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Static and Dynamic Emittance Growth in a Storage Ring

In the design of the storage ring of a synchrotron light source, the natural emittance, ε_o, is a parameter of primary importance. Because the desired value of ε_o is very small (in the order of 10^{-9} m-rad for the APS), any errors of magnetic elements and any external perturbations which lead to distortions in the closed orbit may potentially cause a considerable growth in ε_o. This note is intended to provide some insight into this problem.

The errors of magnetic elements resulting from manufacturing and assembling are usually static (i.e., time-independent), while external perturbations are often dynamic (i.e., varying with time). Correspondingly, one can define two kinds of emittance growth: static and dynamic, denoted by $\Delta\varepsilon_s$ and $\Delta\varepsilon_d$, respectively. The total effective emittance will then be

$$\varepsilon_{eff} = \varepsilon_o + \Delta\varepsilon_s + \Delta\varepsilon_d$$

$$\equiv \varepsilon_s + \Delta\varepsilon_d \, , \qquad (1)$$

in which $\varepsilon_s = \varepsilon_o + \Delta\varepsilon_s$ is the total static emittance. The characteristics of $\Delta\varepsilon_s$ and $\Delta\varepsilon_d$ are quite different. In the following we give a brief discussion for each of them.

A. The static emittance growth $\Delta\varepsilon_s$

Some static machine errors, such as the dipole integrated field errors, the dipole roll errors or the quadrupole misalignment, will give rise to static closed orbit distortions, which in this section we will denote by $\bar{X}$ in the horizontal plane and by $\bar{Y}$ in the vertical plane, respectively. These deviations lead to larger horizontal/vertical dispersions, which, in turn, will produce the static emittance growth $\Delta\varepsilon_s$. Figure 1 shows, for the CG5 lattice, which is a candidate for the storage ring of the APS, how ε_s (the sum of ε_x and ε_y) grows as errors increase. The ratio between ε_y and ε_x, which is sometimes called the "coupling" and is denoted by K, is also plotted against errors in the same diagram. The data are obtained from

PATRIS. It is apparent that ε_s depends on errors quadratically. This behavior can be understood by the following argument.

Because we will only be interested in the closed orbit distortions, the effects of sextupoles can, to the first order, be ignored. The equation of the horizontal closed orbit is then of the form

$$\bar{X}''(s) + K_x \bar{X}(s) = b_x(s), \tag{2}$$

in which the right-hand side, $b_x(s)$, stands for the equivalent horizontal dipole kicks due to the various kinds of errors mentioned above. The equation of the horizontal dispersion has a similar form

$$\eta''(s) + K_x \eta(s) = -\frac{1}{\rho} + K_x \bar{X}(s). \tag{3}$$

The solutions to Eqs. (2) and (3) are well-known and can be expressed in the integral forms:

$$\bar{X}(s) = \frac{\sqrt{\beta(s)}}{2\sin\pi\nu_x} \int_s^{s+C} \sqrt{\beta(s')}\, b_x(s') \cos(\mu' - \mu + \pi\nu_x)\, ds' , \tag{4}$$

$$\eta(s) = \frac{\sqrt{\beta(s)}}{2\sin\pi\nu_x} \int_s^{s+C} \sqrt{\beta(s')} \left(-\frac{1}{\rho} + K_x\bar{X}(s')\right) \cos(\mu' - \mu + \pi\nu_x)\, ds'. \tag{5}$$

One can see that the closed orbit distortion $\bar{X}$ is proportional to the errors. So is the dispersion η to $\bar{X}$. Thus, η is proportional to the errors. The same argument holds for η'. Meanwhile, the horizontal natural emittance can be obtained through

$$\varepsilon_x = \frac{C_o\left(\frac{E}{E_o}\right)^2}{J_x\,\rho} < \gamma\eta^2 + 2\alpha\,\eta\eta' + \beta\eta'^2 >_{\text{bending magnet}}. \tag{6}$$

In other words, ε_x is a quadratic function of η and η'. Also note that the dependence of the Twiss parameters, β, α and γ, on the closed orbit deviation is a 2nd-order effect. Thus, roughly speaking, ε_x is a quadratic function of $\bar{X}$ and, consequently, of the errors. By the same token, the vertical natural emittance, ε_y, as well as the total natural emittance $\varepsilon_s = \varepsilon_x + \varepsilon_y$, depend on errors quadratically. This explains the behavior of ε_s in Fig. 1.

