The math of the hexapod system

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ABSTRACT

In this note we will calculate the forces and movements of the hexapod system. We will demonstrate that the hexapod system is not over constrained, that the maximum force in any hexapod actuator is about 2/3 of the total DECam weight and that an actuator resolution of 3 micrometers is enough to meet the current positioning requirements. We will also study the effect of loads on the actuators and argue that a position measurement feedback system close to the focal plane will be needed.

1. INTRODUCTION

Currently two possibilities are being considered to position the DECam camera relative to the telescope mirror. One involves a set of six actuators arranged in an hexapod configuration. In this note we will spend some time "doing the math" for the hexapod system and calculating loads on the actuators and the effect of actuator length changes on the camera motion. We will consider two hexapod configurations. Configuration 1 which is difficult (or impossible) to build but it is conceptually simple and will allow us to easily visualize how the hexapod forces and movements act on the DECam camera. Configuration 2 is essentially the current hexapod design and as we will see it behaves very close to Configuration 1, showing that the hexapod arrangement constitutes a very stable system.

In Section 2 we will specify hexapod Configurations 1 and 2 and discuss the transformations that will be used to study the motion and forces in the hexapod system. In Section 3 we will calculate the forces on the actuators and study how these forces change as the camera and the cage are moved as a unit through space (which means keeping the actuators lengths fixed). In Section 4 we will calculate how each actuator length has to be changed to move the motion ring relative to the fixed one. The effect of moving one actuator at a time and the errors introduced by the fact that this can only be done in finite steps will also be studied in Section 4. At last in Section 5 we will calculate the effect of elastic deformations due to loads on the hexapod actuators.

2. THE HEXAPOD SYSTEM

The hexapod system consists of six actuators and two plates which are referred to as the motion and fixed plates or rings. The motion ring is rigidly attached to the camera while the fixed ring is rigidly attached to the cage. One end of the actuators insert in the fixed plate, the other end insert in the motion plate.

For our so called Configuration 1, one end of the actuators insert in the motion ring as is shown in Figure 1 left, the other actuator end inserts in the fixed ring as is shown in the center figure. Two actuators are attached in each of the three ring insertion points using a ball joint, therefore at all joints each actuator is allowed to freely rotate in all three dimensions. The camera can be held in space by applying the right set of forces at each of the three motion ring insertion points. At each point each actuator can independently push or pull by the right amount to create an arbitrary force in the plane of the two actuators attached to that point. This means two arbitrary force components at each insertion point, for a total of six independent forces which is enough to satisfy the six static equations that come from the sum of all forces and all moments.

The three insertion points in the motion ring can be arbitrarily positioned with respect to the three insertion points in the fixed ring. Once this relative position is fixed the distances between the points can be calculated.

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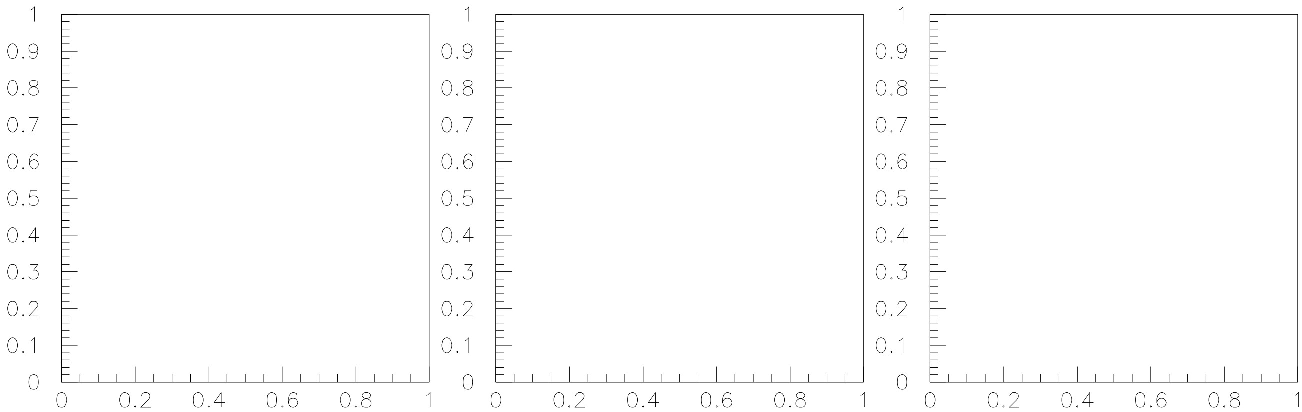

Figure 1. Motion ring (left), fixed ring (center), hexapod system (right).

This set of distances is unique, that is different positions of the motion ring relative to the fixed one will create a different set of six distances. This one-to-one correspondence means that the relation can be inverted, or that specifying a set of six distances will create a unique position of the motion ring relative to the fixed one. If one wishes to do so, only one of the six distances can be changed at a given time which guarantees that the hexapod system is not over-constrained. We will mathematically prove in Section 4 that this is the case, but before doing that we have to define the vectors for each of the ring insertion points and their rotations and translations.

The insertion points will be labeled as $\overrightarrow{r}_{0_i}$ for the motion ring and $\overrightarrow{R}_{0_i}$ for the fixed ring. They are calculated as

$$\overrightarrow{r}_{0_i} = [r\ cos(\alpha_i + \delta_0), r\ sin(\alpha_i + \delta_0), 0.5(A + A')] \tag{1}$$
$$\overrightarrow{R}_{0_i} = [R\,cos(\beta_i + \delta_0), R\,sin(\beta_i + \delta_0), 0.5(A - A')] \tag{2}$$

The angles α_i and β_i are given in Table 1. The motion ring radius r and the fixed ring radius R and the distances A and A' are given in Table 2. The angle δ_0 is arbitrary and allows us to rotate the insertion points around the z-axis. When the camera is pointing up the plane of the fixed plate forms the x-y plane of the coordinate system, the z-axis points up and the origin of the coordinate system is centered on the fixed ring.

Table 1. Angles, in degrees, corresponding to the points where the actuators insert into the motion and fixed plates.

		i	1	2	3	4	5	6
Configuration 1	motion ring	α_i	0	120	120	240	240	360
	fixed ring	β_i	60	60	180	180	300	300
Configuration 2	motion ring	α_i	10	110	130	230	250	350
	fixed ring	β_i	50	70	170	190	290	310

Table 2. Motion and fixed ring insertion points radius r and R. Distances A, A', B and C are described in the text and shown in Figure 2. All numbers are in millimeters.

	r	R	A	A'	B	C
Configuration 1	612.5	715.0	697	697	100	578
Configuration 2	612.5	715.0	697	500	100	578

Two planes will be selected to define rotations and translations. One will be the plane of the motion ring, and the other will be the focal plane. The translations on these planes will be defined as $(\Delta x, \Delta y, \Delta z)$. The rotations can be defined by the Euler angles $(\Delta\phi, \Delta\theta, \Delta\gamma)$ or by the rotations around the x, y and z axis (tip,tilt,twist)=$(\Delta\theta_x, \Delta\theta_y, \Delta\theta_z)$. The mathematics to perform these rotations is described in Appendix A.

Figure 2 shows distances between different parts of the DECam camera. The distance between the fixed and motion rings is labeled A. The distance between the motion plate and the camera's Center of Mass (CM) is B, and the distance between the CM and the focal plane is labeled C. Figure 2 also shows the planes where the actuators insert into the fixed and motion plates. The distance between these planes is labeled A'. In Configuration 1 we assumed that A'=A. The values of the previous distances are given in Table 2.

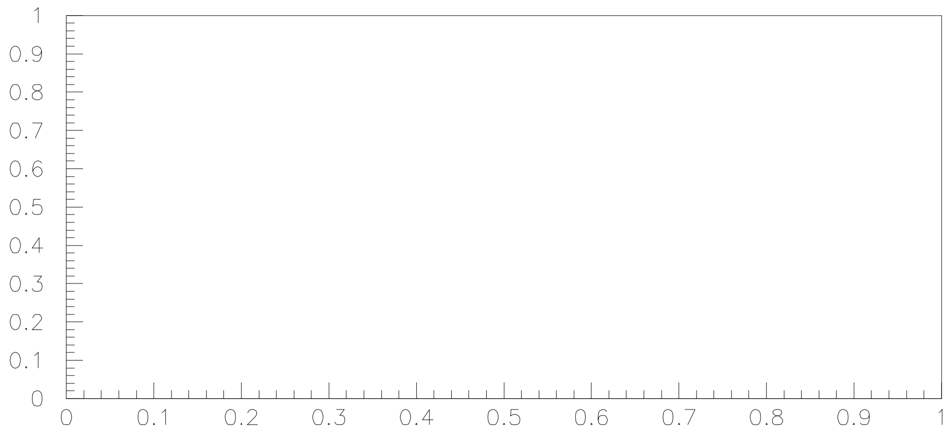

Figure 2. Side view of hexapod system and focal plane. Also shown is the camera Center of Mass (CM). The different distances are listed in Table 2 and described in the text.

