Imprecision and Uncertainty Quantification for the Problem of Mobile Robot Localization

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Abstract

This paper describes the use of a set inversion algorithm to solve the problem of mobile robot localization. The method is based on the formalism of interval analysis. In this formalism, an imprecise number is represented by an interval which contains it in a guaranteed way. This enables to naturally manage the imprecision linked to the mobile robot configuration. Indeed, we show that imprecision is not correlated to uncertainty, that's why we have quantify imprecision independently from uncertainty. So, from a matching between the sensorial map and the theoretical map, our method gives the robot configuration bracketed between two 3-D subpavings.

1. Introduction

The autonomy of a mobile robot is highly dependent on its capacity to know its location. Self-localization is then a relevant problem in mobile robotics. In order to accomplish their task in a robust way and to increase the reliability in operation, the decisions should be made considering an uncertainty and an imprecision about the robot localization. The management of uncertainty and imprecision during the localization process is then a key element for the success of a mobile robotic mission.

Imprecision and uncertainty are two distinct notions. The imprecision results from unavoidable imperfections of the sensors and of the environment map. So the imprecision represents the error associated to the data. On the other hand, the uncertainty represents the belief or the doubt we have on the existence or on the validity of a data.

Concerning imprecision, many localization methods use statistical state estimation techniques. The most widely used method is the Extended Kalman Filter [4][5]. This method provides a point estimate associated with a confidence region which quantifies the imprecision estimation. If we assume small variations and noise statistical modeling, this method is simple to use. But a major problem concerns the observation equation linearization made with the dead-reckoning prediction: the convergence of the Extended Kalman filtering estimation is assured only if the odometric error is not important. Besides, the EKF method has to know the initial location of the mobile robot.

Other statistical methods such as Markov localization [11] or Monte Carlo localization [12] are also used. These two approaches show good localization performances but they include heavy computational loads.

So an attractive alternative to these methods is set-membership estimation [6]. This formalism allows a natural representation of imprecision by way of intervals: an imprecise number is represented by an interval. This paper presents a localization method based on the interval analysis. So this method manages naturally imprecision.

This paper is organized as follows. In a first part, we will sum up the first part of our work which has concerned uncertainty management and propagation during the localization process. We will particularly analyze the link between uncertainty and imprecision. Then, in a second part, we will deal with our mobile robot configuration determination method based on interval analysis. We will notably show in this part that the localization problem can be treated as a set inversion problem. The paper will end with the presentation of the experimental results.

2. Localization uncertainty and imprecision

2.1. Localization uncertainty estimation

The first part of our work has concerned the uncertainty management [9][10]. The originality of our study is its ability to propagate uncertainties from low level data in order to obtain a global uncertainty about the robot configuration. To this aim, we have built an uncertainty propagation architecture shown Figure 1. The key tool used is the Transferable Belief Model (TBM) of Smets [13]. The TBM is a variant of the belief functions theory [8] which do not assume the existence of any underlying probability functions. This formalism enables to treat uncertainty easily since it permits to attribute mass not only on single hypothesis, but also on union of hypothesis. This is the main difference with Bayesian theory. We can thus express ignorance. This is why this theory is used in several problems of uncertain data fusion [14][15].

