Chapter 7, the first on cooling, of lectures to be given at Acc School in Vanderbilt in Jan 99, by R.B.Palmer.

7 Solenoid Cooling

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7.1 The Angular Momentum Problem

or: Why we reverse the Solenoids

In the absence of external forces and energy loss in materials, the angular momentum of a particle, about any defined axis, is conserved. But a non paraxial particle in a fixed solenoidal field, or a paraxial particle in a varying solenoidal field will cross radial field components and experience forces that will change its azimuthal (p_ϕ) and angular ($r\ p_\phi$) momentum. If the field is varying slowly, relative to the particles radius, and there is no material present, then the induced changes in azimuthal momentum are given by:

$$\Delta(p_\phi) \ = \ \frac{e}{2}\, \Delta(B_z\ r)$$

If we add an angular momentum ($(p_\phi\ r)_c$) equal to the value a particle would have before entering, or after leaving, the solenoidal field, then this addition will be conserved and:

$$p_\phi\ r \ = \ (p_\phi\ r)_c \ + \ \frac{e}{2}\, \Delta(B_z\ r)\ r$$

This additional term is defined as the "canonical" angular momentum:

$$(p_\phi\ r)_c\ r \ = \ p_\phi\ r \ - \ \frac{e}{2}\, \Delta(B_z\ r)\ r$$

If material is introduced then, neglecting scattering and straggling, all momenta, both longitudinal and transverse, will be attenuated. Acceleration can be introduced to restore the longitudinal momentum thus leading to a reduction of transverse momenta and cooling. But if this process takes place in a solenoidal field, then besides reducing the random initial transverse momenta, the process will reduce the coherent field-induced transverse momenta, and this will generate an average canonical momentum.

Consider the case of almost complete transverse cooling: all transverse momenta are reduced to near zero leaving the beam streaming parallel to the axis: down the field lines of the solenoid.

$$p_\phi \, r \; = \; 0$$

but

$$(p_\phi \, r)_c \; = \; p_\phi \, r \; - \; \frac{e}{2} \, (B_z \, r) \, r \; = \; -\frac{e}{2} \, (B_z \, r) \, r$$

where the r is the radius in the solenoid where the cooling took place. If the initial emittance is large, r is also large and the resulting angular momentum is large. When the beam exits the solenoid, to enter a quadrupole focused accelerator or collider, then this canonical angular momentum becomes a real angular momentum and represents an effective emittance. In practice, after a few cooling stages, this effective emittance becomes greater than the true emittance and effective cooling ends.

The problem is to find a way to avoid generating, or a way to remove, this canonical angular momentum. Cooling in a quadrupole channel, if strong enough focusing were possible, would not have the problem. Nor is it present in a lithium lens focused channel. This latter alternative is reasonable to be use in the final cooling stages, but it has disadvantages: it must be pulsed, and lithium has relatively more coulomb scattering than the hydrogen that can be used in solenoid channels. We have therefore explored ways of removing this coherent angular momentum in the solenoid case.

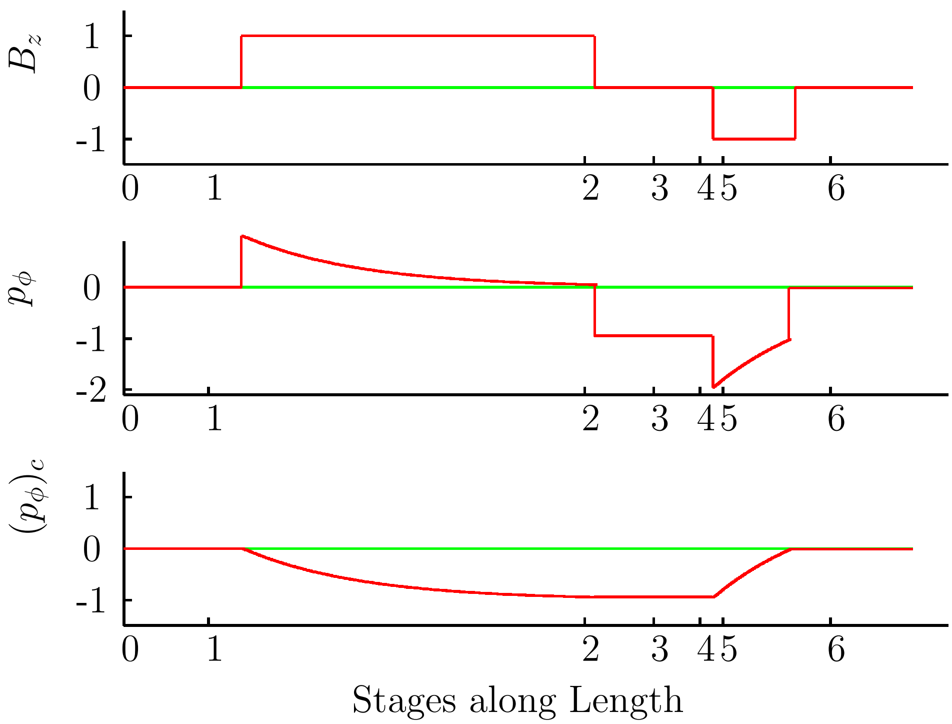
We know of only two ways of doing this, and one way to try and duck the problem:

7.1.1 Metal Grid

The first is to end the solenoidal field on an iron grid. The field lines end on the grid and are carried radially in the iron. Muons that pass through the iron are strongly deflected and lost, but those that pass through the openings emerge with relatively little deflection, and thus relatively little angular momentum. The method only works for low magnetic fields and is not very efficient.

7.1.2 Single Field Reversal Method

The second way involves, at the least, applying one further stage of ionization cooling after a field reversal.



Axial Field, Angular Momentum, and Canonical Angular Momentum, in an Ideal, Single Field Reversal, Solenoid Cooling System.

Consider a beam with initially no angular momentum:

$$(p_\phi \; r)_0 = \; 0$$

After it has entered the solenoid:

$$(p_\phi\ r)_1 \ = \ \frac{e}{2}\,(B_z\ r)\ r$$

After almost complete transverse cooling: all transverse momenta are reduced to near zero leaving the beam streaming parallel to the axis: down the field lines of the solenoid.

$$(p_\phi\ r)_2 \ = \ 0$$

After exiting the solenoid, we have real coherent angular momentum:

$$(p_\phi\ r)_3 \ = \ (p_\phi\ r)_c \ = \ -\frac{e}{2}\,\Delta(B_z\ r)\ r$$

If the beam now enters a solenoid with opposite field $B_{z2} = -B_{z1}$ then the canonical angular momentum remains the same, but the real angular momentum is further increased to exactly twice this value.

$$(p_\phi\ r)_4 \ = \ -\frac{e}{2}\,(B_z\ r)\ r + \frac{e}{2}\,(B_{z\ 2}\ r)\ r \ = \ -2\frac{e}{2}\,\Delta(B_z\ r)\ r$$

If now we introduce material enough to halve the transverse field components then

$$(p_\phi\ r)_5 \ = \ -\frac{e}{2}\,(B_z\ r)\ r$$

but this is now inside the field $-B_z$. The canonical momentum, and the angular momentum on exiting, is now:

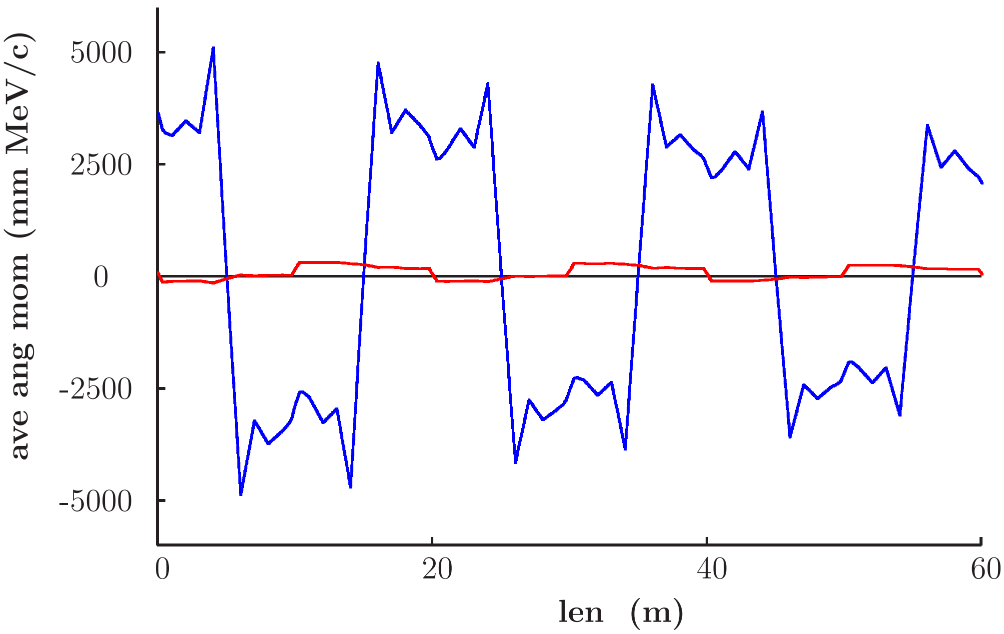
$$(p_\phi\ r)_6 \ = \ -\frac{e}{2}\,(B_z\ r)\ r - \frac{e}{2}\,(B_{z\ 2}\ r)\ r \ = \ 0$$

7.1.3 Alternating Solenoid Method

The above is not an efficient way to do it. The second "cooling" is used solely to reduce the coherent angular momentum of an already cold beam. There are other problems: the beam in the first solenoid, though it is cooled, does not experience any reduction of its radius, and the solenoid diameters have to be larger than they would be if the solenoids are reversed more frequently. In addition, simulations suggest that the alternations should have a spacing small compared to the synchrotron wavelength. If not one has difficulties keeping particle in the rf buckets because of large longitudinal velocity changes of some particles at each field reversal.

The Figure below shows the angular momenta and canonical angular momenta in a simulation of a 1.2 T initial cooling stage. It is seen that while the coherent angular momenta are large, the canonical angular momentum (in red) remains very small.

If there had been no field reversals in this example, then the angular momentum would have been reduced by about a factor of 2 (from 3 m MeV/c to approximately 1.5 m MeV/c) and the resulting canonical momentum of 1.5 m MeV/c would have been approximately equal to the final random angular momenta. In later stages without reversals, the coherent canonical momenta would continue to rise and soon far exceed the falling random values.



