Q, Break-even and the $n\tau_E$ Diagram for Transient Fusion Plasmas

Dale M. Meade
Princeton University
P.O. Box 451, Princeton, N. J. 08543

Abstract - Q, break-even and the $n\tau_E$ diagram are well defined and understood for steady-state fusion plasma conditions. Since many fusion experiments are transient, it is necessary to clarify the definitions for instantaneous Q values and break-even so that the $n\tau_E$ diagram can be interpreted for transient plasma conditions. This discussion shows that there are two mathematically correct methods to describe the $n\tau_E$ diagram for a transient plasma. The Lawson/TFTR method which is consistent with previous analyses of the Lawson cycle, and prior definitions for Q and break-even describes a transient fusion plasma in terms of Q = P_{fusion}/P_{aux} with the plasma energy confinement time for the $n\tau$ diagram given by $\tau_E{}^* = W_p / P_{heat}$ where W_p is the total plasma kinetic energy and $P_{heat} = P_{aux} + P_{alpha} - P_{brem}$ is the net power heating the plasma. In the Lawson/TFTR definition break-even ($P_{fusion} = P_{aux}$) occurs at Q=1, ignition occurs at Q = infinity and the $n\tau_E{}^*$ values required to achieve a given Q are the same in transient and steady-state plasmas. The JET/JT-60 method uses the definitions of $Q^* = P_{fusion}/(P_{aux} - dWp/dt)$ and $\tau_E = W_p/(P_{heat} - dWp/dt)$. This method produces the confusing result that break-even requires $Q^* = P_{aux}/(P_{aux} - dWp/dt)$ which is >1 for many cases of interest. In addition, the $n\tau_E$ value required to achieve break-even depends on dWp/dt and therefore experimental data points with different dWp/dt must be compared to different Q* curves on the Lawson diagram. For a pulsed plasma, this issue can be avoided by using the definition of fusion gain first introduced by Lawson, namely Q = fusion energy per pulse divided by auxiliary plasma heating energy supplied per pulse.

I. INTRODUCTION

The original paper by J. D. Lawson [1] analyzed the requirements for producing net fusion power from a driven fusion system including transient plasma conditions. Lawson defined a fusion gain parameter R = "ratio of the (fusion) energy released in the hot gas to the energy supplied" for a pulsed system where the energy released related to that released for the entire pulse. During the mid 1970's the detailed requirements for producing fusion power by utilizing specific steady-state driven plasma systems were analyzed [2-4]. A typical system is illustrated schematically in Fig. 1 where auxiliary power injected into the plasma is the input power and the fusion power produced is the output power. These analyses defined a steady-state fusion power gain, Q = $P_{output}/P_{input} = P_{fusion}/P_{auxiliary}$, similar to the original definition used by Lawson [1].

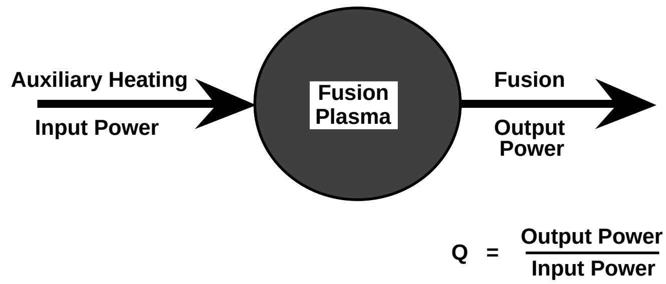

Fig. 1. Standard power balance definitions.

During the mid to late 1970s specific proposals were made for the construction of several large tokamaks that were to produce and study reactor-like plasma conditions. The JET Project Design Proposal [5] defined Q as "Ratio of total fusion power released to the additional power (e.g. injected by neutral beam); Q = 1 corresponds to the "break-even" condition." TFTR also used the same definition for Q and "break-even" in the TFTR Reference Design Report [6]. JT-60 also defined Q and break-even in the traditional way [7]. In the JET and TFTR proposals Q referred explicitly to the actual fusion power produced and did not address the issues associated with using a fusion power projected from different fuels or conditions.