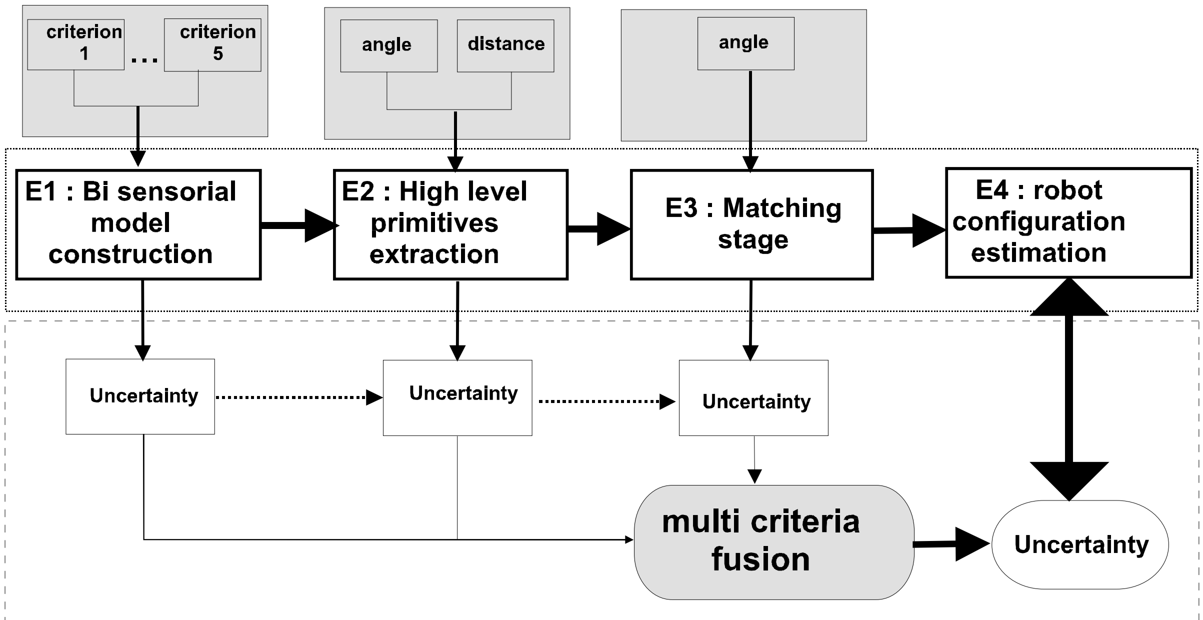

Figure 1. The uncertainty propagation architecture.

This uncertainty propagation architecture is divided into four steps which are directly issued of the classical perception/navigation paradigm commonly used in mobile robotics. In the first step (step **E1** in Figure 1), we compute an uncertainty about the segments that compose the sensorial model. This sensorial model of the environment is built from a multi-sensor cooperation approach between an omnidirectional vision system and a panoramic range finder [9]. This two sensors association provides some complementary and redundant data. So it enables to construct a robust sensorial model which integrates an important number of significant primitives. The segment uncertainty computation is done by considering a binary frame of discernment [10] and by taking into account several criteria fused with the TBM.

The next step is to classify these segments in order to get high level primitives such as "corner", "edge", etc. (step **E2** in Figure 1). These primitives are interested since they are easily observable in an indoor environment and some of them do not suffer to occultation phenomena [10]. The segments uncertainty is propagated to deduce the uncertainty of these primitives [10]. These significant landmarks are then used in our localization method based on multi-target tracking (step **E3** in Figure 1). It uses the TBM in a framework called *extended open world* [7]. This framework is interesting in our case of multi-target tracking since it allows to treat the problem of target apparition and momentary disappearance. This module naturally integrates our uncertainty propagation architecture and enables us to manage an uncertainty for each target.

The last step concerns the localization uncertainty computation (step **E4** in Figure 1). This uncertainty takes notably into account the targets uncertainties [10].

2.2. Study of the correlation between the uncertainty and the imprecision

In order to try to establish a correlation between localization uncertainty and localization imprecision, we have first computed in a basic way the robot's configuration. This is done by considering the matchings we have performed in the multi-target tracking module between the sensorial primitives and the theoretical ones (the robot has in its possession a theoretical map of the environment). To this aim, we basically determine the translation and the rotation between the two maps. This enables us to get a configuration error between the "true" configuration and the computed configuration.

On 80 experimental results performed in an indoor environment (Figure 9), we have tried to determine if the error (i.e. the imprecision) is linked with the localization uncertainty computed in the previous paragraph. To this purpose, we have computed the correlation coefficient between the uncertainty and the localization error (Cartesian error, error in x, in y and in orientation). If the correlation coefficient is close to 1 or –1, this means that the two variables are correlated. If it is close to zero, the two variables are not correlated.

Besides, we have analyzed several others criterion which can influence the imprecision. These criterion are :

- The number of primitives used in the localization process, i.e. the primitives which have been matched with a theoretical one in the multi-target tracking module. If few primitives are used, we can think that the localization will be inaccurate and imprecise [9].
- The number of high level primitives "corner' and "edge" used in the localization process. We want to see if these "strong" and significant primitives influent the localization accuracy [9].
- The angular repartition of the primitives used to localize the robot. If these primitives are placed in a homogenous way around the robot, the localization may be more precise.
- The mean distance between the robot and the primitives. Indeed, our depth sensor becomes less accurate when the distance increases [9].

