Cross-fertilization of QCD and statistical physics

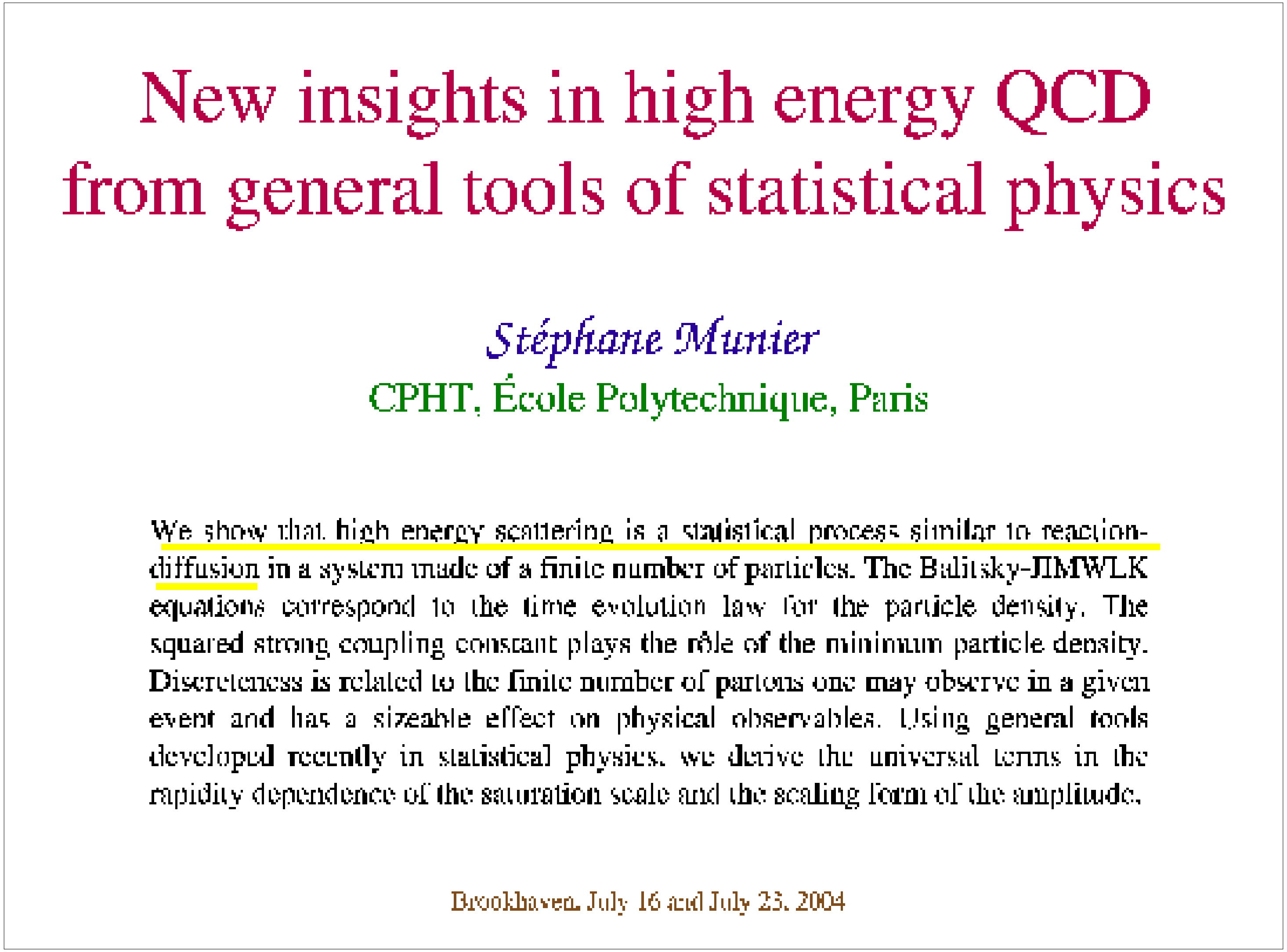
Stéphane Munier

CPHT, École Polytechnique

Lisbon, June 30, 2006

Two years of reaction-diffusion

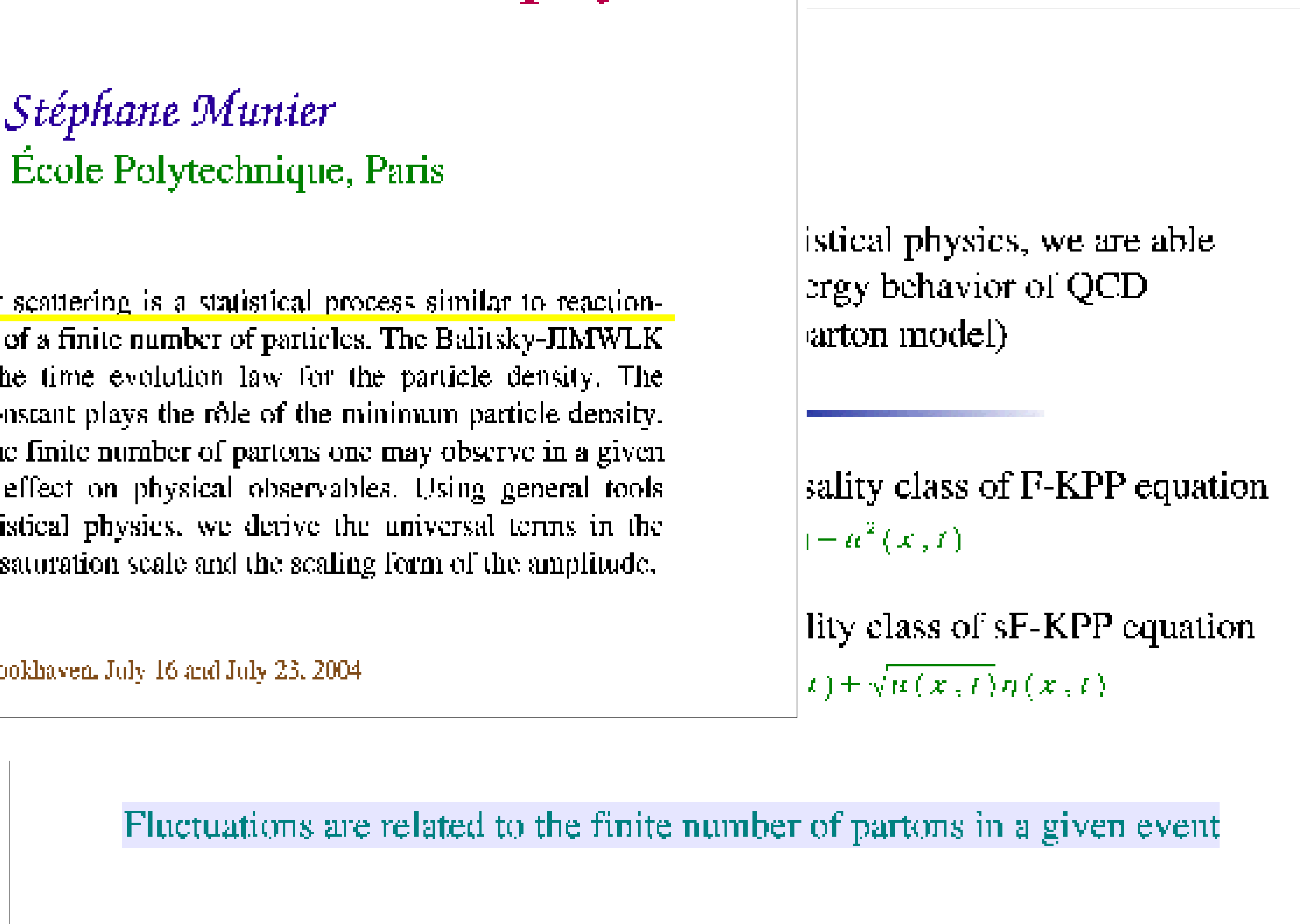
SM, **talk at Brookhaven (2004);**
Iancu, Mueller, SM (2004)
Enberg, Golec-Biernat, SM (2005)


New insights in high energy QCD from general tools of statistical physics

Stéphane Munier
CPHT, École Polytechnique, Paris

We show that high energy scattering is a statistical process similar to reaction-diffusion in a system made of a finite number of particles. The Balitsky-JIMWLK equations correspond to the time evolution law for the particle density. The squared strong coupling constant plays the rôle of the minimum particle density. Discreteness is related to the finite number of partons one may observe in a given event and has a sizeable effect on physical observables. Using general tools developed recently in statistical physics, we derive the universal terms in the rapidity dependence of the saturation scale and the scaling form of the amplitude.

Brookhaven, July 16 and July 23, 2004



Two years of reaction-diffusion

New insights in high energy QCD from general tools of statistical physics

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Summary

Using general results obtained recently in statistical physics, we are able to derive non-trivial properties on the high-energy behavior of QCD scattering amplitudes **from first principles** (parton model)

Balitsky-Kovchegov equation is in the universality class of F-KPP equation

$$\partial_t u(x,t) = \partial_x^2 u(x,t) - u(x,t) - u^2(x,t)$$

Balitsky-JIMWLK equation is in the universality class of sF-KPP equation

$$\partial_t u(x,t) = \partial_x^2 u(x,t) + u(x,t) - u^2(x,t) + \sqrt{u(x,t)}\,\eta(x,t)$$

Fluctuations are related to the finite number of partons in a given event

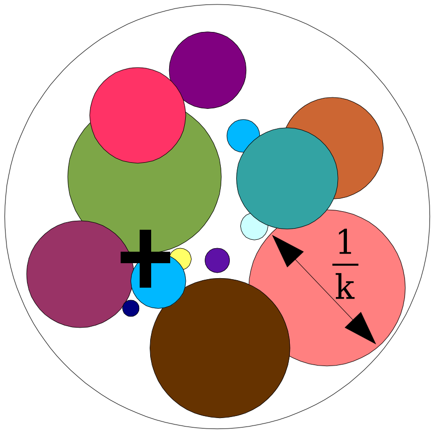
Outline

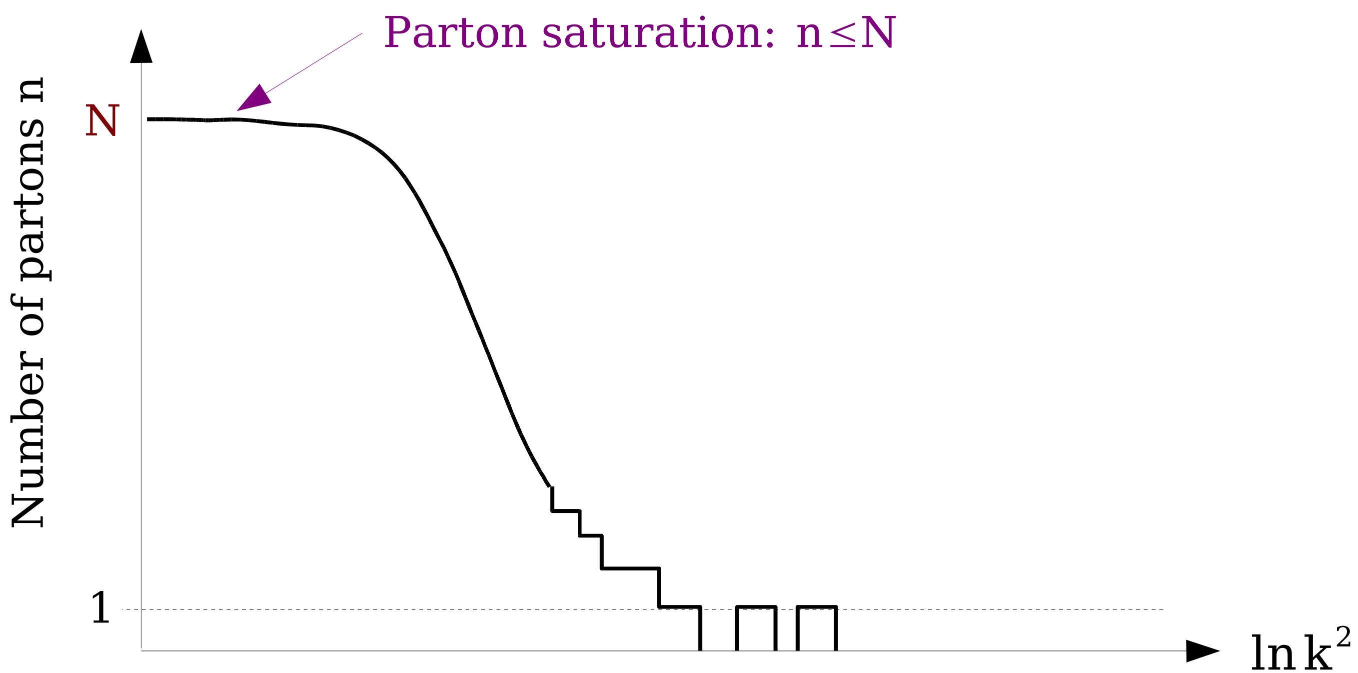
- Why the stochastic FKPP equation is relevant to high energy QCD

