Amplification of pionic instabilities in high-energy collisions?

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Considering a variety of dynamical scenarios within the linear σ model, we examine the conditions for pionic modes to be amplified as the chiral field relaxes towards the normal vacuum following the transient restoration of approximate chiral symmetry in a high-energy collision. While Bjorken-type longitudinal expansions appear to be insufficient for amplification for occur, analoguos expansions in two or three dimensions may enhance the low-energy power spectrum by up to an order of magnitude.

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The possibility of forming disoriented chiral condensates (*DCC*) in high-energy collisions has generated considerable research activity in recent years (for a recent review, see ref. [1]). The basic premise is that such collisions produce extended hot regions within which approximate chiral symmetry is temporarily restored. The subsequent non-equilibrium relaxation may then lead to the formation of coherent sources of low-energy pions and associated anomalous multiplicity distributions [2–4]. Important insights into the dynamics of *DCC* formation have been gained with analytical approaches [5–8] and, in the most elaborate studies, the evolution of the chiral degrees of freedom has been simulated numerically [9–16]. We present here a simple and instructive framework within which we analyze the conditions for the occurrence of the *DCC* phenomenon.

Most dynamical studies of disoriented chiral condensates have been based on the linear σ model in which the chiral degrees of freedom are described by the real $O(4)$ field $\boldsymbol{\phi}(\boldsymbol{r},t)=(\sigma,\boldsymbol{\pi})$ having the equation of motion

$$\left[\Box+\lambda(\phi^2-v^2)\right]\boldsymbol{\phi}=H\hat{\boldsymbol{\sigma}}\ . \tag{1}$$

The three parameters in the model can be fixed by specifying the pion decay constant, $f_\pi = 92$ MeV, and the meson masses, $m_\pi = 138$ MeV and $m_\sigma = 600$ MeV, leading to the values $\lambda = (m_\sigma^2 - m_\pi^2)/2f_\pi^2 = 20.14$, $v = [(m_\sigma^2 - 3m_\pi^2)/(m_\sigma^2 - m_\pi^2)]^{1/2} f_\pi = 86.71$ MeV, and $H = m_\pi^2 f_\pi = (120.55\ \text{MeV})^3$, with $\hbar, c$=1 [17].

As is apparent from eq. (1), the vacuum configuration is aligned with the σ direction, $\boldsymbol{\phi}_{\text{vac}} = (f_\pi, \mathbf{o})$, and at low temperature the fluctuations represent nearly free σ and π mesons. In the other extreme, at temperatures well above v, the field fluctuations are centered near zero and approximate $O(4)$ symmetry prevails.

In order to analyze the situation, it is instructive to decompose the chiral field, $\boldsymbol{\phi}(\boldsymbol{r},t)=\underline{\boldsymbol{\phi}}(t)+\delta\boldsymbol{\phi}(\boldsymbol{r},t)$. The first term, $\underline{\boldsymbol{\phi}}$, is the average over a suitable region in space (of dimensions larger than the correlation length) and can be identified with the (local) order parameter, while the fluctuations, $\delta\boldsymbol{\phi}(\boldsymbol{r})$, may be considered as elementary quasi-particle excitations. We may simplify the discussion without affecting the conclusions by assuming that the order parameter is always fully aligned, $\underline{\boldsymbol{\phi}}=(\sigma_0,\mathbf{o})$. The fluctuations are then $\delta\boldsymbol{\phi}(\boldsymbol{r},t)=(\delta\sigma,\delta\boldsymbol{\pi})$.

Applying a Hartree-type factorization [15,17], we may replace the full equations of motion (1) by a set of self-consistent approximate equations,

$$\left[\Box+\mu_0^2\right]\sigma_0 = H\ , \tag{2}$$

$$\left[\Box+\mu_\sigma^2\right]\delta\sigma = 0\ , \tag{3}$$

$$\left[\Box+\mu_\pi^2\right]\delta\boldsymbol{\pi} = \mathbf{o}\ , \tag{4}$$

where the associated effective masses are given by

$$\mu_0^2 = \lambda[\ \sigma_0^2 + <\delta\phi^2> + 2<\delta\sigma^2> - v^2]\ , \tag{5}$$