From the 80 experimental acquisitions, we have obtained the correlation coefficients summarized in Table 1.

	Cartesian error	Error in X	Error in Y	Orientation error
Number of. primitives	-0.20	-0.66	0.35	0.30
Number of. primitives corner-edge	-0.21	0.09	-0.30	-0.06
Angular repartition	-0.11	-0.28	0.05	0.15
Mean distance	-0.40	0.06	-0.55	0.07
Localization uncertainty	-0.15	-0.55	0.30	0.11

Table 1: Correlation coefficients between the imprecision and several criterion.

So we can note that the uncertainty and the criterion are not strongly correlated to the error. This conclusion is not very surprising since the criteria used to quantify localization uncertainty are qualitative ones: they are not linked to any measurement error, they only denote the existence and the reliability of the data. So, we have decided to use an imprecision quantification formalism which is independent of the uncertainty. This formalism has to be able to determine a localization imprecision from the measurements imprecision. As we will see in the next paragraph, the formalism of interval analysis is adequate.

3. Localization by set inversion

3.1. Introduction

We consider here the localization problem of a mobile robot in a 2D-mapped environment. Its configuration vector $q=(x_r, y_r, \theta_r)$ is defined by the coordinates of the robot together with its orientation in a world reference frame (Xe, Ye).

The world map consists on four maps: a map of corners, of edges, of other primitives and a map of segments. These segments, which compose the high level primitives describe before, are defined in the world reference frame by their endpoints.

The problem is to find the robot configuration q considering the matching realized at the previous step (in the multi-target tracking module) between the sensorial primitives and the theoretical ones, and considering an imprecision on the sensor measurements.

In the next paragraph, we will firstly deal about the set inversion problem in the general case. Then, we will show that the localization problem is a set inversion problem.

3.2. Set inversion and interval analysis

Consider a continuous computable function f from IR^n to IR^p. Consider Y a set in the image space IR^p. The set inversion problem consists in determining the set X in IR^n so that X is the reciprocal image of Y by f (Figure 2 for an example from IR^2 to IR^2). This set X is defined by :

$$X = f^{-1}(Y) = \{x \in IR^n \mid f(x) \in Y\}$$

- The f^{-1} function is the reciprocal image of the function f,
- Y is the set to be inverted,
- X is the solution set of the set inversion problem.

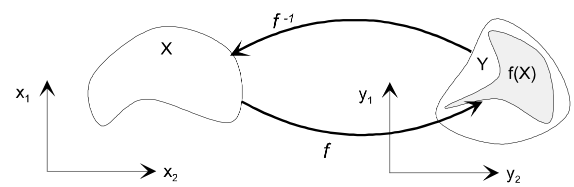

Figure 2: a set inversion problem from IR^2 to IR^2.

The interval analysis is a way to solve this problem. In this formalism, an imprecise number is represented by an interval which contains it in a guaranteed way. In particular, the SIVIA (Set Inversion Via Interval Analysis) algorithm developed by Jaulin and Walter [1] uses interval analysis to solve the set inversion problem and approximates the solution set by an union of boxes

Before the explanation of the SIVIA algorithm, we have to recall the basic notions of interval analysis.

An interval $[x]$ is a closed , bounded and connected set of real numbers [2].

$$[x] = [x^-, x^+] = \{x \in IR \mid x^- \leq x \leq x^+\}$$

The set of all intervals of IR is denoted by II IR.

All classical arithmetic operations can be performed on intervals [2][3].

A box [S] is the Cartesian product of n intervals of II IR. The set of n-dimensional boxes is denoted by II IR^n.

Consider a function f from IR^n to IR^p. The image of a box [S] of II IR^n by f is a set f([S]) in II IR^p which cannot be computed exactly. By using an inclusion function f^I, we can compute a box in II IR^p that contains in a guaranteed way the set f([S]) (Figure 3). The function f^I is an inclusion function of f if it verifies :

$$\forall [S] \in II\ IR^n \quad f([S]) \subset f^I([S])$$

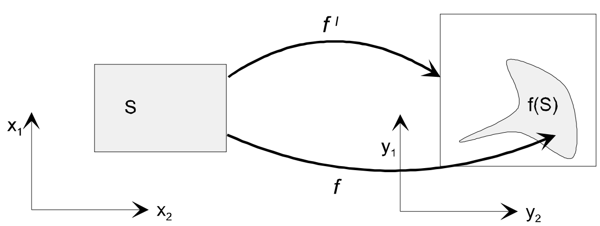

Figure 3: inclusion function.