- A new perspective on its solutions:

  selective population evolution and statistics of genealogies

How a high rapidity hadron looks





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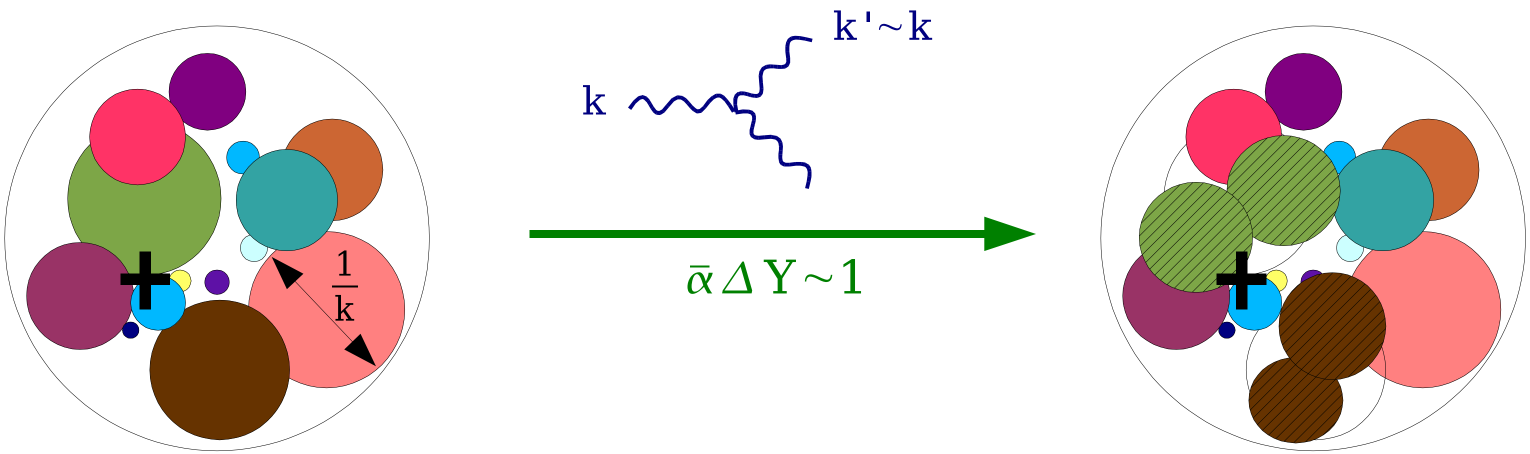
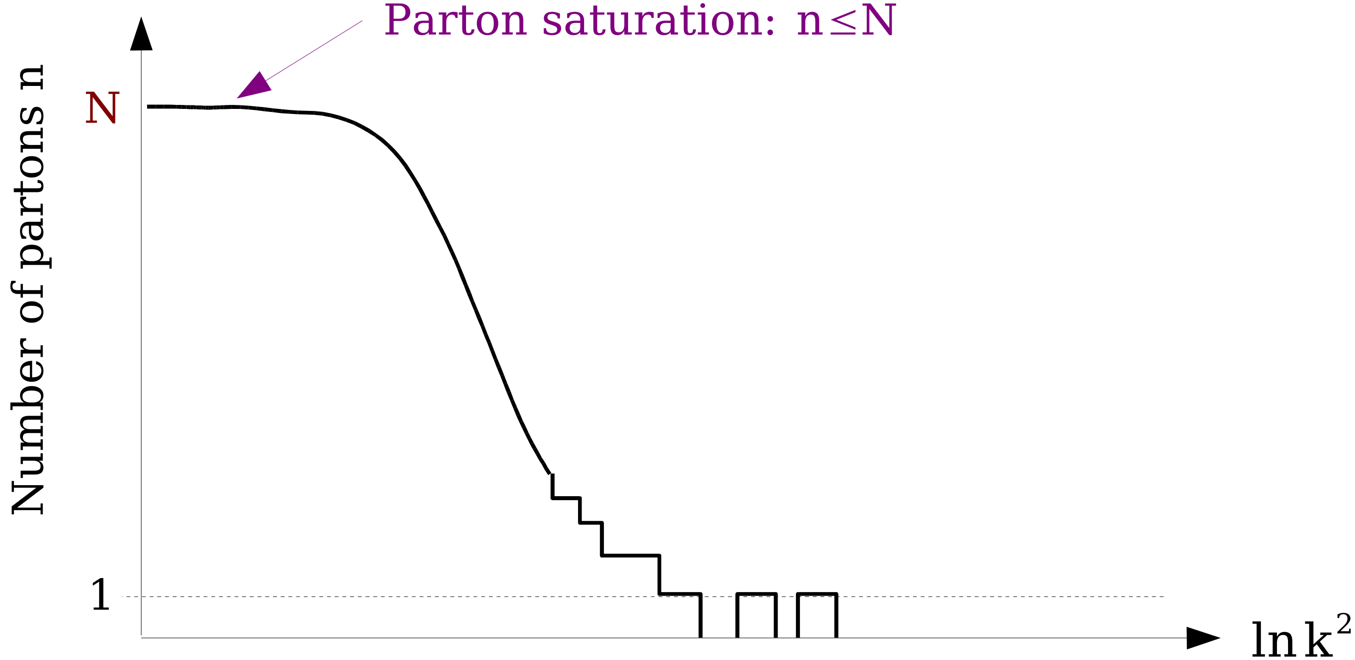
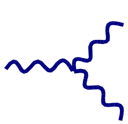




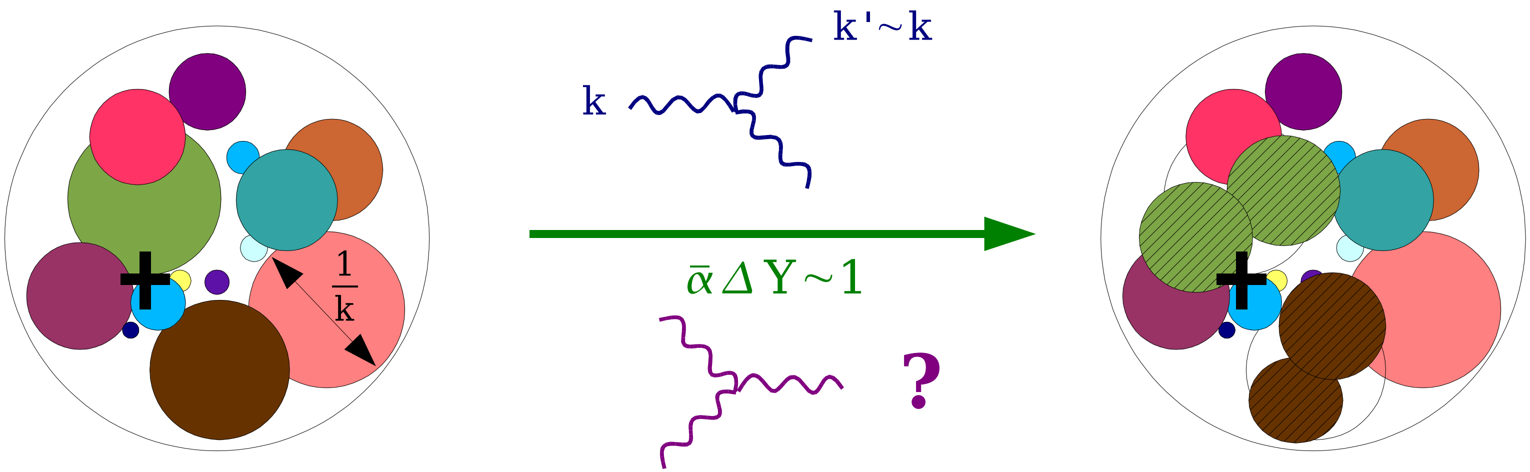
$$\partial_{\bar{\alpha} Y} n = \chi(-\partial_{\ln k^2})\, n \qquad + \sqrt{n}\;\; \nu$$

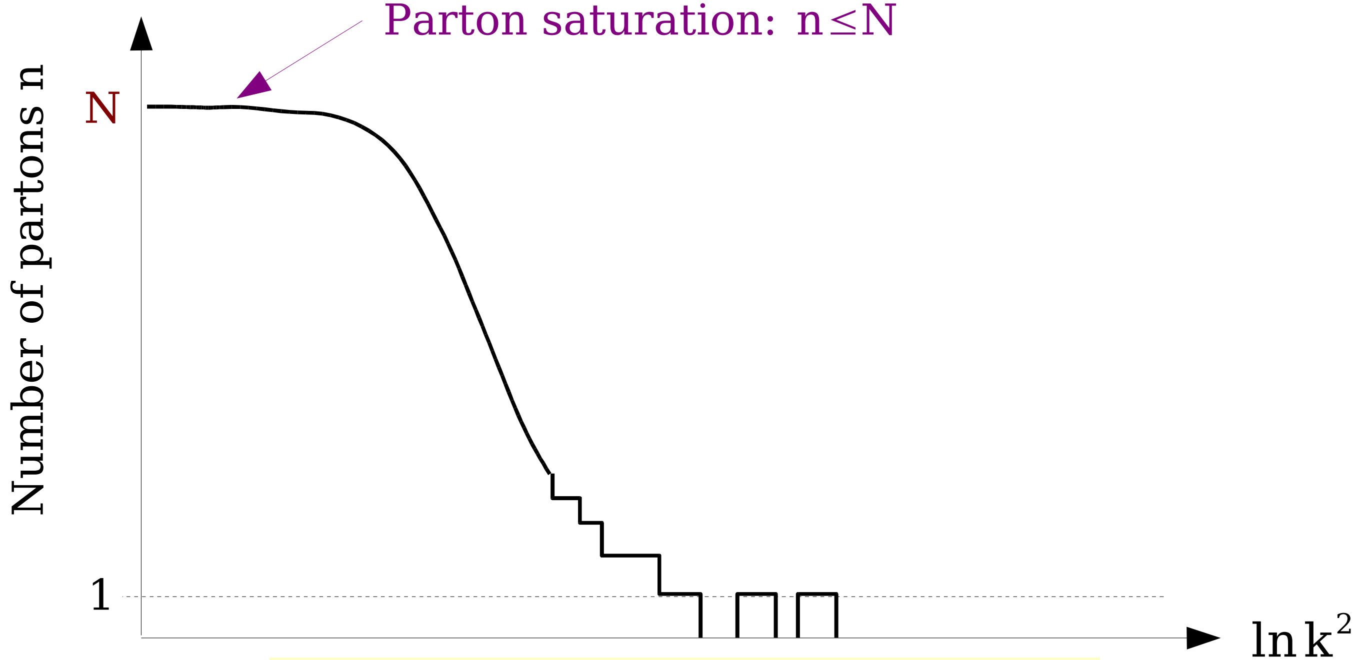


BFKL $\sim \partial^2_{\ln k^2} n + n$

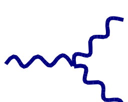
Noise term due to discreteness

How a high rapidity hadron looks





$$\partial_{\bar{\alpha} Y} n = \chi(-\partial_{\ln k^2})\, n - \frac{n^2}{N} + \sqrt{n}\;\nu$$



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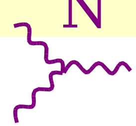

