The Why & How of X-Ray Timing

Z. Arzoumanian
(USRA/NASA-GSFC)
with thanks to C. Markwardt

- Why should I be interested?
- What are the tools?
- What should I do?

Typical Sources of X-Ray Variability

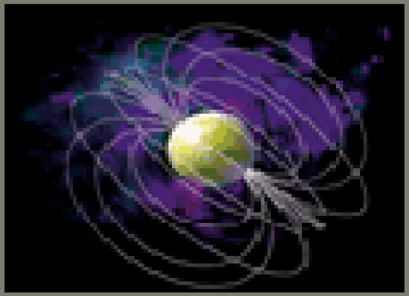
- Isolated pulsars (ms–10 s)
- X-ray binary systems
  - Accreting pulsars (ms–10s s)
  - Eclipses (10s min–days)
  - Accretion disks (~ms–years)
- Flaring stars & X-ray bursts

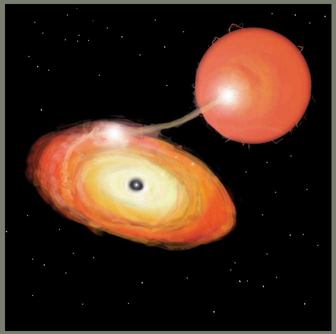
- Probably *not* supernova remnants, clusters, or the ISM
- But there could be variable serendipitous sources in the field, especially in *Chandra* and *XMM* observations

In short, stellar-sized objects (& super-massive black holes?)

Why should I be interested?

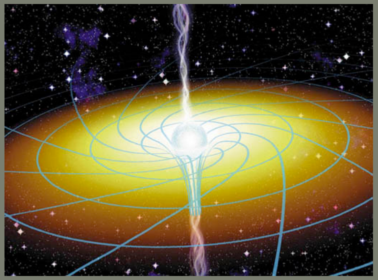
- Rotation of stellar bodies
  - pulsation period
  - stability of rotation



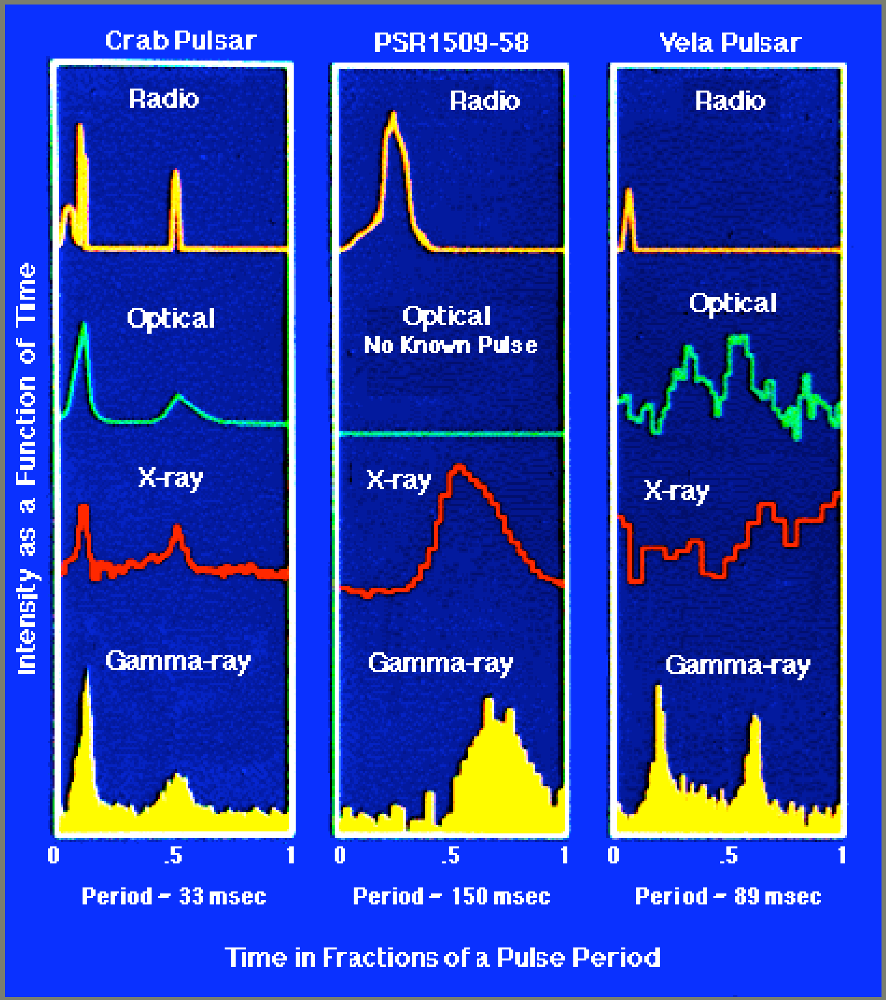


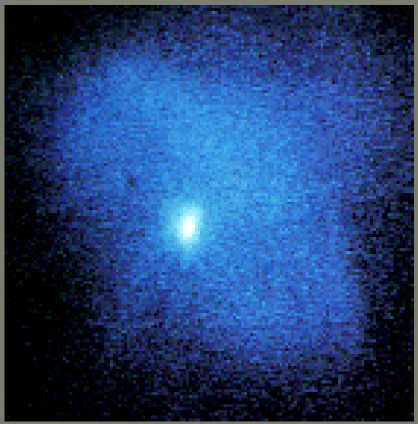
- Binary orbits
  - orbital period
  - sizes of emission regions and occulting objects
  - orbital evolution