In many high performance fusion experiments the plasma does not reach steady-state conditions and the question has been raised regarding the definition of instantaneous values of Q for a transient plasma. Consider the time evolution of fusion power shown in Fig. 2 where constant input power

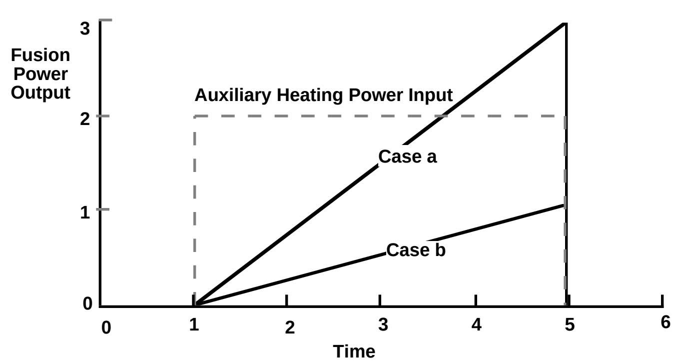

Fig. 2. Fusion power output for constant input power.

produces a rising fusion power output versus time that eventually terminates. The definition for the instantaneous Q value used by TFTR [6] is simply the instantaneous value of $P_{output}/P_{input} = P_{fusion}/P_{auxiliary}$ which involves only quantities external to the plasma. This is the quantity of interest for fusion power applications. This definition could be extended to average over the fusion power pulse, namely $\langle Q \rangle = \langle P_{fusion} \rangle / \langle P_{auxiliary} \rangle$ where the brackets < > indicate a time average over the pulse which would be the same as the original definition used by Lawson [1] and is the quantity of interest for the International Thermonuclear Experimental Reactor (ITER) which is being designed to have a fusion power pulse length much longer than the plasma energy confinement time. Consider the two fusion plasmas illustrated in Fig. 2, case a achieves break-even while case b does not using the TFTR definition. In 1990, JET adopted a revised definition of Q that included transient internal plasma dynamics [8]. In particular, it was argued that input power required to increase the internal plasma energy, dWp/dt, should be subtracted from the input power resulting in $Q^* = P_{output}/(P_{input} - dWp/dt) = P_{fusion}/(P_{auxiliary} - dWp/dt)$. Consider the plasmas in Fig. 2 with $dWp/dt = 0.5\ P_{auxiliary}$, now cases a and b have $Q^* > 1$. Note that case b has not achieved break-even, since for case b break-even requires $Q^* = 2$ to achieve fusion power output equals auxiliary heating power input. Neither of these cases would achieve break-even using the definition of relevance to ITER.

II. DERIVATION OF THE LAWSON ($n\tau_E$) DIAGRAM

The $n\tau_E$ diagram is based on the energy balance equation for the plasma following the analysis first developed by Lawson [1]. A number of papers [3,4,8,9] have assessed the effect of fuel, plasma profiles, ion energy distribution, T_i/T_e, and impurity content on the Lawson diagram for a specific steady-state plasma. In this note the essential features needed to analyze the Lawson diagram for a transient plasma are illustrated by considering a thermal plasma with $T_i = T_e = T$, $Z = 1$, and parameters constant throughout the plasma volume, V_p,. The plasma power balance is given by

$$P_{heat} = P_{aux} + P_{alpha} - P_{brem} = P_{transp} + dWp/dt \quad (1)$$

where P_{heat} is the net power heating the plasma, P_{aux} is the auxiliary plasma heating power, P_{alpha} is the alpha power heating the plasma, P_{brem} is the bremsstrahlung loss, P_{transp} is the transport loss and dWp/dt is the power required to increase the thermal energy of the plasma. Equation (1) can be expanded into the standard form of

$$P_{aux} + n^2 \langle\sigma v\rangle U_\alpha V_p/4 - C_R T^{1/2} n_e^2 V_p = 3nkTV_p/\tau_E + d(3nkTV_p)/dt, \quad (2)$$

where $n_D = n_T = n_e/2 = n/2$, $n^2 \langle\sigma v\rangle U_\alpha V_p/4 = P_\alpha$ is the alpha heating power, $C_R T^{1/2} n_e^2 V_p$ is the radiation loss, Wp $= 3nkTV_p$ and $\tau_E = W_p/(P_{heat} - dW_p/dt)$ is the energy confinement time. Using the conventional definition for fusion power multiplication, $Q = P_{fusion} / P_{aux} = 5P_\alpha / P_{aux}$, eqn. (1) becomes

$$n^2 \langle\sigma v\rangle U_\alpha (Q+5)/4Q - C_R T^{1/2} n^2 = 3nkT/\tau_E + d(3nkT)/dt \quad (3)$$