To avoid coupling rotations with translations the rotation of either the motion or focal planes has to be done at the coordinate system origin. After the rotation is performed the plane can be translated to its designed position. This rotation affects the motion plate insertion points, so the final position of these insertion points is obtained as follows:

- Translate $\overrightarrow{r}_{0_i}$ until the motion plate or focal plane is at the coordinate systems origin. This translation will be given by the vector $\overrightarrow{d}$.
- Perform rotation *Rot*.
- Translate insertion points so the motion plate or focal plane is back at their design position. This translation is again given by $\overrightarrow{d}$.
- Performed translations $\overrightarrow{r}_0 = (\Delta x, \Delta y, \Delta z)$

Then the final position of the motion plate insertion points will be

$$\overrightarrow{r}_i = Rot\,(\overrightarrow{r}_{0_i} - \overrightarrow{d}) + \overrightarrow{d} + \overrightarrow{r}_0 \tag{3}$$

When defining the camera rotations and translations relative to the motion ring we have $\overrightarrow{d} = (0, 0, A)$. When the focal plane is used to define rotations and translations we have $\overrightarrow{d} = (0, 0, A + B + C)$. For small angle rotations one can use tip, tilt and twist and *Rot* will be the matrix given in Appendix A Eq. 36. In the general case we can use the matrix given in Appendix A Eq. 28.

The fixed plate insertion points don't move, therefore

$$\overrightarrow{R}_i = \overrightarrow{R}_{0_i} \tag{4}$$

The actuators lie in the line that connects points $\overrightarrow{r}_i$ and $\overrightarrow{R}_i$. The vector difference between the points is

$$\overrightarrow{L}_i = \overrightarrow{r}_i - \overrightarrow{R}_i \tag{5}$$

Therefore the actuator's length is given by $L_i = |\overrightarrow{L}_i|$, and the unit vectors in the direction of the actuators are

$$\hat{f}_i = \frac{\overrightarrow{L}_i}{L_i} \tag{6}$$

The Center of Mass position $\overrightarrow{r}_{0_{CM}} = (0, 0, A + B)$ is also rotated and translated according to Eq. 3 to give $\overrightarrow{r}_{CM} = Rot\,(\overrightarrow{r}_{0_{CM}} - \overrightarrow{d}) + \overrightarrow{d} + \overrightarrow{r}_0$.

When the camera is rotated away from the vertical position we need to rotate all vectors associated with the camera or the cage (except gravity of course). This rotation is performed using the Eq. 28 matrix in Appendix A. That is

$$\overrightarrow{V'} = R(\phi, \theta, \gamma)\,\overrightarrow{V} \tag{7}$$

where $\overrightarrow{V}$ is any of the vectors calculated in this section.

3. FORCES ON THE HEXAPOD ACTUATORS

In this section we will study the forces exerted by the hexapod actuators. We define a force as positive when the actuator pushes on the motion plate which means that the actuator is under compression. When the force is negative the actuator is pulling on the motion plate and it is under tension.

To study the actuator forces both the fixed and motion plates (or cage and camera) will be rotated as a unit between 0 and 90 degrees around the y-axis and between 0 and 360 degrees around the camera axis. In terms of Euler angles (see Appendix A) this means $\phi = 0°$, $0° \leq \theta \leq 90°$ and $0° \leq \gamma \leq 360°$, which covers the full range of actuator forces.

The motion plate will be assumed to be parallel to the fixed plate and sharing the same axis. In the notation of the previous section this means $\Delta x = \Delta y = \Delta z = 0$ and $\Delta\phi = \Delta\theta = \Delta\gamma = 0$ (or $\Delta\theta_x = \Delta\theta_y = \Delta\theta_z = 0$). The cases where the motion plate is rotated or translated relative to the fixed plate can be easily studied too, but since these displacements are small and the motion plate will be oriented in every possible position, for simplicity we decided to just run the parallel and coaxial case.

As explained in the previous section once the positions of the fixed and motion plates (and the actuator insertion points) are given, the unit vectors $\hat{f}_i$ can be easily calculated (see Eqs 1 to 6). With the vectors $\hat{f}_i$, the relations given in Appendix B (Eqs 37 to 44) can be used to calculate the force per unit camera weight (F/w) exerted by the actuators.

Because it is conceptually simpler we will start the discussion using Configuration 1. This is the case in which pairs of actuators come to a point and can freely rotate in three dimensions (ball joint). In this case there are three insertion points in the motion plate and we will organize the discussion concentrating on them. All plots in Figures 3 to 5 correspond to Configuration 1. The plots in Figure 6 where made with the realistic case of Configuration 2.

When the camera is vertical all actuator forces will be equal and independent of γ. As the telescope rotates the force on the actuators will change. Perhaps the most interesting case to start the discussion with is the case in which the camera is horizontal ($\theta = 90°$) and rotates around its axis ($0° \leq \gamma \leq 360°$). Figure 3 shows F/w for $\theta = 90°$ as a function of γ for all six actuators. The actuators are paired according to the way they insert in the motion ring. When looking from the focal plane towards the hexapod, $\gamma = 0$ correspond to the case in which actuators 1 and 6 insert at 6 o'clock, with actuator 6 coming from the left and 1 from the right. Actuators 2 and 3 insert at 2 o'clock with actuator 2 going up and 3 coming down. Actuators 4 and 5 insert at 10 o'clock with actuator 4 coming down and 5 going up.

As we can see from the left plot in Figure 3 the actuators at the six o'clock ($\gamma = 0$) insertion point are pushing with a small force. The main component of this force is outwards and its main purpose is to cancel the moment produce by the fact that the CM is displaced relative to the motion ring. The black curves in Figures 5.1, 5.2 and 5.3 show actuator 1 forces when the CM is at 0, 150 and 200 mm from the motion ring. We can see that the force is zero when the CM is at the same position than the motion ring and increases as the CM moves away from it. There will also be a small up or down force depending on how much bigger the radius of the fixed ring

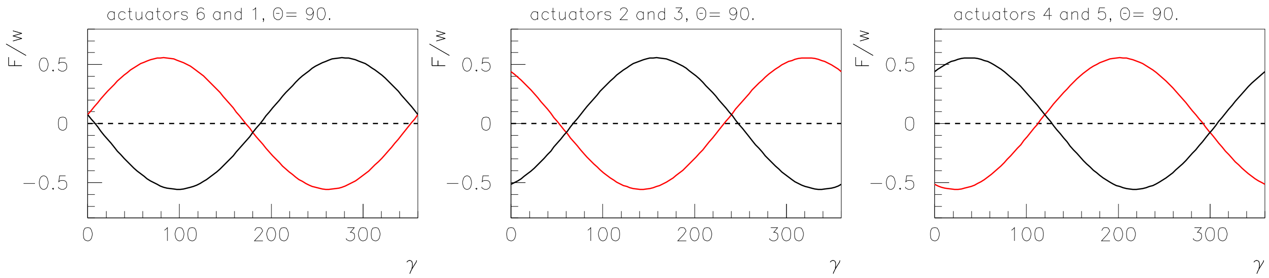

Figure 3. Actuators forces per unit camera weight (F/w) for $\theta = 90°$ as a function of γ. The actuators are paired according to the way they insert in the motion ring: 6 and 1 (left), 2 and 3 (center), 4 and 5 (right). Actuators 2, 4 and 6 are in red, 1, 3 and 5 in black.

is compared with the radius of the motion ring. This force is small because in this position actuators 1 and 6 are almost horizontal, so the weight of the camera is being supported by the 2 and 10 o'clock insertion points.

The actuator forces for the 2 and 10 o'clock insertion points can be red out from the $\gamma = 0$ point at the center and right plots in Figure 3. We can see that the going up actuators (2 and 5) are pushing while the downward ones (3 and 4) are pulling. The pushing and pulling forces are almost equal with the pulling force being slightly bigger than the pushing ones. This produces a total force whose main component is on the vertical plane with a small component pointing opposite to the focal plane. This is needed to support the camera weight and to cancel (together with the outward force at 6 o'clock) the moment produce by the fact that the CM is displaced from the motion ring.

As the angle γ increase the camera rotates counterclockwise. At $\gamma = 60°$ the 2 o'clock point has moved to 12 o'clock. At this point actuators 2 and 3 are almost horizontal and pulling (see center plot in Figure 3). This produces a force that only serves to cancel moments and the weight of the camera is supported by the now at 8 and 4 o'clock points. The maximum forces are close to the 3 (or 9) o'clock point because here there is one insertion point between 12 and 6 o'clock but 2 between 6 and 12 o'clock. So we can study all the actuator forces by just following one insertion point around the clock or if we only want the maximum force by following just one actuator.