- Accretion phenomena
  - broadband variability
  - “quasiperiodic” oscillations (QPOs)
  - bursts & “superbursts”

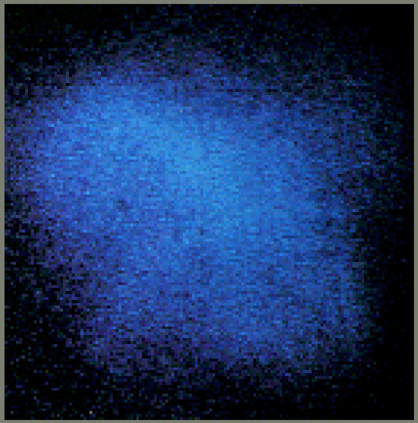


Rotational modulation: Pulsars



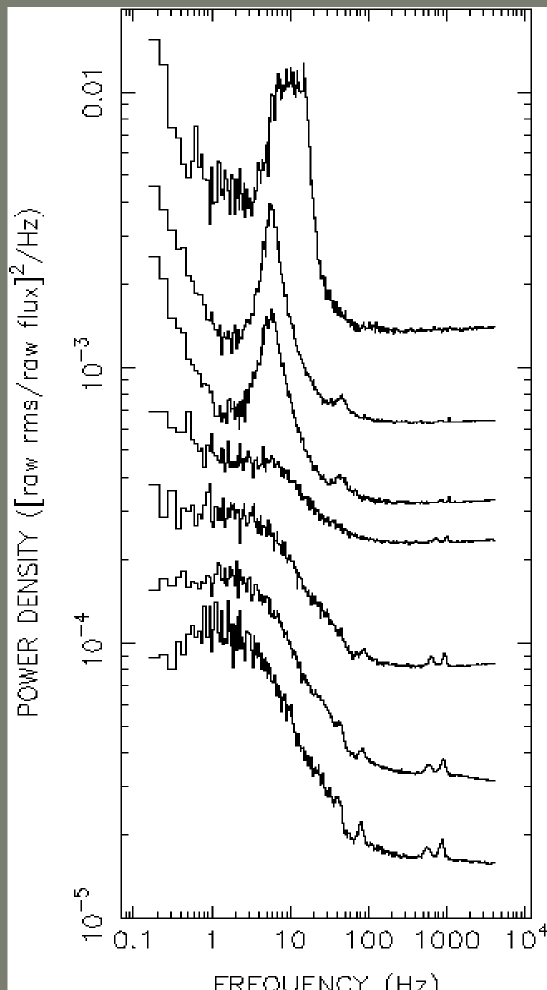


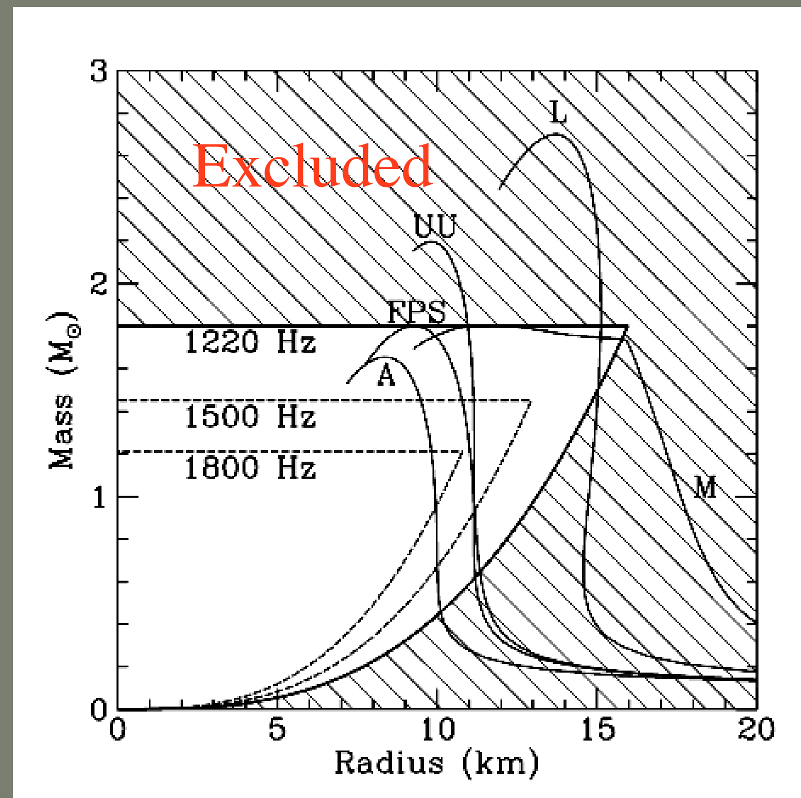
Crab pulsar



Accretion Science: QPOs from Neutron Star Binaries

Sco X-1

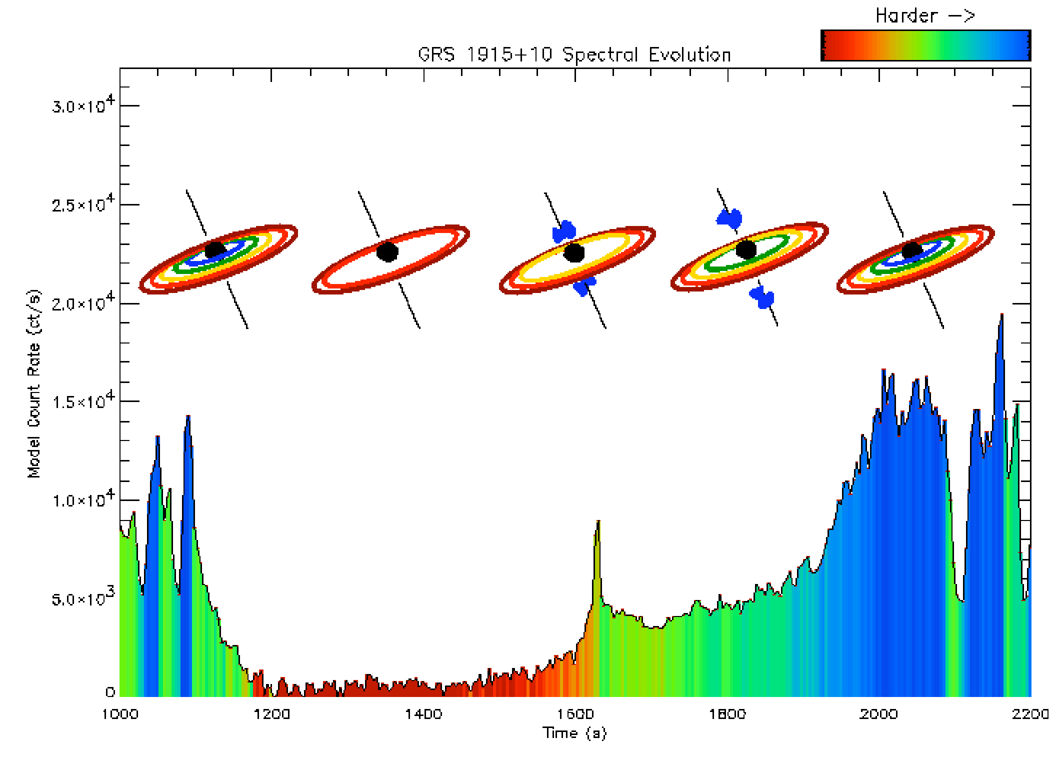




kHz QPO maximum frequency constrains NS equations of state

