Booster, AGS, and RHIC Parameters for the 2002–2003 RHIC Run

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February 12, 2003

The Tables in this note contain the nominal parameter values for the 2002–2003 RHIC Run.

1 Basic Formulae

1.1 Mass, Energy, Momentum, Rigidity, and Frequency

A Gold ion with charge eQ has $N = 197$ Nucleons, $Z = 79$ Protons, and $(Z - Q)$ electrons. (Here Q is an integer and e is the charge of a single proton.) The mass and energy are

$$m = au - Qm_e + E_b/c^2, \quad E = \sqrt{p^2c^2 + m^2c^4} \tag{1}$$

where $a = 196.966552$ is the atomic mass [1, 2] of the neutral Gold atom, $u = 931.494013$ MeV/c^2 is the unified atomic mass unit [3], $m_ec^2 = .510998902$ MeV is the electron mass [3], and p is the momentum. E_b is the binding energy of the Q electrons removed from the neutral Gold atom. This amounts to 0.327 MeV for the fully stripped Gold ion as calculated by Trbojevic [4]. The deuteron mass [3] is 1875.612762(75) MeV/c^2. The kinetic energy is defined to be

$$W = E - mc^2. \tag{2}$$

In terms of W, the momentum and energy are

$$cp = \sqrt{W^2 + 2mc^2W}, \quad E = mc^2 + W. \tag{3}$$

The magnetic rigidity of the ion in units of Tm is

$$B\rho = kp/Q \tag{4}$$

where $k = 10^9/299792458$ and p is the momentum in units of GeV/c. The relativistic parameters β and γ, and the revolution frequency of the ion are

$$\beta = cp/E, \quad \gamma = E/(mc^2), \quad f = c\beta/(2\pi R) \tag{5}$$

where R is the machine radius (defined to be the closed orbit circumference divided by 2π). The angular frequency is $\omega = 2\pi f$. We also define the phase-slip factor

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \tag{6}$$

where γ_t is the transition gamma.

1.2 RF Parameters

Various RF Bucket and Bunch parameters are as follows. The half-height of the bucket is

$$\Delta E = \left(\frac{h\omega_s}{8}\right) A_S \left|\frac{(\pi - 2\phi_s)\sin\phi_s - 2\cos\phi_s}{2}\right|^{1/2} \tag{7}$$

where the subscripts "s" indicate parameter values for the synchronous particle, h is the RF harmonic number, ϕ_s is the synchronous phase, and

$$A_S = 8\,\frac{R_s}{hc}\left\{\frac{2eQV_gE_s}{\pi h|\eta_s|}\right\}^{1/2}. \tag{8}$$

Here V_g is the total RF gap voltage per turn and A_S is the area of the corresponding Stationary bucket. The synchronous phase is given by

$$V_g \sin\phi_s = 2\pi R_s \rho_s \dot{B}/c \tag{9}$$

where ρ_s is the radius of curvature, B is the magnetic field and $\dot{B} = dB/dt$. Employing Gaussian units (R_s and ρ_s in cm, $c = 2.99792458 \times 10^{10}$ cm/s, and $\dot{B}$ in G/s) gives $V_g \sin\phi_s$ in Statvolts. Multiplying by 299.792458 then gives $V_g \sin\phi_s$ in Volts.

The width of the bucket is

$$\Delta t = \frac{|\pi - \phi_s - \phi_e|}{h\omega_s} \tag{10}$$

where the phase ϕ_e satisfies

$$\cos\phi_e - \cos(\pi - \phi_s) = -\{\phi_e - (\pi - \phi_s)\}\sin\phi_s. \tag{11}$$

The synchrotron frequency for small-amplitude oscillations about ϕ_s is

$$F_s = \frac{c}{2\pi R_s}\left\{\frac{-h\eta_s eQV_g\cos\phi_s}{2\pi E_s}\right\}^{1/2} \tag{12}$$

and the corresponding synchrotron tune is $Q_s = 2\pi F_s/\omega_s$.