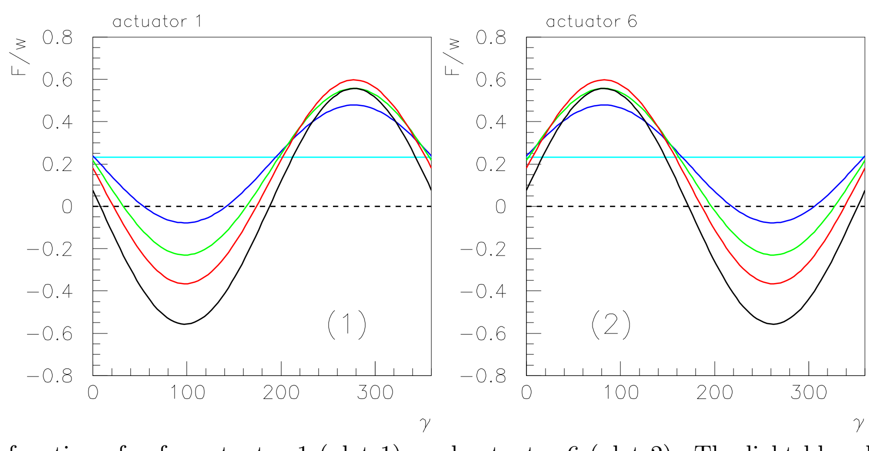

Figure 4. F/w as a function of γ for actuator 1 (plot 1), and actuator 6 (plot 2). The light blue, blue, green, red and black curves correspond to θ equals to 0, 30, 45, 60 and 90 degrees.

Figure 4 shows the forces on actuators 1 and 6 for different values of θ. The light blue, blue, green, red and black curves correspond to θ equals to 0, 30, 45, 60 and 90 degrees. We can see that the maximum force exerted by an actuator is about 0.6 of the total camera weight.

Of course the question now is how stable are these forces per unit weight with respect to changes in the

Table 3. Parameter values for plots in Figure 5. All numbers are in millimeters.

	r	R	A	B
figure 5.1	612.5	715.0	697	0
figure 5.2	612.5	715.0	697	150
figure 5.3	612.5	715.0	697	200
figure 5.4	612.5	715.0	460	100
figure 5.5	612.5	715.0	350	100
figure 5.6	612.5	612.5	697	100

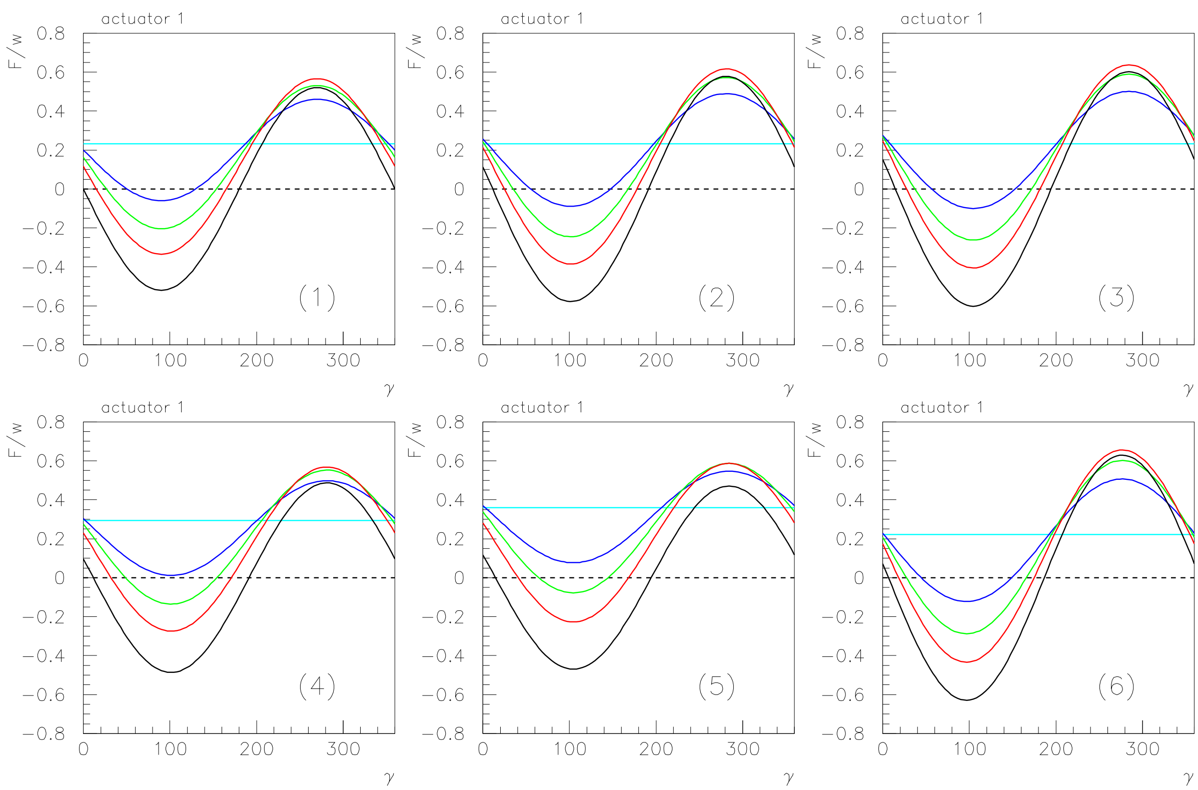

Figure 5. Same curves as in Figure 4 but for different values of: the distance between the motion plate and the CM (plots 1 to 3), different distances between the fixed and motion plates (plots 4 and 5) and for the fixed ring radius equal to the motion ring one (plot 6). The actual distances are given in Table 3.

hexapod parameters. To study that we calculated F/w changing the distance between the CM and the motion plate from 100 millimeters to 0, 150 and 200 millimeters. We also changed the distance between the fixed and motion plates from 697 millimeters to 460 and 350 millimeters (about 2/3 and 1/2 of the original value). Finally we also made the radius of the fixed ring equal to the radius of the motion ring. All these values are summarized in Table 3 and the plots are shown in Figure 5. As we can see the forces don't change very much, showing the stability of the hexapod configuration.

The forces per unit camera weight F/w for the realistic Configuration 2 are shown in Figure 6.1. We see that there is very little change between the forces in Configurations 1 and 2. We also changed in this case the distance between the CM and the motion plate from 100 millimeters to 0, 150 and 200 millimeters. The distance between the fixed and motion plates was changed from 697 millimeters to 460, with the distance between the fixed and motion plate insertion points changed from 500 to 300 millimeters. This distance was chosen to avoid changing the insertion point positions at each plate by much. Finally the radius of the fixed ring was made equal to the radius of the motion ring. All these changes are summarized in Table 4 and the plots are shown in Figure

Table 4. Parameter values for plots in Figure 6. All numbers are in millimeters.

	r	R	A	A'	B
figure 6.1	612.5	715.0	697	500	100
figure 6.2	612.5	715.0	697	500	0
figure 6.3	612.5	715.0	697	500	150
figure 6.4	612.5	715.0	697	500	200
figure 6.5	612.5	715.0	460	300	100
figure 6.6	612.5	612.5	697	500	100

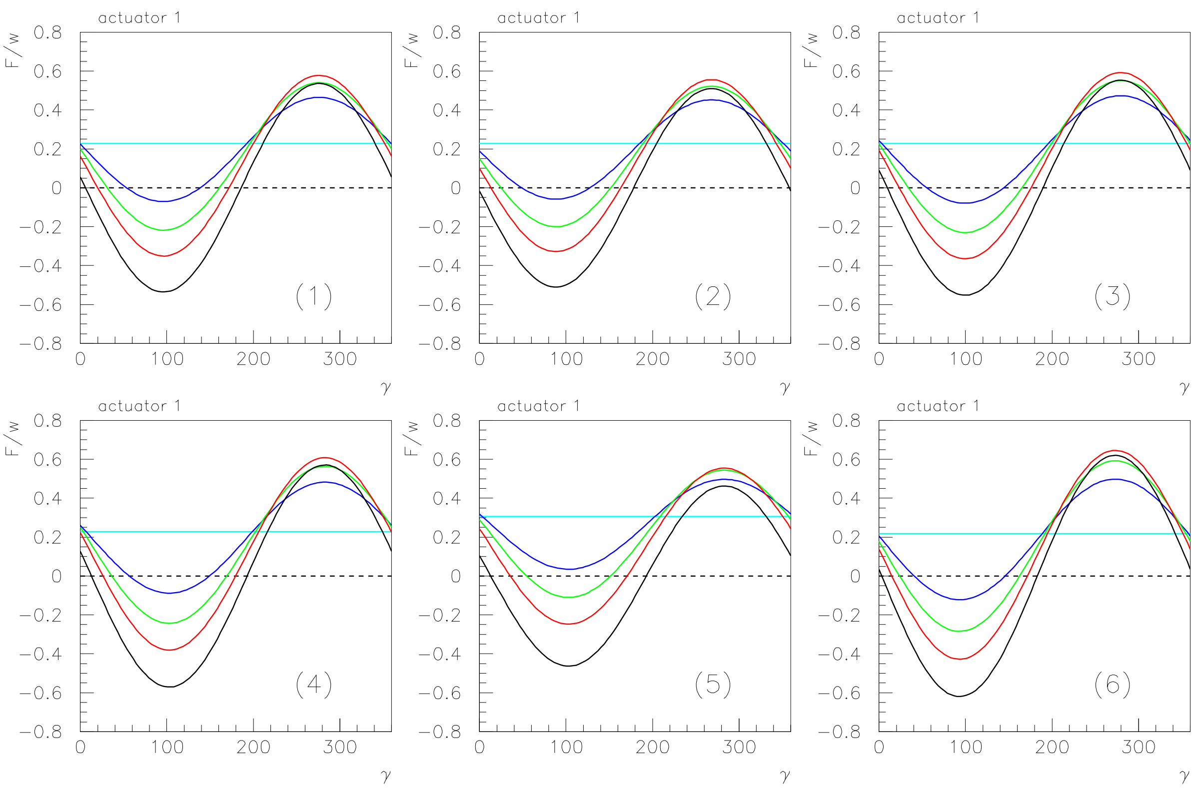

Figure 6. Forces per unit camera weight F/w for actuator 1 in Configuration 2. As in previous figures different curves correspond to different values of θ. The values of θ and the color coding is the same as in Figures 4 and 5. F/w is shown for different distances between the motion plate and the CM (plots 2 to 4), a different distance between the fixed and motion plates (plot 5) and for the fixed ring radius equal to the motion ring one (plot 6). The actual distances are given in Table 4.