Questions that timing analysis should address

- Does the X-ray intensity vary with time?
- On what timescales?
  - Periodic or *a*periodic? What frequency?
  - How coherent? (Q-value)
- Amplitude of variability
  - (Fractional) RMS?
- Any variation with time of these parameters?
- Any correlated changes in spectral properties or emissions at other wavelengths?



Basics

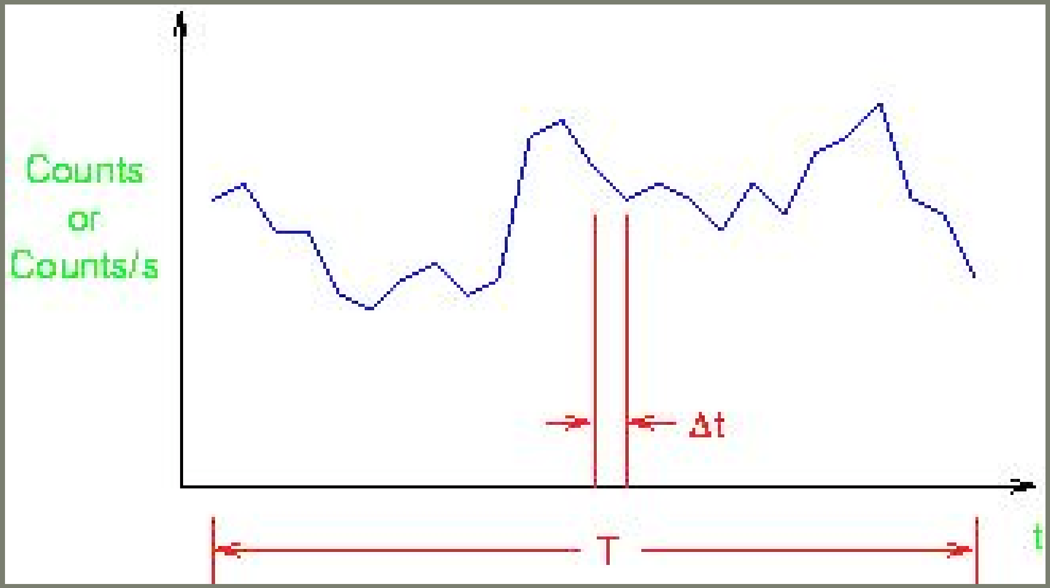
A *light curve* (for each source in the FOV) is a good first step