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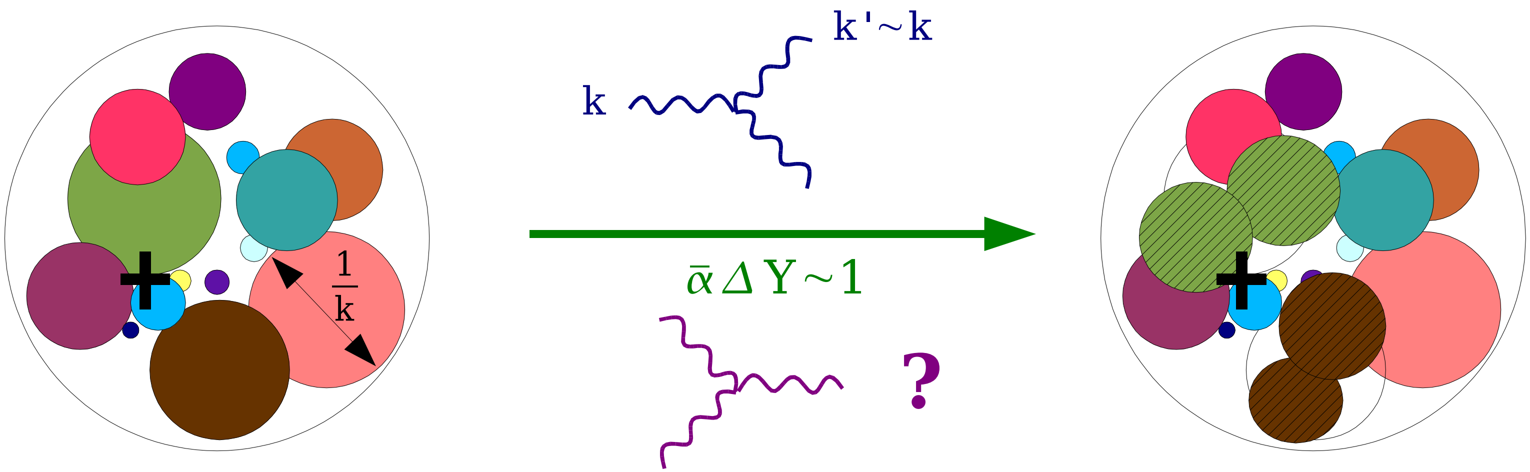

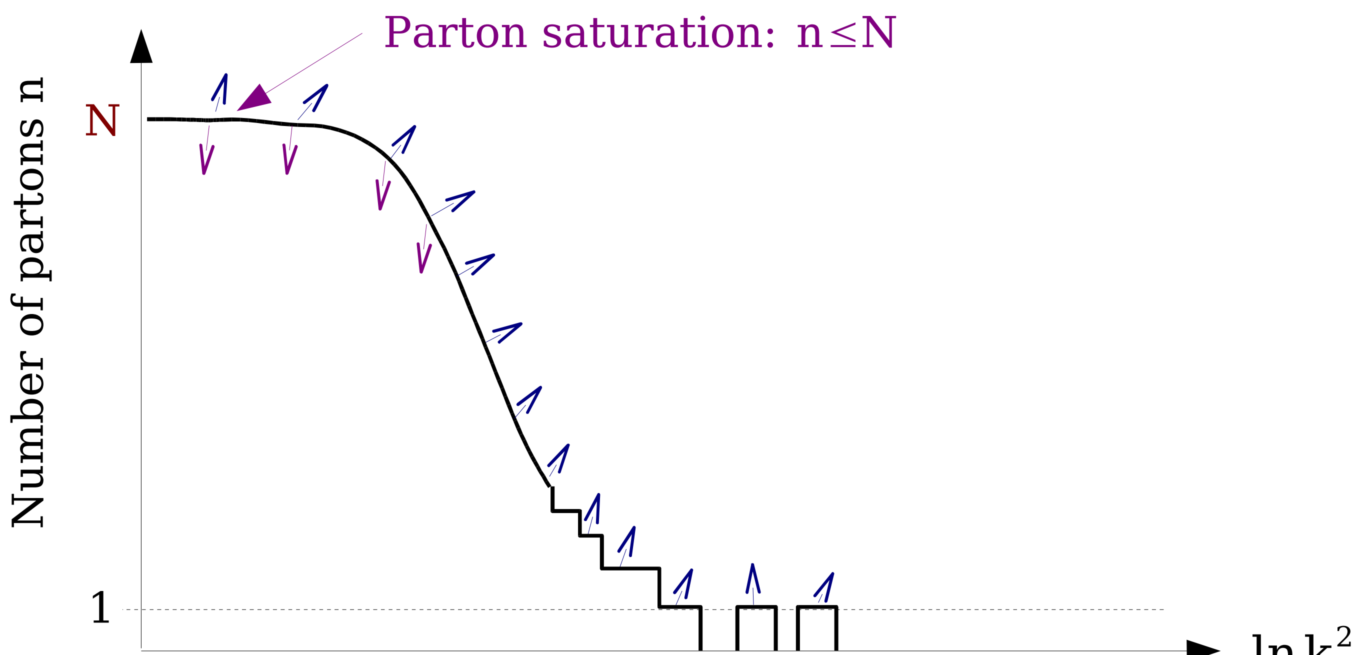

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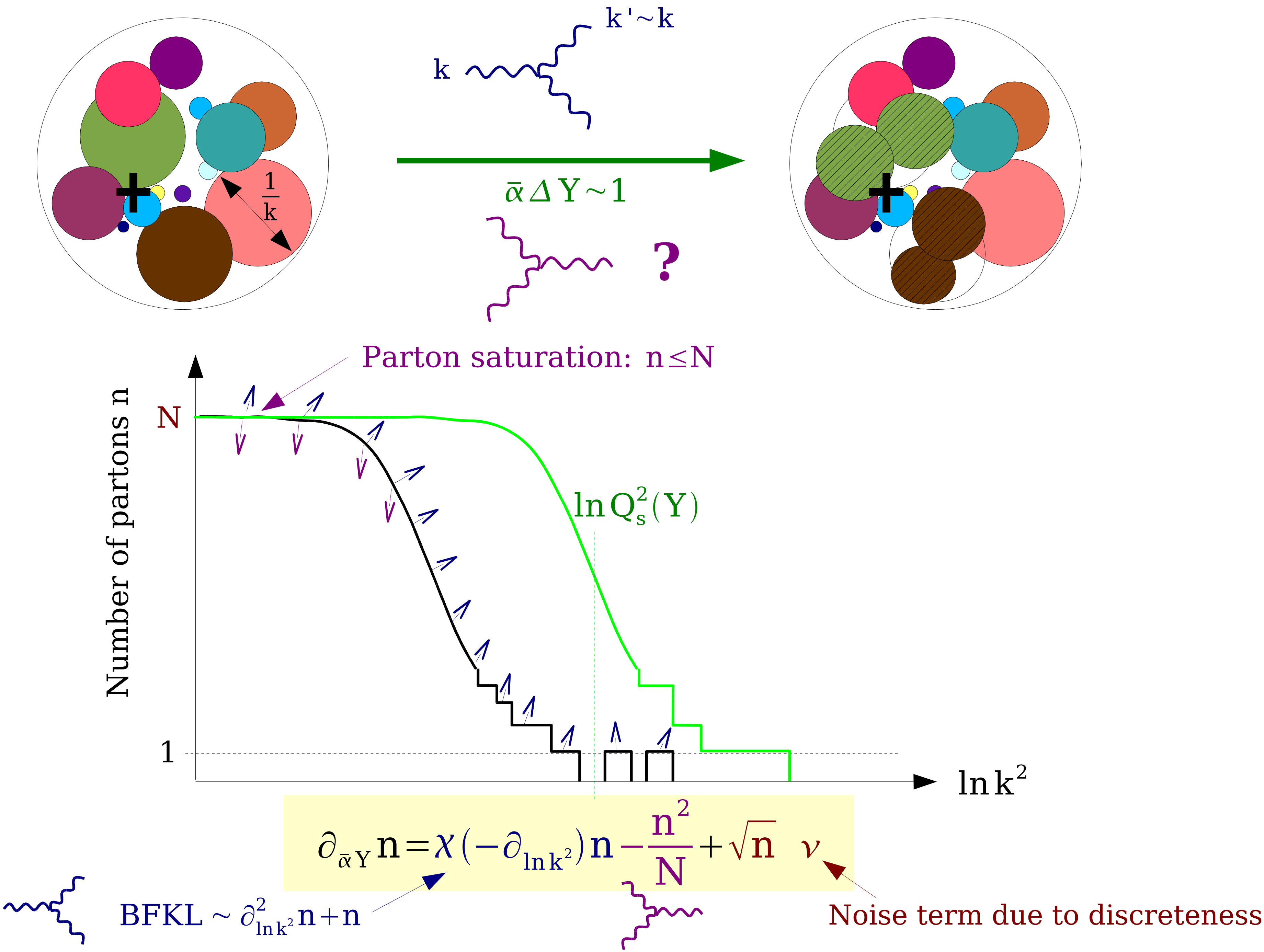

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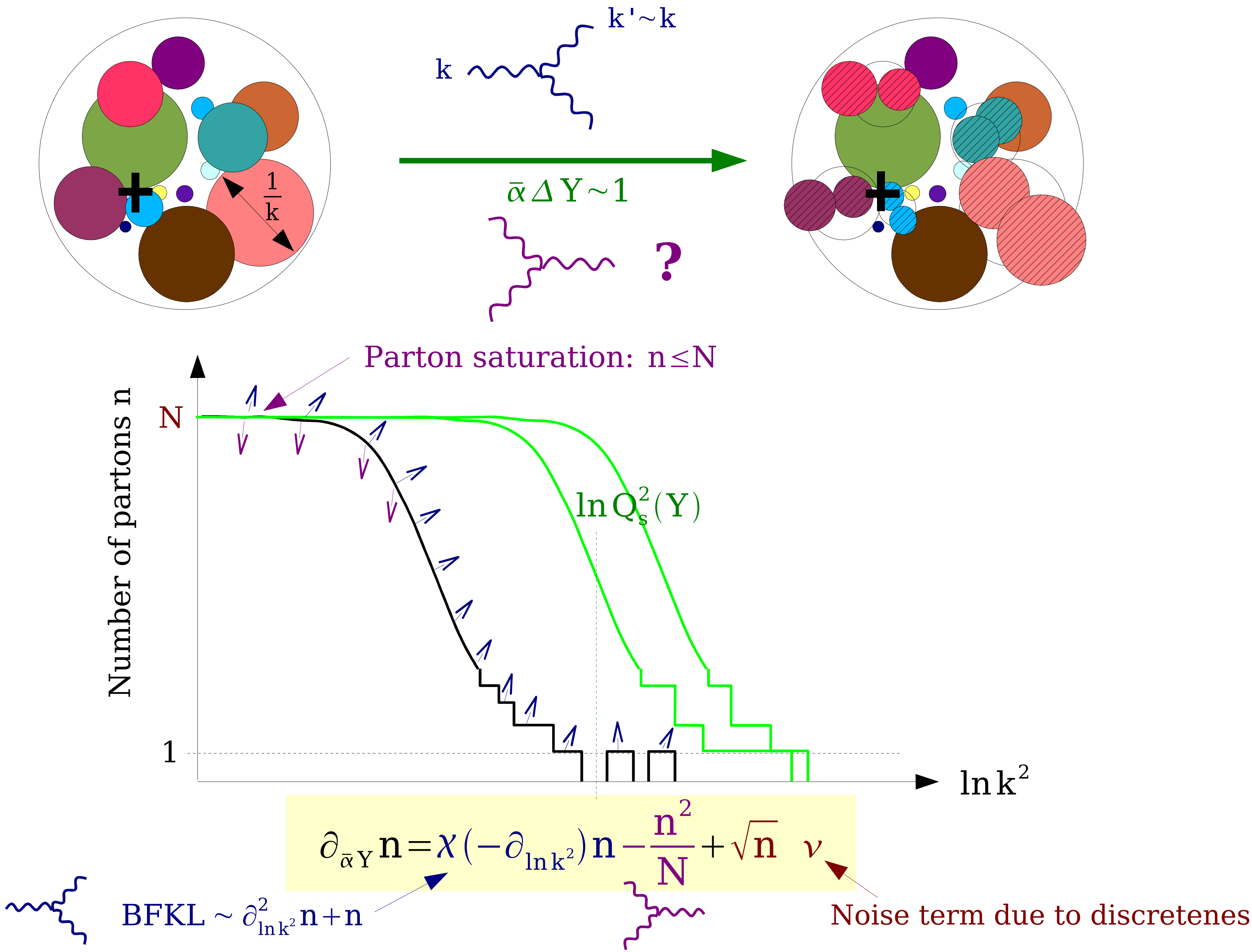