6. Again we see a high degree of stability.

We currently don't know the parameters of the final hexapod design but the studies in this section clearly show two things: 1) the forces are very stable with respect to the design parameters, and 2) as a rule of thumb we can say that the maximum force in the actuators is about 2/3 of the camera weight.

4. MOTION AND ACCURACY STUDIES

Now we will turn our attention to the study of how the hexapod system moves. Again for conceptual simplicity we will start with Configuration 1. Since we are trying to understand the accuracy with which the hexapod system can position the camera we will restrict ourselves to small motions (of the order of a millimeter at the focal plane) which, as explained in Appendix A.1, have the advantage that rotations commute and movements can be treated in a linear way.

The restriction to small motions will not change the generality of our conclusions for two reasons: 1) almost inevitably after large movements there will be small adjustments to reach the final camera position, so the final accuracy of the hexapod system relies on the ability to make small movements, and 2) given a change in linear translations and in angles the change in actuators length can be easily calculated in a very general way (see Section 2), so except for the inversion to go from actuator lengths to motion, every thing else calculate in this section can be easily generalized.

For small motions the relation between translations and rotations with respect to the three coordinate axis and the actuators length will be given by a 6x6 matrix M. Equation 36 in Appendix A.1 shows that for small rotations we can calculate each rotation independently and then add the effect of each one to obtain the final rotation. For translations this is also the case, so the columns of M can be calculated by performing one motion at a time. For example a translation along the x-axis by an amount Δx will induce a change in the length of the six actuators $(\Delta L_1^x, \Delta L_2^x, \Delta L_3^x, \Delta L_4^x, \Delta L_5^x, \Delta L_6^x)$ and this will constitute the first column of our matrix. A translation along the y-axis by an amount Δy will induce a change in the length of the six actuators $(\Delta L_1^y, \Delta L_2^y, \Delta L_3^y, \Delta L_4^y, \Delta L_5^y, \Delta L_6^y)$. And the effect of both translations will be $(\Delta L_1^x + \Delta L_1^y, \Delta L_2^x + \Delta L_2^y, \Delta L_3^x + \Delta L_3^y, \Delta L_4^x + \Delta L_4^y, \Delta L_5^x + \Delta L_5^y, \Delta L_6^x + \Delta L_6^y)$. So we calculate the 6x6 matrix M by performing small motions using the general equations given in Section 2. We define translations and rotations in two different places: 1) the motion plate, and 2) the focal plane. For hexapod Configuration 1 and translations and rotations defined at the motion plane we obtain

$$\begin{pmatrix} \Delta L_1 \\ \Delta L_2 \\ \Delta L_3 \\ \Delta L_4 \\ \Delta L_5 \\ \Delta L_6 \end{pmatrix} = \begin{pmatrix} 0.264 & -0.640 & 0.721 & 0.000 & -2.141 & -1.902 \\ -0.686 & -0.091 & 0.721 & 1.855 & 1.070 & 1.903 \\ 0.423 & 0.549 & 0.721 & 1.854 & 1.071 & -1.902 \\ 0.423 & -0.548 & 0.721 & -1.855 & 1.071 & 1.903 \\ -0.686 & 0.092 & 0.721 & -1.854 & 1.070 & -1.902 \\ 0.264 & 0.641 & 0.721 & 0.000 & -2.141 & 1.903 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} \tag{8}$$

where Δx, Δy and Δz are translations along the x, y and z axis in micrometers, $\Delta\theta_x$, $\Delta\theta_y$ and $\Delta\theta_z$ are rotations around the x, y and z axis in arcseconds and ΔL_1, ΔL_2, ΔL_3, ΔL_4, ΔL_5 and ΔL_6 are actuators length changes in micrometers.

Equation 8 can be inverted to give a relation between the change in actuators length and the camera translations and rotations:

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} = \begin{pmatrix} 0.000 & -0.450 & 0.451 & 0.450 & -0.451 & 0.000 \\ -0.520 & -0.260 & 0.260 & -0.260 & 0.260 & 0.520 \\ 0.231 & 0.231 & 0.231 & 0.231 & 0.231 & 0.231 \\ 0.064 & 0.167 & 0.103 & -0.103 & -0.167 & -0.064 \\ -0.156 & 0.022 & 0.133 & 0.133 & 0.022 & -0.156 \\ -0.088 & 0.088 & -0.088 & 0.088 & -0.088 & 0.088 \end{pmatrix} \begin{pmatrix} \Delta L_1 \\ \Delta L_2 \\ \Delta L_3 \\ \Delta L_4 \\ \Delta L_5 \\ \Delta L_6 \end{pmatrix} \tag{9}$$

The fact that Eq. 8 inverts without a problem proves that the hexapod system is not over-constraint. In other words we can move one actuator at a time and the entire system will move without problems (except of course for friction in the actuator to plate joints that will have to be designed carefully). For example if the length of actuator 1 changes by $\Delta L_1 = 10\ \mu$m then the camera will move in y and z by -5.2 and 2.3 micrometers and it will rotate around x, y and z by 0.6, -1.6 and -0.9 arcseconds. As expected there is a lot of symmetry in Eq. 9. For example in Configuration 1 actuators 1 and 6 insert in the motion plate at a point along the x-axis, so they will both push up in z in the same direction but they will push in opposite directions in y, so the translation in z and rotations around the y-axis will have the same sign and translations in y and rotation around the x and z-axis will have opposite signs. And this is what we see in Eq. 9.

Equations 8 and 9 are essential to calculate motion but no particular number in the two matrices is very important, specially because these numbers depend on how we define the coordinate system with respect to the insertion points. So we need other numbers to understand how precisely the hexapod system can move the camera. We see for example from Eq. 8 that if we want to move by $\Delta x = 10\ \mu$m then the actuators lengths

ΔL_1 to ΔL_6 will have to change by 2.64, -6.86, 4.23, 4.23, -6.86 and 2.64 micrometers respectively. If the minimum actuator step size is 1 micrometer then the changes will be 3, -7, 4, 4, -7 and 3 steps, so there will be errors due to the finite size of the actuators steps. Over many motions these errors will act randomly and generate a distribution of errors in all three translations and rotations. The rms of these distributions can be calculated by adding the numbers in Eq. 9 in quadratures. If we write Eq. 8 as $\overrightarrow{\Delta L} = M\,\overrightarrow{\Delta x}$ and Eq. 9 as $\overrightarrow{\Delta x} = M^{-1}\,\overrightarrow{\Delta L}$ and assume that the step size S_L is the same in all actuators then the rms of the distributions will be $\sigma_i = \sqrt{\sum_j (M^{-1}_{i,j})^2}\,\sigma_L$, with $\sigma_L = S_L/\sqrt{12} \approx 0.29\,S_L$ (the factor $1/\sqrt{12}$ is just the rms of a square distribution of width 1). For Eq. 9 these number are:

$$(\sigma_x, \sigma_y, \sigma_z, \sigma_{\theta_x}, \sigma_{\theta_y}, \sigma_{\theta_z}) = (0.90, 0.90, 0.57, 0.29, 0.29, 0.22)\,\sigma_L \tag{10}$$

As with every distribution we may wonder how far the tails of the above mentioned distributions go. So another useful number is the maximum error that can be introduced due to the finite actuator step size. If h is the minimum interval that contains all errors, and if we assume that all step size errors conspire to give the largest possible deviation then $h_i = (\sum_j |M^{-1}_{i,j}|)\,S_L$. For Eq. 9 these gives

$$(h_x, h_y, h_z, h_{\theta_x}, h_{\theta_y}, h_{\theta_z}) = (1.80, 2.10, 1.39, 0.67, 0.62, 0.53)\,S_L \tag{11}$$