A simple way to obtain an inclusion function is to replace all elementary operators (+, -, /, etc…) by their interval counterparts. The result of this substitution is called a natural extension of the function f.

3.3. The SIVIA algorithm

The algorithm SIVIA (Set Inversion Via Interval Analysis) [1] can resolve the problem of set inversion. From a set Y to inverse, it enables to approximate the solution set X by two subpavings (union of boxes). It can work with any function f which has an inclusion function f^I.

Before the explanation of the algorithm, we have to precise that any box [S] can be in three different states in comparison with the solution set X :

- [S] is *feasible* if [S] $\subset X$
- [S] is *unfeasible* if [S] $\cap X = \varnothing$
- else [S] is *ambiguous*.

SIVIA uses two tests to decide if any box [S] is feasible or not :

- If f^I ([S]) $\subset$ Y, then [S] $\subset X$: the box [S] is feasible (Figure 4).
- If f^I ([S]) $\cap$ $Y = \varnothing$, then [S] $\cap$ $X = \varnothing$: the box [S] is unfeasible (Figure 5).

In the other cases, the box [S] is ambiguous.

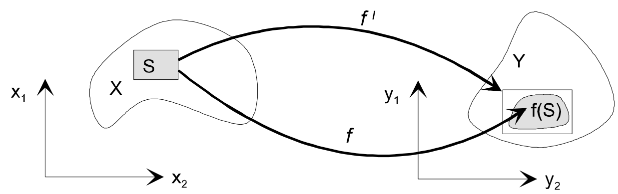

Figure 4. An example of feasible box.

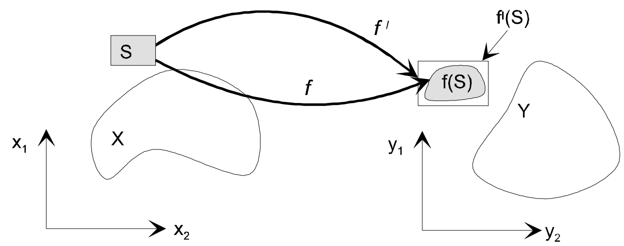

Figure 5. An example of unfeasible box.

From these two tests, SIVIA splits the search domain (i.e. a domain which contains the solution set X in a guaranteed way) into three subpavings :

- [X_{in}] consists of all boxes which have been proved to belong to X, i.e. the feasible boxes.
- [X_{out}] consists of all boxes which have been proved to have an empty intersection with X, i.e. the unfeasible boxes.
- [X_{ind}] consists of all the ambiguous boxes.

The strategy used by SIVIA is to recursively split the boxes in [X_{ind}]. The splitting occurs until a predefined threshold (this implies that the algorithm is finite) [1].

At the end of the algorithm, the solution set X is bracketed between two subpavings :

- The inner subpaving [X_{in}] which contains the feasible boxes, i.e. the boxes which belong to X
- The outer subpaving [X_{in}] $\cup$ [X_{ind}] which contains the feasible boxes and the ambiguous boxes

For more details about SIVIA, please refer to [1] and [3].

3.4. Localization is a set inversion problem

Problem statement. The robot configuration estimation can be seen as a set inversion problem. Indeed, the localization problem from exteroceptive data is the inverse problem of the sensor simulation.

The sensor simulation problem is the following :

Knowing the evolution world of the robot, its configuration q=(x_r, y_r, θ_r) and a modeling function f of the sensor, compute the set M of the sensor measurements m_i, image of q by the function f

$$\begin{pmatrix} x_r \\ y_r \\ \theta_r \end{pmatrix} \xrightarrow{f} \begin{pmatrix} m_1 \\ m_2 \\ \dots \\ m_n \end{pmatrix}$$

From this statement, the localization problem can be seen as follow:

Knowing a set M of sensors measurement which are matched with their corresponding primitives of the theoretical map, compute the set Q of the configurations q whose image by the function f belongs to M

$$Q = \{ q \mid f(q) \in M \} = f^{-1}(M)$$

This is a set inversion problem:

- The set to inverse is M
- The function is f
- The solution set is Q

Problem resolution. Consider the robot configuration q=(x_r, y_r, θ_r) defined by the coordinates of the robot together with its orientation in a world reference frame (*Xe*, *Ye*). The robot detects and matches n landmarks *Bi* (i=1..n) in the robot reference frame (X_R,Y_R). In order to take into account the sensor inaccuracy, the polar coordinates of these landmarks are expressed as intervals:

- [d_i] for the distance from the sensor to the landmark.
- [ϕ_i] for the azimuth angle of the landmark.