- Sampling interval Δt and frequency $f_{samp} = 1/\Delta t$

- *Nyquist* frequency,

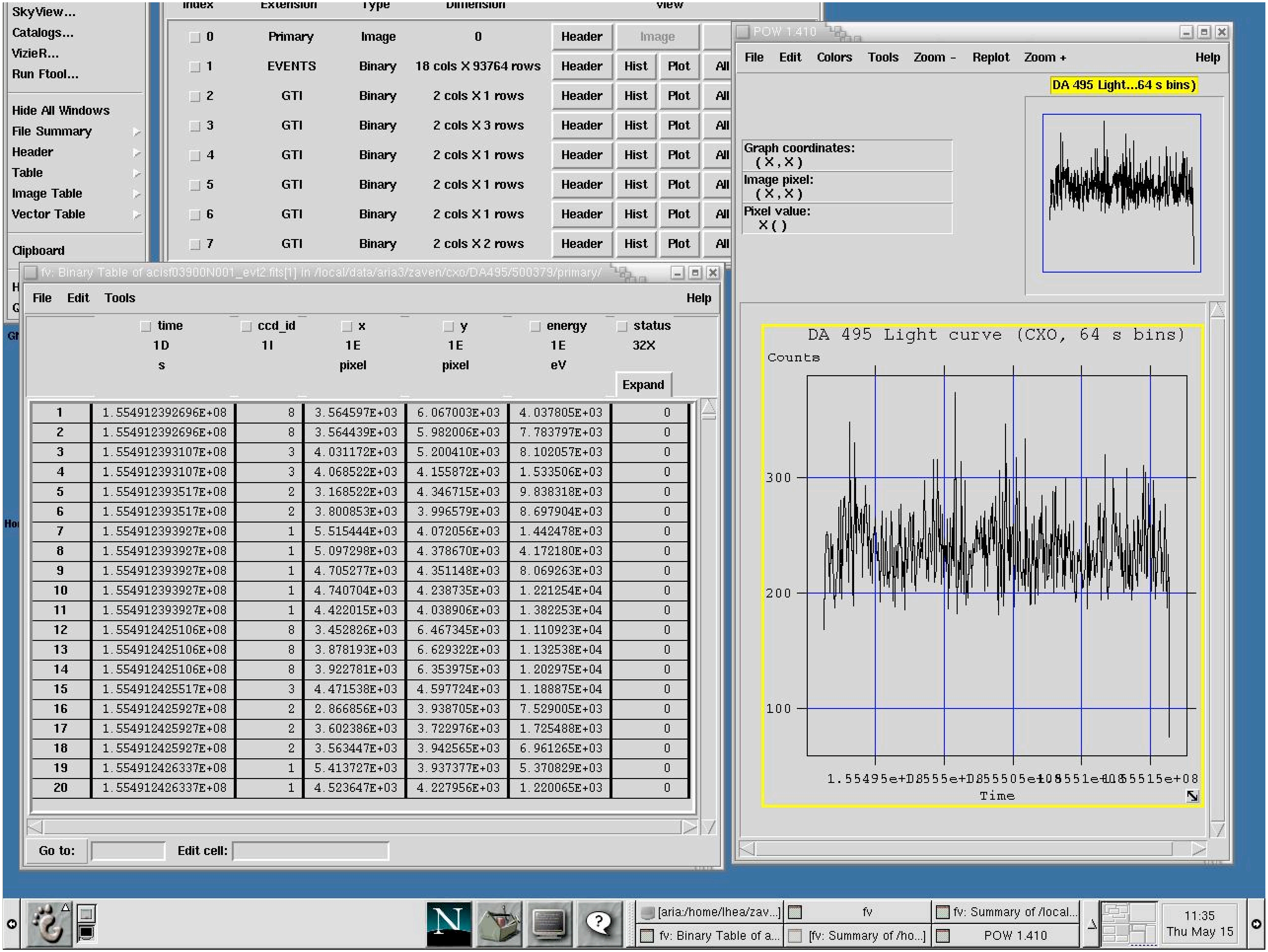
  $f_{Nyq} = {}^{1}/_{2}\, f_{samp}$,

  is the highest signal frequency that can be accurately recovered



- Basic variability measure, variance: $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$

  $\sigma \equiv$ Root Mean Square



SkyView...
Catalogs...
VizieR...
Run Ftool...
Hide All Windows
File Summary
Header
Table
Image Table
Vector Table
Clipboard

Index	Extension	Type	Dimension	View			
0	Primary	Image	0	Header	Image		
1	EVENTS	Binary	18 cols X 93764 rows	Header	Hist	Plot	All
2	GTI	Binary	2 cols X 1 rows	Header	Hist	Plot	All
3	GTI	Binary	2 cols X 3 rows	Header	Hist	Plot	All
4	GTI	Binary	2 cols X 1 rows	Header	Hist	Plot	All
5	GTI	Binary	2 cols X 1 rows	Header	Hist	Plot	All
6	GTI	Binary	2 cols X 1 rows	Header	Hist	Plot	All
7	GTI	Binary	2 cols X 2 rows	Header	Hist	Plot	All

fv: Binary Table of acisf03900N001_evt2.fits[1] in /local/data/aria3/zaven/cxo/DA495/500379/primary/
File Edit Tools Help

	time	ccd_id	x	y	energy	status
	1D	1I	1E	1E	1E	32X
	s		pixel	pixel	eV	Expand
1	1.554912392696E+08	8	3.564597E+03	6.067003E+03	4.037805E+03	0
2	1.554912392696E+08	8	3.564439E+03	5.982006E+03	7.783797E+03	0
3	1.554912393107E+08	3	4.031172E+03	5.200410E+03	8.102057E+03	0
4	1.554912393107E+08	3	4.068522E+03	4.155872E+03	1.533506E+03	0
5	1.554912393517E+08	2	3.168522E+03	4.346715E+03	9.838318E+03	0
6	1.554912393517E+08	2	3.800853E+03	3.996579E+03	8.697904E+03	0
7	1.554912393927E+08	1	5.515444E+03	4.072056E+03	1.442478E+03	0
8	1.554912393927E+08	1	5.097298E+03	4.378670E+03	4.172180E+03	0
9	1.554912393927E+08	1	4.705277E+03	4.351148E+03	8.069263E+03	0
10	1.554912393927E+08	1	4.740704E+03	4.238735E+03	1.221254E+04	0
11	1.554912393927E+08	1	4.422015E+03	4.038906E+03	1.382253E+04	0
12	1.554912425106E+08	8	3.452826E+03	6.467345E+03	1.110923E+04	0
13	1.554912425106E+08	8	3.878193E+03	6.629322E+03	1.132538E+04	0
14	1.554912425106E+08	8	3.922781E+03	6.353975E+03	1.202975E+04	0
15	1.554912425517E+08	3	4.471538E+03	4.597724E+03	1.188875E+04	0
16	1.554912425927E+08	2	2.866856E+03	3.938705E+03	7.529005E+03	0
17	1.554912425927E+08	2	3.602386E+03	3.722976E+03	1.725488E+03	0
18	1.554912425927E+08	2	3.563447E+03	3.942565E+03	6.961265E+03	0
19	1.554912426337E+08	1	5.413727E+03	3.937377E+03	5.370829E+03	0
20	1.554912426337E+08	1	4.523647E+03	4.227956E+03	1.220065E+03	0