$$\mu_\sigma^2 = \lambda[3\sigma_0^2 + <\delta\phi^2> + 2<\delta\sigma^2> - v^2]\ , \tag{6}$$

$$\mu_\pi^2 = \lambda[\ \sigma_0^2 + <\delta\phi^2> + 2<\delta\pi_i^2> - v^2]\ , \tag{7}$$

with $\delta\pi_i$ being the particular cartesian component of $\delta\boldsymbol{\pi}$. Thus the field fluctuations provide an additional stiffnes resisting the growth of the order parameter σ_0. The effective masses are degenerate for σ_0=0 and they vanish at the temperature $T_c=\sqrt{2}v$, in the mean-field treatment. Moreover, we always have $\mu_0^2 \leq \mu_\pi^2 \leq \mu_\sigma^2$.

Eqs. (2) and (5) were first derived in ref. [18]. The term $<\delta\phi^2>$ in eqs. (5-7) is the sum of the field fluctuations in each of the N=4 chiral directions and is of leading order in $1/N$. These are the 'direct' terms that have been included in previous *DCC* treatments or discussions in terms of effective masses [5,9,10,15,16]. The next term in eqs. (5-7) is twice the fluctuation along the particular direction considered (either parallel or perpendicular to the order parameter) and arises from the 'exchange' terms. A recent analysis of the statistical properties of the linear σ model [17] suggests that their effect is significant ($\sim 2/N$=50%) and that their inclusion leads to a quite good approximation to the full eq. (1).

The approximate equations (2-7) provide a convenient framework for developing a qualitative understanding of the dynamics generated by the full equation (1). Imagine that the system is initially created in thermal equilibrium at a temperature T_0 well above T_c. The field fluctuations are then sufficiently large to ensure μ^2>0 in all three eqs. (5-7). The system is expected to experience a cooling resulting from expansion and radiation, so the fluctuations decrease in the course of time. This reduces μ^2 which allows the order parameter to grow larger, thus counteracting the decrease of the effective masses. The resulting behavior of μ^2 is then a delicate balance: for slow cooling the induced growth of σ_0 is approximately adiabatic and the system relaxes towards the vacuum through metastable configurations; however, if the fluctuations diminish rapidly a compensating growth of the order parameter can no longer occur quickly enough and one or more of the effective masses may turn imaginary, $\mu^2<0$, indicating that the system has entered a regime exhibiting exponential growth of some modes.

Fig. 1 shows the equilibria and the unstable region. As T is increased, the fluctuations grow steadily and the equilibrium value of σ_0 decreases from f_π. The most rapid change occurs at $T \approx 220$ MeV, above which σ_0 tends to zero. The effective pion mass μ_π increases monotonically with T from its free value m_π towards $\approx 1.6T$ for $T \gg T_c$, while μ_σ first decreases, then displays a minimum at $T \approx 240$ MeV, and finally becomes degenerate with μ_π. The border of the unstable region intersects the σ axis at f_π and extends up to $T_c/\sqrt{3}$ at σ_0=0.

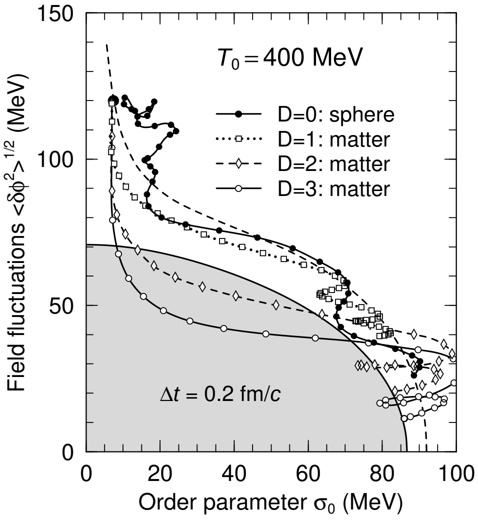

FIG. 1. Dynamical trajectories.