For the steady-state case d/dt = 0, and eqn. (3) yields

$$n\tau_E = \frac{3kT}{\langle\sigma v\rangle U_\alpha (Q+5)/4Q - C_R T^{1/2}} \quad (4)$$

$n\tau_E$, with $\tau_E = W_p/P_{heat}$, is traditionally plotted as a function of T with Q as a parameter. Steady-state plasmas with the same value of T and Q have the same $n\tau_E$ and break-even occurs at Q = 1 by definition of Q. This formulation is the standard description of the $n\tau$ diagram for a steady-state plasma.

For a transient plasma, there are two mathematically correct ways to rearrange the terms in eqn. (1) to include the dWp/dt term. However, these two methods result in different definitions for Q and $n\tau$, and break-even does not occur at Q =1 in one method.

III. $n\tau_E$ DIAGRAM FOR A TRANSIENT PLASMA USING THE LAWSON/TFTR METHOD

The transient plasma case using the Lawson/TFTR method follows by noting that the right hand side of eqn. (3) has the properties of a net plasma loss consisting of the normal transport energy loss, $3nkT/\tau_E$, and the d(3nkT)/dt "loss" acting in parallel. The right hand side of eqn. (3) can be rewritten in terms of an effective Lawson confinement time, τ_E^*, for the net loss given by

$$3nkT/\tau_E^* = 3nkT/\tau_E + d(3nkT)/dt \quad (5)$$

$$= 3nkT/(W_p/(P_{heat} - dW_p/dt)) + d(3nkT)/dt$$

Noting that $W_p = 3nkTV_p$, eqn. (4) becomes

$$3nkT/\tau_E^* = 3nkT/(W_p/P_{heat}) \quad (6)$$

and therefore $\tau_E^* = W_p / P_{heat}$. (7)

Using definition given by eqn. (7), the power balance for a transient plasma given by eqn. (3) can be rewritten as

$$n^2 \langle\sigma v\rangle U_\alpha (Q+5)/4Q - C_R T^{1/2} n^2 = 3nkT/\tau_E^* \quad (8)$$

and the Lawson diagram for a transient plasma is given by

$$n\,\tau_E^* = \frac{3kT}{\langle\sigma v\rangle U_\alpha (Q+5)/4Q - C_R T^{1/2}} \quad (9)$$

with the following definitions

$$Q = P_{fusion}/P_{aux} \quad \text{and} \quad \tau_E^* = W_p / P_{heat} \,. \quad (10)$$

Since eqn. (9) has the same form as the steady-state Lawson condition given by eqn. (4), the steady-state Lawson diagram can be used to describe a transient plasma if the definitions given by eqn. (10) are followed. The Lawson/TFTR method has the advantage that break-even occurs at Q =1, and plasmas with the same T and $n\tau_E^*$ but different transient effects will have the same Q. Since the fusion performance parameter, $n\tau_E T$, relates to the $n\tau_E$ on the $n\tau_E$ diagram, this derivation gives the expression for the instantaneous Lawson fusion parameter, namely $n\tau_E^* T$, for transient plasmas. Table I contains the parameters for high $n\tau_E T$ transient plasmas in several tokamaks. The points for deuterium plasmas in JET, JT-60U and DIII-D and a D-T plasma in TFTR are plotted (Fig. 3) on the steady-state Lawson diagram for a D-T plasma with reference to the curve for break-even using the Lawson/TFTR method.