Go to: Edit cell:

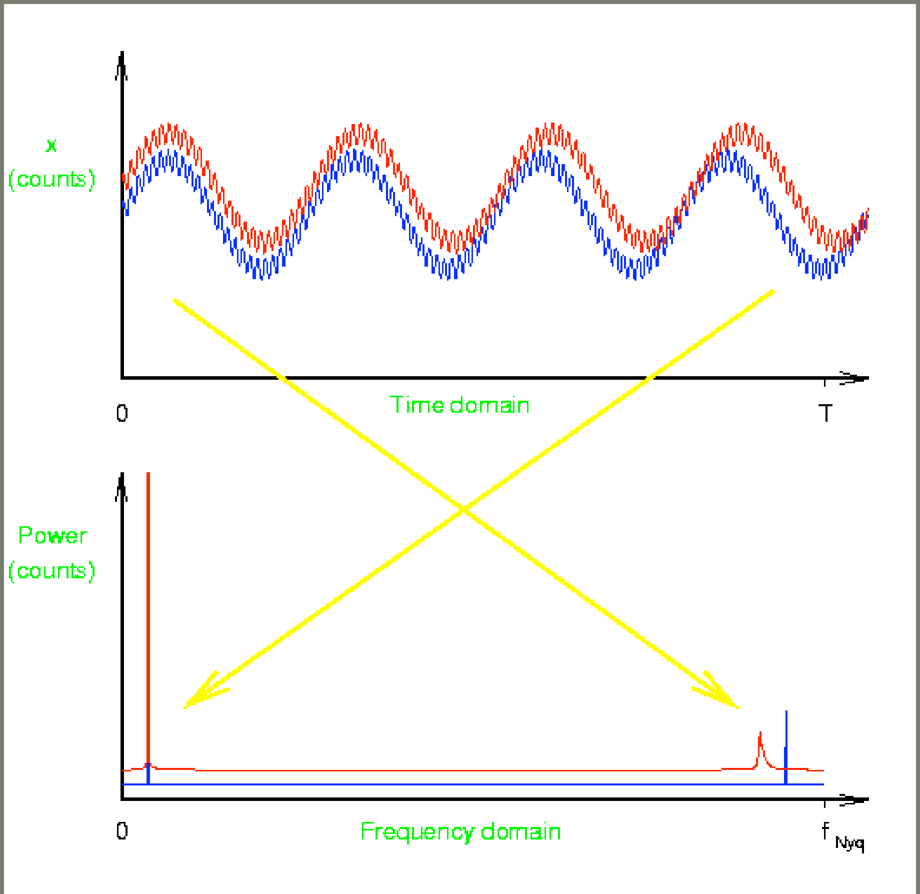
POW 1.410
File Edit Colors Tools Zoom - Replot Zoom + Help
DA 495 Light...64 s bins)
Graph coordinates: (X , X)
Image pixel: (X , X)
Pixel value: X ( )
DA 495 Light curve (CXO, 64 s bins)
Counts
300
200
100
1.55495e+08 1.555e+08 1.55505e+08 1.5551e+08 1.55515e+08
Time

[aria:/home/lhea/zav...] fv fv: Summary of /local...
fv: Binary Table of a... [fv: Summary of /ho...] POW 1.410
11:35
Thu May 15

Fourier Analysis

Answers the question: how is the variability of a source distributed in frequency?

- Long-timescale variations appear in low-frequency spectral bins, short-timescale variations in high-frequency bins
- If time-domain signal varies with non-constant frequency, spectral response is smeared over several bins