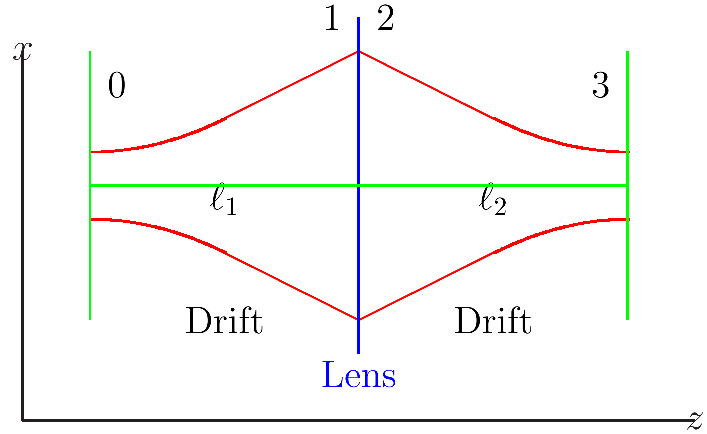
Average Angular momentum (blue), and Canonical Angular Momentum (red) in a 1.2 T Alternating Solenoid Cooling Channel.

7.1.4 FOFO

7.2 Linear Dynamics

7.2.1 Introduction

We will consider the variations in beam size, and thus β, as a function of drifts and a thin focusing lens. In particular, we will consider the sequence: drift-lens-drift, in which at both the start and end, the ellipse is upright.

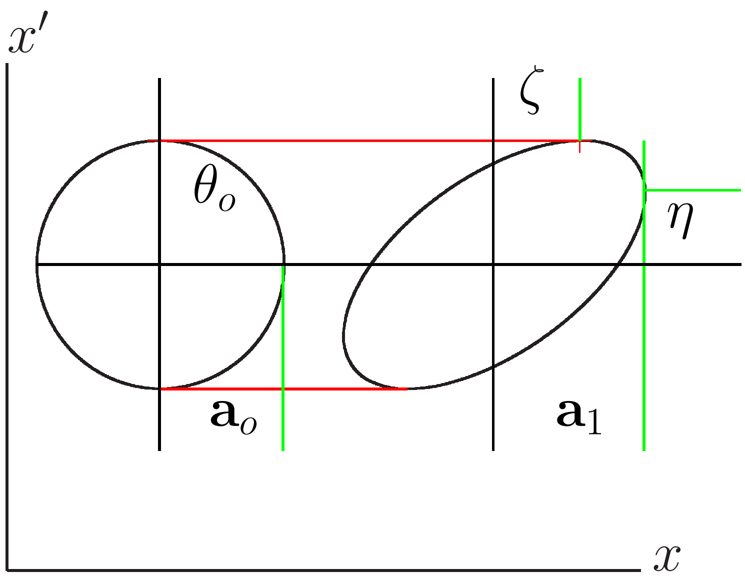


Let emittance = 1, then for an upright ellipse:

$$\theta_o = \sqrt{\frac{1}{\beta_o}}$$

$$a_o = \sqrt{\beta_o}$$

7.2.2 Drift



After a drift of length ℓ_1, the top of the ellipse has moved by ζ:

$$\zeta = \ell_1 \; \theta_o$$

The maximum width of the ellipse is the rms sum of ζ and a_o:

$$a_1 = \sqrt{a_o^2 + \zeta^2}$$

and thus

$$\beta_1 = a_o^2 + \ell_1^2 \theta_o^2$$

$$\beta_1 = \beta_o + \frac{\ell_1^2}{\beta_o}$$

Defining $\alpha_o = 0$ at z=0, when the ellipse is upright, and

$$\alpha_1 = \alpha_o + \frac{\ell_1}{\beta_o}$$

then

$$\beta_1 = \beta_o(1 + \alpha_1^2)$$

The angle η_1 of the point of maximum size can be obtained by symmetry considerations:

$$\frac{\eta_1}{\theta_o} = \frac{\zeta}{a_1}$$

so

$$\eta_1 = \frac{\zeta\theta_o}{a_1}$$

7.2.3 Thin Lens

The ratio κ_1 of the angle η_1 to its amplitude a_1 is a measure of the divergence of the beam. It gives, for instance, the strength of a focus system that would set the ellipse upright again:

$$\kappa_1 = \frac{\eta_1}{a_1} = \frac{\theta_o^2 \ell_1}{(a_1)^2} = \frac{\alpha_1}{\beta_1}$$

At a thin lens, the size a_1, and thus β_1, cannot change, but the angles change by the focus strength k times the amplitude a_1. For a particle at the maximum amplitude a_1

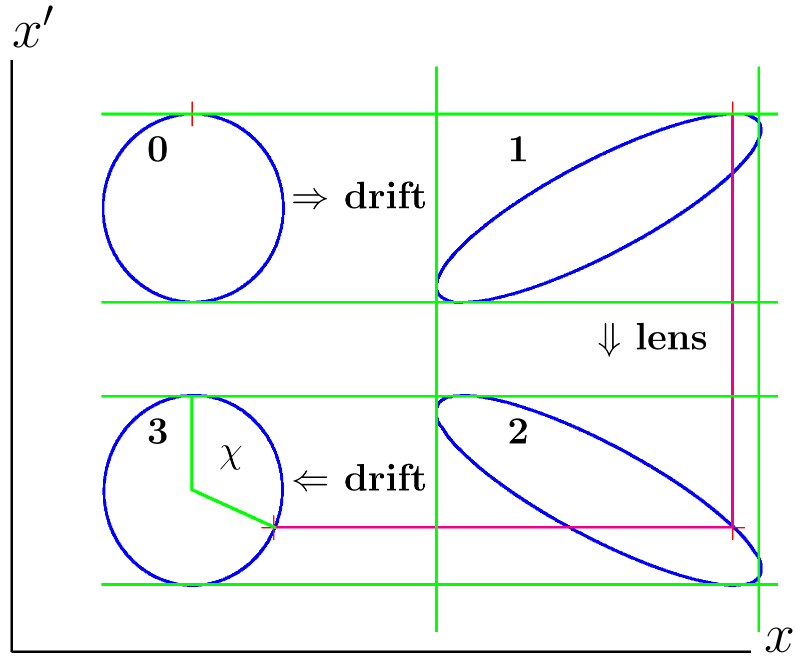
$$\eta_2 = \eta_1 + k\ a_1$$

$$\kappa_2 = \kappa_1 + k$$

$$\alpha_2 = \alpha_1 + k\beta_1$$

$$\beta_2 = \beta_1$$

7.2.4 Another Drift



As before, we can give β in the following drift by:

$$\beta = \beta_3(1 + (\alpha)^2)$$

where

$$\alpha = \frac{z - z_3}{\beta_3}$$

and β_3 is the beta at the following waist, if focused, or at an imaginary waist before the thin lens, if not. In particular,

$$\alpha_2 = \frac{-\ell_2}{\beta_3}$$

β_3 it can be obtained from the values of α_2 and β_2 after the thin lens:

$$\beta_2 = \beta_3 \, (1 + \alpha_2^2)$$

so:

$$\beta_3 = \frac{\beta_2}{(1 + (\alpha_2)^2)}$$

and

$$\ell_2 \;=\; \beta_3 \; \alpha_2$$

7.2.5 Momentum Dependences

At the end of a matched solenoid, i.e. one in which the beta function remains constant,

$$\beta_o(solenoid) \propto p$$

but at the end of a matched lithium lens:

$$\beta_o(lens) \propto \sqrt{p}$$

The strength of a thin solenoid:

$$k_{solenoid} \propto p^2$$

But for a thin lithium lens or horn:

$$k_{lens} \propto p$$

7.3 Matching

7.3.1 Adiabatic matching

If,in a long solenoid, the field is changed adiabatically, then the beta functions also change in proportion and maintain the same dependences on momentum. There is no emittance growth.

But we have shown that filed reversals are required, and these cannot be achieved adiabatically, because as the field passes through zero, the beam will blow up and be lost.

We must therefore address the question asto whether systems of solenoids can be designed that can match a range of momenta from one solenoid to another through a field reversal.

Conventionally, such matching is achieved by removing the chromaticity, so that the phase advance for different momenta are the same. Such removal is done using dispersion (transverse position dependent on momentum) and sextupoles (whose focusing quadrupole strength depends on lateral beam position). With chromaticity removed, matching is trivial, but it is not necessary, as the adiabatic case shows.

7.3.2 with Thin Lenses

A continuous solenoid an be well approximated by a series of thin lenses, whose dynamics we have described above. Where the field reverses, it must go to zero at one point, and be low over a distance comparable with the beam pipe diameter. We can approximate this situation by requiring a finite gap between a central pair of such solenoids. We will find that other gaps will also be required.

Consider the matching from one solenoid to another identical one, with a gap in the center where the field reverses.

I am given the initial β, initial $d\beta/dp$, $\alpha = 0$ and $d\alpha/dp = 0$; and require all four to have the same value at the entry to the following solenoid: 4 constraints.

But if I can ignore the energy changes in the hydrogen and reaccelerators, then I have symmetry about the middle of the gap. In this case the matching requirement reduces to just two requirements at the symmetry plane:

$$\alpha = 0$$

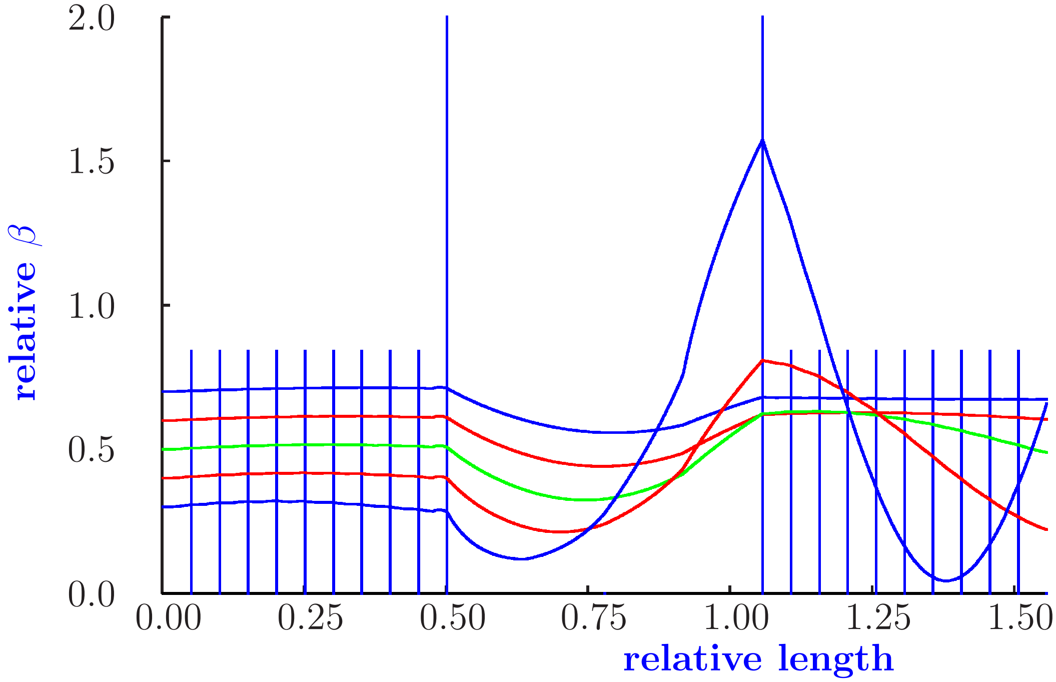
$$\frac{d\alpha}{dp} = 0 \quad i.e. \quad \frac{d\beta}{z} \ = \ 0$$