The half-height and full width of the bunch matched to the bucket are given by

$$\Delta E_m = \left(\frac{h\omega_s}{8}\right) A_S \left|\frac{\cos\phi_m - \cos\phi_s + (\phi_m - \phi_s)\sin\phi_s}{2}\right|^{1/2} \tag{13}$$

and

$$\Delta t = \frac{|\phi_m - \phi_e|}{h\omega_s} \tag{14}$$

where the phase ϕ_e satisfies

$$\cos\phi_m - \cos\phi_e + (\phi_m - \phi_e)\sin\phi_s = 0. \tag{15}$$

For a bunch matched to a stationary bucket the half-height and width are given by

$$\Delta E_m = \left(\frac{h\omega_s}{8}\right) A_S \left|\frac{\cos\phi_m \mp 1}{2}\right|^{1/2}, \quad \Delta t = \frac{|2\phi_m|}{h\omega_s} \tag{16}$$

where the "$-$" and "$+$" signs are for buckets below and above transition respectively. The area of a small bunch in a stationary bucket is approximately

$$A_b = \left(\frac{\pi A_S}{16}\right)\phi_m^2. \tag{17}$$

2 Lattice Parameters

Parameter	Booster	AGS	RHIC	Unit
C_I	C_b	C_a	$C_r + \Delta C_I$	m
C_E	$C_a/4$	$4(C_r + \Delta C_I)/19$	$C_r + \Delta C_E$	m
ρ	13.8656	85.378351	242.7806	m
$\gamma_{\rm tr}$	4.806	8.5	22.89	
Q_H, Q_V	4.757, 4.777	8.78, 8.72	28.19, 29.18	
Max β_H, β_V	13.5, 13.2	22.3, 22.2	48.6, 47.4	m
Max D_H	2.90	2.17	1.81	m

Here C_I and C_E are the circumferences of the closed orbits in the machines at injection and extraction (or store) respectively. C_b, C_a, and C_r are the circumferences of the "design" orbits in Booster, AGS, and RHIC respectively. These are

$$C_b = 201.780, \quad C_a = 2\pi(128.4526), \quad C_r = 3833.845181 \tag{18}$$

meters. The radius is defined to be the orbit circumference divided by 2π.

In order to have deuteron-gold collisions in RHIC, deuterons in the blue ring must have the same revolution frequency as gold ions in the yellow ring. This implies beam trajectories in the interaction regions that differ from those for gold-gold collisions. As a result, the orbit circumference for deuterons is 2.420 mm greater than C_r and the circumference of the gold orbit is 2.154 mm less than C_R. (These numbers have been obtained by Steve Tepikian using the MAD code.)

Note that $4(C_r/19) = 2\pi(128.4580)$ m which gives an AGS radius at extraction approximately 5 mm larger than the "design" AGS radius (128.4526 m) reported by Bleser [5]. The other Booster and AGS parameters were obtained from MAD runs. The RHIC parameters are taken from Ref. [6] and from MAD runs by Steve Tepikian.

3 Gold and Deuteron Parameters in Booster, AGS, and RHIC

The parameters values in the following tables are calculated assuming that:

1. The revolution frequency of the Au^{32+} ion at injection in Booster is 66.290 kHz; the magnetic rigidity of deuterons is the same as that of the gold ions.

2. At Booster extraction, the magnetic rigidity of the Au^{32+} ions is $B\rho = 9.152950$ Tm; the magnetic rigidity of deuterons is 7.322360 Tm.

3. The energy lost by a gold ion in the stripping foil between the Booster and AGS is $\delta E = 4.43$ MeV per nucleon. This number is calculated in Ref. [7]. (Deuterons do not pass through a stripping foil.)

4. The magnetic rigidity of the Au^{79+} ion at RHIC injection is 90 Tm. The revolution frequency of deuterons at RHIC injection is the same as that of the gold ions.

5. The orbit circumference for deuterons in the blue ring at injection is 2.420 mm greater than C_r. The orbit circumference for gold in the yellow ring at injection is 2.154 mm less than C_r.

6. The energy of the Au^{79+} ion at RHIC Store is 100 GeV per nucleon. The revolution frequency of the deuteron is the same as that of the gold ion. The orbit circumference for gold in the yellow ring is 2.154 mm less than C_r. The orbit circumference for deuterons in the blue ring is 2.420 mm greater than C_r.

The Bunch and Bucket parameters were obtained from the Computer Program "bbat".