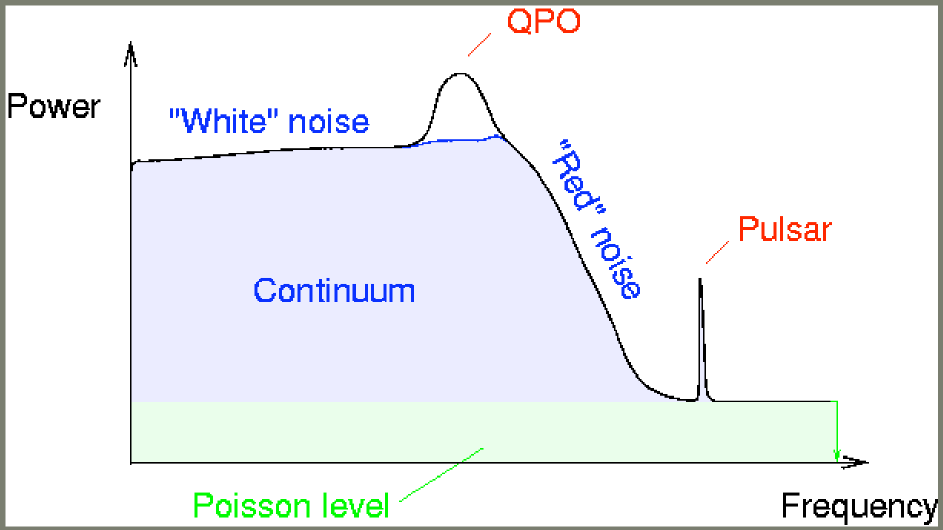


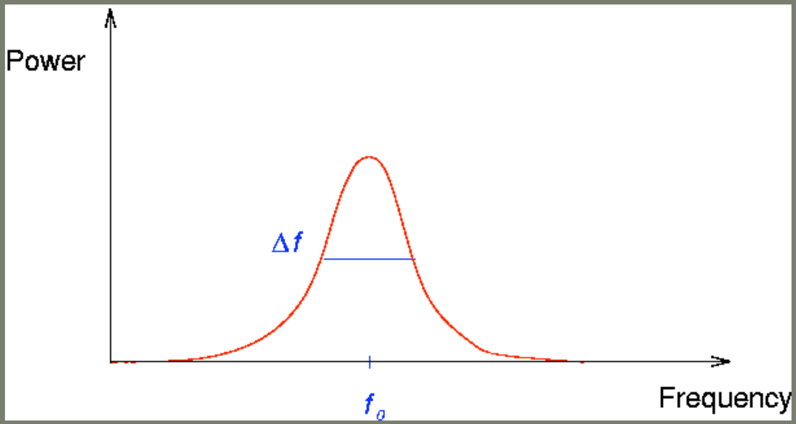
What is a QPO?

A *quasiperiodic oscillation* is a “sloppy” oscillation—can be due to:

- intrinsic frequency variations
- finite lifetime
- amplitude modulation

Q-value = $f_o / \Delta f$





Fourier Transform & FFT

- Given a light curve $\{x\}$ with N samples, Fourier coefficients are:

  $$a_j = \sum_k x_k \exp(2\pi i jk/N),$$
  $$[\, j = -{}^{N}/_{2},\ldots,0,{}^{N}/_{2}-1],$$

  usually computed with a Fast Fourier Transform (FFT) algorithm, e.g., with the `powspec` tool.

- Power density spectrum (PDS):

  $$P_j = {}^{2}/_{N_{\mathrm{ph}}}\, |a_j|^2$$
  [Leahy normalization]

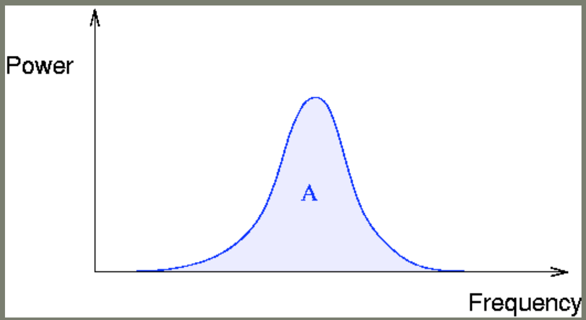
  Use ${}^{Pj}/_{\langle CR\rangle}$ (fractional RMS normalization) to plot $({}^{\mathrm{rms}}/_{\mathrm{mean}})^2\ \mathrm{Hz}^{-1}$, often displayed and rebinned in a log-log plot.

Estimating Variability from observations

- Find "area" A under curve in power spectrum,

  $$A = \int P\, d\nu \approx \sum_j P_j\, \Delta\nu,$$

  where P_j are the PDS values, and $\Delta\nu = {}^1/_T$ is the Nyquist spacing.



- Fractional RMS is

  $$r = \sqrt{{}^A/_{<CR>}}$$

- For coherent pulsations,

  $$f_p = \sqrt{{}^{2(P-2)}/_{<CR>}}$$

  is the *pulsed fraction*, i.e., (peak–mean)/mean

Estimating Variability for Proposals

To estimate amplitude of variations, or exposure time, for a desired significance level…

- Broadband noise:

$$r^2 = {2 n_\sigma \sqrt{\Delta\nu}}/{I\sqrt{T}}$$

where

r — RMS fraction

n_σ — number of "sigma" of statistical significance demanded

$\Delta\nu$ — frequency bandwidth (e.g., width of QPO)

I — count rate

T — exposure time

- Coherent pulsations:

$$f_p = {4\, n_\sigma}/{I\, T}$$

- Example:

X-ray binary, 0–10 Hz, 3σ detection, 5 ct/s source, 10 ks exposure

➾ 3.8% threshold RMS

Power Spectrum Statistics

- Any form of noise will contribute to the PDS, including Poisson (counting) noise
- Distributed as χ^2 with 2 degrees of freedom (d.o.f.) for the Leahy normalization
- Good—Hypothesis testing used in, e.g., spectroscopy also works for a PDS
- Bad —mean value is 2, variance is 4!
  ⇨ Typical noise measurement is 2±2
- Adding more lightcurve points won't help: makes more finely spaced frequencies

Statistics: Solutions

- Average adjacent frequency bins
- Divide up data into segments, make power spectra, average them (essentially the same thing)
- Averaging M bins together results in noise distributed as χ^2/M with $2M$ d.o.f. ⇒ for hypothesis testing, still chi-squared, but with more d.o.f.