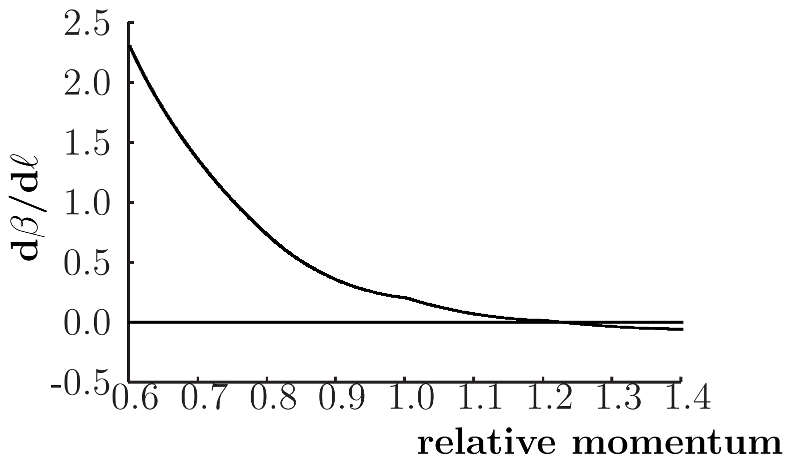
7.3.3 Simple solenoid-solenoid

In the first example I place a thin lens at the exit of the solenoid, followed by the gap where the field would be reversed.

In the solenoid, alpha=0 and the β's are constant. The thin lens drives α negative and the β falls to a minimum and then rises to meet the thin lens at the entrance to the opposite solenoid. For a fixed gap I have only one variable (the thin lens strength). These thin lens strengths can be chosen to put the minimum at the center for one momentum, and at this momentum a good match is achieved. But $\frac{d\alpha}{dp}$ can not be set to zero, and at other momenta the match is bad.

I show below: a: tracings at relative momenta of .6 to 1.4; followed by b: $d\beta/dz$ vs. relative p


2.0
1.5
1.0
0.5
0.0
relative β
0.00
0.25
0.50
0.75
1.00
1.25
1.50
relative length

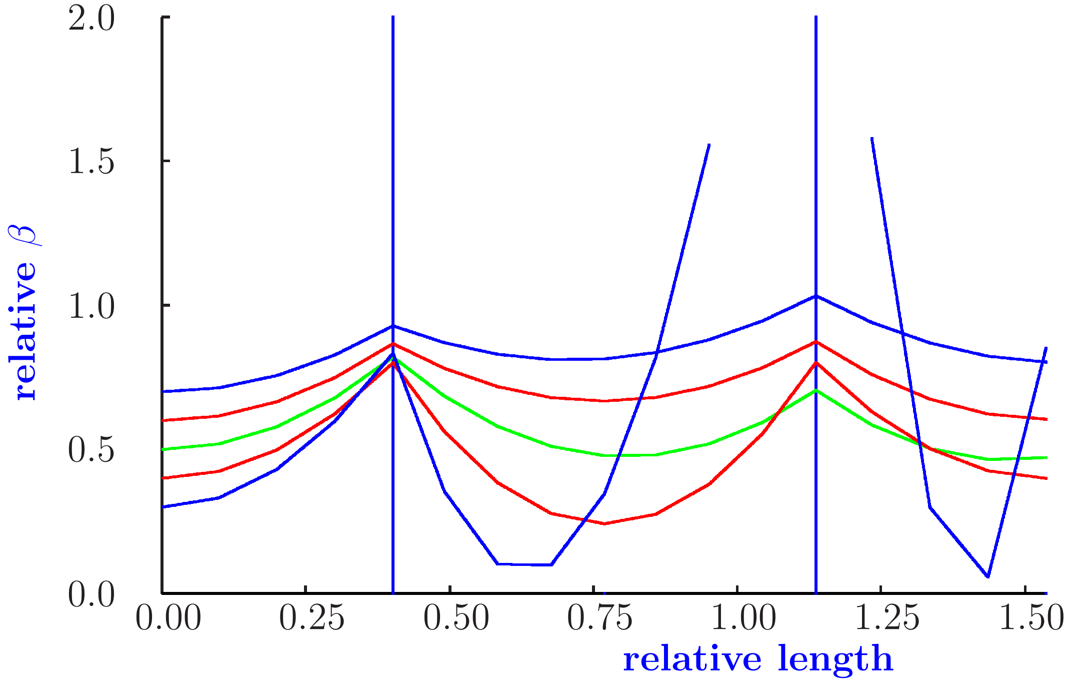

2.5
2.0
1.5
1.0
0.5
0.0
-0.5
dβ/dℓ
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
relative momentum

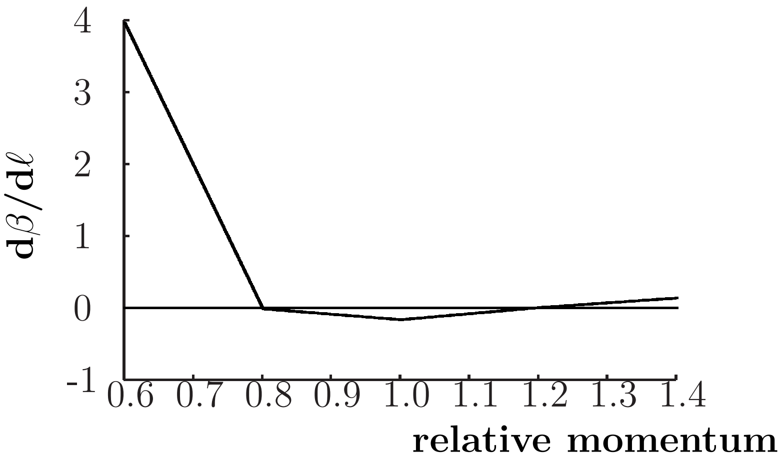
7.3.4 Simple solenoid after a gap

Again I match from one long solenoid to another, but now with gaps between the end of the solenoids and the thin lenses (two thin lenses and three gaps altogether)

Now I have two variables (the first gaps and the thin solenoid strengths, and should thus be able to satisfy both constraints. In fact, I looked for, and found, solutions with perfect matching at two separated momenta - probably a best solution for wide momentum acceptance. Clearly I could have satisfied the two other constraints.

Again, I show below: a: tracings at relative momenta of .6 to 1.4; followed by b: $d\beta/dz$ vs. relative p


2.0
1.5
1.0
0.5
0.0
relative β
0.00
0.25
0.50
0.75
1.00
1.25
1.50
relative length


4
3
2
1
0
-1
$d\beta/d\ell$
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
relative momentum

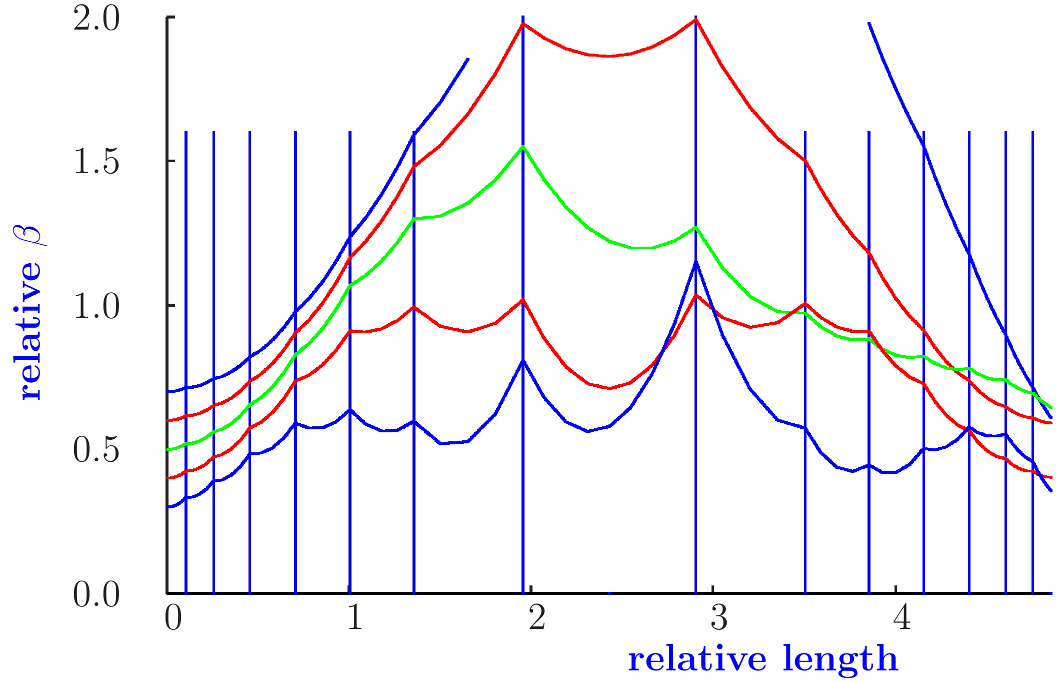
7.3.5 solenoid-solenoid with tapers

The above solution is nice because it tells us that solutions are possible, but 1) the length of the match and is constrained; 2)The matching thin lens would have to have a field greater than the cooling solenoid; and 3) Such a high filed thin lens solenoid would need to have a small bore that would not accept the required acceleration cavities.

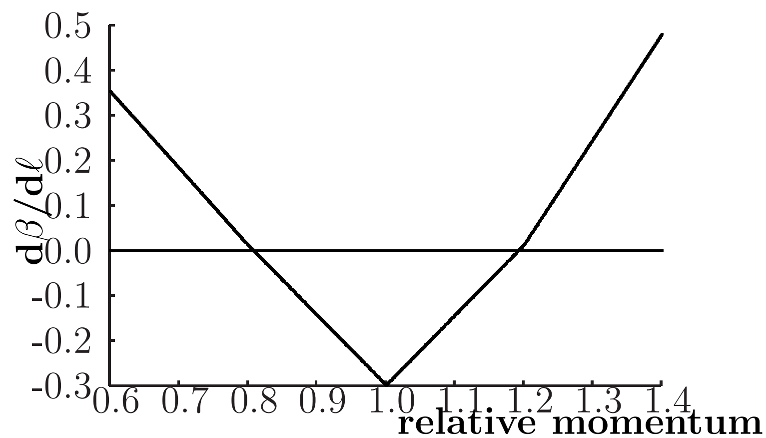
So in the second example I keep the symmetry and again match between solenoids, but introduce an adiabatic section to allow the β to grow to a much larger value at the symmetry point.