In the following tables, more digits are given for some parameters than would be warranted by the measurement precision; this is done for computational convenience. The notation "$/N$" in the Units column means "per nucleon".

3.1 Gold in Booster

Parameter	Injection	Extraction	Unit
Q	32	32	
m	183.456812	183.456812	GeV/c^2
W	182.8790/197	101.1721	MeV/N
cp	41.59161	445.7235	MeV/N
E	0.9321812	1.0324250	GeV/N
$B\rho$	0.854085	9.152950	Tm
β	0.04461752	0.43172485	
$\gamma - 1$	0.996850/1000	0.108641	
η	-0.955	-0.770	
ϵ_H (95%)	8.3π	8.3π	mm mrad
ϵ_V (95%)	3.9π	3.9π	mm mrad
h	6	6	
hf	0.397740	3.848719	MHz
R	$201.780/(2\pi)$	128.4526/4	m

Here ϵ_H and ϵ_V are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during multi-turn injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are 185π and 87π mm mrad (un-normalized) respectively.

Parameter	Injection	Extraction	Unit
No. of Bunches	6	6	
Bunch Spacing	2514.205	259.827	ns
Ions/Bunch	3.0/6	2.4/6	10^9
Bunch Area	0.045/6	0.045/6	eV s/N
Bunch Δt	1500	48	ns
Bunch ΔE	0.65	20	MeV
Bucket ΔE	0.81	51	MeV
Gap Volts	0.5	30	KV
Bucket Area	0.079/6	0.350/6	eV s/N

Capture of the injected beam occurs on a 6 ms porch at constant field. During this time the gap voltage is increased from 0 to 0.5 kV. The bunch area is determined from the measured bunch width at extraction with $\dot{B} = 37$ G/ms and $V_g = 30$ kV.

3.2 Deuterons in Booster

Parameter	Injection	Extraction	Unit
W	17.3965/2	505.8673	MeV/N
cp	0.12802418	1.0975942	GeV/N
E	0.9465046	1.4436737	GeV/N
$B\rho$	0.854085	7.322360	Tm
β	0.13525997	0.76027856	
$\gamma - 1$	0.927509/100	0.539416	
η	-0.938	-0.379	
ϵ_H (95%)	25π	25π	mm mrad
ϵ_V (95%)	12π	12π	mm mrad
h	2	1	
hf	0.401922	1.129616	MHz
R	$201.780/(2\pi)$	128.4526/4	m

Note that the normalized emittances are significantly larger than those for gold; this is due to $\beta\gamma$ at injection being three times larger for deuterons

than it is for gold.

Parameter	Injection	Extraction	Unit
No. of Bunches	2	1	
Bunch Spacing	2488.045	885.257	ns
Ions/Bunch	10.0/2	8.0	10^{10}
Bunch Area	0.160/2	0.200	eV s/N
Bunch Δt	1770	133	ns
Bunch ΔE	0.062	1.95	MeV
BucketΔE	0.069	8.2	MeV
Gap Volts	0.4	24	KV
Bucket Area	0.217/2	4.62	eV s/N

Capture of the injected beam occurs on a 12 ms porch at constant field. During this time the gap voltage is increased from 0 to 0.4 kV.

3.3 Gold in AGS

Parameter	Injection	Transition	Extraction	Unit
Q	77	77	77	
m	183.434144	183.434144	183.434144	GeV/c^2
W	0.0967296	6.983533	9.928835	GeV/N
cp	0.4353089	7.859708	10.819981	GeV/N
E	1.0278674	7.914671	10.859973	GeV/N
$B\rho$	3.714945	67.075078	92.338177	Tm
β	0.42350685	0.99305547	0.99631752	
γ	1.103883	8.5000	11.663121	
η	-0.807	0.0	0.00649	
ϵ_H (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
h	24	12	12	
hf	3.775458	4.426421	4.440777	MHz
R	128.4526	128.4526	128.45791	m

Parameter	Injection	Extraction	Unit
No. of Bunches	24	4	
Bunch Spacing	264.868	676.132	ns
Ions/Bunch	1.4/6	1.3	10^9
Bunch Area	0.180/6	0.270	eV s/N
Bunch Δt	62	15.4	ns
Bunch ΔE	62	2206	MeV
Bucket ΔE	172	20600	MeV
Gap Volts	320	260	KV
Bucket Area	1.76/6	30	eV s/N

Four batches of six bunches are injected into AGS at constant field. Stationary Rf buckets at harmonic 24 receive the bunches. The beam is then debunched adiabatically and rebunched at harmonic 4. Acceleration to top energy occurs on harmonic 12. The bunches are extracted on flat-top at constant field.