We can now calculate the equivalent of Eqs 8 to 11 for Configuration 2 with translations and rotation defined in the motion plate. They are

$$\begin{pmatrix} \Delta L_1 \\ \Delta L_2 \\ \Delta L_3 \\ \Delta L_4 \\ \Delta L_5 \\ \Delta L_6 \end{pmatrix} = \begin{pmatrix} 0.211 & -0.647 & 0.733 & 0.070 & -2.243 & -1.999 \\ -0.665 & -0.140 & 0.733 & 1.980 & 1.062 & 2.002 \\ 0.456 & 0.506 & 0.733 & 1.908 & 1.184 & -1.999 \\ 0.456 & -0.505 & 0.733 & -1.909 & 1.184 & 2.002 \\ -0.665 & 0.142 & 0.733 & -1.976 & 1.062 & -1.999 \\ 0.211 & 0.647 & 0.733 & -0.068 & -2.243 & 2.002 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} \tag{12}$$

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} = \begin{pmatrix} 0.016 & -0.449 & 0.434 & 0.433 & -0.450 & 0.016 \\ -0.510 & -0.241 & 0.269 & -0.269 & 0.241 & 0.510 \\ 0.228 & 0.227 & 0.227 & 0.227 & 0.228 & 0.227 \\ 0.048 & 0.151 & 0.103 & -0.103 & -0.151 & -0.048 \\ -0.147 & 0.032 & 0.115 & 0.115 & 0.032 & -0.147 \\ -0.083 & 0.083 & -0.083 & 0.083 & -0.083 & 0.083 \end{pmatrix} \begin{pmatrix} \Delta L_1 \\ \Delta L_2 \\ \Delta L_3 \\ \Delta L_4 \\ \Delta L_5 \\ \Delta L_6 \end{pmatrix} \tag{13}$$

$$(\sigma_x, \sigma_y, \sigma_z, \sigma_{\theta_x}, \sigma_{\theta_y}, \sigma_{\theta_z}) = (0.88, 0.88, 0.56, 0.27, 0.27, 0.20)\,\sigma_L \tag{14}$$

$$(h_x, h_y, h_z, h_{\theta_x}, h_{\theta_y}, h_{\theta_z}) = (1.80, 2.04, 1.36, 0.60, 0.59, 0.50)\,S_L \tag{15}$$

We can see that Eq. 12 inverts without any problem, which again proves that the hexapod system is not over-constraint over a large range of parameters. As briefly discussed in Section 2 this is a general property and it goes as follows. In general we have six actuator insertion points in the motion plate and six in the fixed plate. The six insertion points in the motion plate can be arbitrarily positioned with respect to the six insertion points in the fixed plate. Once this relative position is fixed the distances between the insertion points can be calculated. This set of distances is unique, that is different positions of the motion plate relative to the fixed one will create a different set of six distances. This is true as long as we don't line up two points in the motion ring with two points in the fixed ring identically. This will create degeneracies but also will make the camera unable to stand in space, because six non-degenerate parameters are needed to position a body in three dimensional space. So if the camera is to stand in space, for a given position of the motion and fixed plates the distances between the actuator insertion points is unique. This one-to-one correspondence means that the relation can be inverted, or that specifying a set of six distances will create a unique position of the motion ring relative to the fixed one. Therefore only one of the six distances can be changed at a given time which guarantees that the hexapod system is not over-constrained. Therefore as long as the actuators can rotate freely in three dimensions at the insertion points the hexapod system is not constrained at all. The only constraining will come from the binding at the actuator insertion points, so these insertions will have to be designed carefully to make sure that the binding is small enough so that the system will move when any of the actuators is moved by one step.

Finally the equations for Configuration 2 when the translations and rotations are defined at the focal plane are

$$\begin{pmatrix} \Delta L_1 \\ \Delta L_2 \\ \Delta L_3 \\ \Delta L_4 \\ \Delta L_5 \\ \Delta L_6 \end{pmatrix} = \begin{pmatrix} 0.211 & -0.647 & 0.733 & -2.052 & -2.929 & -1.999 \\ -0.665 & -0.140 & 0.733 & 1.522 & 3.253 & 2.002 \\ 0.456 & 0.506 & 0.733 & 3.575 & -0.305 & -1.999 \\ 0.456 & -0.505 & 0.733 & -3.568 & -0.305 & 2.002 \\ -0.665 & 0.142 & 0.733 & -1.505 & 3.253 & -1.999 \\ 0.211 & 0.647 & 0.733 & 2.064 & -2.929 & 2.002 \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} \tag{16}$$

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} = \begin{pmatrix} -0.467 & -0.343 & 0.812 & 0.811 & -0.345 & -0.467 \\ -0.667 & -0.739 & -0.071 & 0.071 & 0.739 & 0.667 \\ 0.229 & 0.226 & 0.225 & 0.226 & 0.228 & 0.229 \\ 0.048 & 0.151 & 0.103 & -0.103 & -0.151 & -0.048 \\ -0.147 & 0.032 & 0.115 & 0.115 & 0.032 & -0.147 \\ -0.083 & 0.083 & -0.083 & 0.083 & -0.083 & 0.083 \end{pmatrix} \begin{pmatrix} \Delta L_1 \\ \Delta L_2 \\ \Delta L_3 \\ \Delta L_4 \\ \Delta L_5 \\ \Delta L_6 \end{pmatrix} \tag{17}$$

$$(\sigma_x, \sigma_y, \sigma_z, \sigma_{\theta_x}, \sigma_{\theta_y}, \sigma_{\theta_z}) = (1.41, 1.41, 0.56, 0.27, 0.27, 0.20)\,\sigma_L \tag{18}$$

$$(h_x, h_y, h_z, h_{\theta_x}, h_{\theta_y}, h_{\theta_z}) = (3.24, 2.96, 1.36, 0.60, 0.59, 0.50)\,S_L \tag{19}$$

We can see that the only differences between Eqs 12 and 16 are in the columns corresponding to $\Delta\theta_x$ and $\Delta\theta_y$. These are the only expected differences and they are due to the fact that rotations around the x and y axis at the motion plate produce translations at the focal plane.

Table 5 summarizes the rms errors σ_i and the minimum intervals containing all possible errors h_i. As expected from the effect of x and y rotations just mentioned, the numbers for x and y translations increase when going from the motion plate to the focal plane. Other than that the numbers don't change much between configurations.

Table 5. Summary of rms errors σ_i and the minimum intervals containing all possible errors h_i.

		σ_x/σ_L	σ_y/σ_L	σ_z/σ_L	$\sigma_{\theta_x}/\sigma_L$	$\sigma_{\theta_y}/\sigma_L$	$\sigma_{\theta_z}/\sigma_L$
Configuration 1	motion plate	0.90	0.90	0.57	0.29	0.29	0.22
Configuration 2	motion plate	0.88	0.88	0.56	0.27	0.27	0.20
Configuration 2	focal plane	1.41	1.41	0.56	0.27	0.27	0.20
		h_x/S_L	h_y/S_L	h_z/S_L	h_{θ_x}/S_L	h_{θ_y}/S_L	h_{θ_z}/S_L
Configuration 1	motion plate	1.80	2.10	1.39	0.67	0.62	0.53
Configuration 2	motion plate	1.80	2.04	1.36	0.60	0.59	0.50
Configuration 2	focal plane	3.24	2.96	1.36	0.60	0.59	0.50

The only point that remains to be studied is the stability with respect to the selection of the coordinate system. The angle δ_0 in Eqs 1 and 2 rotates the insertion points around the z-axis. We studied the dependence of the numbers in Table 5 as a function of δ_0. All six rms's σ_i are completely independent of δ_0, and the values of h_i only change slightly. Figure 7 shows the values of h_x/S_L to h_{θ_z}/S_L as a function of δ_0. We can see that the numbers are very stable.

The tolerances specified in the current hexapod RFI * are: $\Delta_x = \Delta_y = \pm\ 25\ \mu$m, $\Delta_z = \pm\ 5\ \mu$m and $\Delta\theta_x = \Delta\theta_y = \pm\ 1$ arcseconds (1 arcsec = 4.8 μrad). Using Table 5 for Configuration 2 and the focal plane as a

*See "Request for Information (RFI). DECam Position Adjustment System". R. French Leger, Dave McGinnis, Andy Stefanik, July 12, 2007 - Revision 2.

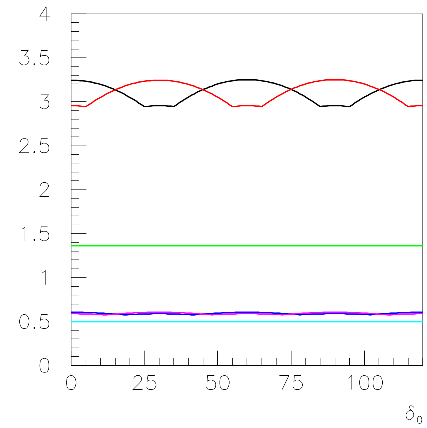

Figure 7. This figure shows $(h_x, h_y, h_z, h_{\theta_x}, h_{\theta_y}, h_{\theta_z})/S_L$ (black, red, green, blue, magenta and light blue) as a function of δ_0 for Configuration 2 and with displacements and rotations defined at the focal plane. The angle δ_0 is defined in Eqs 1-2.

reference for rotations and translations the previous specifications translate into the following step sizes S_L: 16 μm (50/3.2) for lateral motion, 6 μm (10/1.36) for focusing and 3 μm (2/0.6) for tip and tilt. So we see that the specifications for tip and tilt set the actuators resolution to 3 μm.