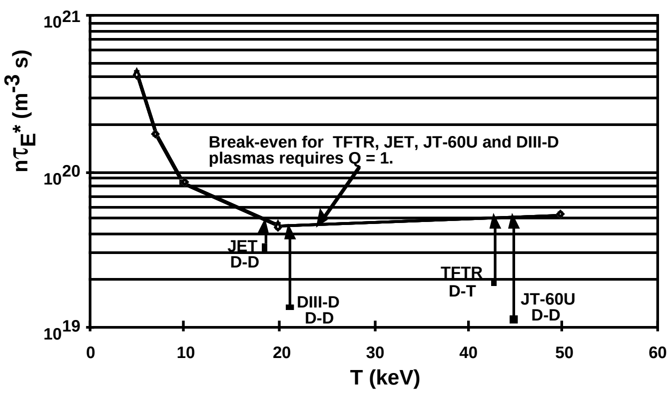

Fig. 3. $n\tau_E$ diagram for break-even using the Lawson/TFTR method.

IV. $n\tau_E$ DIAGRAM FOR A TRANSIENT PLASMA USING THE JET/JT-60 METHOD

The transient plasma case using the JET/JT-60 method [8] follows by combining the dWp/dt term with the P_{aux} term in eqn. (1)

$$(P_{aux} - dWp/dt) + P_{alpha} - P_{brem} = P_{transp} \qquad (11)$$

and defining $Q^* = P_{fusion}/(P_{aux} - dWp/dt)$ to yield

$$(1 + 5/Q^*)P_{alpha} - P_{brem} = P_{transp}. \qquad (12)$$

The Lawson diagram for a transient plasma is given by

$$n\tau_E = \frac{3kT}{\langle\sigma v\rangle\, U_\alpha\, (Q^*+5)/4Q^* - C_R T^{1/2}} \qquad (13)$$

with the following definitions

$Q^* = P_{fusion}/(P_{aux} - dWp/dt)$ and

$$\tau_E = W_p / (\, P_{heat} - dWp/dt). \qquad (14)$$

Since eqn.(13) has the same form as the steady-state Lawson condition given by eqn. (4), the steady-state Lawson diagram can be used to describe a transient plasma if the definitions given by eqn. (14) are followed. However, this method produces the confusing result that the $n\tau_E$ values required to achieve break-even depend on dWp/dt and therefore experimental data points with different dWp/dt must be compared to different Q* curves on the Lawson diagram. This dilemma is illustrated in Fig. 4 where the $n\tau_E T$ points from Table I are plotted relative to their corresponding break-even curves using the JET/JT-60 method. Also, break-even requires $Q^* = P_{aux}/(P_{aux} - dWp/dt) > 1$ for the many cases of interest which have dWp/dt > 0. For example, in cases which have dWp/dt = 0.5 P_{aux}, the curve on the $n\tau_E$ diagram for break-even requires Q* = 2. Similar confusion arises in the interpretation of plasmas near ignition using the JET/JT-60 method.

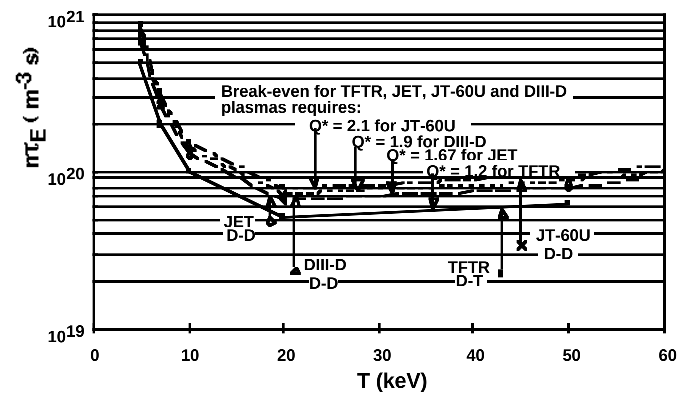

Fig. 4. $n\tau_E$ diagram for break-even using JET/JT-60 method.

V. LAWSON CYCLE

The original paper by J. D. Lawson [1] defined a fusion gain parameter R = “ratio of the (fusion) energy released in the hot gas to the energy supplied” for his analysis that led to the first Lawson diagrams for a transient plasma. In fact, he analyzed a case with no plasma transport losses (τ_E = infinity) during a pulse of duration t and derived the Lawson condition for an energy multiplication R in terms of nt and T. The same result can be obtained by integrating the instantaneous power balance equation from t = 0 to time t with no plasma transport losses (τ_E= infinity) and the dWp/dt term included.