- *However*, in detecting a source, you examine many Fourier bins, perhaps all of them. Thus, the significance must be reduced by the number of trials. Confidence is

$$C = 1 - N_{\text{bins}} \times \text{Prob}(MP_j, 2M),$$

where N_{bins} is the number of PDS bins (i.e., trials), and $\text{Prob}(\chi^2, \nu)$ is the hypothesis test.

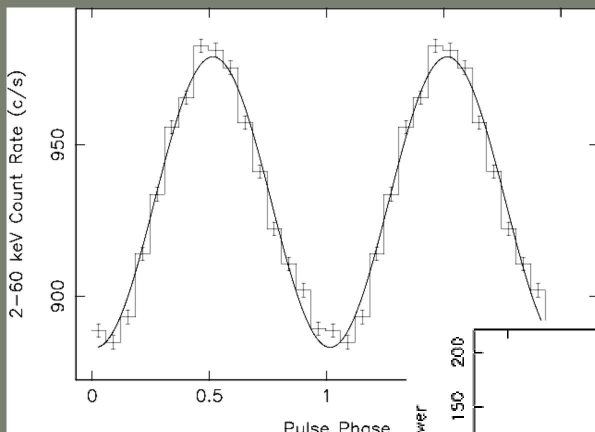
Tips

- Pulsar (coherent pulsation) searches are most sensitive when *no rebinning* is done
- QPO searches need to be done with *multiple rebinning* scales
- Beware of signals introduced by
  - instrument, e.g., CCD read time
  - dead time
  - orbit of spacecraft
  - rotation period of Earth (and harmonics)

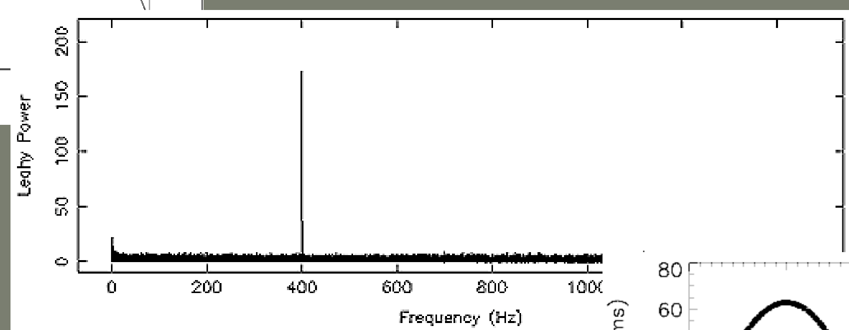
What To Do

- Step 1. Create light curves for each source in your field of view ➪ inspect for features, e.g., eclipses. Usually, this is enough to know whether to proceed with more detailed analysis, but *you can't always see variability by eye!*
- Step 2. Power spectrum. Run `powspec` or equivalent and search for peaks. A good starting point, e.g., for RXTE, is to use FFT lengths of ~ 500 s.

SAX J1808: The First Accreting Millisecond Pulsar

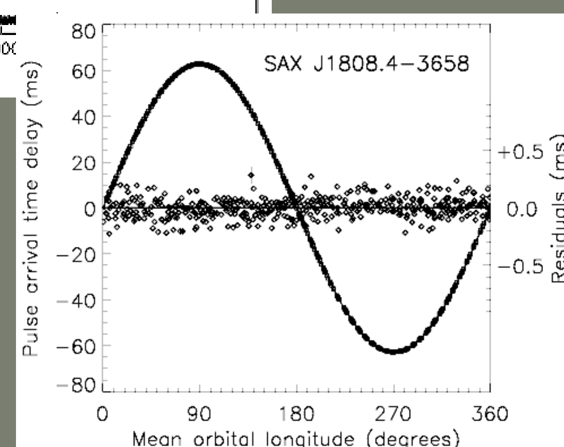


Lightcurve



PDS

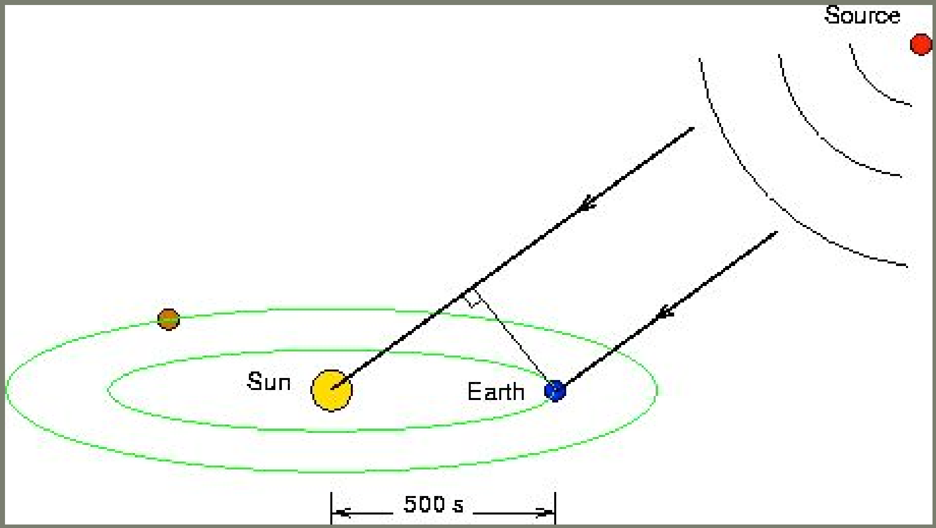
Residuals



Step 3. Pulsar or eclipses found?