It should be pointed out that in a real machine, one always uses correctors to correct $\bar{X}$ and $\bar{Y}$ to small acceptable values. The actual value of $\Delta\varepsilon_s$ will then be reduced accordingly.

B. The dynamic emittance growth $\Delta\varepsilon_d$

External perturbations are usually time-dependent (e.g., the ground vibrations, the coolant flow, the varying dipole field due to fluctuating power supplies, etc.). The resulting closed orbit deviations will change with time as well. This leads to the dynamic emittance growth and is illustrated in Fig. 2.

Assume that the dynamic growth $\Delta\varepsilon_d$ is small compared with the static emittance ε_s. Then it is easily seen that

$$\frac{\Delta\varepsilon_d}{\varepsilon_s} \approx \begin{cases} 2\,\dfrac{\bar{x}}{\sigma_x}, & \text{horizontal,} \\ 2\,\dfrac{\bar{y}}{\sigma_y}, & \text{vertical,} \end{cases} \tag{7}$$

in which $\bar{x}$ and $\bar{y}$ are the <u>time-dependent</u> components of the closed orbits, and σ_x and σ_y are the horizontal and the vertical beam sizes, respectively. Because most of the experiments on the APS will make use of insertion devices (IDs), we take the values of the σ's in the ID region to estimate the dynamic emittance growth. Assuming that

$$(\varepsilon_s)_y = 0.1\ (\varepsilon_s)_x \tag{8}$$

and that $\Delta\varepsilon_s$ is negligible, then for the CG5 lattice, one has

$$\begin{cases} \sigma_x = 321\ \mu\text{m}, \\ \sigma_y = 89\ \mu\text{m} \end{cases} \tag{9}$$

at the center of the ID region. Plugging Eq. (9) into Eq. (7), we get

$$\frac{\Delta\varepsilon_d}{\varepsilon_s} \approx \begin{cases} 0.6\%, & \text{for } \bar{x} = 1\ \mu\text{m}, \\ 2.2\%, & \text{for } \bar{y} = 1\ \mu\text{m}. \end{cases} \tag{10}$$

In other words, each 1 μm of horizontal closed orbit oscillation will give rise to a 0.6% increase in ε_x; each 1 μm vertical one, to a 2.2% increase in ε_y. In order to keep the emittance growth below a certain amount, say 10%, one has to control the oscillating closed orbits such that

$$\begin{cases} \bar{x} < \frac{10}{0.6} = 16\ \mu\text{m}, \\ \bar{y} < \frac{10}{2.2} = 4.5\ \mu\text{m}. \end{cases} \tag{11}$$

There exist various kinds of time-dependent perturbations. For each of them, one can define a magnification factor as the ratio between the magnitude of the closed orbit oscillation and that of the perturbation. These magnification factors for the storage ring of the APS have been calculated elsewhere. The results are listed in Tables 1, 2, and 3 for three types of perturbations. The notations A_x and A_y in the tables represent the horizontal and vertical magnification factors, respectively.

By combining Eq. (10) and Tables 1, 2, and 3, one can readily estimate the dynamic emittance growth $\Delta\varepsilon_d$ due to a certain time-dependent perturbation or, vice versa, for a given value of $\Delta\varepsilon_d$, one can calculate the allowed magnitude of the perturbation.