The combined dynamical evolution of the order parameter $\sigma_0 =< \sigma >$ and the field fluctuations $< \delta\phi^2 >^{1/2}$ (sampled for r <2.5 fm in the case of D=0). The dashed curve connects the equilibria from T=0 to above 500 MeV and the unstable region within which $\mu_\pi^2 < 0$ is shown by the shaded region. Each system has been prepared in thermal equilibrium at $T_0 = 400$ MeV, using a periodic box (20 fm side length). The irregular solid trajectory (D=0) was obtained by solving the standard eq. (1) after applying a spherical Saxon-Woods modulation factor (5 fm radius and 0.5 fm width) to the hot matter, thereby producing a hot sphere embedded in vacuum [19]. The other three trajectories have been obtained without a spatial modulation but with the term $-(D/t)\partial_t\boldsymbol{\phi}$ added in the equation of motion to emulate uniform expansions in D=1,2,3 dimensions. The marks along the trajectories are positioned at time intervals of $\Delta t = 0.2$ fm/c.

Also shown in fig. 1 is the dynamical trajectory of the central part of a Ni-sized spherical source prepared at T_0=400 MeV. The system keeps away from the unstable regime, exhibiting an approximately adiabatic evolution. (The windings reflect the damped oscillations of σ_0 in the relaxing effective potential, in the familiar manner [11,12].) This behavior is rather robust, as it occurs for a wide range of initial temperatures and for rod or slab geometries as well. It thus appears that initially static field configurations in local equilibrium do not develop any instabilities during their subsequent expansion.

However, it is expected that the early parton dynamics causes the chiral field to be in a state of rapid expansion. The subsequent evolution may then lead to a supercooled configuration situated inside the unstable region, thus effectively producing a "quench". A number of quenched scenarios have been considered [9,11–16] but they were imposed by *fiat*, thereby reducing the predictive power of the dynamcial calculations (essentially any degree of magnification can be achieved by suitable adjustment of the initial conditions). It is our aim here to reduce the degree of arbitrariness by elucidating under which conditions a quench-like early scenario may develop dynamically from plausible initial configurations.

Simple Bjorken-like pictures can be invoked to emulate expanding scenarios in D dimensions, either longitudinal (D=1) [11,13,16], transverse (D=2), or isotropic (D=3) [12,20]. We have considered such scenarios in the following approximate manner. The essential effect of the expansion is the appearance of an additional term on the right-hand side of eq. (1), $-(D/t)\partial_t\boldsymbol{\phi}$, where the time variable should now be reinterpreted as the elapsed proper time in a comoving frame (starting at $t_0 = 1$ fm/c, usually). The corresponding Lorentz transformation of the (scaled) spatial coordinates is less essential for our present discussion and has therefore been ignored.

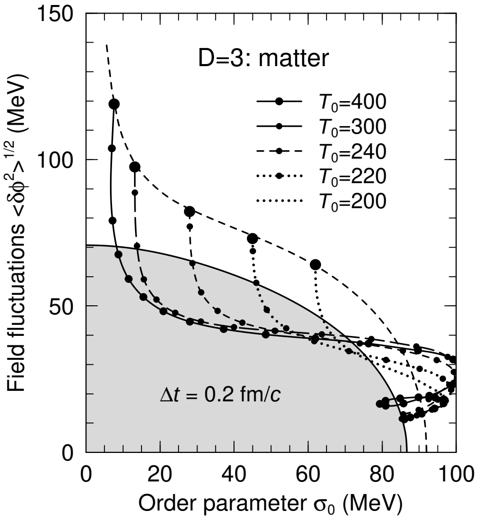

FIG. 2. Idealized isotropic expansions.

Similar to fig. 1, but for idealized isotropic expansions using D=3 in the modified equation of motion.

The additional term in the equation of motion, $-(D/t)\partial_t\boldsymbol{\phi}$, acts as a time-dependent damping, its form being akin to the Rayleigh dissipation function in classical mechanics, and it is significantly more effective in reducing the fluctuations than self-generated expansions. In order to examine its effect, we ignore the spatial geometry and consider a macroscopically uniform configuration within a large box. The discussion is then simplified and the resulting scenarios can be regarded as idealized representations of chiral matter subjected to an externally prescribed cooling rate, and so the results will have a corresponding general applicability.