In the multi-target tracking module, these detected landmarks *Bi* have been matched with their corresponding primitives *Bc* of the theoretical map whose coordinates in the world reference frame are (x_c, y_c) (see Figure 6 for the example of one landmark).

The goal is to compute a subpaving which contains the robot configuration. This configuration is represented by a 3D box ([x_r], [y_r], [θ_r]).

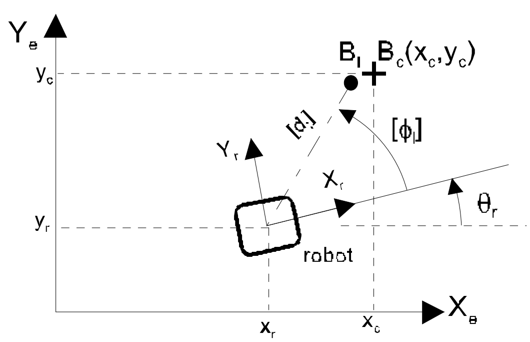

Figure 6. The data of the problem.

To solve this problem, we will first argue in the ideal case (i.e. perfect sensor) for one landmark. Then we will add the interval formalism, always for one landmark. Finally, we will consider all the landmarks.

Ideal case, one landmark

In the world reference frame (*Xe*, *Ye*), the distance d_c between the robot and the theoretical landmark *Bc* is:

$$d_c = \sqrt{(x_r - x_c)^2 + (y_r - y_c)^2}$$

Always in the world reference frame, the angle ϕ_c between the robot and the theoretical landmark *Bc* in the robot reference frame is (Figure 7):

$$\phi_c = \arctan\left(\frac{y_c - y_r}{x_c - x_r} \right) - \theta_r$$

Since (x_r, y_r, θ_r) is the robot configuration and since the sensorial primitive *Bi* has been matched with the theoretical one *Bc*, we have d_c=d_i and ϕ_c=ϕ_i. This observation will be the test used by SIVIA to determine if the boxes are feasible or not.

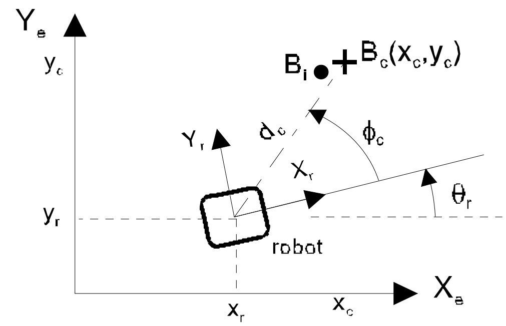

Figure 7: the localization problem in the perfect case.

Imprecise case, one landmark

The robot configuration is now represented by a 3D box ([x_r], [y_r], [θ_r]). The distance d_i between the robot and the landmark and its azimuth angle ϕ_i are not known with precision. They are expressed in an interval way [d_i] and [ϕ_i] (Figure 8).

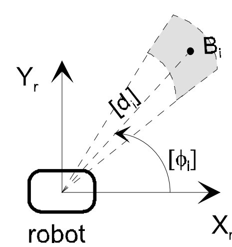

Figure 8: landmark coordinates in the interval case.

In the world reference frame (*Xe*, *Ye*), the distance [d_c] between the robot and the theoretical landmark *Bc* is now an interval:

$$[d_c] = \sqrt{([x_r] - x_c)^2 + ([y_r] - y_c)^2}$$

Always in the world reference frame, the angle [ϕ_c] between the robot and the theoretical landmark *Bc* in the robot reference frame is also an interval :

$$[\phi_c] = \arctan\left(\frac{y_c - [y_r]}{x_c - [x_r]}\right) - [\theta_r]$$

Since the box ($[x_r]$, $[y_r]$, $[\theta_r]$) contains the robot position and since the sensorial primitive *Bi* has been matched with the theoretical one *Bc*, the interval $[d_c]$ is included in the interval $[d_i]$ and the interval $[\phi_c]$ is included in $[\phi_i]$. In other words, the box $[d_c]$ $[\phi_c]$ is included in the box $[d_i]$ $[\phi_i]$.