Once again, it should be emphasized that the dynamic closed orbit distortions will be corrected in a real machine. Therefore, the actual dynamic emittance growth will ultimately depend on the dynamic accuracy of monitors and correctors. Assuming that the oscillating frequency of the closed orbit is not too high (say, below 20 Hz), so that the EM field of the correctors can effectively penetrate into the vacuum chamber and that the repeatability of the monitoring and correcting system is about 30 μm, then, in view of Eq. (10), the final value of $\Delta\varepsilon_d$ due to all kinds of perturbations will be about

$$\begin{cases} \left(\frac{\Delta\varepsilon_d}{\varepsilon_s}\right)_{\text{horizontal}} = 18\%, \\ \left(\frac{\Delta\varepsilon_d}{\varepsilon_s}\right)_{\text{vertical}} = 66\%. \end{cases} \tag{12}$$

Reference

1. For calculations, see W. Chou, ANL Light Source Note LS-88 (1987).

Table 1
Magnification Factors of Ground Vibrations with Frequency f[1]

		A_x		A_y
Support Effects Neglected†	f<62 Hz:	1	f<18 Hz:	1
	62<f<110 Hz:	$1<A_x<17.9$	18<f<38 Hz:	$1<A_y<15.6$
	f>110 Hz:	17.9	f>38 Hz:	15.6
Additional Magnification Due to Supports*,**	f<41 Hz:		$A_m = 3.5$	
	f~41 Hz:		first resonant peak	
	41<f<128 Hz:		$A_m \lesssim 1$	
	f~128 Hz:		second resonant peak	
	f>128 Hz:		$A_m \to 0$	

† Propagation velocity v of plane wave in the ground is assumed to be 2500 m/s. For different values of v, the values of f should be scaled by using $\frac{v}{f}$ = const.

* Calculation is done for the long girder supporting four quads and three sextupoles in the dispersive straight section of the storage ring. The values of f in this part are functions of the mechanical structure of the girder only and are independent of v.

** The total magnification should be the product $A_m \cdot A_x$ (or $A_m \cdot A_y$).

Table 2
Magnification Factors of Local Excitations†

	A_x	A_y
ID region	7.2	3.4
Maximum	9.3	4.6

†For example, the quad vibrations caused by coolant flow.

Table 3
Magnification Factors of Random Excitations[†]

	A_x	A_y
Dipole field fluctuation	$\dfrac{\bar{x}_{rms}}{\frac{\Delta B}{B} \times 10^{-6}} = 1.3\ \mu m$	$\dfrac{\bar{y}_{rms}}{\frac{\Delta B}{B} \times 10^{-6}} = 3\ \mu m$
Quad vibration with amplitude δ	$\dfrac{\bar{x}_{rms}}{\delta} = 47$	$\dfrac{\bar{y}_{rms}}{\delta} = 37$

[†]The results are obtained from 21 runs of PATRIS and are taken from the ID region.

