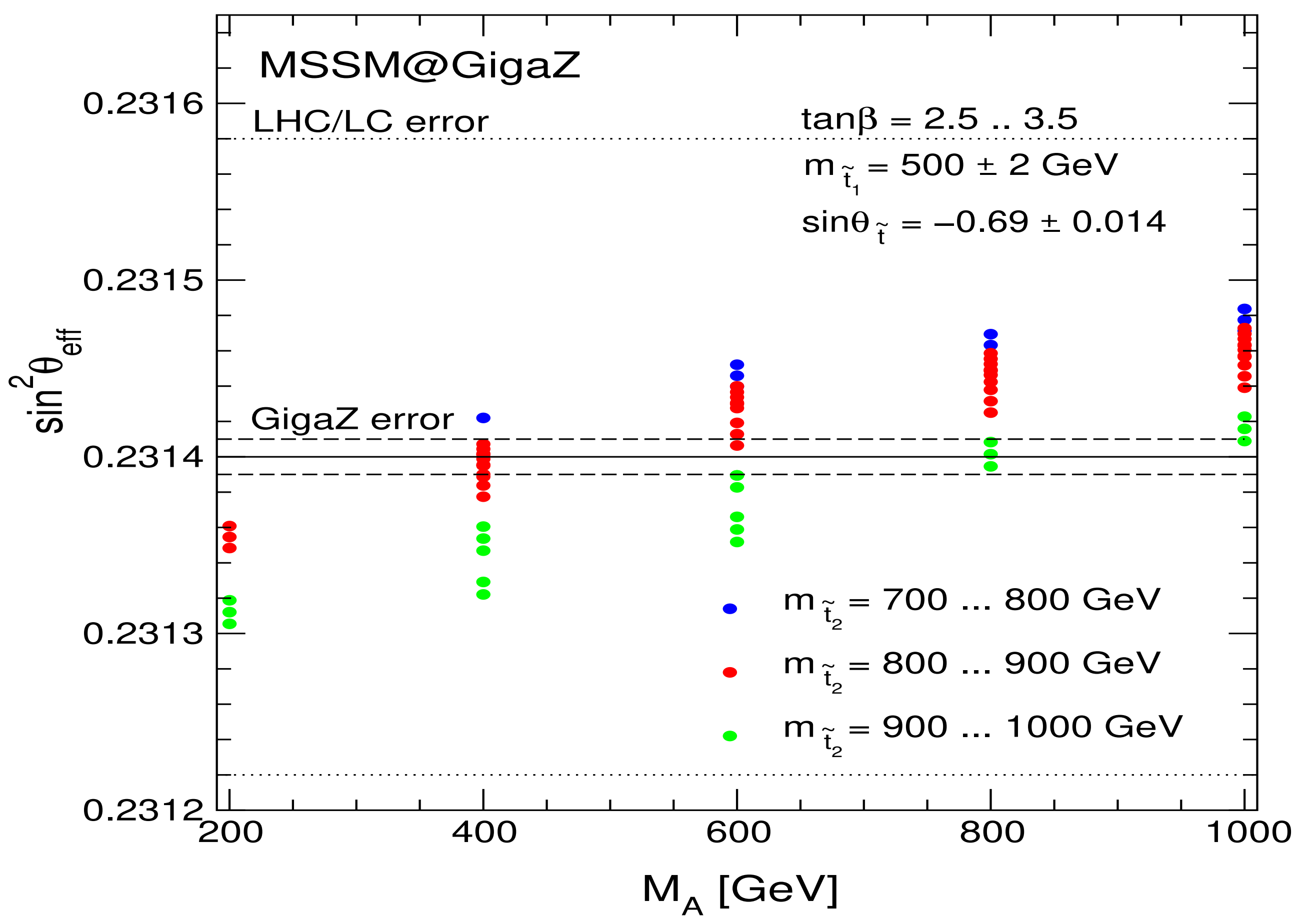
[*J. Erler, S.H., W. Hollik, G. Weiglein and P. Zerwas '00*]

no experimental information on $m_{\tilde{t}_2}$, M_A

$\sin^2\theta_{\rm eff} = 0.23140 \pm 0.00001$

$m_{\tilde{t}_1} = 500 \pm 2$ GeV, $\sin\theta_{\tilde{t}} = -0.69 \pm 2\%$, $\tan\beta = 3 \pm 0.5$

Constraints for M_A only from $\sin^2\theta_{\rm eff}$:



⇒ Logarithmic M_A dependence

⇒ no constraints on M_A from LHC/LC measurement

Now: use all PO's

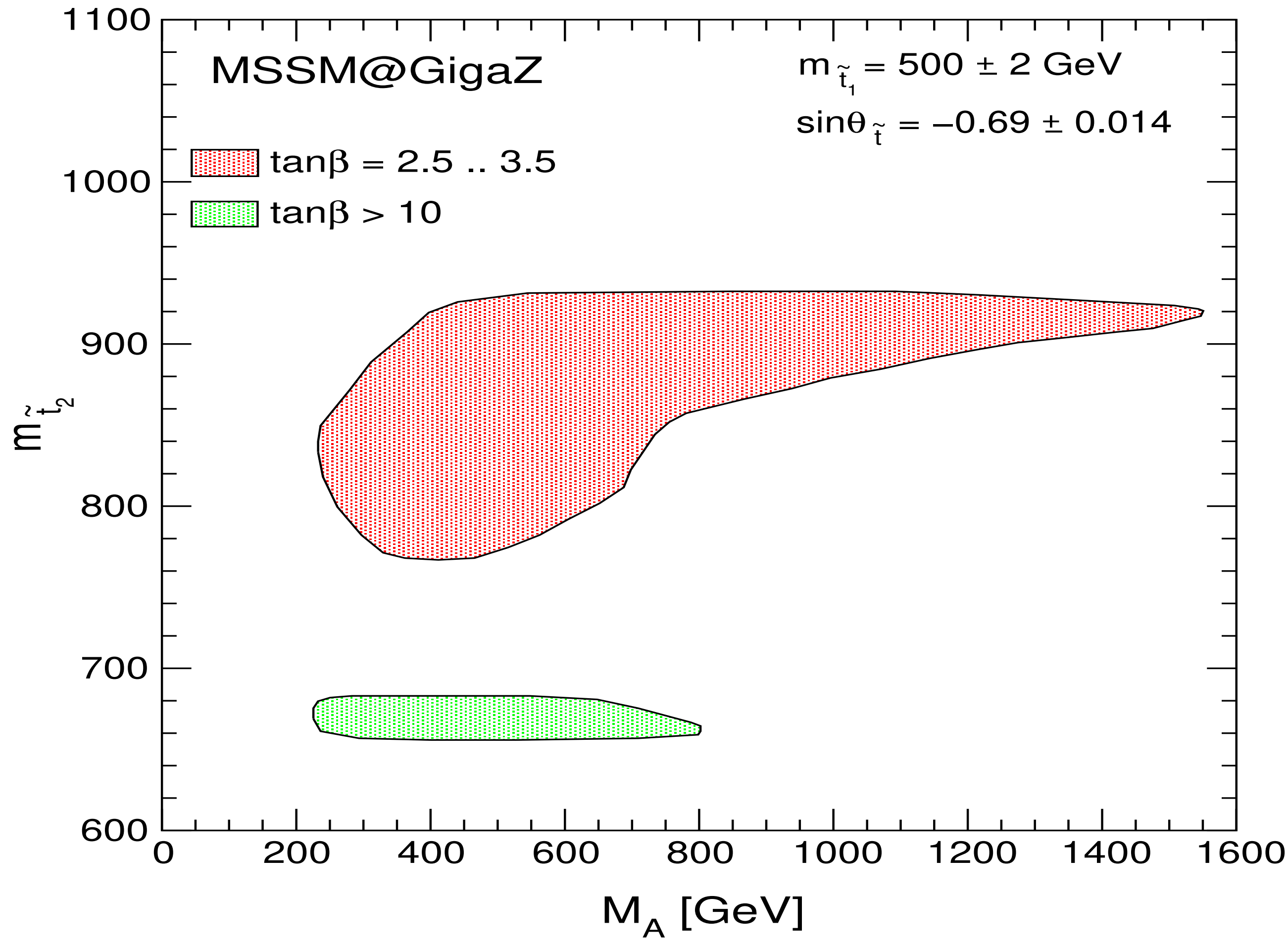
$\to M_W, \sin^2\theta_{\rm eff}, m_h$:

$M_W = 80.4 \pm 0.006$ GeV, $\sin^2\theta_{\rm eff} = 0.23140 \pm 0.00001$
$m_h = 115 \pm 0.05$ GeV, $\delta m_h^{\rm theo} = \pm 0.5$ GeV

$2.5 < \tan\beta < 3.5$ or $\tan\beta \geq 10$
(from measurements in gaugino sector)

$m_{\tilde{t}_1} = 500 \pm 2$ GeV, $\sin\theta_{\tilde{t}} = -0.69 \pm 2\%$, $A_b = A_t \pm 10\%$,
$\mu = -200 \pm 1$ GeV, $M_2 = 400 \pm 2$ GeV, $m_{\tilde{g}} = 500 \pm 10$ GeV

Allowed region in M_A-$m_{\tilde{t}_2}$ plane:



$\Rightarrow$ Constraints on $m_{\tilde{t}_2}$, upper bound on M_A

Scenario III:

[J. Erler, S.H., W. Hollik, G. Weiglein and P. Zerwas '00]

no experimental information on $\tan\beta$, M_A

$m_{\tilde{t}_2} = 520 \pm 1$ GeV or $m_{\tilde{t}_2} = 640 \pm 10$ GeV

$M_W = 80.4 \pm 0.006$ GeV, $\sin^2\theta_{\rm eff} = 0.23138 \pm 0.00001$

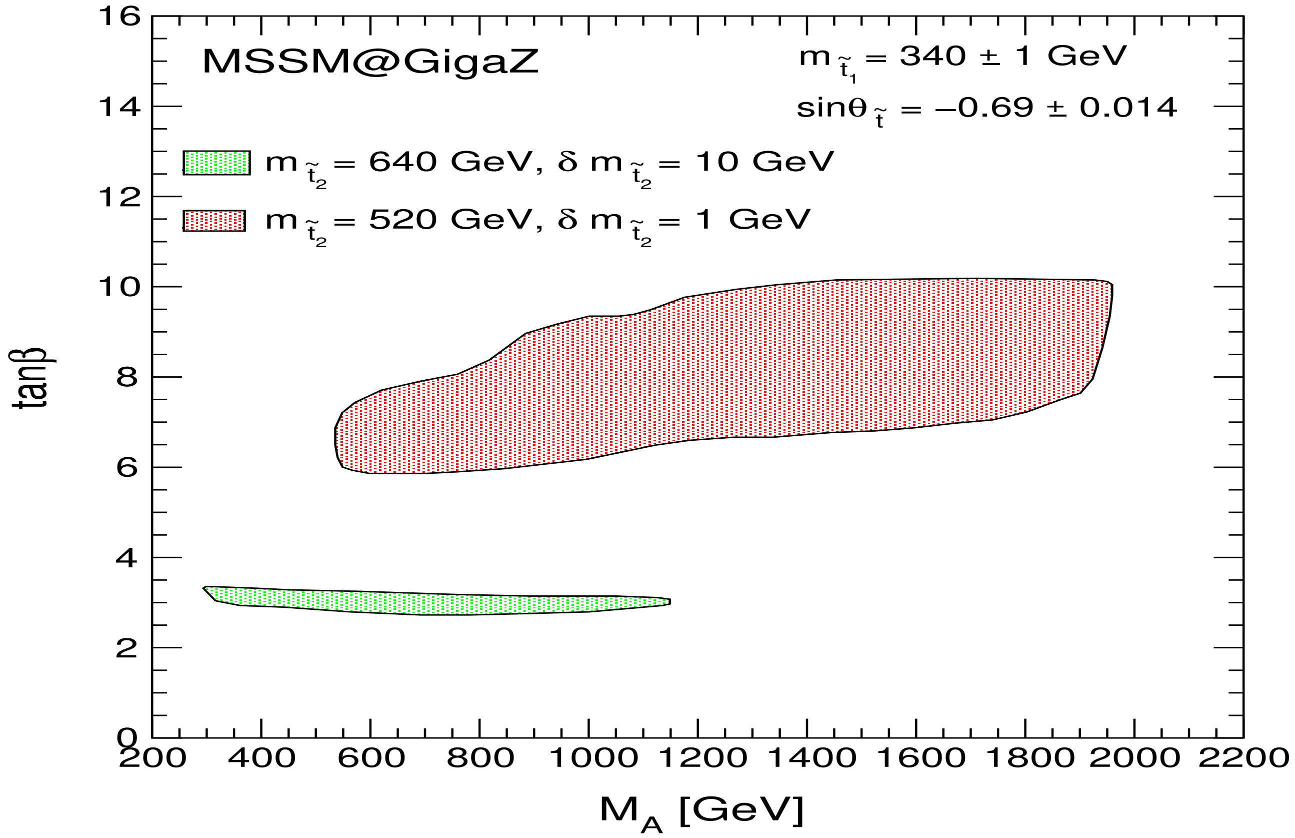
$m_h = 110 \pm 0.05$ GeV, $\delta m_h^{\rm theo} = \pm 0.5$ GeV

$m_{\tilde{t}_1} = 340 \pm 1$ GeV, $\sin\theta_{\tilde{t}} = -0.69 \pm 2\%$,

$A_b = -640 \pm 60$ GeV, $\mu = 316 \pm 1$ GeV,

$M_2 = 152 \pm 2$ GeV, $m_{\tilde{g}} = 496 \pm 10$ GeV

Allowed region in M_A-$\tan\beta$ plane:



⇒ Constraints on $\tan\beta$, upper bound on M_A

4. Conclusinos

- **GigaZ:** $\rightarrow$ $\delta \sin^2\theta_{\rm eff} = 0.00001$
  $\delta M_W = 6$ MeV,

  Combined with high precision of LC for
  m_h , m_t, $m_{\tilde{t}}$, . . .

  $\Rightarrow$ Highly sensitive test of SM and MSSM

- SM:
  Indirect determination of M_H:

  $$\delta M_H/M_H \approx 7\%$$

- MSSM:
  m_h is precision observable
  - Combination of direct and indirect constraints in the $\tilde{t}$ sector:
    determination of $m_{\tilde{t}_2}$
  - Constraints on $\tan\beta$
  - upper bound on M_A possible

- High sensitivity to scales beyond SM and MSSM

What would YOU do
with 1 billion Z bosons ?