Figure 1 includes dynamical trajectories obtained in this manner for D=1,2,3. The effect increases with D, since the dimensionality of the expansion effectively acts as the strength of the damping term. The isotropic expansion leads to a significant incursion into the unstable region, while the longitudinal one is too slow for that.

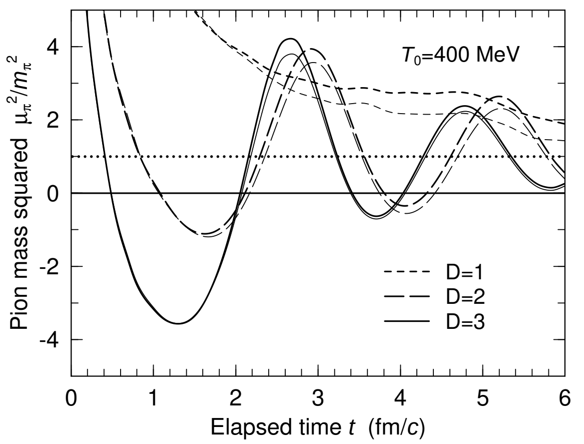

FIG. 3. Time evolution of the pion mass.

The time evolution of μ_π^2, the square of the effective pion mass, eq. (7), for idealized D-dimensional expansions starting from thermal equilibrium at T_0=400 MeV. The thin curves show the corresponding evolution of μ_0^2 (recall $\mu_0^2 \leq \mu_\pi^2 \leq \mu_\sigma^2$). All curves approach the free pion mass m_π (dotted line) in the course of time.

TABLE I. Amplification coefficients/correlation lengths.

T_0 (MeV)	$D=1$	$D=2$	$D=3$
200	0.00 /1.4	0.02 /1.8	0.11 /2.0
220	0.00 /1.3	0.50 /1.9	0.55 /2.5
240	0.01 /1.3	1.20 /2.0	1.19 /2.7
300	0.00 /0.9	1.84 /1.7	2.06 /2.7
400	0.00 /0.6	1.67 /1.3	2.49 /2.1
500	0.00 /0.5	1.31 /1.1	2.61 /1.6

The enhancement factor $G^\pi_{k=0}$ in macroscopically uniform chiral matter and the resulting correlation length, expressed as the full width at half maximum (in fm) of the spatial two-point function $\prec \delta\boldsymbol{\pi}(x)\cdot\delta\boldsymbol{\pi}(y) \succ$, for pions emerging after idealized expansions with D=1,2,3 starting from thermal equilibrium at T_0.

Figure 2 shows trajectories for D=3 starting from various temperatures. If the initial temperature is lower than 200 MeV or so, the initial value of σ_0 is already fairly large (over 60 MeV) and the dynamical trajectories will miss the unstable region. A wide range of higher temperatures lead into the unstable region, provided the supercooling occurs sufficiently fast. Ultimately, at very high temperatures (above those shown) the system will again stay stable throughout, because it takes so long to reduce the fluctuations down to critical size that the order parameter has meanwhile had time to start its growth.

In order to quantify the analysis, it is useful to consider the time evolution of the effective masses. Since $\mu_\sigma^2 > \mu_\pi^2$ we concentrate on μ_π^2 which is illustrated in fig. 3. It is noteworthy that $\mu_0^2 \approx \mu_\pi^2$ throughout the evolution. (Early on σ_0 is small so $O(4)$ symmetry holds approximately, and later on the fluctuations are less important than σ_0.) The amplification of the lowest pionic modes is then practically identical to that of the order parameter itself. This simple feature makes it an easier task to analyze more complicated scenarios. As was already evident from figs. 1-2, multiple incursions into the unstable region are possible, especially for large initial cooling rates (since a rapid quench brings σ_0 into larger oscillations around f_π), but the first one is always dominant.

