DETERMINATION OF STANDARD MODEL PARAMETERS FROM $K^+ \to \pi^+\nu\bar{\nu}$

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1. Introduction

A new way of representing a measurement of the decay $K^+ \to \pi^+\nu\bar{\nu}$ in the Standard Model has been developed. Using the Wolfenstein representation of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix, a measurement of $K^+ \to \pi^+\nu\bar{\nu}$ determines a region in the $\rho - \eta$ plane which can be combined with other measurements to ascertain the consistency of the Standard Model. This graphic display of the expected branching ratio also gives additional insight into how the predicted branching ratio arises in the Standard Model.

2. $K^+ \to \pi^+\nu\bar{\nu}$ in the Standard Model

Wolfenstein[1] has expressed the CKM matrix

$$\begin{pmatrix} V_{ud}V_{us}V_{ub} \\ V_{cd}V_{cs}V_{cb} \\ V_{td}V_{ts}V_{tb} \end{pmatrix}$$

in the following parameterization in powers of the Cabibbo angle, λ:

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

where the CP violating phase is represented by the (ρ, η) point in the complex plane.

The branching ratio for $K^+ \to \pi^+\nu\bar{\nu}$ is given in the Standard Model for three light neutrino types as[2,3]

$$\frac{3\alpha^2 B_{K^+\to\pi^0 e^+\nu}}{8\pi^2 \sin^4\theta_W} \frac{|\, V_{cs}^* V_{cd} D_c + V_{ts}^* V_{td} D_t \,|^2}{V_{us}^2}$$

In the Wolfenstein parameterization of the CKM matrix, some algebraic manipulation can show that this implies

$$\frac{B_{K^+\to\pi^+\nu\bar{\nu}}}{B_{K^+\to\pi^0 e^+\nu}} \cdot \frac{8\pi^2 \sin^4\theta_W}{3\alpha^2} \cdot \frac{1}{A^4\lambda^8 D_t^2}$$
$$= \eta^2 + \left(1 + \frac{(1-\lambda^2/2)}{A^2\lambda^4}\frac{D_c}{D_t} - \rho\right)^2$$

which is a circle in the $\rho - \eta$ plane, with center on the ρ axis at

$$\rho = 1 + \frac{(1-\lambda^2/2)}{V_{ts}^2}\frac{D_c}{D_t}$$

and radius

$$\frac{1}{V_{ts}^2 D_t}\sqrt{\frac{B_{K^+\to\pi^+\nu\bar{\nu}}}{2.11\times10^{-6}}}$$

where $\sin^2\theta_W = 0.23$, $\alpha = 1/128$, and $B_{K^+\to\pi^0 e^+\nu} = 0.0482$, and the kinematic function D is given by

$$D(x) = \left(1 + \frac{3-(4-x)^2}{(1-x)^2}\right)\frac{x\ln x}{8} + \frac{x}{4} - \frac{3x}{4(1-x)}$$

with $x = m_{q_{2/3}}/m_W$, and D_c and D_t are values for the charm and top quark, respectively.

Thus, a measurement of $K^+ \to \pi^+\nu\bar{\nu}$ determines a circle in the $\rho - \eta$ plane which is centered on the ρ axis, displaced slightly from the point (1,0) by the charm quark contribution to the branching ratio, with a distance from that point that decreases with increasing $|\, V_{ts} \,|$ or increasing top quark mass (since the function D is monotonically increasing with top quark mass). The radius grows as the square root of the measured branching ratio, and varies with the same dependence on $|\, V_{ts} \,|$ and top quark mass as the displacement of the center.

The CKM parameter with the largest uncertainty after V_{td} is V_{ts}, which is assigned a 20% uncertainty in the 1990 *Review of Particle Properties*.[4] If V_{ts} is chosen as the vertical axis, the region determined by a measurement of $K^+ \to \pi^+\nu\bar{\nu}$ is the surface of a cone-like object, which decreases in radius and approaches (1,0) as $|\, V_{ts} \,|$ increases.

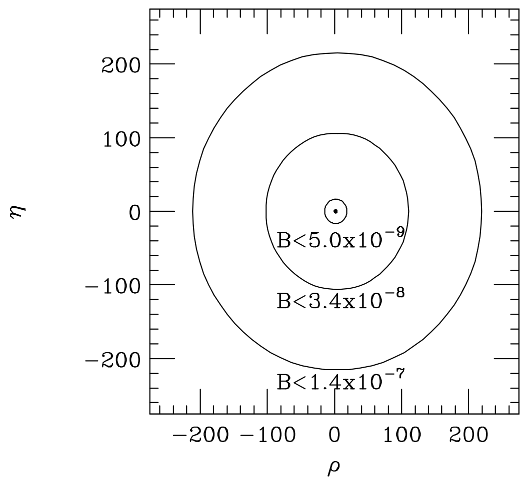

Figure 3.1: Regions of the $\rho - \eta$ plane determined by the KEK experiment,[7] the published Brookhaven result,[8] and a preliminary result from Brookhaven. The region expected in the standard model is in the small circle near the origin.