Refine timing analysis to boost signal-to-noise.

- *Barycenter* the data: corrects to arrival times at solar system's center of mass (tools: `fxbary/axbary`)



- Refined timing
  - Epoch folding (`efold`)
  - Rayleigh statistic (Z^2)
  - Arrival time analysis (Princeton `TEMPO?`)

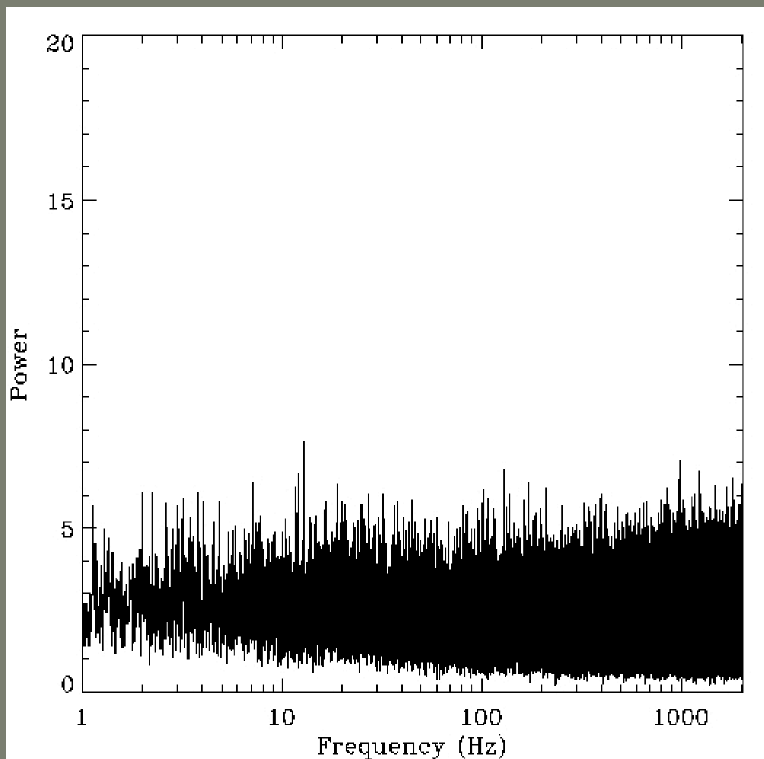
- Hint: best to do timing analysis (e.g., epoch folding especially) on segments of data if they span a long time baseline, rather than all at once.

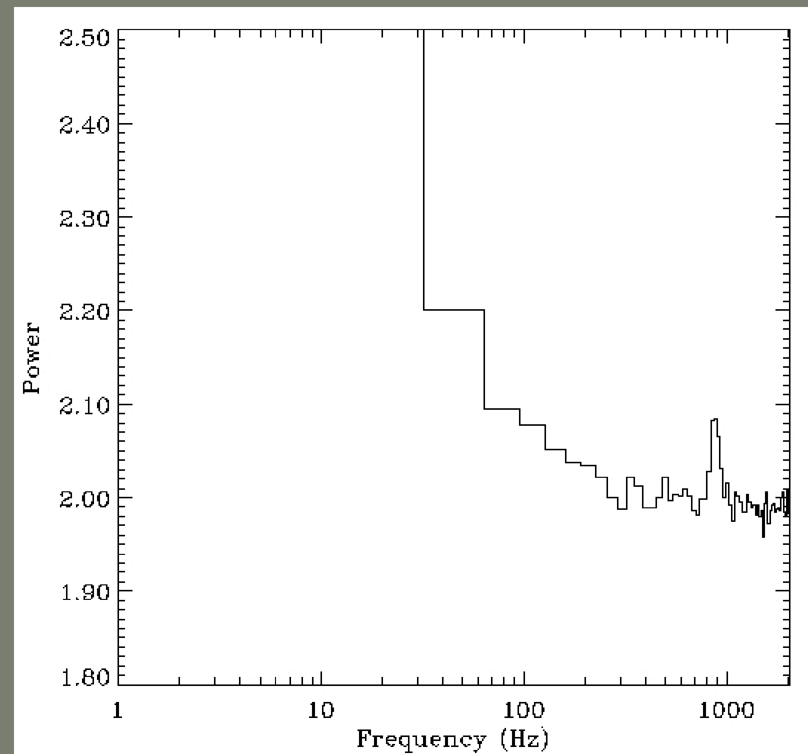
Step 3. Broadband feature(s) found?

Refined analysis best done interactively (IDL? MatLab?).

- Plot PDS
- Use χ^2 hypothesis testing to derive significance of features
- Rebin PDS as necessary to optimize significance
- If detected with good significance, fit to simple-to-integrate model(s), e.g., gaussian or broken power-law
- Compute RMS

Rebinning to find a QPO: 4U1728–34





- To detect a weak QPO buried in a noisy spectrum, finding the right frequency resolution is essential!

Suggested Reading

- van der Klis, M. 1989, *Fourier Techniques in X-ray Timing*, in Timing Neutron Stars, NATO ASI 282, Ögelman & van den Heuvel eds., Kluwer
- Press et al., *Numerical Recipes*
  - power spectrum basics
  - Lomb-Scargle periodogram
- Leahy et al. 1983, ApJ 266, 160
  - FFTs; power spectra; statistics; pulsars
- Leahy et al. 1983, ApJ 272, 256
  - epoch folding; Z^2
- Vaughan et al. 1994, ApJ 435, 362
  - noise statistics