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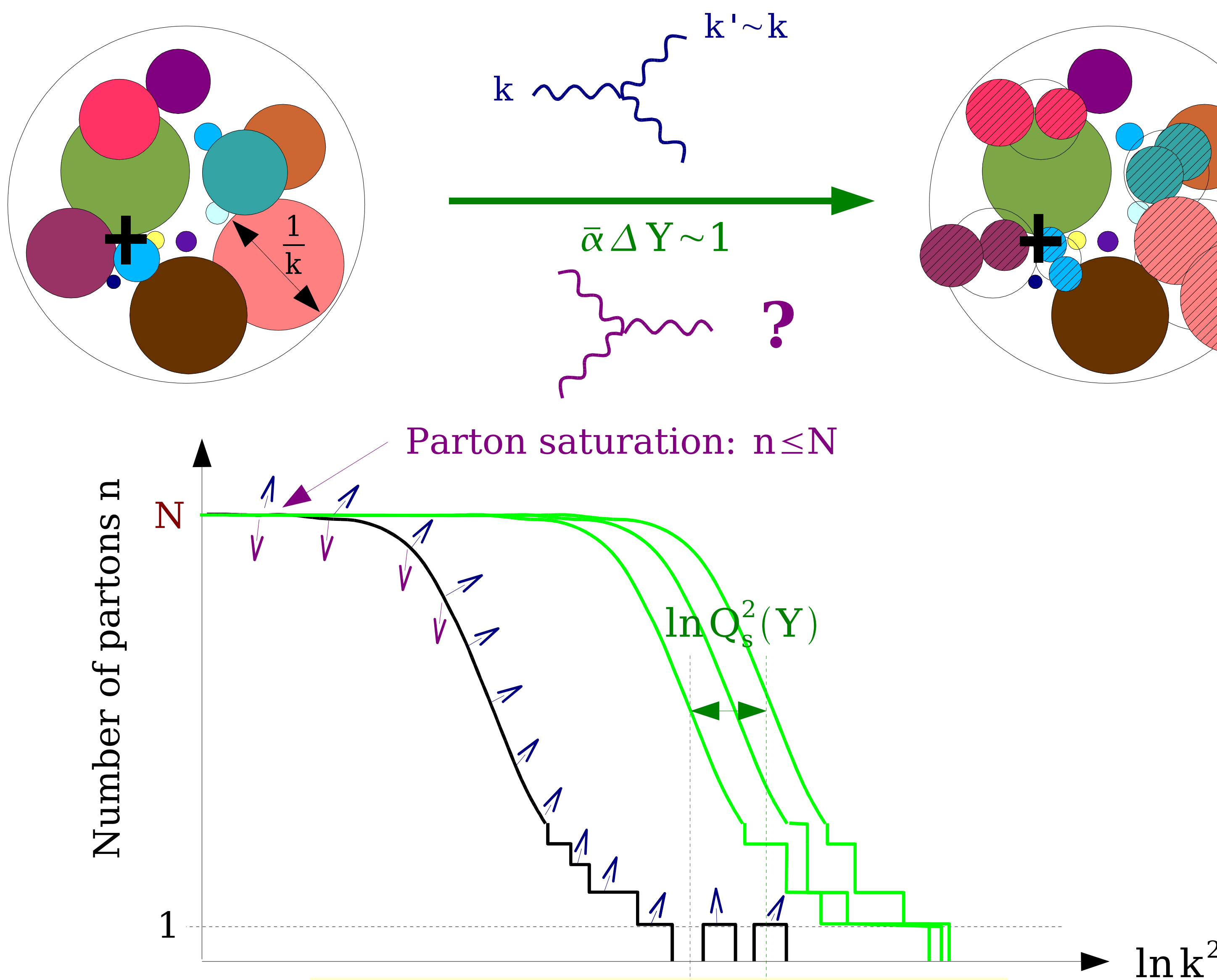

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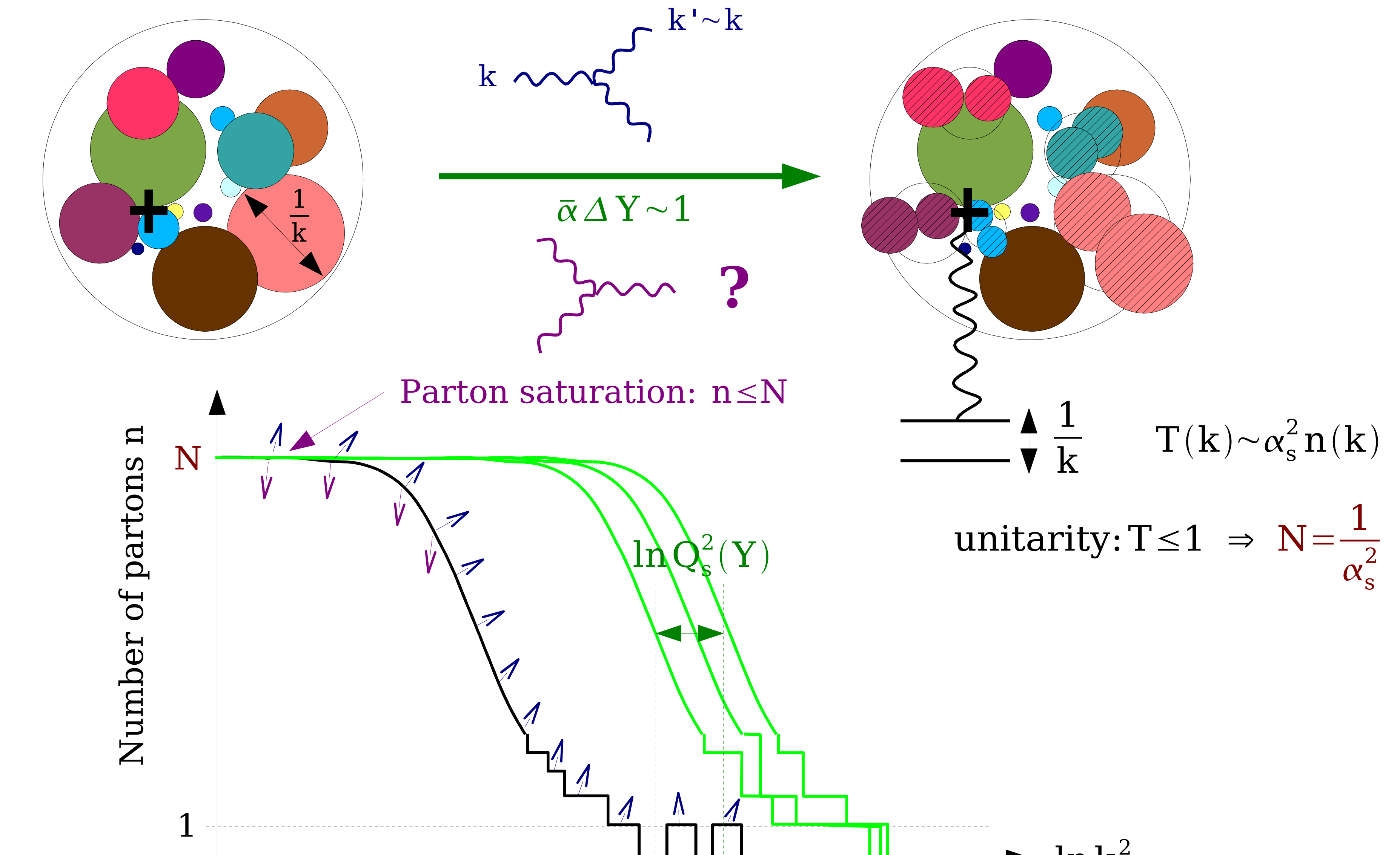

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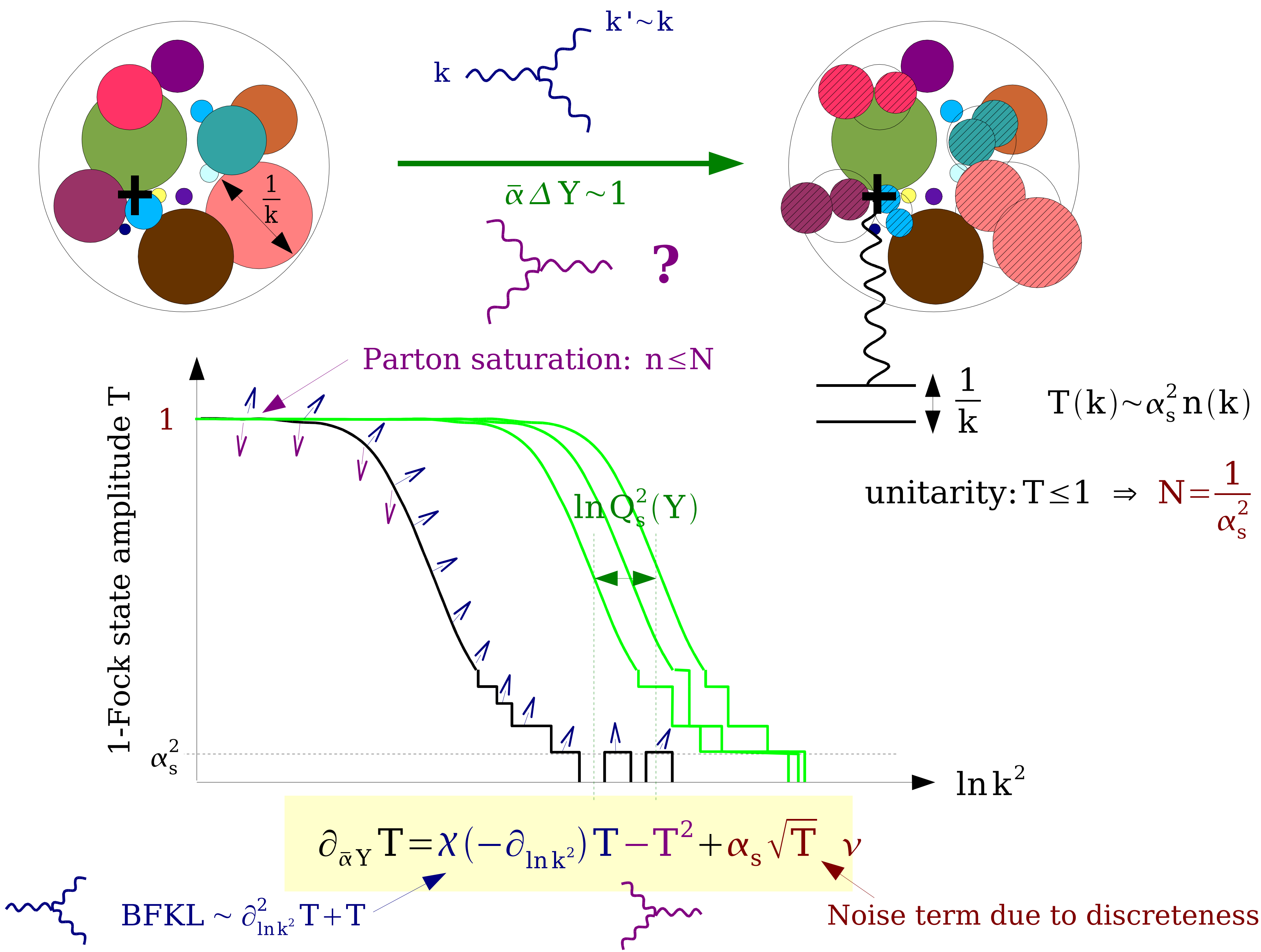