It is convenient to express the resulting enhancement of a mode in terms of its amplification coefficient [21],

$$G_k^\pi \equiv \int_{\omega_k^2<0} dt\, \sqrt{-\omega_k(t)^2}\,, \tag{8}$$

where the frequency is given by the dispersion relation, $\omega_k^2 = k^2 + \mu_\pi^2$, using the time-dependent effective pion mass (7). [The dispersion relations for the quasiparticles are in principle affected by the presence of the expansion term, as is ordinarily the case for a damped oscillator. This effect may be significant for damping rates large enough to bring the system into the unstable region, but has been ignored in the present study.] The quantity $\exp(G_k^\pi)$ expresses approximately the factor by which the amplitude of pions having the wave number k has been magnified due to the incursion(s) into the unstable regime. Since largest magnification occurs for k=0 (a finite k adds a positive amount to ω_k^2), we concentrate on this quantity as an upper bound. Moreover, we note that the minimum in $\mu_\pi^2(t)$ (see fig. 3) determines the maximum wave number for which magnification occurs. Amplification coefficients obtained for various idealized expansion scenarios are shown in table I, together with the resulting width of the pion correlation function.

The purely longitudinal expansions (D=1) miss the unstable region, except for insignificant incursions near T=240 MeV, while a significant degree of magnification occurs for the transverse and isotropic expansions, amounting to over a factor of ten in the most favorable cases. It should be recalled that G_0^π applies to k=0 only, so it provides an upper bound on the enhancement.

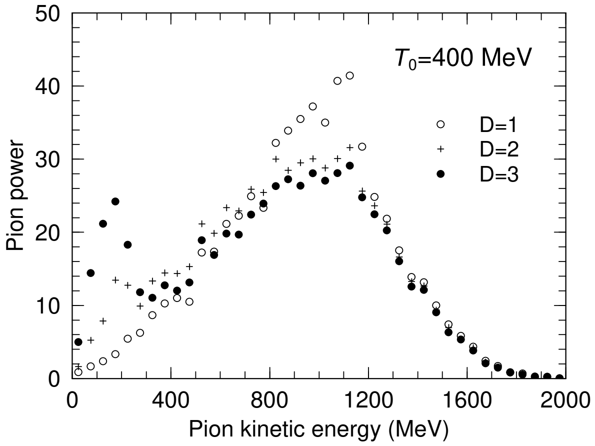

FIG. 4. Pion power spectrum.

The relative power spectrum of the pions, $\sim \omega_k^2 \pi_k^2$, where $\boldsymbol{\pi}_k$ is the Fourier amplitude of the pion field, plotted as a function of the pion kinetic energy ω_k-m_π. The extraction is made at large times when the asymptotic scenario of free evolution has been reached. The plots are based on samples of 20 field configurations prepared at T_0=400 MeV and subjected to idealized expansions with either D=1 (open dots), D=2 (crosses), or D=3 (solid dots). The irregularities are primarily due to the shell structure in the level density of the cube.

Figure 4 gives an impression of the net effect on the power spectrum of the emerging free pions. As expected, the transient instabilities present for D=2,3 lead to significant enhancements of the power carried off by soft pions. The effect amounts to about an order of magnitude for D=3 (relative to the smooth spectrum obtained for D=1), in accordance with the amplification coefficient given in Table I. Although these results were calculated for idealized expansion scenarios, they do support the suggestion that such enhancements may provide an observable signal of *DCC* formation [22].

It should be emphasized that the dynamical evolution of the chiral field was obtained by solving the full equation of motion (1). The simple Hartree approximation (2-7) has been invoked only as a convenient framework for the discussion since it brings out so clearly the conditions for amplification.

Our analysis shows that the occurrence of instabilities, and the associated amplification of pionic modes, depends sensitively on the cooling rate, which in turn is intimately related to the character of the expansion. Our idealized scenario for D=3 corresponds closely to the isotropic expansion considered in refs. [12,20] and our results corroborate the conclusion in [12] that such a scenario leads to amplification. Furthermore, our analysis suggest that a longitudinal expansion alone is insufficient to cause a quench, if the initial fluctuations are of thermal magnitude. This is consistent with what was found in refs. [11,16] for effectively one-dimensional expansions.

This qualitative sensitivity to the collision dynamics highlights the importance of employing realistic initial conditions for the dynamical simulations of *DCC* formation. Ultimately, the appropriate initial field configurations must be calculated on the basis of the early partonic evolution, a task which is thus crucial for our ability to assess the prospects of forming disoriented chiral condensates in high-energy collisions.

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