This means that, if any box $[S]=([x], [y], [\theta])$ contains the robot localization, we must have :

$$[d_c]\,[\phi_c] \subset [d_i]\,[\phi_i]$$

$$\text{with : } [d_c] = \sqrt{([x]-x_c)^2 + ([y]-y_c)^2} \qquad (1)$$

$$[\phi_c] = \arctan\left(\frac{y_c - [y]}{x_c - [x]}\right) - [\theta] \qquad (2)$$

The inclusion functions of our problem are the equations (1) and (2).

The algorithm starts with an initial box $[S_0]$ equal to the theoretical map. This enables us to be sure that the solution set is in this initial box. Then this initial box $[S_0]$ and the following boxes $[S]$ are split up according to the equations (1) and (2) by SIVIA. To this aim, SIVIA uses the two tests detailed in paragraph 3.3:

- If $f^{-1}([S]) \subset Y$, i.e. if $[d_c]\,[\phi_c] \subset [d_i]\,[\phi_i]$, then $[S] \subset X$: the box $[S]$ is feasible.
- If $f^{-1}([S]) \cap Y = \varnothing$, i.e. if $[d_c]\,[\phi_c] \cap [d_i]\,[\phi_i] = \varnothing$, then $[S] \cap X = \varnothing$: the box $[S]$ is unfeasible.

In the other cases, the box $[S]$ is ambiguous.

Imprecise case, several landmarks

If n landmarks have been detected and matched, the test given by the equations (1) and (2) is performed for each landmark, i.e. n times. The box $[S]$ under analyze is then feasible if all the n tests conclude that it is feasible.

The drawback of this strategy is that it doesn't allow any outliers. Our method to manage outliers is the following: if, after the algorithm, no feasible box is found, we restart it with one outlier allowed. This means that, if the box under analyze is unfeasible for one landmark, this box is not declared as unfeasible. But if a second landmark gives a conclusion "unfeasible", the landmark is now declared unfeasible. If no feasible box is found, the method is restarted with two outliers allowed, etc… If no feasible box is found considering $n/2$ outliers, we consider that no configuration can be found.

4. Experimental results

We have tested our algorithm on several acquisitions made in an indoor environment (two trajectories in the end of a corridor shown Figure 9 whose theoretical map in possession of the robot is on Figure 10). The omnidirectional acquisitions and the localization algorithm are computed in a Pentium PC located on our mobile robot. The sensor which is on the robot is shown Figure 9.

Figure 9: the experimental environment and the sensor.

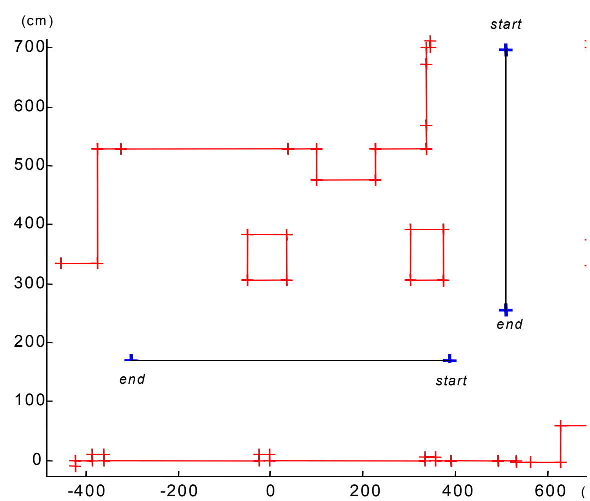

Figure 10: the theoretical map of the environment and the two paths performed by the robot.

The sensor imprecision on orientation γ is fixed at one degree (Figure 11). The imprecision in distance α*d* is proportional to the landmark distance. Indeed, our depth sensor is less precise when the distance increases [9].

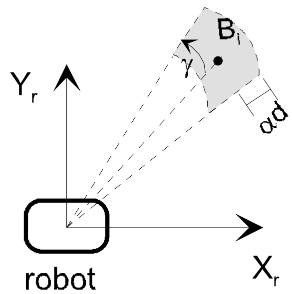

Figure 11. The sensor imprecision.