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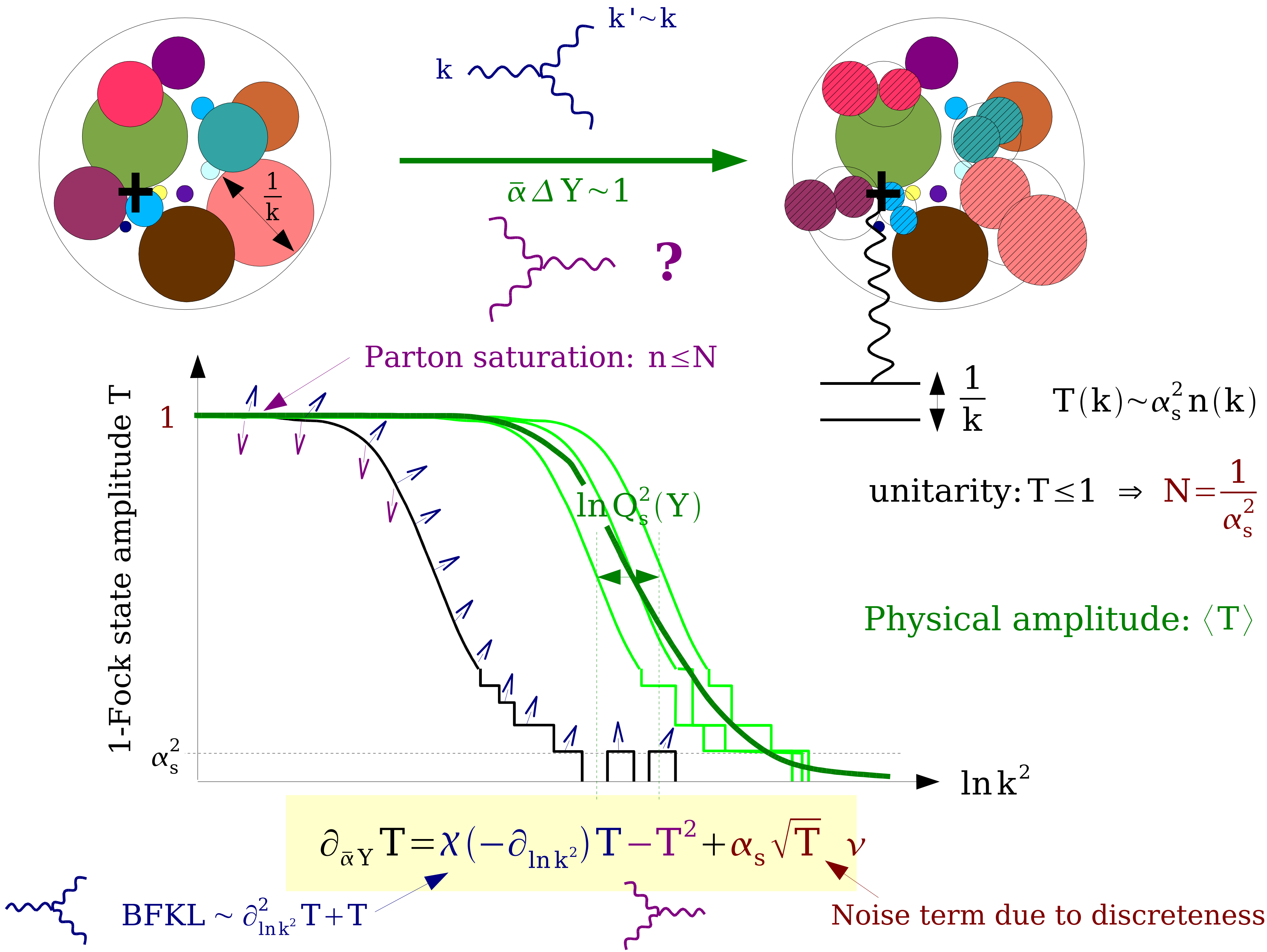

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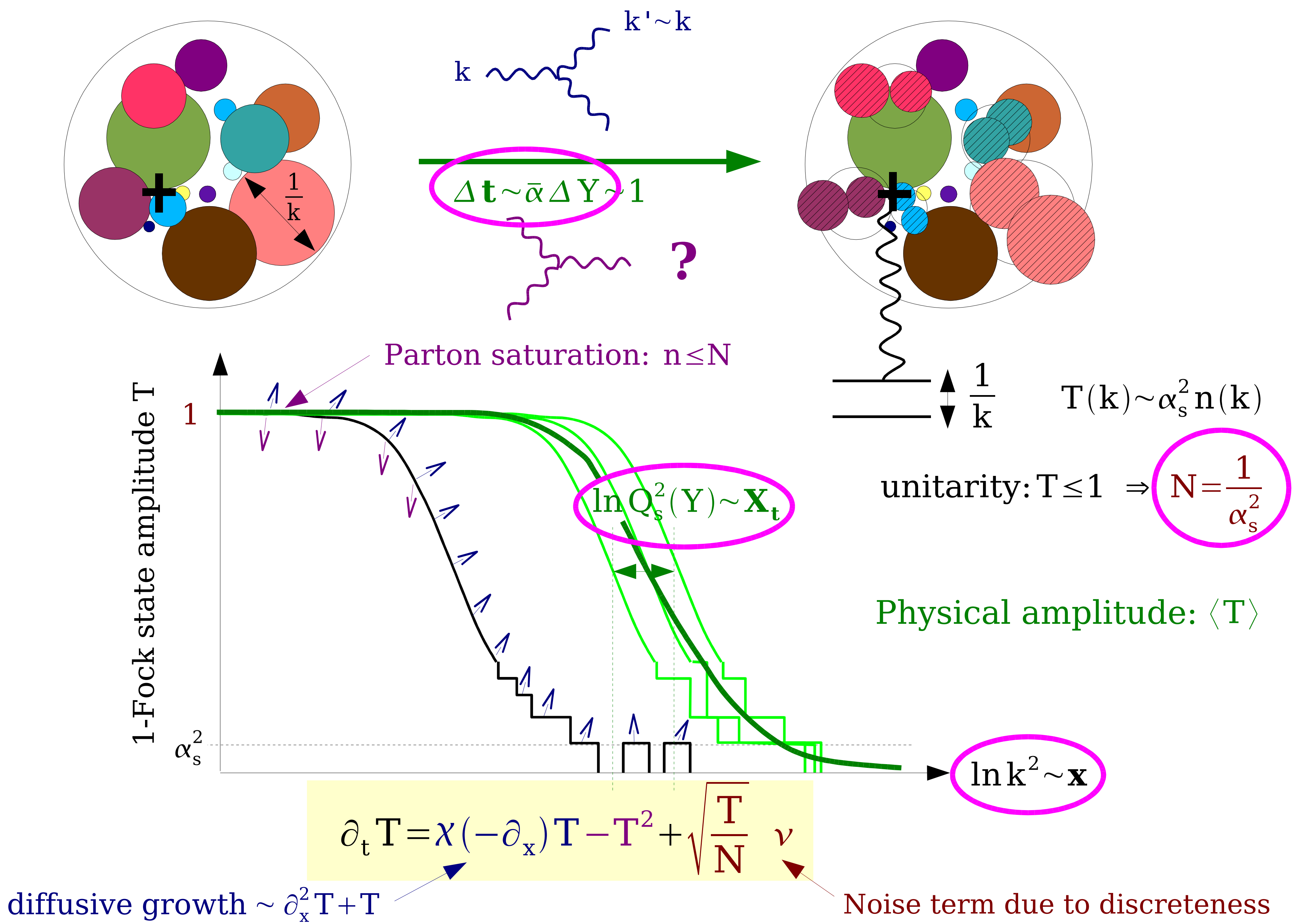

Statistics of the position of the front

$$\partial_t T = \chi(-\partial_x) T - T^2 + \sqrt{\frac{T}{N}}\ \nu$$

... Bramson (1986)
Gribov, Levin, Ryskin (1980)

Brunet, Derrida (1997)
Mueller, Shoshi (2004)

Brunet, Derrida, Mueller, S.M. (cond-mat/2005, Phys.Rev.E)

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$(\ln N \gg 1)$

with $\frac{\chi(\gamma_0)}{\gamma_0} = \chi'(\gamma_0)$

Dictionary to QCD

$t = \bar{\alpha} Y \qquad x = \ln k^2$

$X_t = \ln Q_s^2(Y)$

$\chi = \text{BFKL kernel}$

$N = \frac{1}{\alpha_s^2}$

Universal results for sFKPP-like equations: independent of the details, in particular of the saturation mechanism

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Universal results for sFKPP-like equations: independent of the details, in particular of the saturation mechanism

The cumulants are all
- of the same order: long tail distribution!
- proportional to $\kappa = \frac{t}{\ln^3 N}$

X_t is the sum of κ independent random variables

Outline

- Why the stochastic FKPP equation is relevant to high energy QCD

- A new perspective on its solutions:

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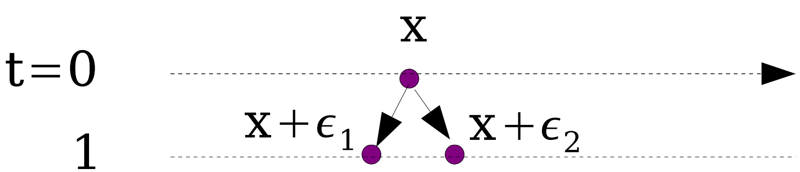
Brunet, Derrida, Mueller, S.M., Letter on cond-mat/2006
Extended version to appear

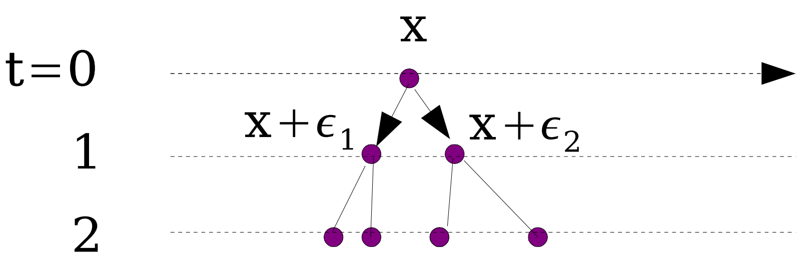
Selective population evolution



The individual at position x has
2 descendants at positions $x+\epsilon_1$ and $x+\epsilon_2$

distribution $\rho(\epsilon)$

Selective population evolution



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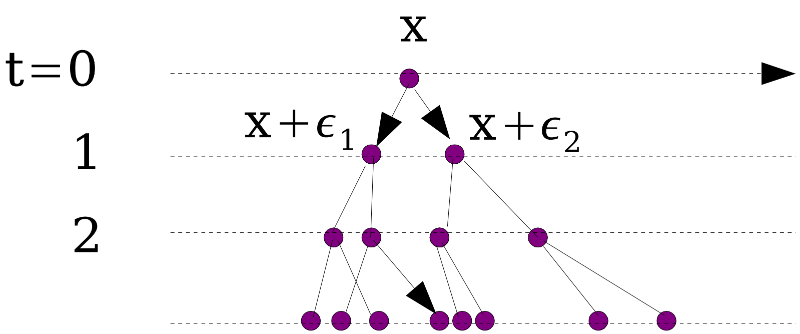

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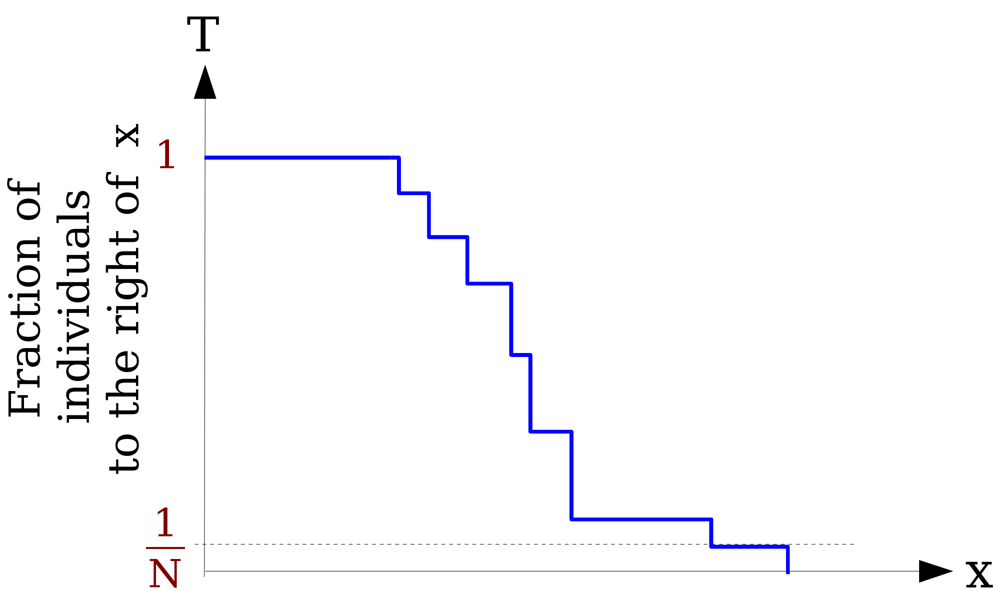
Keep only the **N=8 rightmost** individuals



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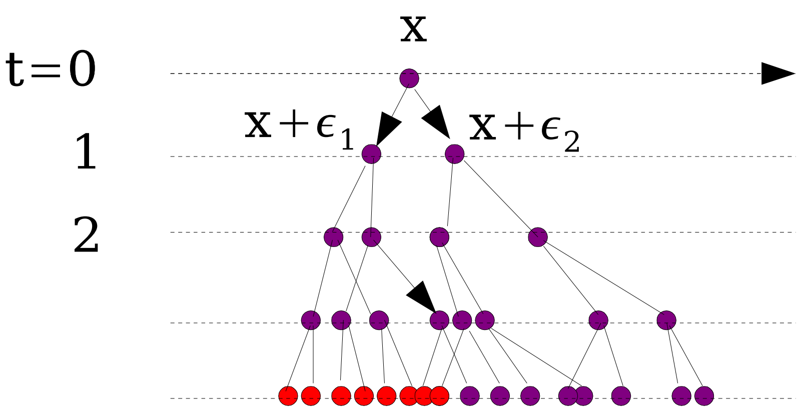