The QCD radiative corrections to $K^+ \to \pi^+\nu\bar{\nu}$ have been calculated recently.[5] The effect is roughly to scale the charm quark contribution represented by D_c by 0.71. This does not change the geometric interpretation of the branching ratio, but does directly reduce the displacement of the center from (1,0) by that factor. In the calculations that follow, D_c has been scaled by 0.71 to take account of QCD radiative corrections.

3. Present and Future Constraints in the $\rho - \eta$ Plane from $K^+ \to \pi^+\nu\bar{\nu}$

3.1. Present Limits

Upper limits on the branching ratio of $K^+ \to \pi^+\nu\bar{\nu}$ define the boundary of a circular region in the $\rho - \eta$ plane which must contain the value of ρ and η chosen by nature. The lower limit on the top quark mass of 89 GeV/c^2 from CDF[6] and the minimum value of $\mid V_{ts} \mid$ are also necessary assumptions to define the circumscribed region. Since the present upper limit is more than an order of magnitude from the prediction of the Standard Model, it is not surprising to find that the region provides no new constraint. Fig. 3.1 shows the regions allowed by Asano at KEK,[7] the Brookhaven E-787 published limit from data taken in 1988,[8] and a preliminary result from data taken by E-787 in 1989,[9]. The region predicted by the Standard Model is just a dot near the center on this scale.

3.2. Measurement of $K^+ \to \pi^+\nu\bar{\nu}$

In this section, an observed signal with a branching ratio of 2×10^{-10} will be imagined. Since such a small branching ratio is not likely to be observed for several years, it will be assumed that the top quark will have been discovered, with a mass of 140 GeV/c^2, which is Marciano and Rosner's[10] present day prediction based on electroweak radiative corrections to $\sin^2\theta_{\mathrm{W}}$.

When the branching ratio of $K^+ \to \pi^+\nu\bar{\nu}$ is measured, the region defined will be subject to the following uncertainties:

- Statistical and systematic error in the measurement of the branching ratio of $K^+ \to \pi^+\nu\bar{\nu}$. A measurement with 25% uncertainty in the branching ratio should be possible in the mid-1990's.
- Uncertainty in the top quark mass. Assuming that the top quark is discovered between 89 and 200 GeV/c^2, it has been estimated for the SSC that the uncertainty in the mass would be about 1%.[11]
- Uncertainty in the value of $\mid V_{ts} \mid$, which is equivalent to the magnitude of V_{cb}. This is determined by the b-quark lifetime and the semileptonic branching ratio to charm, or from exclusive B$_{e3}$ decays. The error is dominated by theoretical uncertainties either way. Currently, the uncertainty is estimated to be 10 to 20%[4,12], which directly translates into the error on the radius and displacement.

Fig. 3.2 show the annular region determined by a measurement of $K^+ \to \pi^+\nu\bar{\nu}$, assuming a top quark mass of 140 GeV/c^2. The parameter $\mid V_{ts} \mid$ has been allowed to vary between 0.044 and 0.053 in the five nearly concentric circles. The annular region centered on the origin is determined[12] by present knowledge of $b \to u$ compared to $b \to c$ transitions.

4. Other Constraints in the $\rho - \eta$ Plane

Kim, Rosner, and Yuan[12] have recently reviewed previously known constraints on the allowed region in the $\rho - \eta$ plane. When the top quark mass is known, the measurement of $K^+ \to \pi^+ \nu\bar{\nu}$ can be displayed along with those results to see whether the three generation Standard Model is adequate to explain the results of all those experiments. The constraint most similar to $K^+ \to \pi^+ \nu\bar{\nu}$ is from $B - \bar{B}$ mixing. $K^+ \to \pi^+ \nu\bar{\nu}$ has a theoretical advantage over $B - \bar{B}$ mixing, in that mixing requires knowledge of the form factor f_B which is the subject of intense theoretical scrutiny.

5. Conclusion

An interpretation of $K^+ \to \pi^+ \nu\bar{\nu}$ has been developed in the Wolfenstein parameterization of the CKM matrix, which graphically shows what that process tells us about the Standard Model, and how a discrepancy with the model or with other experiments would be revealed. It also more clearly shows the origin of the prediction in the Standard Model, and the contribution of charm and top to the predicted branching ratio.

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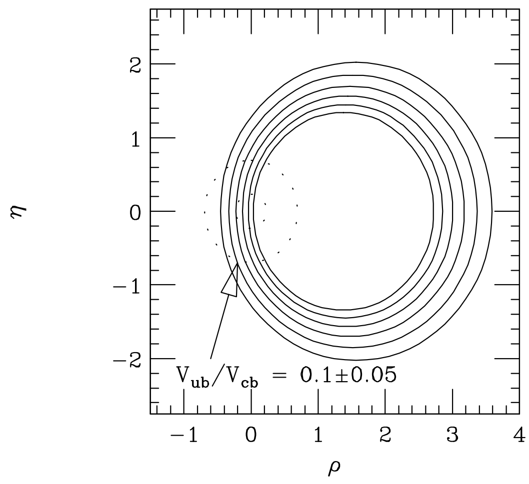

Figure 3.2: Region which would be determined by a measurement of a branching ratio of 2×10^{-10} for $K^+ \to \pi^+ \nu\bar{\nu}$, assuming a top quark mass of 140 GeV/c^2, and allowing $0.044 <| V_{ts} |< 0.053$.

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