Again, we look for, and find, solutions with perfect matching at two separated momenta.

a: tracings at relative momenta of .6 to 1.4; followed by b: $d\beta/dz$ vs. relative p



2.0
1.5
1.0
0.5
0.0
relative β
0
1
2
3
4
relative length



0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
dβ/dℓ
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
relative momentum

7.4 Real Coil Matching

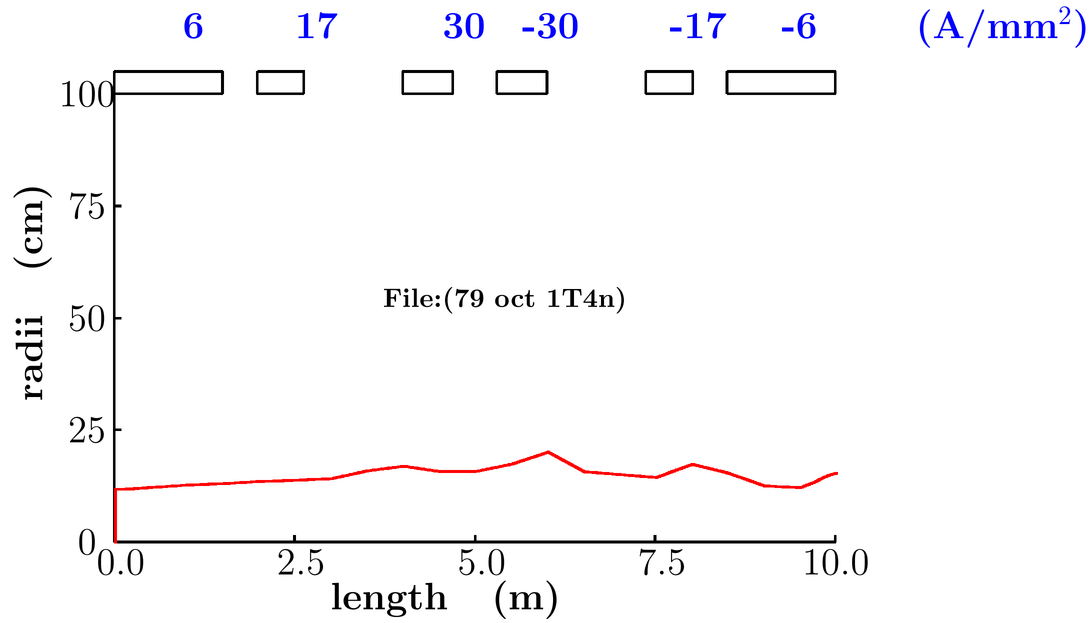
7.4.1 Without Energy loss and Gain

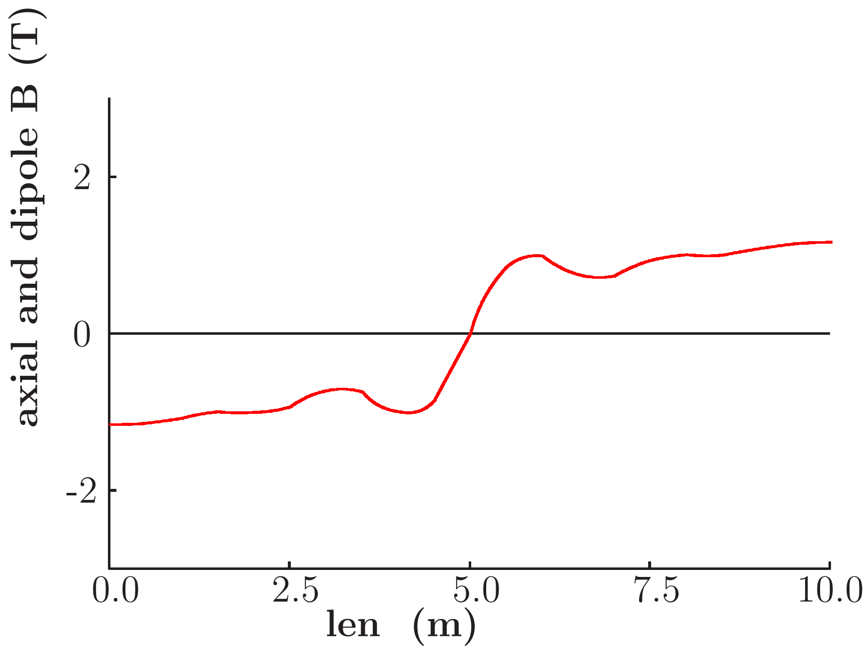
s an example, we now give the design of a "first cooling stage". The emittances, both transverse and longitudinal are large. High magnetic fields are not needed (1.2 T is used), but a very large momentum acceptance (+/-40%) is essential.

Coil positions, thicknesses and currents are now chosen to generate focusing approximately scaled from the above adiabatic plus thin lens example. A minimizing program "amoeba" was then used to minimize the rms central slope for 9 momenta spaced at intervals of 10 % on either side of the central value (186 MeV/c in this case).

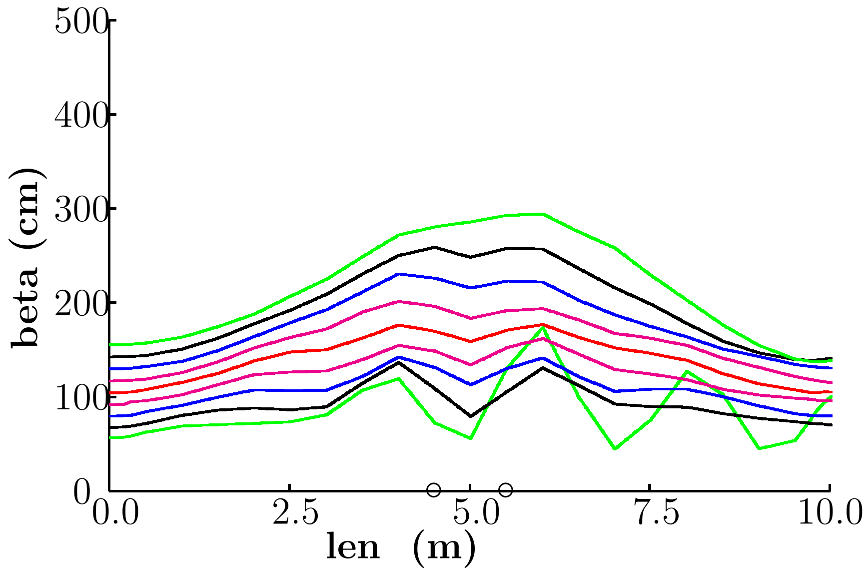
The coil layouts are shown in the first figure. The cell, that is repeated, is 10 m long. The first and last elements shown are the back and front end of the cooling solenoid (3 m long, 1m inside rad, and 5 cm thick). There is a pair of "bucked" coils near the center (each 70 cm long, 1 m inside radius and 5 cm thick) and another pair of "matching" coils between the cooling solenoid and the bucked pair. The minimizer was allowed to vary the currents and longitudinal positions of the bucked and matching coils (i.e. 4 variables).

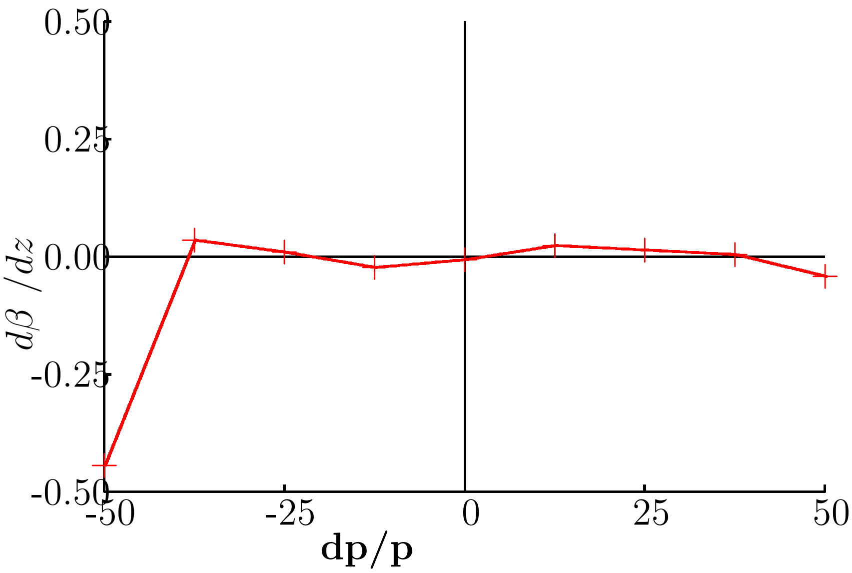
The positions shown are those obtained after minimization. The resulting axial magnetic field is shown in the next plot. The β's vs. length are then shown for 9 momenta spanning -50% to +50% in dp/p, and the slopes at the center for this range of momenta given in the last plot. It is seen that 4 momenta at which perfect matching (i.e. dβ/dz=0), and that the slope is maintained close to zero over a very wide range of momenta: -40 % to +50 % in dp/p.