Upon passing through the stripping foil between the Booster and AGS, the longitudinal emittance increases by about a factor of four due to energy straggling in the foil. An additional increase of 50% results from filamentation in the $h = 24$ buckets on the AGS injection porch.

3.4 Deuterons in AGS

Parameter	Injection	Transition	Extraction	Unit
W	0.5058673	7.033548	10.001705	GeV/N
cp	1.0975942	7.915997	10.899240	GeV/N
E	1.4436737	7.971354	10.939512	GeV/N
$B\rho$	7.322360	52.809847	72.711904	Tm
β	0.76027856	0.99305547	0.99631871	
γ	1.539416	8.5000	11.665000	
η	-0.408	0.0	0.00649	
ϵ_H (95%)	$\leq 25\pi$	$\leq 25\pi$	$\leq 25\pi$	mm mrad
ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
h	12	12	12	
hf	3.388847	4.426421	4.440777	MHz
R	128.4526	128.4526	128.45806	m

Parameter	Injection	Extraction	Unit
No. of Bunches	8	4	
Bunch Spacing	885.257/2	675.069	ns
Ions/Bunch	8.0	16.0	10^{10}
Bunch Area	0.2	0.5	eV s/N
Bunch Δt	134	21	ns
Bunch ΔE	2.0	30	MeV
Bucket ΔE	3.0	205	MeV
Gap Volts	41	260	KV
Bucket Area	0.56	29.4	eV s/N

3.5 Gold in RHIC

Parameter	Injection	Transition	Store	Unit
Q	79	79	79	
m	183.433122	183.433122	183.433122	GeV/c^2
W	9.928780	20.382493	99.068867	GeV/N
cp	10.819921	21.293276	99.995665	GeV/N
E	10.859912	21.313625	100.000000	GeV/N
$B\rho$	90.000000	177.117274	831.763013	Tm
β	0.99631752	0.99904526	0.99995665	
γ	11.663121	22.8900	107.396090	
η	-0.00544	0.0	0.00182	
ϵ_H (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
h	360	360	2520	
hf	28.047015	28.123787	197.0462113	MHz
$2\pi R$	3833.8430	3833.8452	3833.8430	m

Parameter	Injection	Store	Unit
No. of Bunches	60	60	
Bunch Spacing	214.108	213.148	ns
Ions/Bunch	1.3	1.3	10^9
Bunch Area	0.270	0.7	eV s/N
Bunch Δt	15.5	3.0	ns
Bunch ΔE	2240	30300	MeV
Bucket ΔE	3570	36000	MeV
Gap Volts	340	6000	KV
Bucket Area	0.82	1.18	eV s/N

3.6 Deuterons in RHIC

Parameter	Injection	Transition	Store	Unit
W	10.001705	20.528582	101.194014	GeV/N
cp	10.899240	21.445893	102.127514	GeV/N
E	10.939512	21.466388	102.131820	GeV/N
$B\rho$	72.711904	143.071599	681.321438	Tm
β	0.99631871	0.99904526	0.99995784	
γ	11.665000	22.8900	108.905017	
η	-0.00544	0.0	0.00182	
ϵ_H (95%)	$\leq 21\pi$	$\leq 21\pi$	$\leq 21\pi$	mm mrad
ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
h	360	360	2520	
hf	28.047015	28.123787	197.0462113	MHz
$2\pi R$	3833.8476	3833.8452	3833.8476	m

Parameter	Injection	Store	Unit
No. of Bunches	60	60	
Bunch Spacing	213.772	213.148	ns
Ions/Bunch	16	16	10^{10}
Bunch Area	0.5	0.7	eV s/N
Bunch Δt	21	2.8	ns
Bunch ΔE	32	327	MeV
Bucket ΔE	40	412	MeV
Gap Volts	150	6000	KV
Bucket Area	0.91	1.33	eV s/N

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