The initial box [S_0] is fixed to the size of the theoretical map, i.e. [-500 cm, 800 cm][-100 cm, 800 cm][0 degree, 360 degrees]. The Figure 12 shows several localization results. The gray boxes are the feasible ones, the yellow boxes are the ambiguous ones. The graduations on the *x* axis and *y* axis represent one meter.

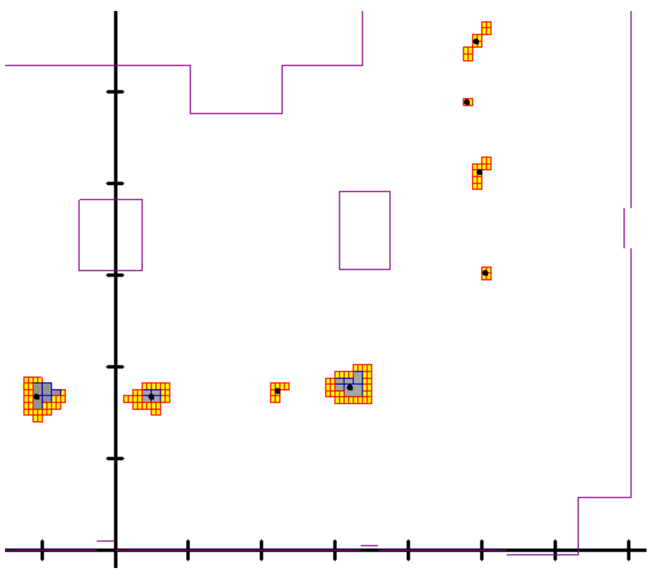

Figure 12: some experimental results.

On the major part of the cases, the subpavings are found considering no outliers. Only few cases admit one or two outliers. In all the cases, a subpaving is found (however, in certain cases, we have only an outer subpaving). The subpavings are coherent with reality. This coherence shows that it is possible to treat the problem of imprecision quantification independently from the problem of uncertainty quantification. Finally, the error (distance between the true position and the center of gravity of the subapving) is acceptable: 15 cm and 6.2 degrees in orientation.

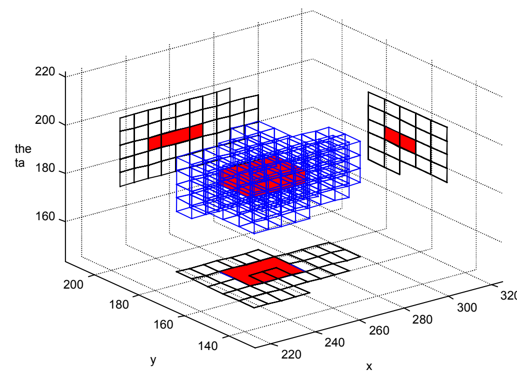

Figure 13: an example of 3D subpavings

We show on Figure 13 a 3D-view of one localization. The red boxes are the feasible ones and the white boxes are the ambiguous ones. The x axis and the y axis are graduated in cm and represent the position of the robot. The "theta" axis is graduated in degree and represents the robot orientation.

5. Conclusion

We have presented in this article a localization method based on interval analysis. This formalism is adequate to quantify in a natural way imprecision. Indeed, we have noted on experimental results that the uncertainty is not correlated to imprecision. That's why we have decided to treat the imprecision in an independent way. The landmarks coordinates are then represented as intervals. We have shown that the localization problem can be seen as a set inversion problem. So we have used the SIVIA algorithm which enables to solve the set inversion problem by the way of interval formalism. The result is a robot configuration bracketed by two 3-D subpavings.

On two paths made in an indoor environment, we have tested our algorithm and we have remarked that the experimental results are coherent. Besides, the localization error is weak. A consequent advantage of this method is to supply a guaranteed error domain of the robot's configuration.

An evolution of this work will consists in adding proprioceptive data from dead-reckoning. The dead reckoning information will enables us to use the two sensors intermittently (indeed, until now, they are always used together). This adding will be able to integrate our uncertainty and imprecision quantifications methods. To this aim, the use of constraint propagation on intervals can be judicious.

An other amelioration could concern the conflict management in our uncertainty propagation architecture. At every step, the conflict, i.e. the mass on empty set, is rejected on the ignorance. This is not very interesting since this conflict “pollute” the upper levels. Besides, we have no information about the conflict: we do not know if it is high or low.

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