dp/p steps of .125

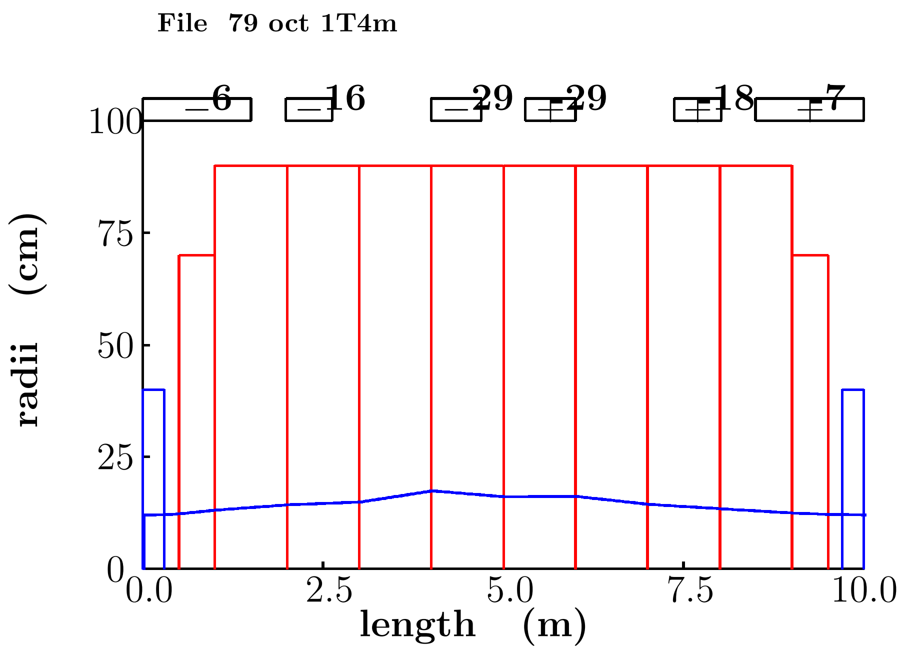


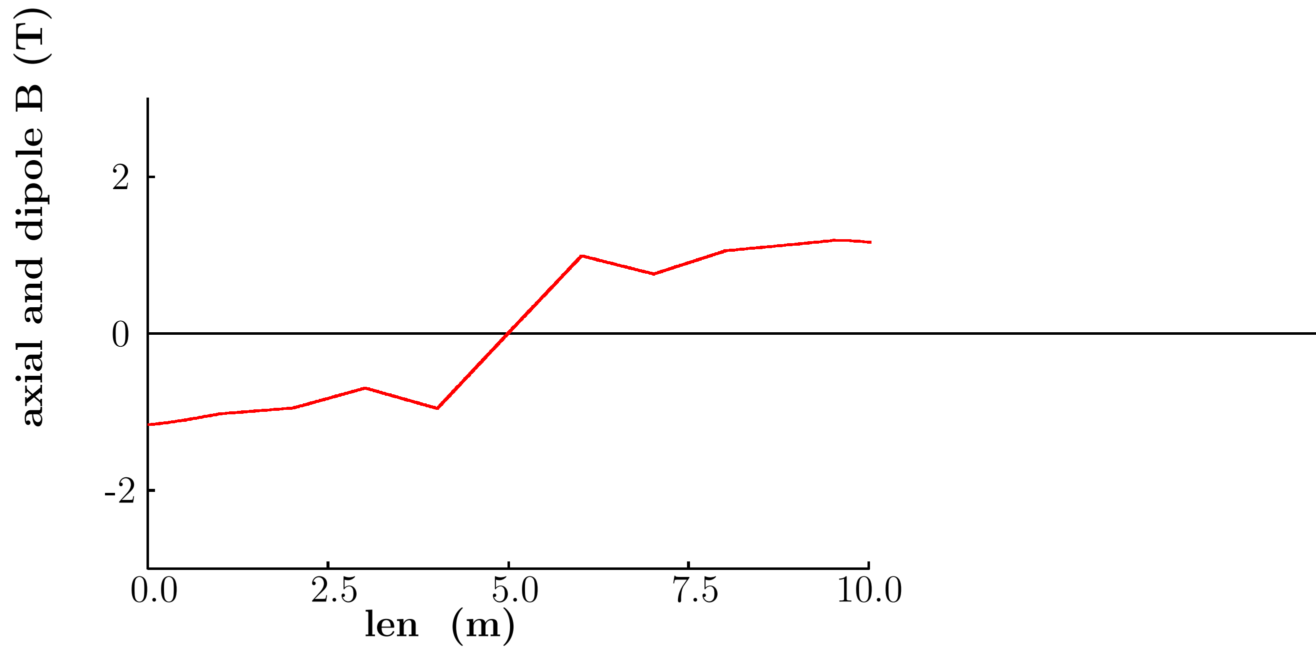

0.50
0.25
0.00
-0.25
-0.50
$d\beta\ /dz$
-50
-25
0
25
50
dp/p

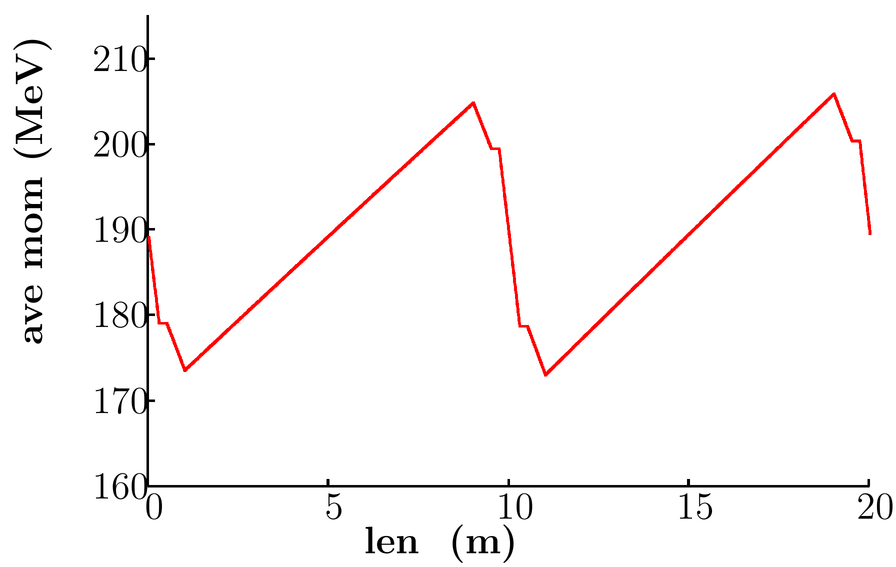
7.4.2 With Energy loss and Gain

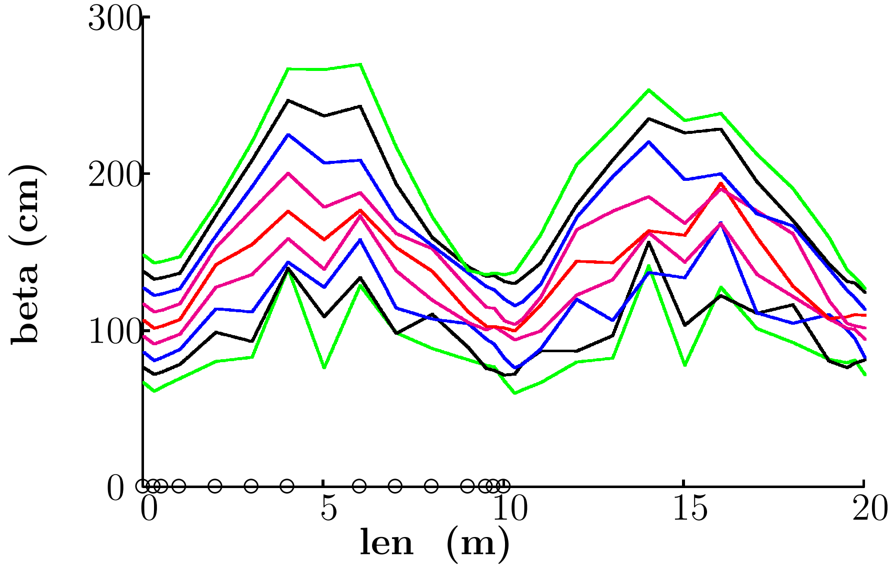
The above cell was designed neglecting the fact that the real momenta are not constant, but fall in the cooling material and are restored in the rf. The minimizing program was therefore used again to adjust the relative currents in the coils before and after the midplane of the cell (3 variables), and the average current in the pair was also adjusted.

The results are shown below:

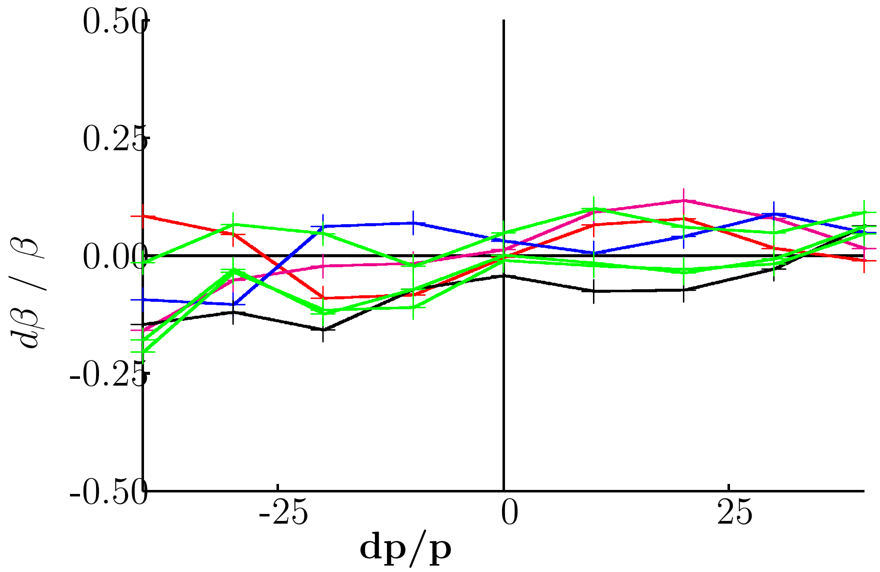



axial and dipole B (T)
2
0
-2
0.0
2.5
5.0
7.5
10.0
len (m)


ave mom (MeV)
210
200
190
180
170
160
0
5
10
15
20
len (m)



300
200
100
0
beta (cm)
0
5
10
15
20
len (m)