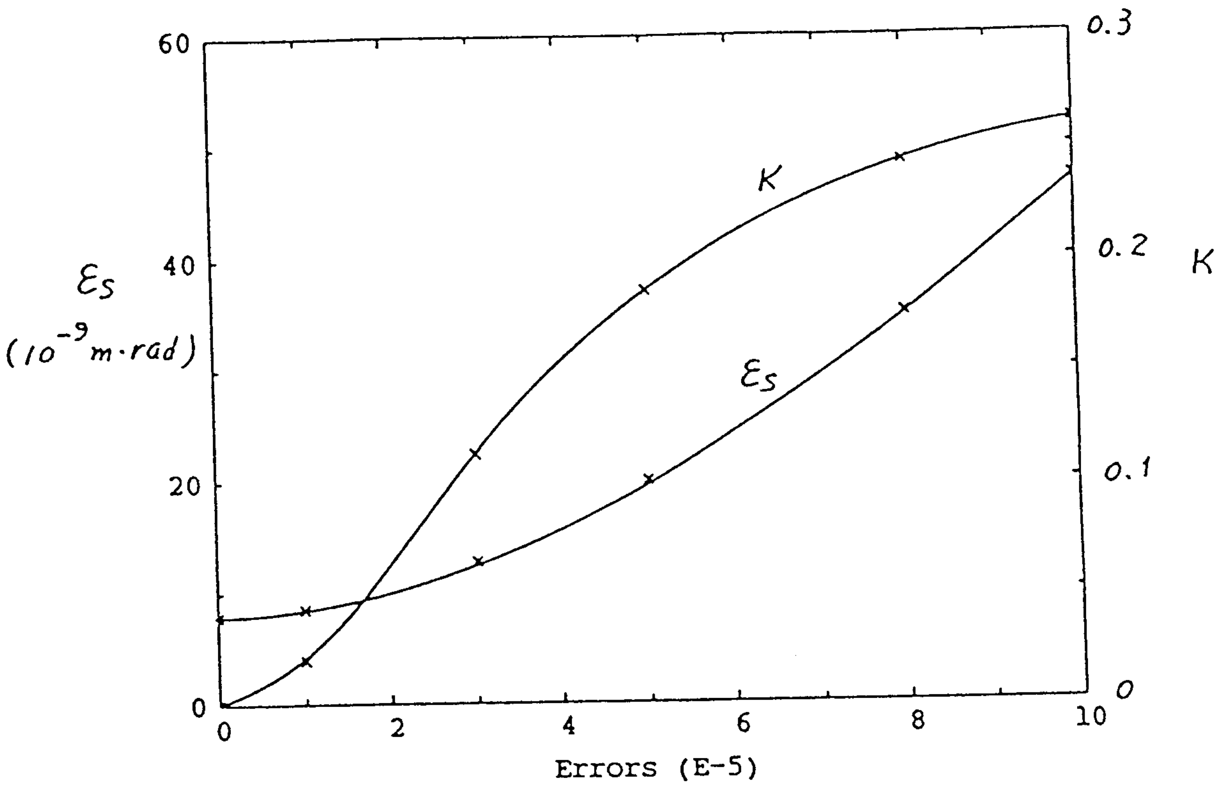

Fig. 1. The static emittance growth in the APS due to errors of magnetic elements. The total static emittance ε_s is the sum of ε_x and ε_y. The "coupling" K is the ratio $\varepsilon_y/\varepsilon_x$. Data are from PATRIS.

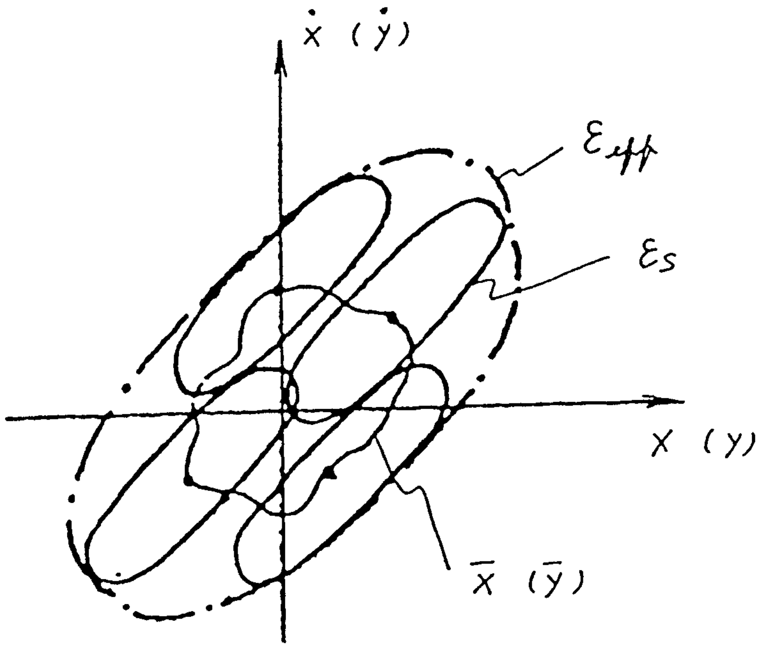

Fig. 2. A time-dependent closed orbit distortion $\bar{x}$ (or $\bar{y}$) leads to a dynamic emittance growth.