5. EFFECT OF LOADS ON THE ACTUATORS

The elastic module of steel is about $E = 200$ Giga-Pascals (GPa). When an axial force F is applied to a bar of length L and an area A the elastic deformation ΔL of the bar is $\Delta L = [L/(A\,E)]\,F$. We will take $L = 0.68$ meters which is the length of the actuators in Configuration 2, and assume that an actuator is equivalent to a one inch diameter bar, or $A = \pi\,(0.0254/2)^2 = 5.07 \times 10^{-4}\,m^2$. Then

$$\Delta L[\mu m] = 0.066\,F[Kg] \tag{20}$$

with ΔL in micrometers and F in kilograms. For a camera weight of 3000 Kg the maximum actuator load will be 2000 Kg, which will produce an actuator deformation of 130 microns. The effects of these deformations can be easily calculated as follow: 1) calculate forces as in Section 3, 2) with Eq. 20 calculate the actuator deformations and 3) with Eq. 17 calculate the focal plane motion due to these deformations. The result of these calculations for Configuration 2 is shown in Figure 8.

Plots 8.1 to 8.6 show the focal plane displacements $(\Delta x, \Delta y, \Delta z, \Delta\theta_x, \Delta\theta_y, \Delta\theta_z)$ as a function of γ due to the elastic deformation of the actuators. The light blue, blue, green, red and black curves in each plot correspond to θ values of 0, 30, 45, 60 and 90 degrees. This range of θ and γ covers the entire motion of the telescope. The displacements shown in the plots exceed our specifications by at least an order of magnitude. This of course means that as the telescope moves we will have to step the actuators to correct for the actuators elastic deformations. One can do this by monitoring the length of the actuators, or by installing devices to measure the position of the camera relative to the cage. In our minds the problem of just controlling the length of the actuators is that most likely there will be other deformations (like at the actuator insertion points) that can not be compensated that way. So it appears to us that installing devices to measure the position of the camera with respect to the cage is the right thing to do, specially if these devices are installed near the focal plane.

We want now to turn our attention to the problem of trying to focus the camera during exposures. As we can see from Eq. 17 if we step all actuators by the same amount $(\Delta L_1 = \Delta L_2 = \Delta L_3 = \Delta L_4 = \Delta L_5 = \Delta L_6 = \Delta L)$ then the focal plane moves in the z direction by the amount $\Delta z = 1.37\,\Delta L$, and all the other motions are zero. Then assuming that all actuators can act equally the motion of focusing the camera should be smooth and should only affect the z-motion. This is true except for elastic deformations. As the actuators move theirs lengths will change and since the loads on the actuators is usually very different they will deform by different amounts. We

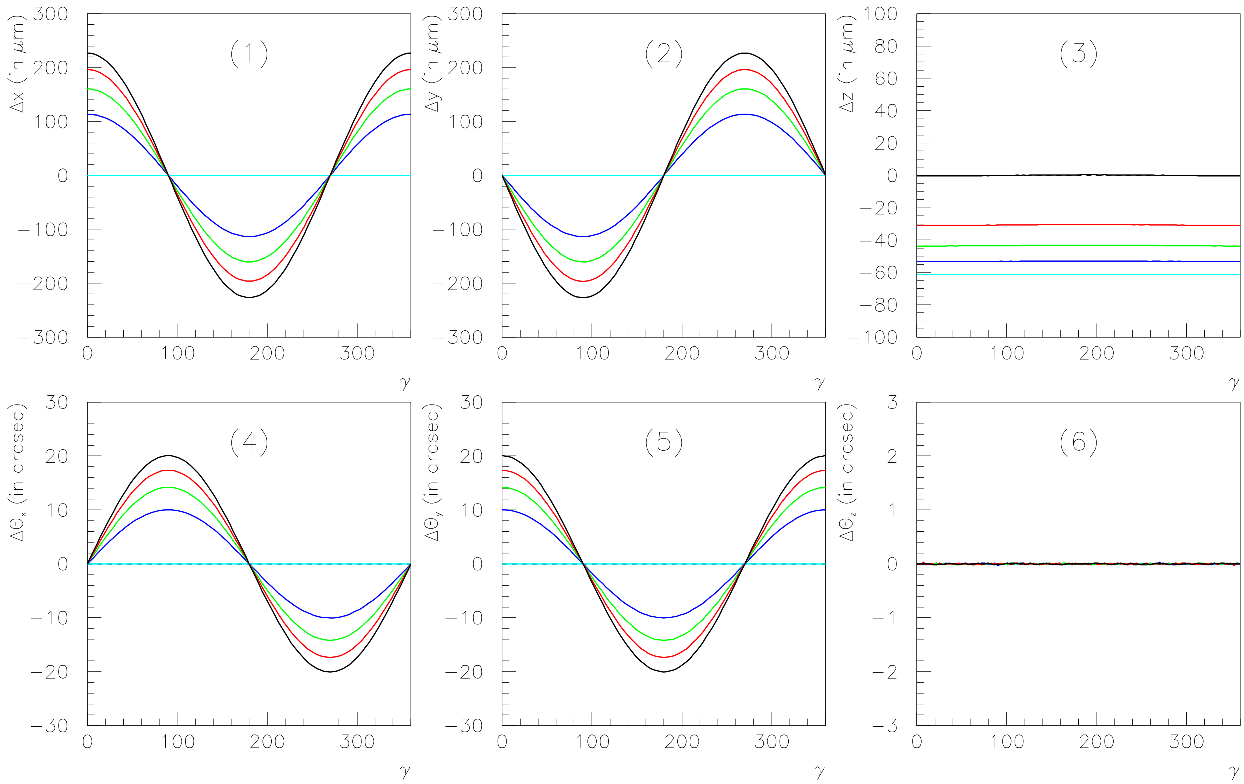

Figure 8. Plots 1-6 show the focal plane motions ($\Delta x, \Delta y, \Delta z, \Delta\theta_x, \Delta\theta_y, \Delta\theta_z$) produced by the elastic deformations of the actuators as the telescope moves. The displacements are plotted as a function of γ and the light blue, blue, green, red and black curves correspond to θ values of 0, 30, 45, 60 and 90 degrees. Lengths are in micrometers, angles in arcseconds.

studied this problem in the following way: 1) for a given camera position we calculated all the actuator loads, 2) we moved the camera axially by 10 millimeters and recalculated all the loads, 3) we calculated the elastic deformation on the actuators due to the change in loads and 4) we used Eq. 17 to calculate all the displacements. The results of this calculation for Configuration 1 are shown in Figure 9. The plots are defined as in Figure 8. We can see that all motions are well within specifications. So we should be able to focus the camera smoothly if all actuators can be made to move in sync by the same amounts.

6. CONCLUSIONS

In this note we have shown that

- Six actuators arranged in a hexapod configuration form a system that is not over-constrained and that is stable with respect to changes in the design parameters.
- Given the space we plan to utilize for our hexapod system and the range of CM positions, the maximum force in any of the actuators is about 2/3 of the camera weight.
- The actuator to plate joints should be designed such that the actuators can rotate in all three dimensions and that the binding in the joints is small enough to allow for a single actuator step motion.
- The actuators should be made as stiff and short as possible in order to minimize camera motion due to elastic deformation in the actuators. Also the stiffer the actuators the larger the one step force exerted on the plates, which will facilitate the design of the actuator to plate joints.