0.50
0.25
0.00
-0.25
-0.50
dβ / β
-25
0
25
dp/p

Longitudinal Complications

7.5 Synchrotron Motion Complications

7.5.1 dz/dt Amplitude Correlation

7.5.2 dz/dt Angular Momentum Correlation

7.6 dz/dt Amplitude Correlation Generation

7.6.1 buttons

7.6.2 rf

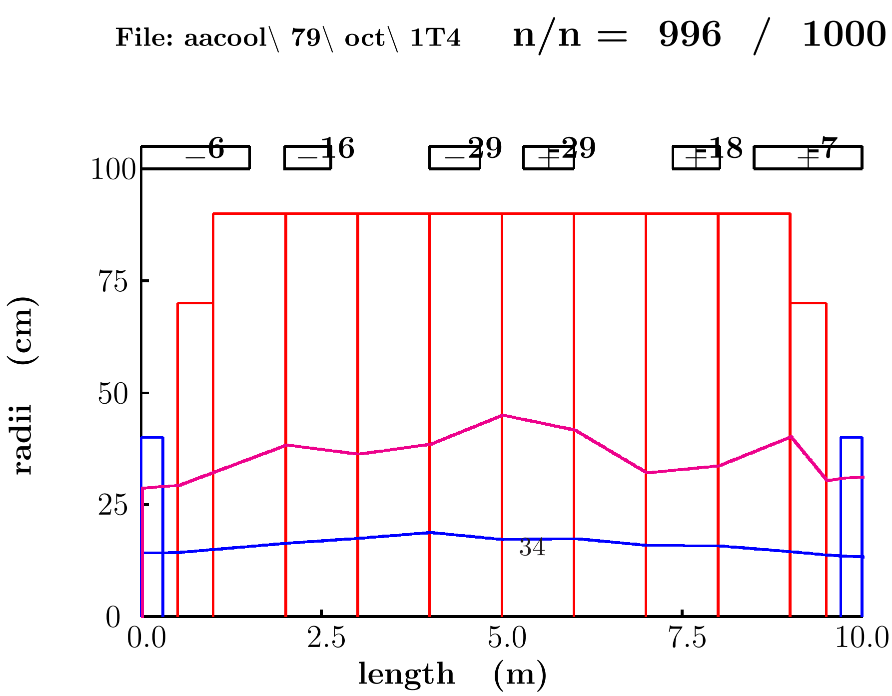
7.7 Additional Heating Terms

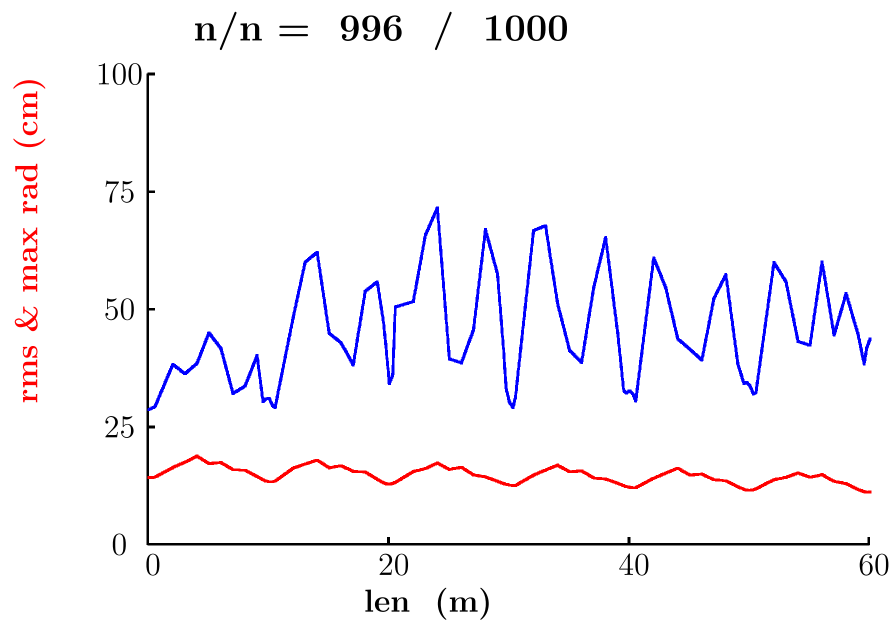
7.7.1 Transverse from straggling

7.7.2 Longitudinal from scattering

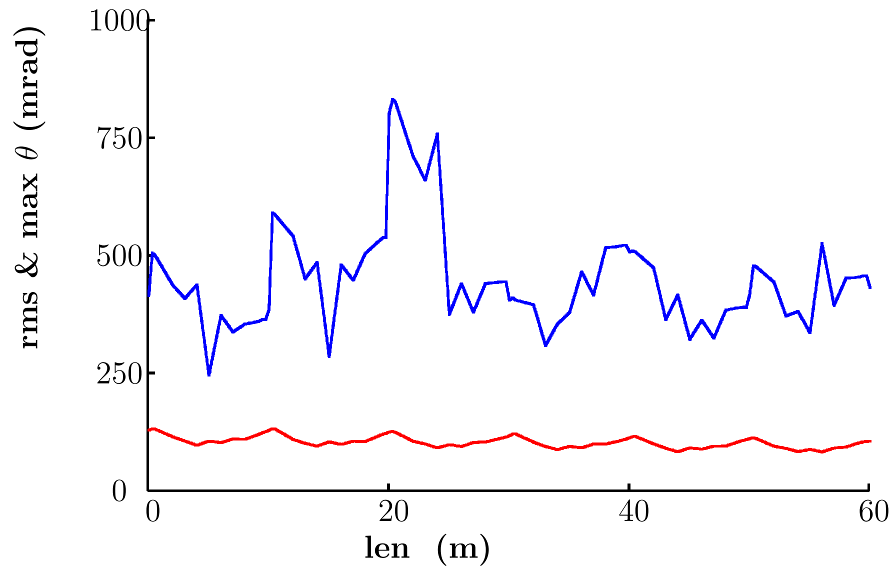
7.8 Simulation

7.8.1 First Cooling Stage

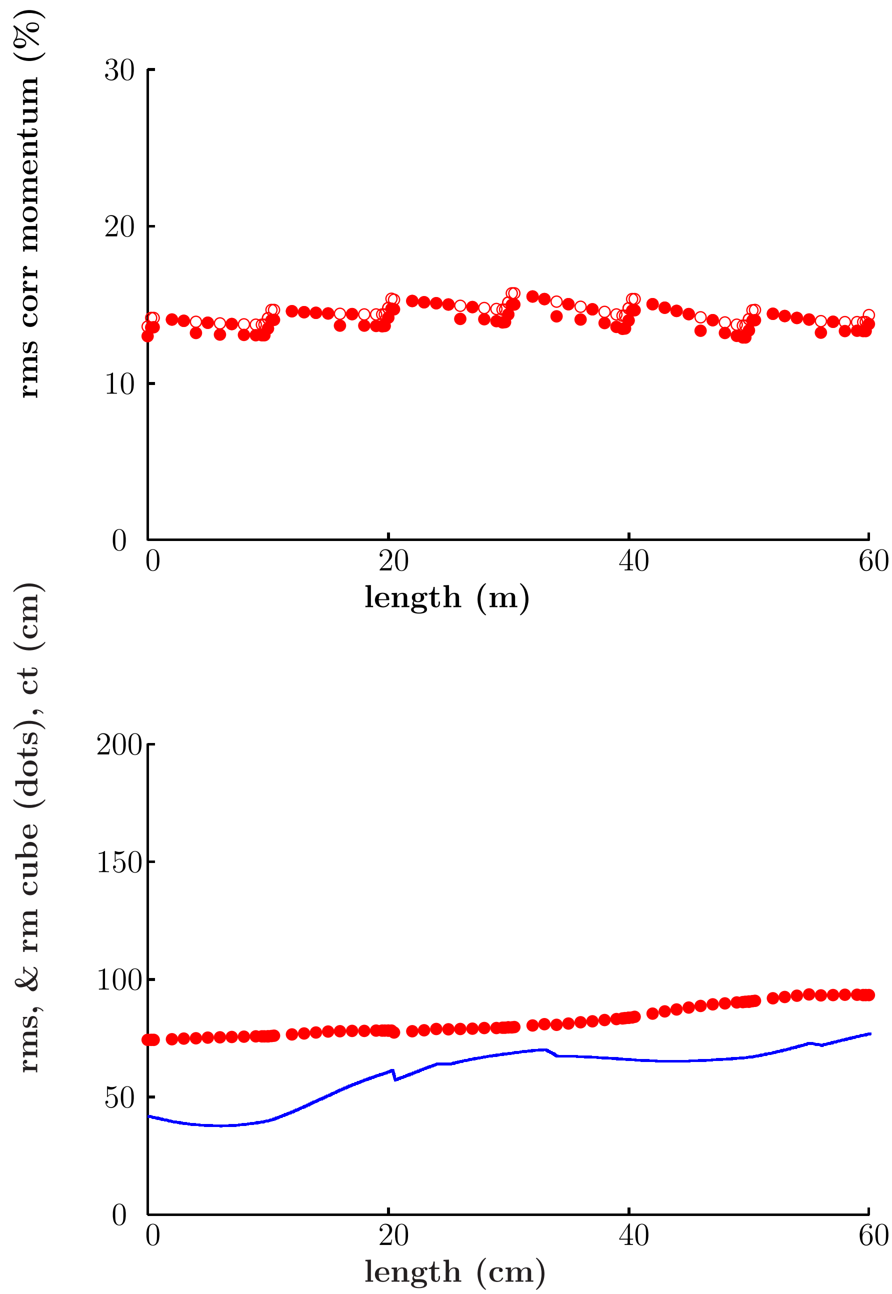


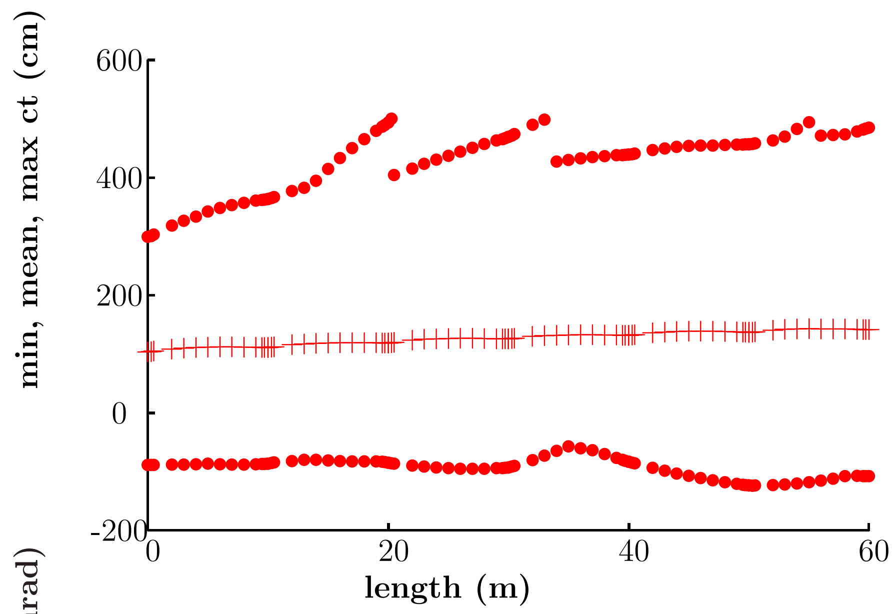


n/n = 996 / 1000
rms & max rad (cm)
100
75
50
25
0
0
20
40
60
len (m)