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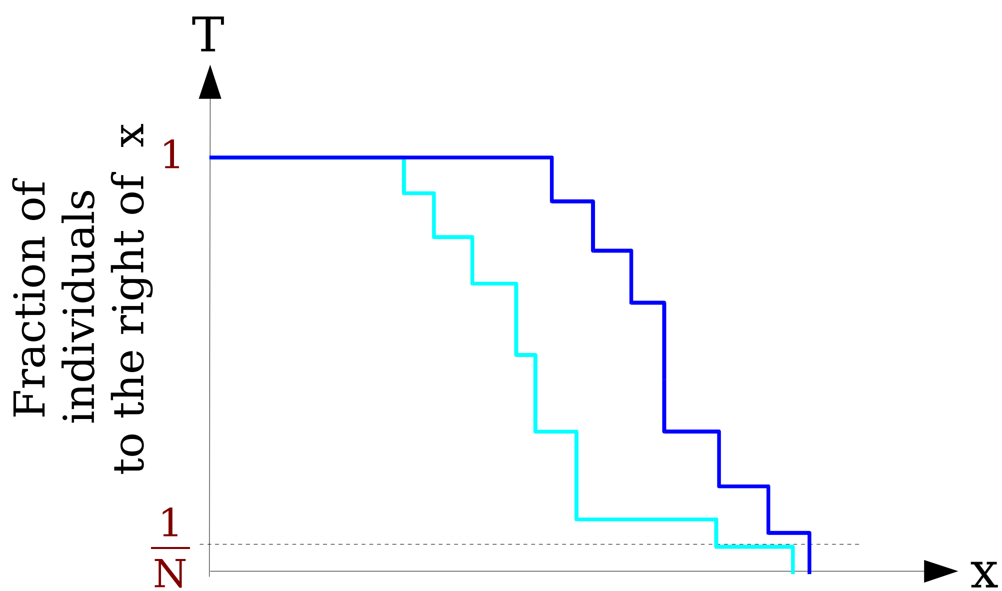

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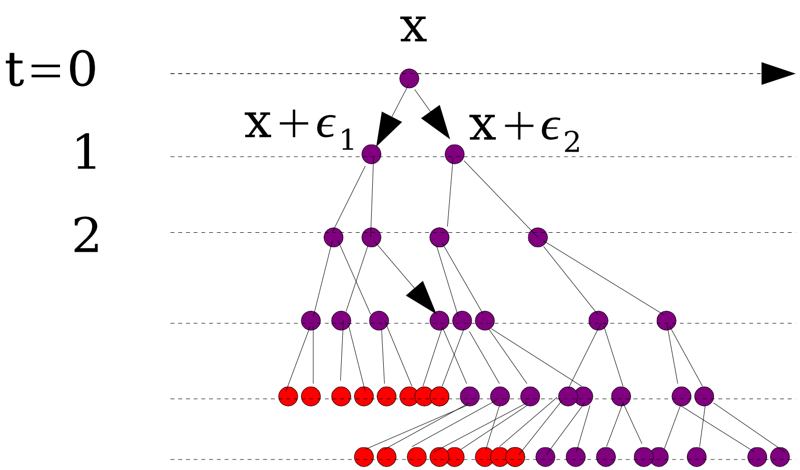

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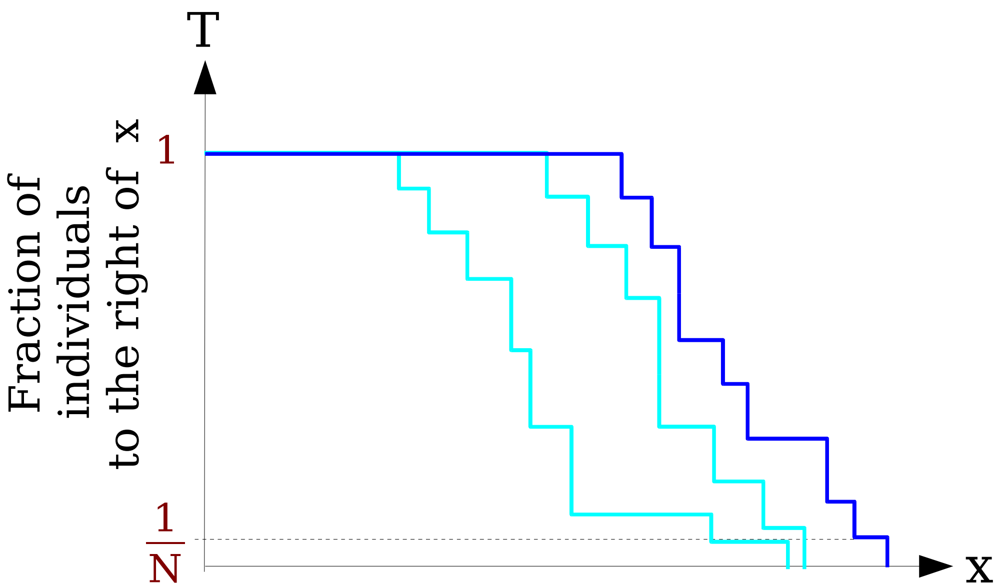

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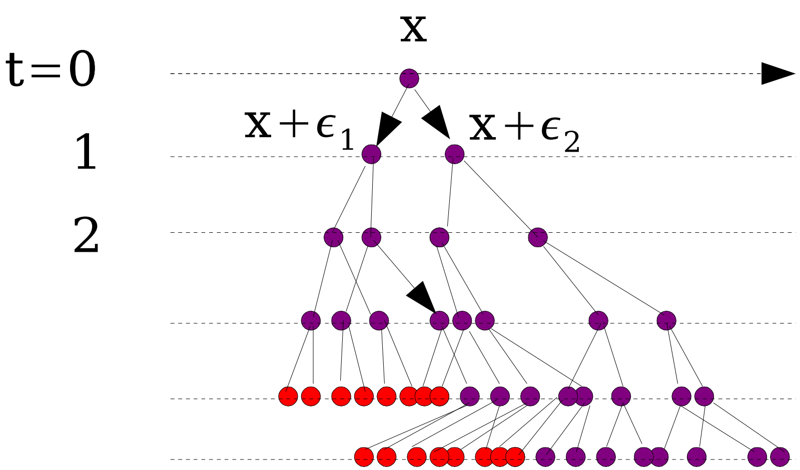

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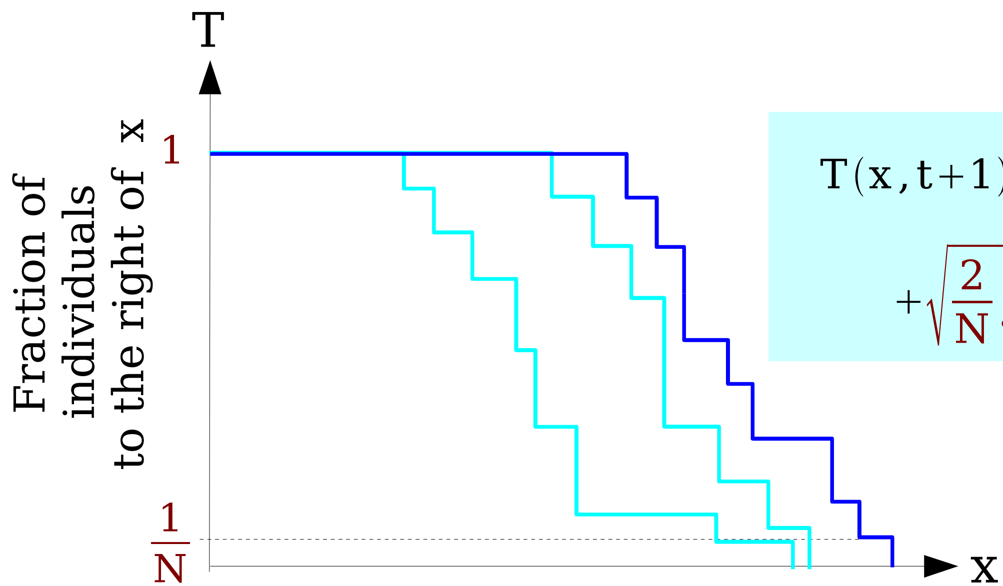

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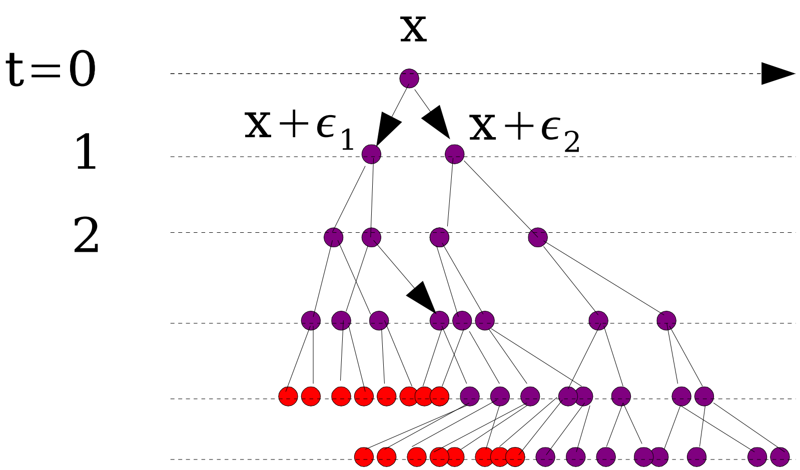

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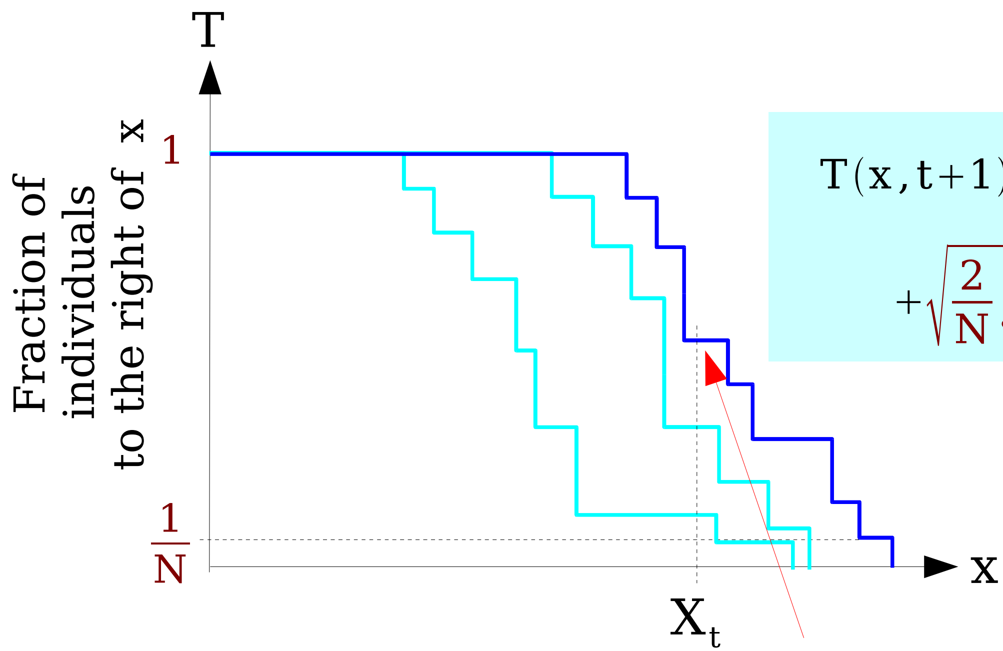

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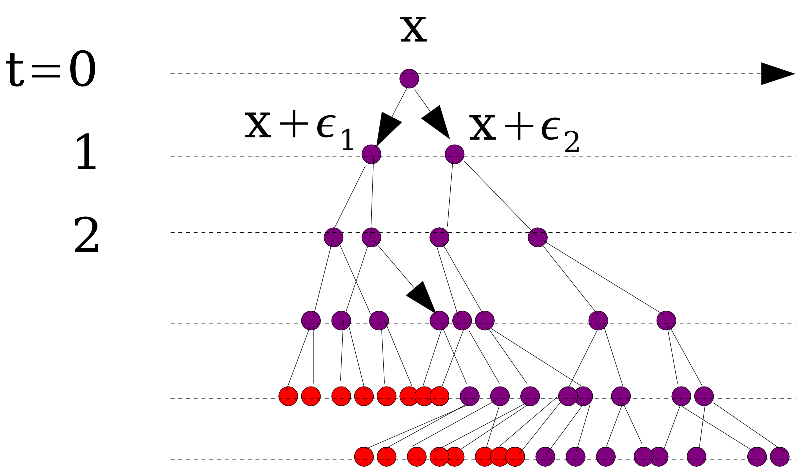
Traveling wave: all universal results on the front are applicable with $\chi(\gamma)=\ln\left(2\int d\epsilon\,\rho(\epsilon)e^{\gamma\epsilon}\right)$