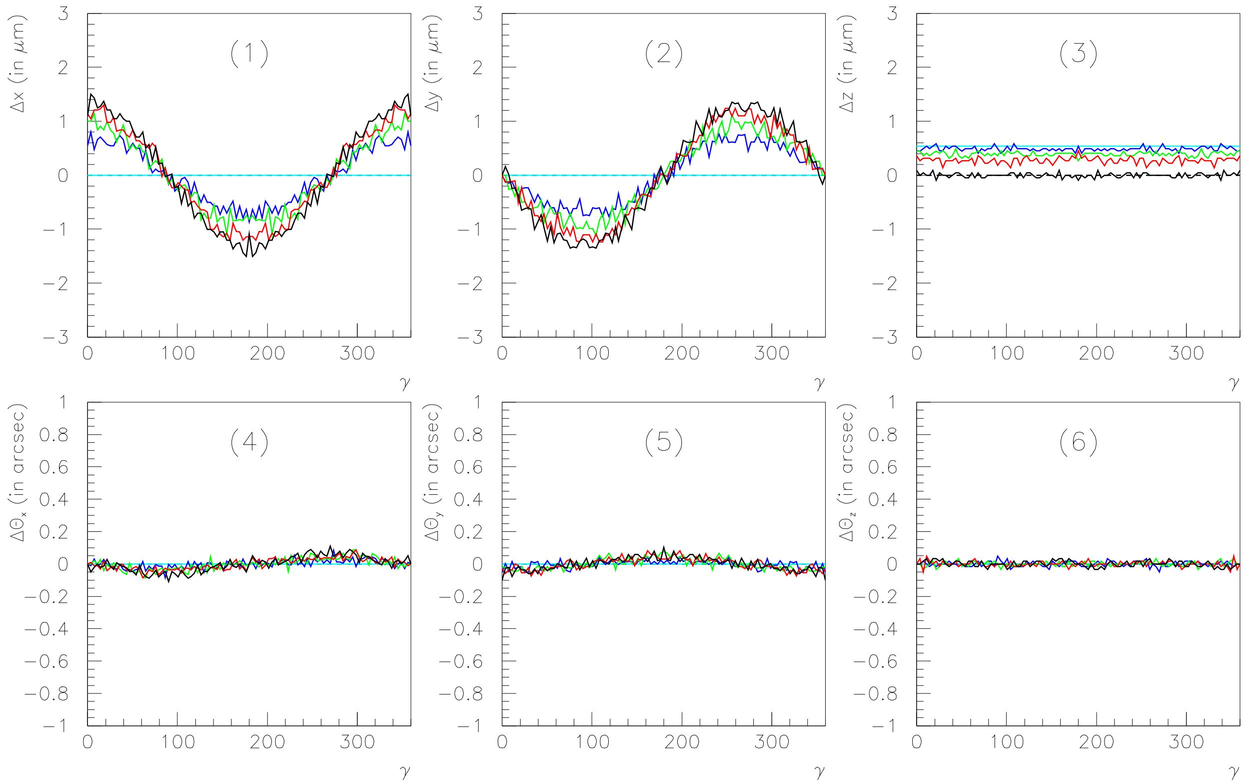

Figure 9. Plots 1-6 show the focal plane motions ($\Delta x, \Delta y, \Delta z, \Delta\theta_x, \Delta\theta_y, \Delta\theta_z$) produced by the elastic deformations of the actuators as the camera is moved by 10 millimeters along its axis. The displacements are plotted as a function of γ and the light blue, blue, green, red and black curves correspond to θ values of 0, 30, 45, 60 and 90 degrees. Lengths are in micrometers, angles in arcseconds.

- We have calculated the rms and the minimum interval that covers all possible motion errors due to the finite actuator step size. We find that an actuator positioning errors of 3 μm is enough to satisfy all the current positioning requirements. We have also shown that the requirement on tip and tilt determine the 3 μm positioning error in the actuators.

Perhaps one last comment on plate design versus actuators length is in order. Using long actuators allows to group the insertion points in pairs, which is equivalent to having three insertion points in each ring. This minimizes the moments on the rings but increases the elastic deformations in the actuators. If the plates can be designed in a very stiff way the insertion points can be opened up which will shorten the actuators length and minimize the camera motion due to elastic deformations.

APPENDIX A. ACTIVE ROTATIONS

This section describes the calculation of the matrix needed to perform active rotations in three dimensions. In an active rotation the objects are rotated while the coordinate system remains fixed. In Figure 10 the vector labeled $\overrightarrow{x}_1$ has been rotated by an angle θ to a new position $\overrightarrow{x}_2$. In terms of the coordinates (x, y) this rotation is written as

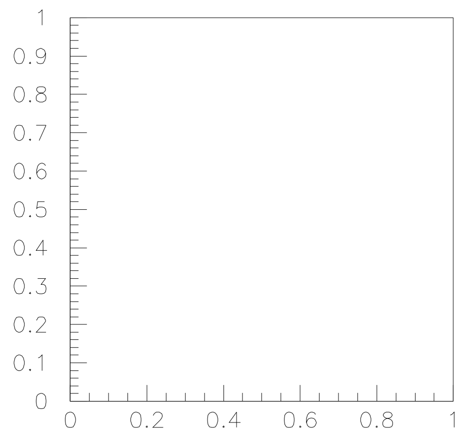

Figure 10. Two dimensional rotation.

$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} cos\,\theta & -sin\,\theta \\ sin\,\theta & cos\,\theta \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \tag{21}$$

It is easy to understand the structure of the above matrix as follows. To preserved lengths the 2x2 matrix responsible for the rotation has to be unitary, which means that the matrix can be written in terms of sines and cosines. For a zero angle rotation the matrix has to be the identity matrix, which means that the cosines have to be along the diagonal and the sines off the diagonal. In order for the scalar product of rows 1 and 2 to be zero one of the sines need to have a negative sign. It is easy to see where to put the negative sign by looking a which coordinate gets smaller. In the rotation shown in Figure 10 the x coordinate gets smaller after the rotation therefore the minus sign is in row 1. The extension to three dimensions is obviously given by:

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} cos\,\theta & -sin\,\theta & 0 \\ sin\,\theta & cos\,\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \tag{22}$$

The above rotation is said to be around the z-axis, and the rotation is defined as positive when the objects rotate around the z-axis as a right handed cork screw. To simplify the notation we can write

$$\overrightarrow{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{23}$$

and

$$\overrightarrow{x}_2 = R_z(\theta)\overrightarrow{x}_1 \tag{24}$$

Writing $c_\theta = cos\,\theta$ and $s_\theta = sin\,\theta$, the rotation $R_z(\theta)$ can simply be written as

$$R_z(\theta) = \begin{pmatrix} c_\theta & -s_\theta & 0 \\ s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{25}$$

In the same way we can define 3-dimensional active rotations around the x-axis and y-axis as

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\theta & -s_\theta \\ 0 & s_\theta & c_\theta \end{pmatrix} \text{ , and } \; R_y(\theta) = \begin{pmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{pmatrix} \tag{26}$$

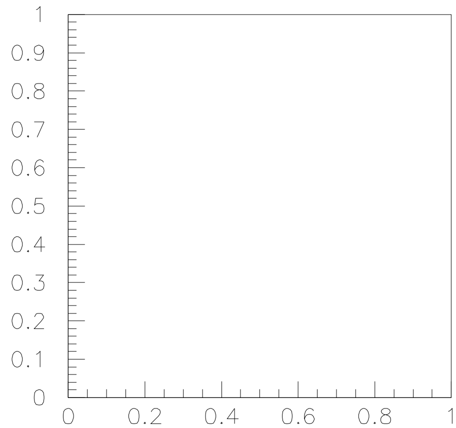

Figure 11. Three dimensional rotation using Euler angles.

As we can see in Figure 11 if a body is rotated around the x-axis the y-coordinate is the one that gets smaller and the minus sign has to be in the second row. For a rotation around the y-axis the z-coordinate gets smaller and the minus sign is in the third row.

Using Euler angles as defined in Figure 11 a general 3-dimensional rotation can be written as

$$R(\phi, \theta, \gamma) = R_z(\phi)\; R_y(\theta)\; R_z(\gamma) \tag{27}$$

Or in matrix form as

$$\begin{aligned} R(\phi,\theta,\gamma) &= \begin{pmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_\theta & 0 & s_\theta \\ 0 & 1 & 0 \\ -s_\theta & 0 & c_\theta \end{pmatrix} \begin{pmatrix} c_\gamma & -s_\gamma & 0 \\ s_\gamma & c_\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_\theta\, c_\gamma & -\, c_\theta\, s_\gamma & s_\theta \\ s_\gamma & c_\gamma & 0 \\ -\, s_\theta\, c_\gamma & s_\theta\, s_\gamma & c_\theta \end{pmatrix} \\ &= \begin{pmatrix} c_\phi\, c_\theta\, c_\gamma - \; s_\phi\, s_\gamma & -\, c_\phi\, c_\theta\, s_\gamma - \; s_\phi\, c_\gamma & c_\phi\, s_\theta \\ s_\phi\, c_\theta\, c_\gamma + \; c_\phi\, s_\gamma & -\, s_\phi\, c_\theta\, s_\gamma + \; c_\phi\, c_\gamma & s_\phi\, s_\theta \\ -\, s_\theta\, c_\gamma & s_\theta\, s_\gamma & c_\theta \end{pmatrix} \end{aligned} \tag{28}$$

Rotations around the x-axis using Euler angles can be produced by

$$R_x(\theta) = R(3\pi/2, \theta, -3\pi/2) \tag{29}$$

This is easy to see by noting that $sin(3\pi/2) = -1$ and $cos(3\pi/2) = 0$, and therefore $c_\phi = c_\gamma = 0$ and $s_\gamma = -s_\phi = 1$. Replacing these values in Eq. 28 we obtain the rotation around the x-axis shown in Eq. 26. It is also easy to see that the rotations around the y-axis and z-axis are given by

$$R_y(\theta) = R(0, \theta, 0) \text{ , and } \; R_z(\theta) = R(\theta, 0, 0) = R(0, 0, \theta) \tag{30}$$