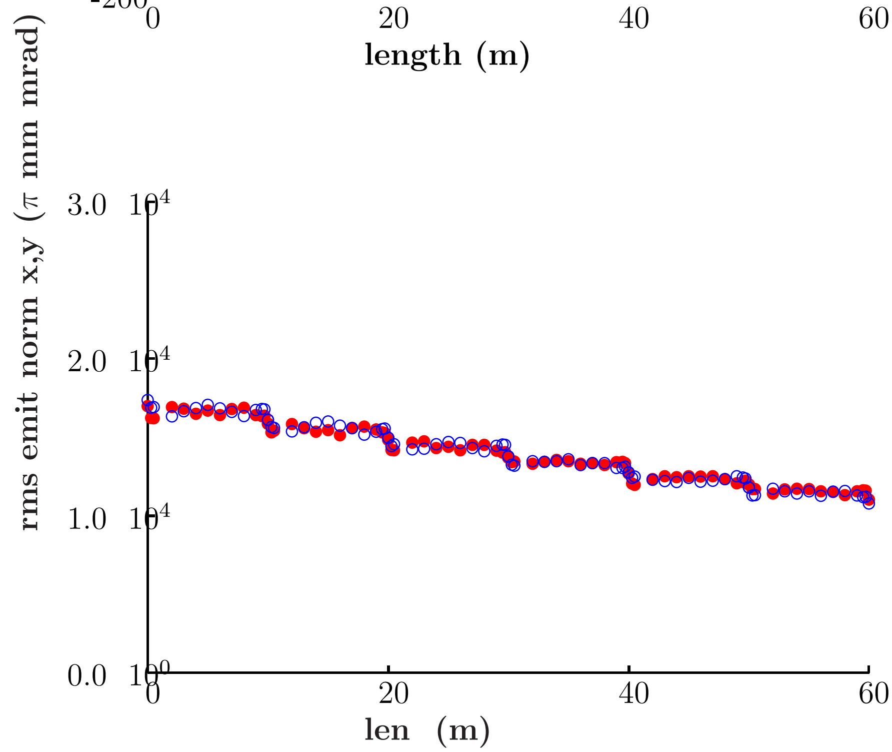


rms & max θ (mrad)
1000
750
500
250
0
0
20
40
60
len (m)

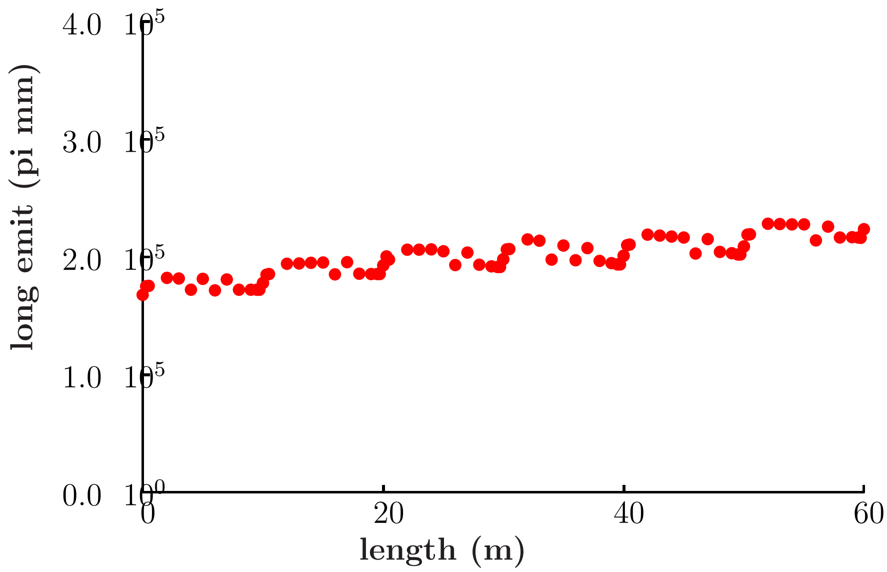

rms corr momentum (%)
30
20
10
0
0
20
40
60
length (m)
rms, & rm cube (dots), ct (cm)
200
150
100
50
0
0
20
40
60
length (cm)



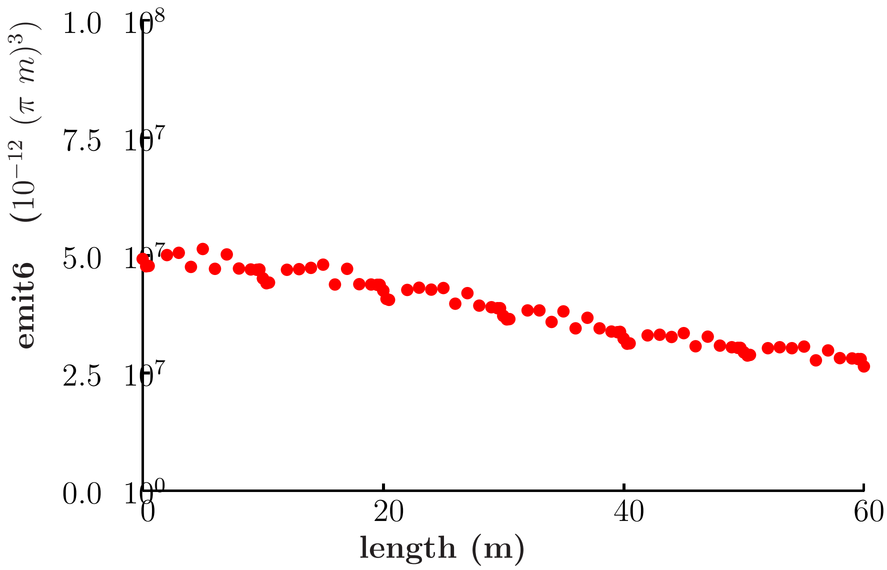
min, mean, max ct (cm)
600
400
200
0
-200
0
20
40
60
length (m)



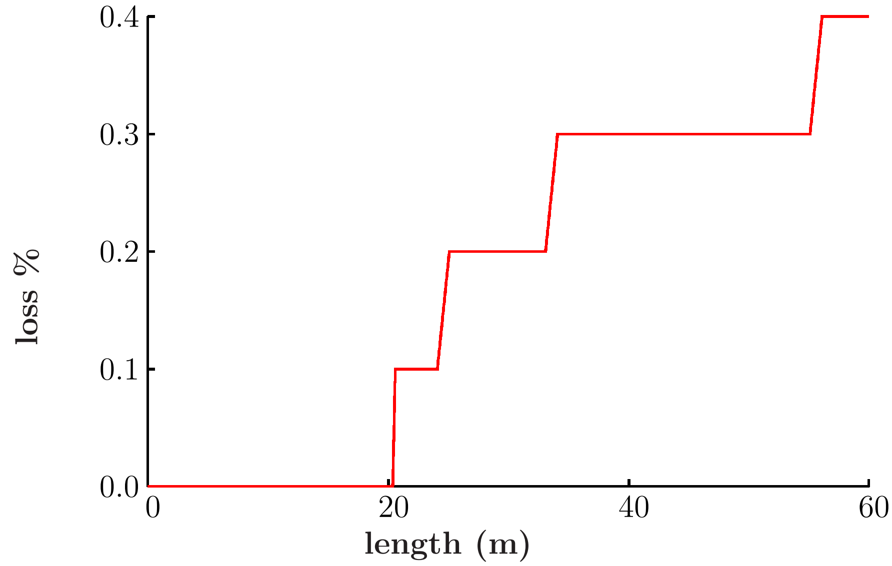
rms emit norm x,y (π mm mrad)
3.0 10^4
2.0 10^4
1.0 10^4
0.0 10^0
0
20
40
60
len (m)

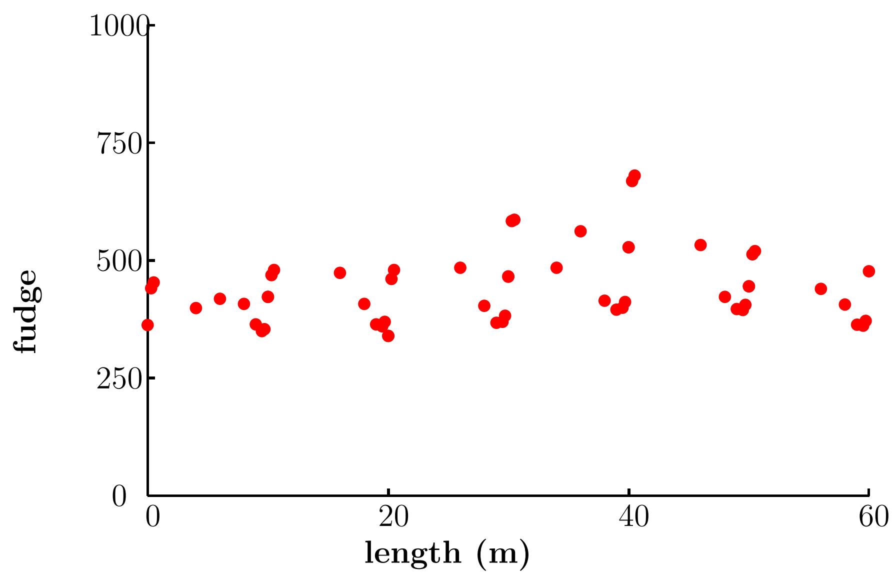


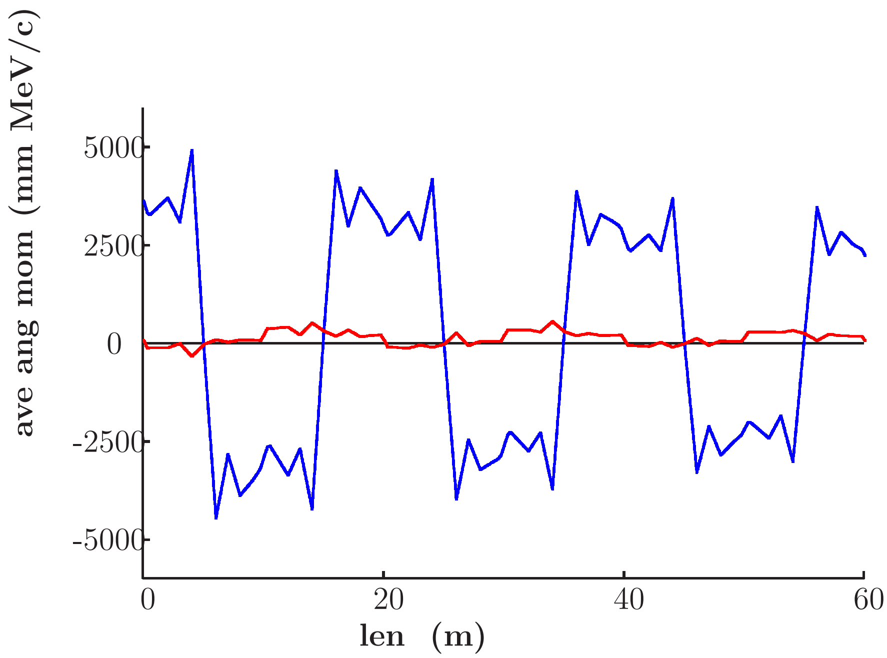
4.0 10^5
3.0 10^5
2.0 10^5
1.0 10^5
0.0 10^0
long emit (pi mm)
0
20
40
60
length (m)