Selective population evolution



$$\langle X_t\rangle=\left(\frac{\chi(\gamma_0)}{\gamma_0}-\frac{\pi^2\gamma_0\chi''(\gamma_0)}{2\ln^2 N}\right)t+\left(\pi^2\gamma_0^2\chi''(\gamma_0)\frac{3\ln\ln N}{\gamma_0\ln^3 N}\right)t$$

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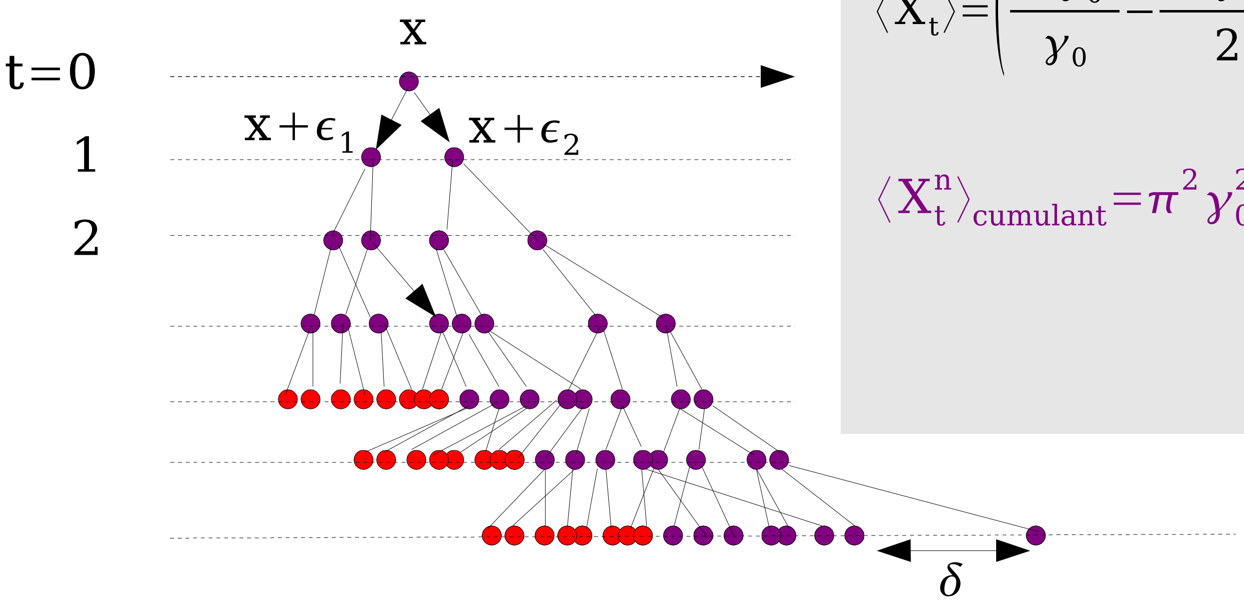

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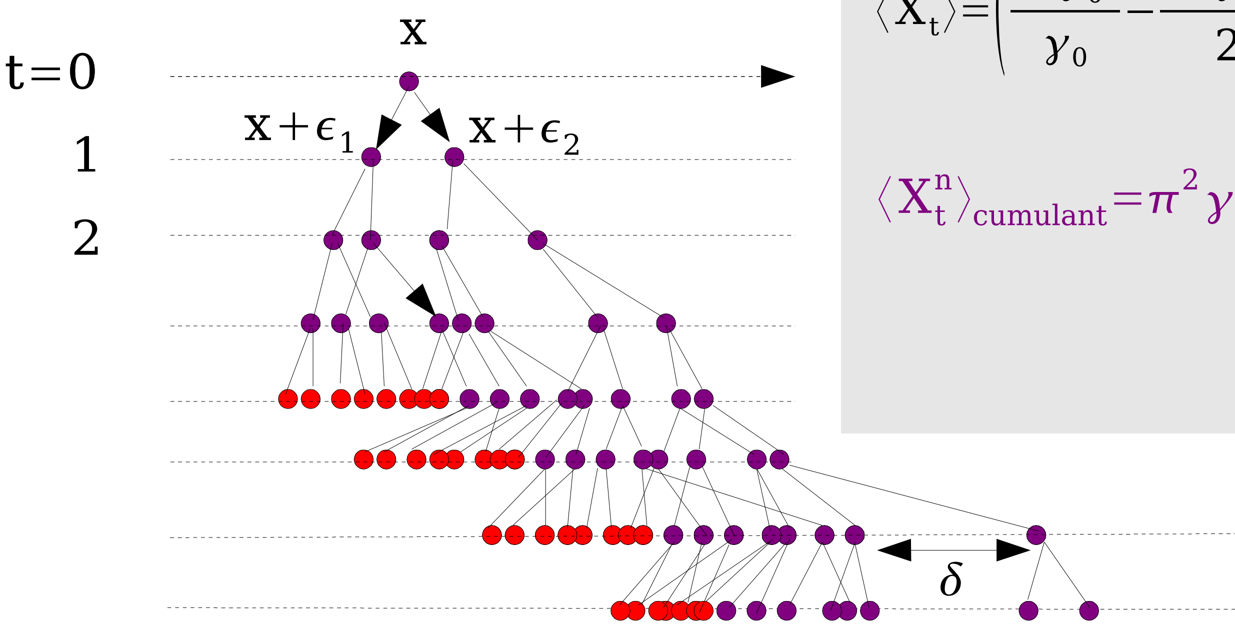

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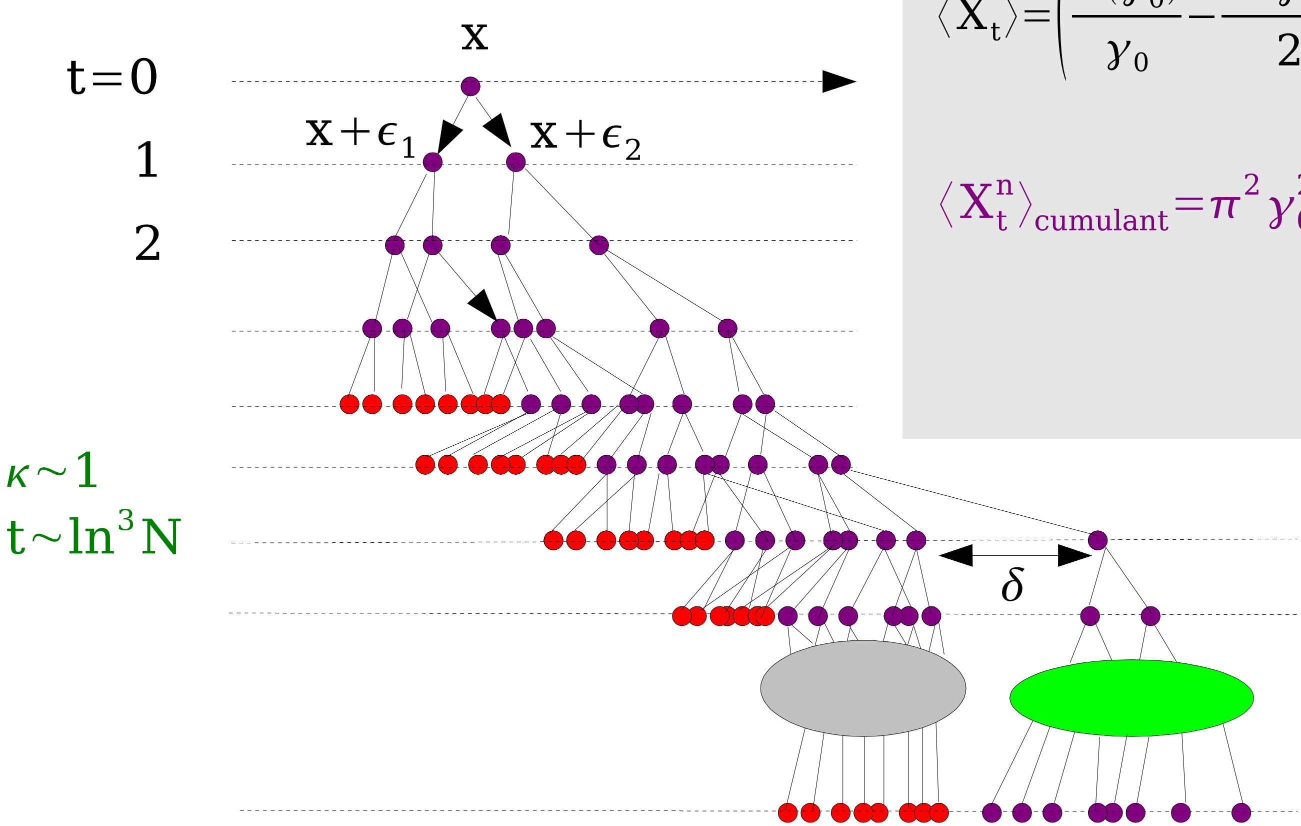

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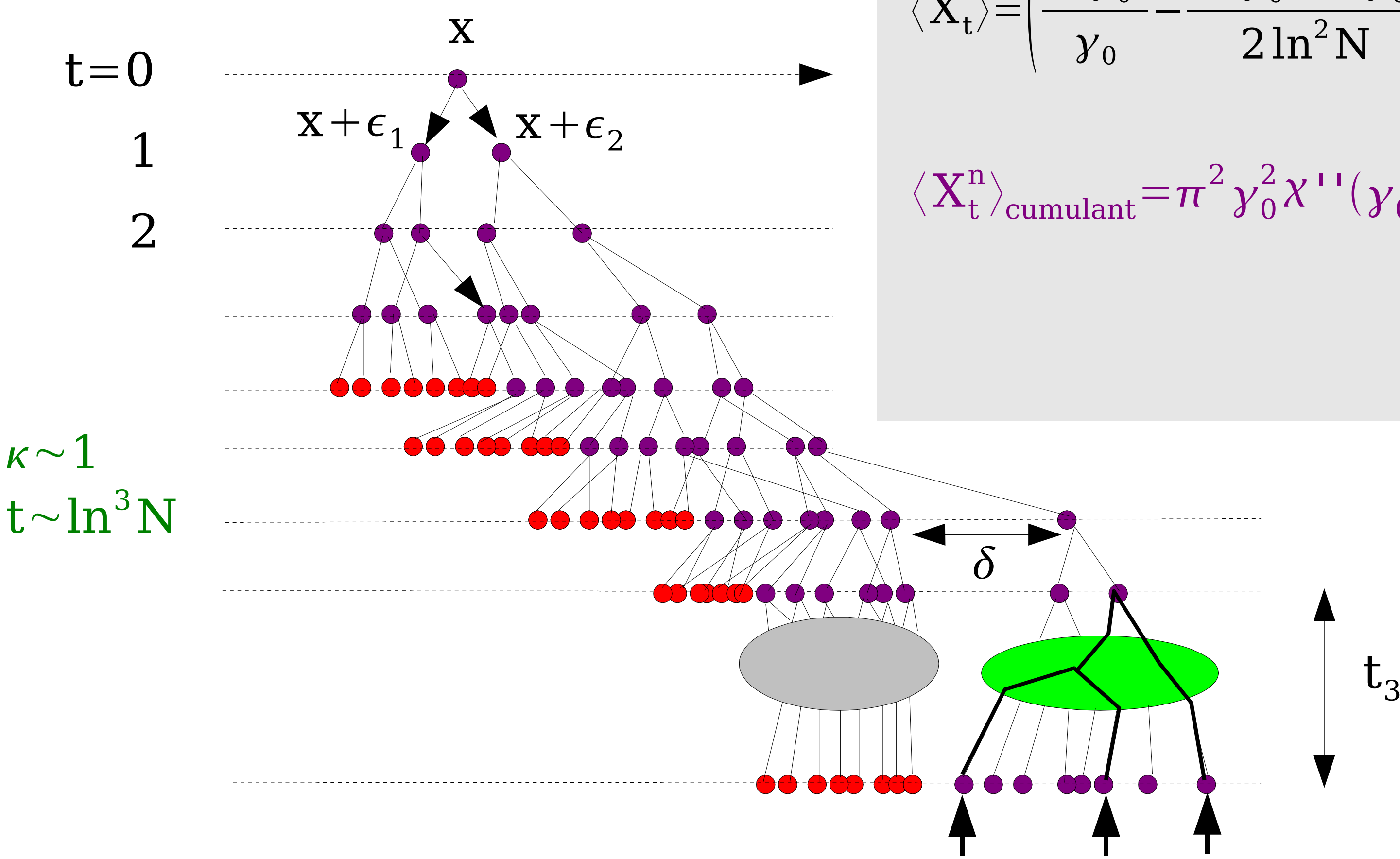

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Average number of generations to the first common ancestor of k=2, 3... randomly chosen individuals?