A.1 Small rotations

Small rotations commute, so it is convenient in this case to use rotations around the x, y and z axis defined as (tip,tilt,twist)=($\theta_x, \theta_y, \theta_z$). For small angles we can write Eqs 25 and 26 as

$$R_i(\theta) = I(1 - \frac{\theta^2}{2}) + M_i\,\theta + O(\theta^3) \tag{31}$$

where I is the identity matrix and

$$M_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \;,\quad M_y = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \;\text{, and}\quad M_z = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{32}$$

Neglecting terms in θ^3 or higher, the result of a tip, tilt and twist rotation will be given by

$$\begin{aligned} R_{xyz}(\theta_x, \theta_y, \theta_z) &= R_x(\theta_x)\,R_y(\theta_y)\,R_z(\theta_z) & (33)\\ &= \left[I(1-\frac{\theta_x^2}{2}) + M_x\,\theta_x\right]\left[I(1-\frac{\theta_y^2}{2}) + M_y\,\theta_y\right]\left[I(1-\frac{\theta_z^2}{2}) + M_z\,\theta_z\right] & (34)\\ &= I + (M_x\,\theta_x + M_y\,\theta_y + M_z\,\theta_z) + (M_xM_y\,\theta_x\theta_y + M_xM_z\,\theta_x\theta_z + M_yM_z\,\theta_y\theta_z) & \\ &\quad -I(\frac{\theta_x^2}{2} + \frac{\theta_y^2}{2} + \frac{\theta_z^2}{2}) & (35) \end{aligned}$$

The required resolution for tip and tilt is 1 arcsec = 4.85 μrad. So we want the non-linear terms in Eq. 35 to be smaller than 1 arcsec. That means $\theta^2 < 4.85$ μrad or $\theta < 2.20$ mrad = 454 arcsec. So as long as the tip, tilt and twist rotations are smaller than 400 arcsec (which translates into about 1.5 mm at the focal plane) we can safely use the approximation

$$R_{xyz}(\theta_x, \theta_y, \theta_z) = I + (M_x\,\theta_x + M_y\,\theta_y + M_z\,\theta_z) \tag{36}$$

APPENDIX B. CALCULATING ACTUATOR FORCES

In this appendix we want to calculate the forces on the hexapod actuators. The static problem we want to solve requires the forces on the DECam camera, so we will calculate the forces exerted by the actuators on the motion ring. The forces on the actuators themselves are just the negative of the forces we will calculate. Since these forces are in the direction of the actuators we will write them as

$$\overrightarrow{F}_i = F_i\,\hat{f}_i \tag{37}$$

where $\hat{f}_i$ is a unit vector in the direction of actuator i, and points from the fixed to the motion plate. With this definition of $\hat{f}_i$ the force F_i will be positive when the actuator is pushing on the motion ring, therefore $F_i > 0$ means that the hexapod is under compression.

The static problem that we have to solve is (forces during acceleration when DECam or the telescope are moved will be calculated in a different note)

$$\sum_{i=1}^{6} F_i\,\hat{f}_i \;+\; \overrightarrow{w} = 0 \tag{38}$$

$$\sum_{i=1}^{6} F_i\,\hat{f}_i \times \overrightarrow{\Delta}_i = 0 \tag{39}$$

where $\overrightarrow{w}$ is the DECam weight applied in the DECam Center of Mass (CM) and the moments in Eq. 39 are calculated relative to the CM, that is

$$\overrightarrow{\Delta}_i = \overrightarrow{r}_i - \overrightarrow{r}_{CM} \tag{40}$$

The previous equations form a 6x6 linear system with the following structure

$$\begin{pmatrix} f_{x_1} & f_{x_2} & f_{x_3} & f_{x_4} & f_{x_5} & f_{x_6} \\ f_{y_1} & f_{y_2} & f_{y_3} & f_{y_4} & f_{y_5} & f_{y_6} \\ f_{z_1} & f_{z_2} & f_{z_3} & f_{z_4} & f_{z_5} & f_{z_6} \\ n_{x_1} & n_{x_2} & n_{x_3} & n_{x_4} & n_{x_5} & n_{x_6} \\ n_{y_1} & n_{y_2} & n_{y_3} & n_{y_4} & n_{y_5} & n_{y_6} \\ n_{z_1} & n_{z_2} & n_{z_3} & n_{z_4} & n_{z_5} & n_{z_6} \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{pmatrix} = \begin{pmatrix} -w_x \\ -w_y \\ -w_z \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{41}$$

with $\overrightarrow{n}_i = \hat{f}_i \times \overrightarrow{\Delta}_i$. If we write Eq. 41 as $M\,\overrightarrow{F} = \overrightarrow{U}$ then the solution is

$$\overrightarrow{F} = M^{-1}\,\overrightarrow{U} \tag{42}$$

The weight is always in the -z direction, $\overrightarrow{w} = (0, 0, -w)$, therefore the force actuator i exerts on the motion ring is given by

$$F_i = M^{-1}_{i3}\,w \tag{43}$$

and the forces per unit weight are

$$F_i/w = M^{-1}_{i3} \tag{44}$$

APPENDIX C. CALCULATING VIBRATIONS

In this appendix we will write down the equations to calculate eigenvalues and eigenvectors for the hexapod vibrations. We will make two (in our minds) very reasonable simplifying assumption. The first one is that the principal axis of the moment of inertia tensor coincide with the camera axis. The second one is that vibrations are dominated by the elongation of the hexapod legs.

Due to the filters and the electronic crates the camera does not have rotational symmetry around the telescope's axis. But we will assume that this deviation from symmetry will have a small effect in the vibration modes. In this case the moment of inertia tensor will be diagonal in the coordinate system of the camera.

The fixed and motion plates can be designed such that they deform very little in comparison to the deformations of the haxapod legs, so it is reasonable to ignore these deformations. Also we believe that the transverse vibrations of the hexapod legs can be ignored. The reason is that the change in length of the hexapos legs due to tranverse vibrations is negligible, and therefore the camera motion should not be affected by these vibrations. We will further assume that the elongation of the hexapos legs is elastic and can be described by a single constant. This constant will be dominated by the weakest point in the leg, and could be either the actuator or the joint.

In the presence of motion equations 38 and 39 have to be modified to read

$$\sum_{i=1}^{6} F_i\,\hat{f}_i \,+\, \overrightarrow{w} = \frac{d(m\overrightarrow{v})}{dt} \tag{45}$$

$$\sum_{i=1}^{6} F_i\,\hat{f}_i \times \overrightarrow{\Delta}_i = \frac{d(I\,\overrightarrow{w})}{dt} \tag{46}$$

where (as before) the translations and rotations are defined with respect to the center of mass.

$$\begin{pmatrix} f_{x_1} & f_{x_2} & f_{x_3} & f_{x_4} & f_{x_5} & f_{x_6} \\ f_{y_1} & f_{y_2} & f_{y_3} & f_{y_4} & f_{y_5} & f_{y_6} \\ f_{z_1} & f_{z_2} & f_{z_3} & f_{z_4} & f_{z_5} & f_{z_6} \\ n_{x_1} & n_{x_2} & n_{x_3} & n_{x_4} & n_{x_5} & n_{x_6} \\ n_{y_1} & n_{y_2} & n_{y_3} & n_{y_4} & n_{y_5} & n_{y_6} \\ n_{z_1} & n_{z_2} & n_{z_3} & n_{z_4} & n_{z_5} & n_{z_6} \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{pmatrix} = \frac{d}{dt^2} \begin{pmatrix} m\,\Delta x \\ m\,\Delta y \\ m\,\Delta z \\ I_x\,\Delta\theta_x \\ I_y\,\Delta\theta_y \\ I_z\,\Delta\theta_z \end{pmatrix} \tag{47}$$

As in Eqs 9, 13 and 17

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{pmatrix} = D \begin{pmatrix} \Delta L_1 \\ \Delta L_2 \\ \Delta L_3 \\ \Delta L_4 \\ \Delta L_5 \\ \Delta L_6 \end{pmatrix} \tag{48}$$

Writing $F_i = -k\,\Delta L_i$ and $\overrightarrow{I} = (1, 1, 1, I_x/m, I_y/m, I_z/m)$ we have

$$-k\,M_{i,j}\,\Delta L_j = m\,I_i\,D_{i,j}\frac{d^2\Delta L_j}{dt^2} \tag{49}$$

With $D^{-1} = (I_i\,D_{i,j})^{-1}$ we have

$$\frac{m}{k}\,\frac{d^2\overrightarrow{\Delta L}}{dt^2} + D^{-1}\,M\,\overrightarrow{\Delta L} = 0 \tag{50}$$

with $\overrightarrow{\Delta L} = \overrightarrow{u}\,e^{i\omega t}$

$$\left(D^{-1}\,M - \lambda\,\delta_{i,j}\right)\overrightarrow{u} = 0 \tag{51}$$

with $\lambda = \omega^2 m/k = (\omega/\omega_0)^2$, $\omega_0^2 = (2\pi f_0)^2 = k/m$