Comment on "Nuclear Arrhenius-Type Plots"

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Tsang and Danielewicz recently argued that the linearity of the Arrhenius plots observed in multifragmentation is due to a special kind of autocorrelation which they call "self-correlation" [1]. However, the derivation of their explanation for linear Arrhenius plots is flawed by *circular algebraic logic*, leading to a trivial identity, regardless of whether or not there exists an autocorrelation.

It is generally observed that the intermediate mass fragment (IMF) multiplicity distribution at any given transverse energy E_t is binomial, so that an elementary probability p and the number of tosses m can be extracted. The Arrhenius plot $\ln 1/p$ vs $1/\sqrt{E_t}$ is observed to be linear in all experimental cases. Since m is frequently constant, we can construct, like the authors, an Arrhenius plot $\log 1/\langle n\rangle$ vs $1/\sqrt{E_t}$ which will be linear, since $\langle n\rangle = mp$. One observes that E_t is constructed with *both* the light charged particles (LPs) and the IMFs. In the authors' notation $E_t = (N_C - \langle n\rangle)\, E_t^{\mathrm{LP}} + \langle n\rangle\, E_t^{\mathrm{IMF}}$, where N_C is the number of charged particles in an event and E_t^{LP} and E_t^{IMF} are the average transverse energies of the LPs and IMFs, respectively. The danger of ordinary autocorrelation in the dependence of $\langle n\rangle$ on E_t is discussed in [2].

The authors have observed that $\langle n\rangle$ and N_C are *globally* correlated. They fit this correlation with the linear form

$$N_C = a' + b'\langle n\rangle \tag{1}$$

and obtain E_t as a linear function of $\langle n\rangle$ only

$$E_t = E_t(\langle n\rangle) = b\left(\frac{a}{b} + \langle n\rangle\right) \tag{2}$$

where $a = a'E_t^{\mathrm{LP}}$ and $b = (b'-1)E_t^{\mathrm{LP}} + E_t^{\mathrm{IMF}}$. The values of a and b from a *direct* fit using Eq. (2) are 100 MeV and 220 MeV, respectively. These values are nearly equal to those extracted by the authors who started with Eq. (1).

Eq. (2) contains the original experimental dependence of $\langle n\rangle$ on E_t, and it does not differ in any significant way, other than in the linear fit, from the data used to construct the Arrhenius plots. *It is just the mathematical inverse of* $\langle n\rangle(E_t)$. Here is the start of the *circular logic.*

The authors then take a linear Arrhenius plot and substitute for E_t the value calculated from Eq. (2). Since Eq. (2) provides a good approximation for E_t, the Arrhenius plot remains approximately linear. This simply means that Eq. (2) is a good approximation to the data.

Yet the authors make the surprising claim that the linearity in such a plot comes from a novel kind of autocorrelation, which they call "self-correlation", because now one has $\langle n\rangle$ both in the ordinate (as $\log 1/\langle n\rangle$) and in the abscissa (as $1/\sqrt{b(a/b+\langle n\rangle)}$). This technique for demonstrating autocorrelations is fundamentally flawed; it reduces any equation to an identity and will give the *same* result for *all* functions, *even for those in which autocorrelation has been ruled out by construction.*

Deviations from linearity of course occur when the parameters a and b are changed from their "optimal" values. This is equivalent to spoiling the goodness of the linear fit that makes Eq. (2) a good inverse function.

The problem can be summarized as follows: given any function $y = y(x)$, substitute x with its value $x(y)$: $y = y(x(y))$. This always gives the identity $y = y$, proving that *any* function is completely "self"-correlated with its own inverse.

To appreciate the true role of autocorrelations, let $\langle n\rangle = f(E_t)$ and $E_t = E_t(E, \langle n\rangle)$ where E is the excitation energy of the system. The dependence of $\langle n\rangle$ on E_t can be explored differentially

$$\Delta\langle n\rangle = \frac{d\langle n\rangle}{dE_t}\Delta E_t. \tag{3}$$

Now, the change in transverse energy can be written

$$\Delta E_t = \left.\frac{\partial E_t}{\partial E}\right|_n \Delta E + \left.\frac{\partial E_t}{\partial\langle n\rangle}\right|_E \Delta\langle n\rangle \tag{4}$$

where the first term represents the change in E_t due to the change in excitation energy and the second term represents the trivial change in E_t due to autocorrelation. The *partial* derivative $\partial E_t/\partial\langle n\rangle|_E$ at *constant* E (not the *total* derivative) tells us about the autocorrelation.

Substituting Eq. (4) in Eq. (3) we have

$$\Delta\langle n\rangle\left(1 - \frac{d\langle n\rangle}{dE_t}\left.\frac{\partial E_t}{\partial\langle n\rangle}\right|_E\right) = \frac{d\langle n\rangle}{dE_t}\left.\frac{\partial E_t}{\partial E}\right|_{\langle n\rangle}\Delta E \tag{5}$$

where the complete left hand side is the true increase in $\langle n\rangle$ induced by ΔE *corrected for the autocorrelation.* Failure to appreciate the difference between $\partial E_t/\partial\langle n\rangle|_E$ and $dE_t/d\langle n\rangle$ leads to the circular reasoning.

In fact, if we write $\Delta E_t = \frac{dE_t}{d\langle n\rangle}\Delta\langle n\rangle$ and substitute this in Eq. (3) we obtain as the authors did

$$\Delta\langle n\rangle = \frac{d\langle n\rangle}{dE_t}\frac{dE_t}{d\langle n\rangle}\Delta\langle n\rangle = \Delta\langle n\rangle. \tag{6}$$

[1] M.B. Tsang and P. Danielewicz, Phys. Rev. Lett. **80**, 1178, (1998).
[2] L.G. Moretto *et al.*, Phys. Rep. **287**, 249 (1997).