One could analyze the case of a more general Lawson cycle where the power lost from the plasma is converted with efficiency, η, into power that heats the plasma. The power balance equation for this case is

$$\eta\ (P_{fusion} + P_{brem} + 3nkT/\tau_E) = 3nkT/\tau_E + P_{brem} + d(3nkT)/dt \quad (15)$$

In this case the flexibility to redefine η using the JET/JT-60 method does not exist and the transient case is treated by combining the P_{transp} and d(3nkT)/dt terms according to the TFTR method which results in

$$\eta\ (P_{fusion} + P_{brem} + 3nkT/\tau_E) = 3nkT/\tau_E^* + P_{brem} \quad (16)$$

VI. SUMMARY

This discussion shows that the least confusing method that connects with previous analyses of the Lawson cycle and prior definitions for Q and break-even is to describe a transient fusion plasma in terms of $Q = P_{fusion}/P_{aux}$ with break-even ($P_{fusion} = P_{aux}$) occurring at Q=1 and the plasma energy confinement time for the $n\tau_E$ diagram given by $\tau_E^* = W_p / P_{heat}$ where W_p is the total plasma kinetic energy and $P_{heat} = P_{aux} + P_{alpha} - P_{brem}$ is the net power heating the plasma.

ACKNOWLEDGMENTS

This work was supported by DOE contract No. DE-AC02-76-CHO-3073.

REFERENCES

1. J. D. Lawson, Proc. Phys. Soc. B, V 70, p 6, (1957).
2. J.M. Dawson, H. P. Furth, F. H. Tenney, Phys. Rev. Lett. V26, p1156, 1971
3. J. Kesner and R. W. Conn, Nuclear Fusion V16, p397, (1976)
4. J. F. Clarke, Nuclear Fusion V20, p563, 1980
5. JET Project Design Proposal, EUR 5791e, p A-4, (1976).
6. TFTR Reference Design Report, PPPL 1312, p 3-58, (1976).
7. JT-60 Design Proposal papers, ~1977 - 1980
8. B. Balet, J. G. Cordey and P. M. Stubberfield, 17th EPS Conference on Controlled Fusion and Plasma Heating V14B, Part I, p 106 (1990)
9. D. M. Meade, Nuclear Fusion V14, p289, (1974)
10. The JET Team, Nuclear Fusion V 32, p 187, (1992).
11. K. M. McGuire, et al, Sixteenth IAEA Fusion Energy Conference, paper F1-CN-64/01-2, 1996.
12. K. Ushigusa, et al, Sixteenth IAEA Fusion Energy Conference, paper F1-CN-64/01-3, 1996.
13. VH mode, T. Simonen, Private Communication

TABLE I.
HIGH $n\tau_E T$ SHOTS ARE COMPARED USING THE JET/JT-60 METHOD AND THE TFTR METHOD.

	TFTR[11]	JET[10]	JT-60U[12]	JT-60U[12]	DIII-D[13]
Shot Number	83546	26087	26949	26939	78136
n_i (0), $10^{20}m^{-3}$	0.66	0.41	0.43	0.43	0.72
T_i (0), keV	43	18.6	35.5	45	21.3
P_{NB}, MW	17.4	14.9	32.9	32.7	14.15
W_p, MJ	4.9	11.6	9.32	8.55	2.52
dW_p/dt, MW	3.0	6	2.3	16.9	6.72
τ_E (confinement), s	0.34	1.2	0.33	0.75	0.34
τ_E^* (Lawson), s	0.28	0.78	0.28	0.26	0.18
JET/JT-60 Method					
$n_i\tau_E T_i$, $10^{20}m^{-3}$ keV s	9.6	9.1	5.1	15.3	5.2
Break-even requires Q* =	1.2	1.67	1.08	2.1	1.9
TFTR Method					
$n_i\tau_E^* T_i$, $10^{20}m^{-3}$ keV s	8	6	4.3	5.0	2.8
Break-even requires Q =	1	1	1	1	1