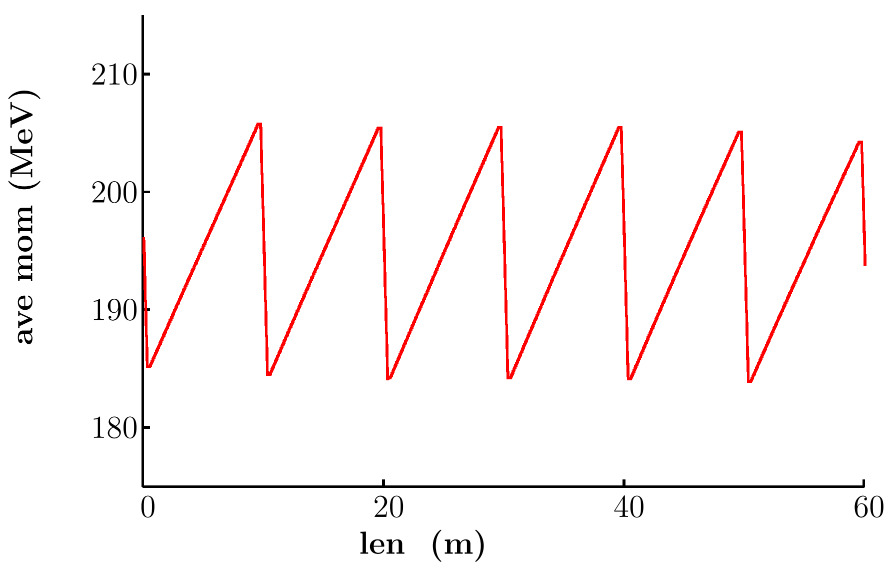
1.0 10^8
7.5 10^7
5.0 10^7
2.5 10^7
0.0 10^0
emit6 $(10^{-12}\ (\pi\ m)^3)$
0
20
40
60
length (m)


0.4
0.3
0.2
0.1
0.0
loss %
0
20
40
60
length (m)


1000
750
500
250
0
fudge
0
20
40
60
length (m)

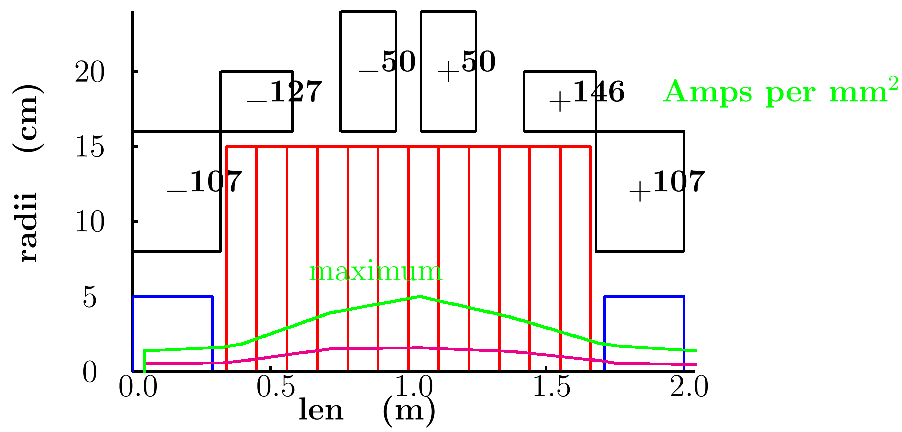


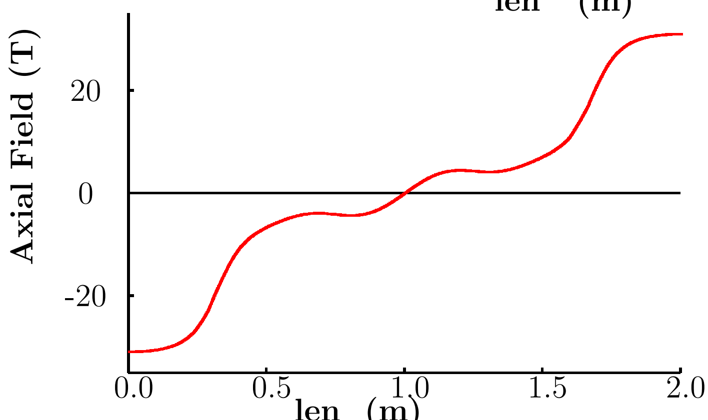
ave ang mom (mm MeV/c)
5000
2500
0
-2500
-5000
0
20
40
60
len (m)

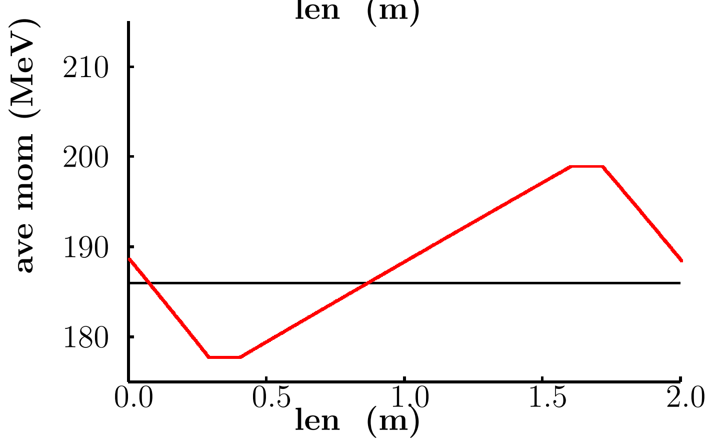


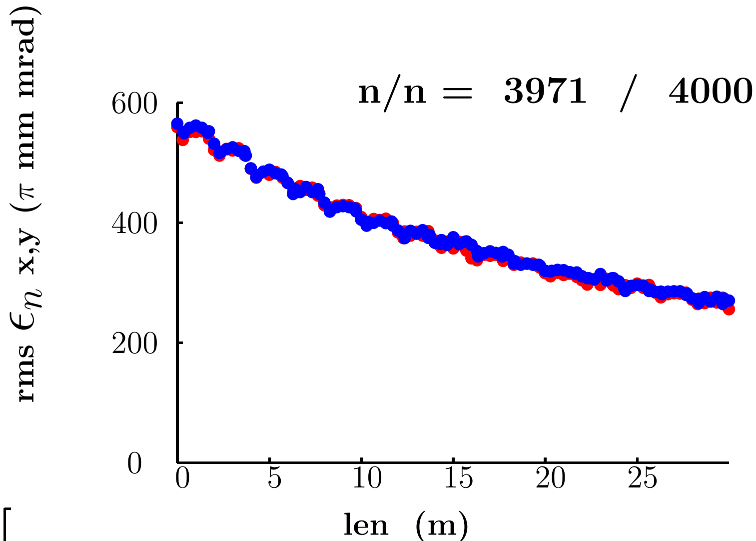
ave mom (MeV)
210
200
190
180
0
20
40
60
len (m)

7.8.2 Last Cooling Stage ?

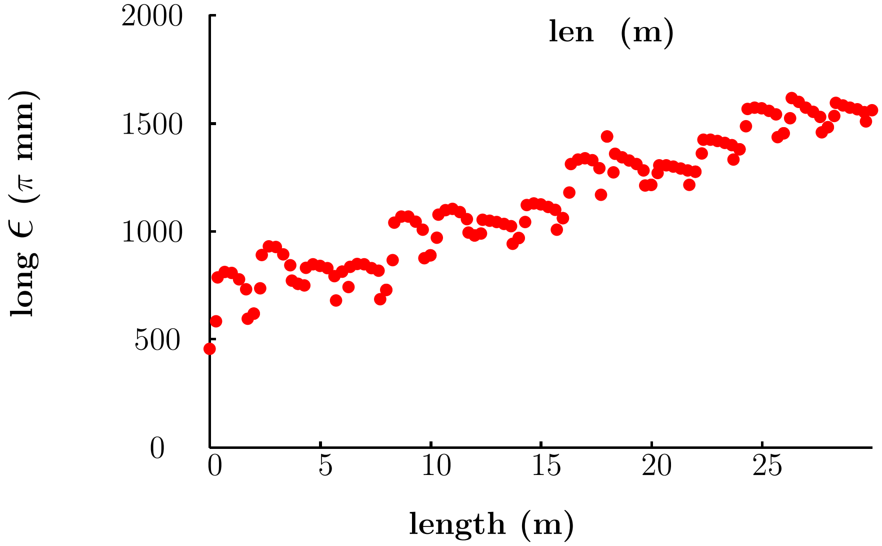




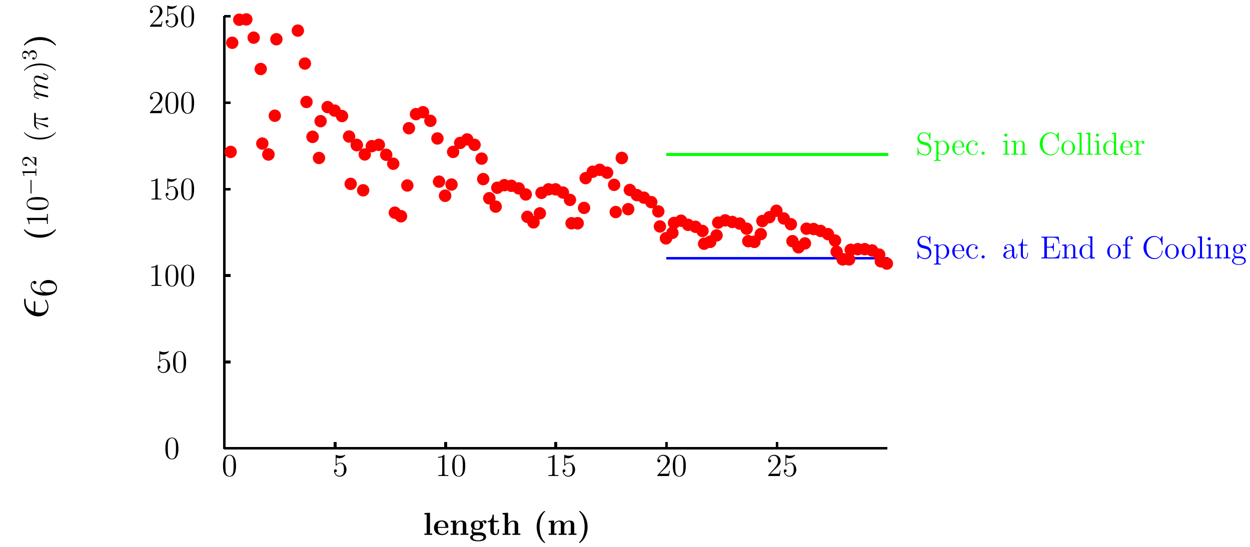




n/n = 3971 / 4000
rms ϵ_η x,y (π mm mrad)
600
400
200
0
0
5
10
15
20
25
len (m)



long ϵ (π mm)
2000
1500
1000
500
0
0
5
10
15
20
25
length (m)



250
200
150
100
50
0
0
5
10
15
20
25
$\epsilon_6 \quad (10^{-12}\ (\pi\ m)^3)$
length (m)
Spec. in Collider
Spec. at End of Cooling