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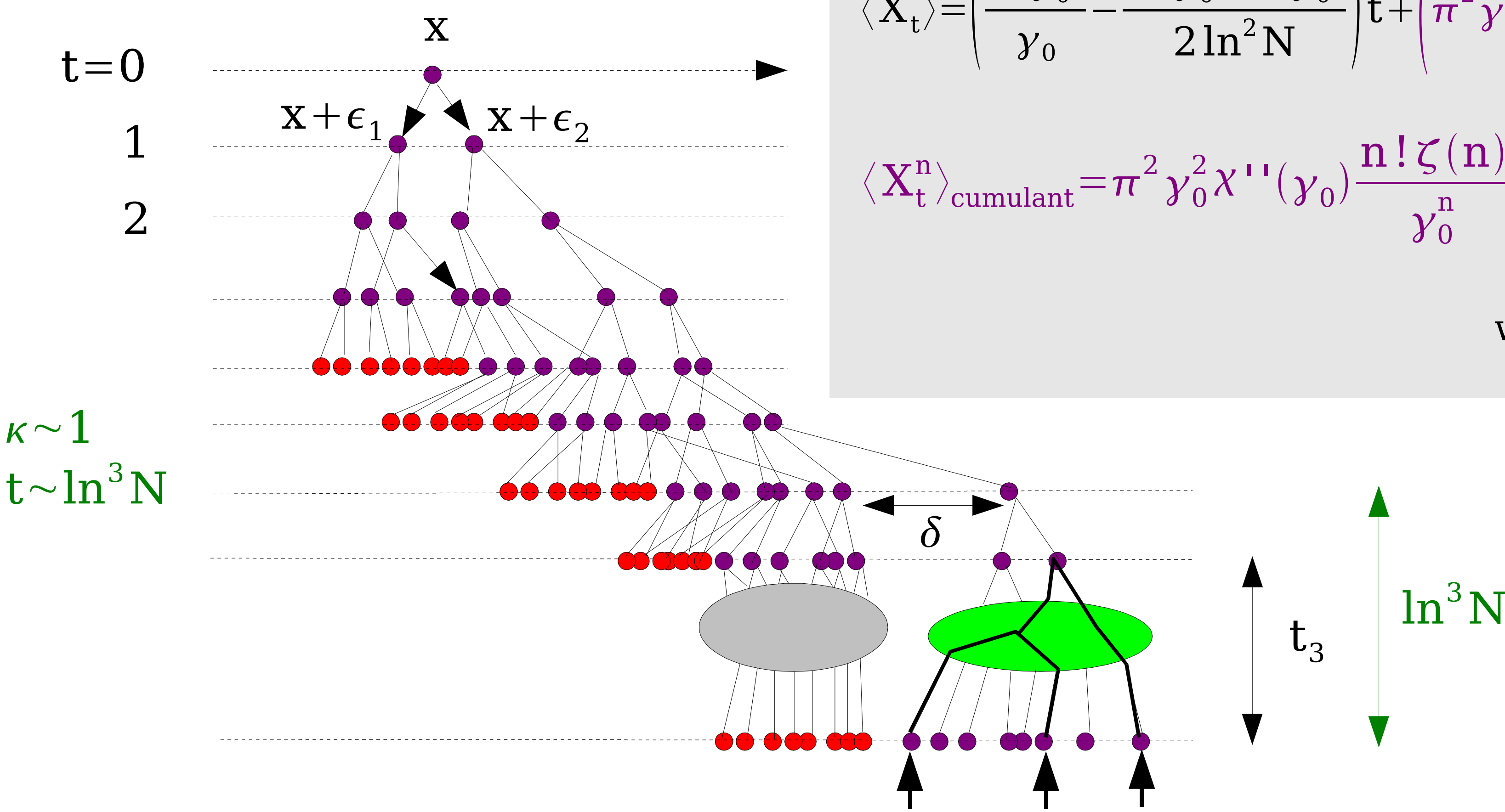

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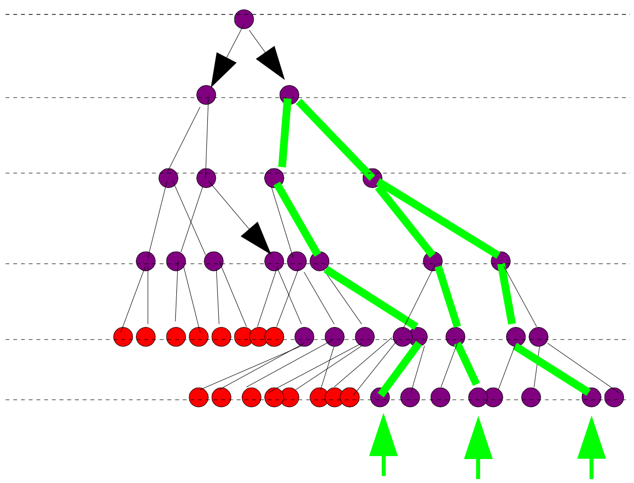
$$\langle t_k \rangle \sim \ln^3 N$$

$$\frac{\langle t_3 \rangle}{\langle t_2 \rangle} = \frac{5}{4} \qquad \frac{\langle t_4 \rangle}{\langle t_2 \rangle} = \frac{25}{18}$$

Selective population evolution

With selection



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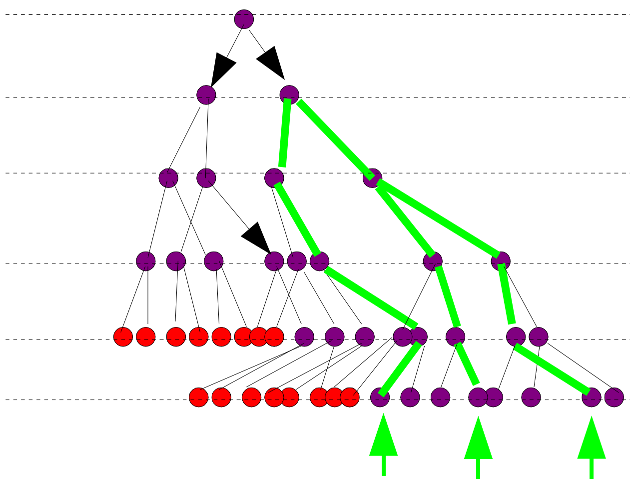
Selective population evolution

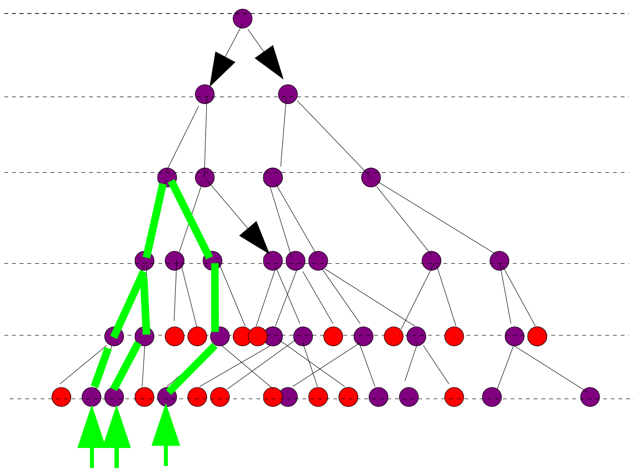
With selection



$$\langle t_k \rangle \sim \ln^3 N$$

$$\frac{\langle t_3 \rangle}{\langle t_2 \rangle} = \frac{5}{4} \qquad \frac{\langle t_4 \rangle}{\langle t_2 \rangle} = \frac{25}{18}$$

Without selection



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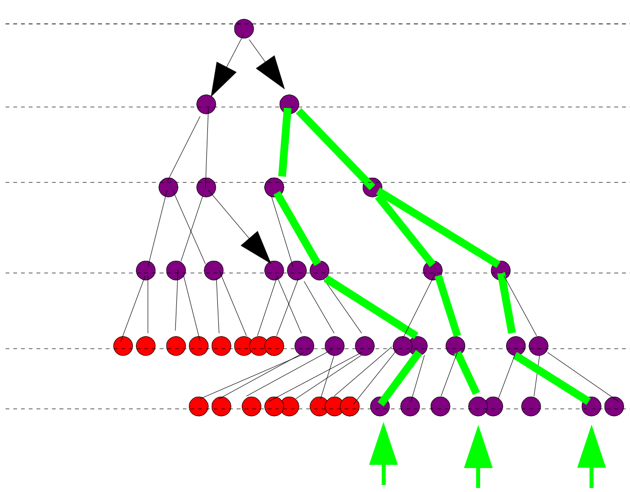

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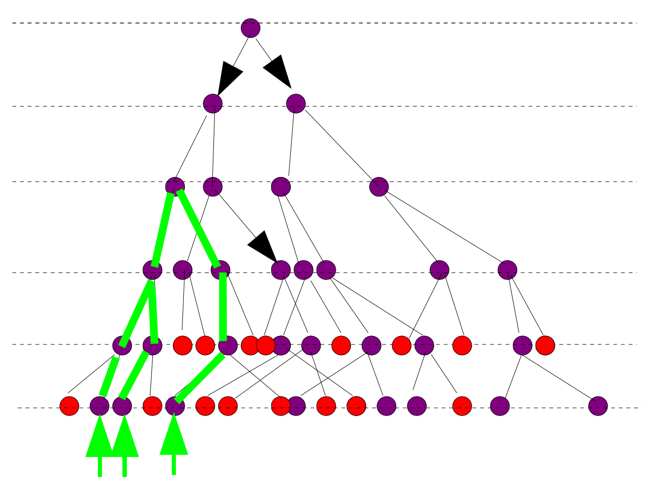
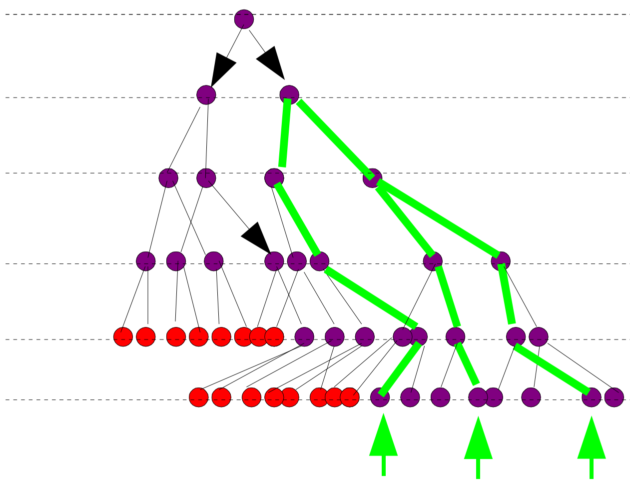


$$\langle t_k \rangle \sim N$$

$$\frac{\langle t_3 \rangle}{\langle t_2 \rangle} = \frac{4}{3} \qquad \frac{\langle t_4 \rangle}{\langle t_2 \rangle} = \frac{3}{2}$$

Selective population evolution

With selection



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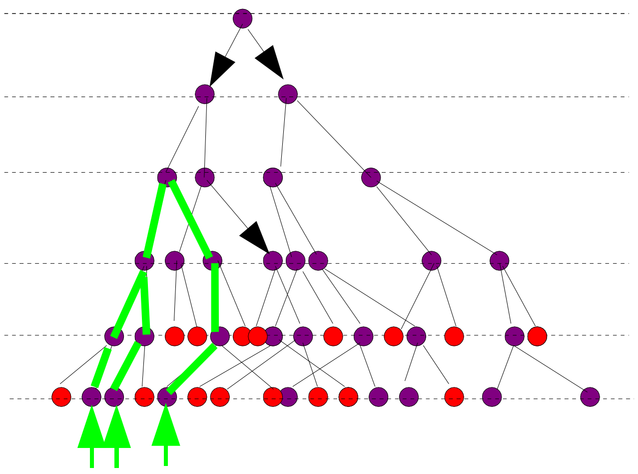
Same statistics as Parisi's trees
that appear in the **theory of spin glasses**!

Without selection



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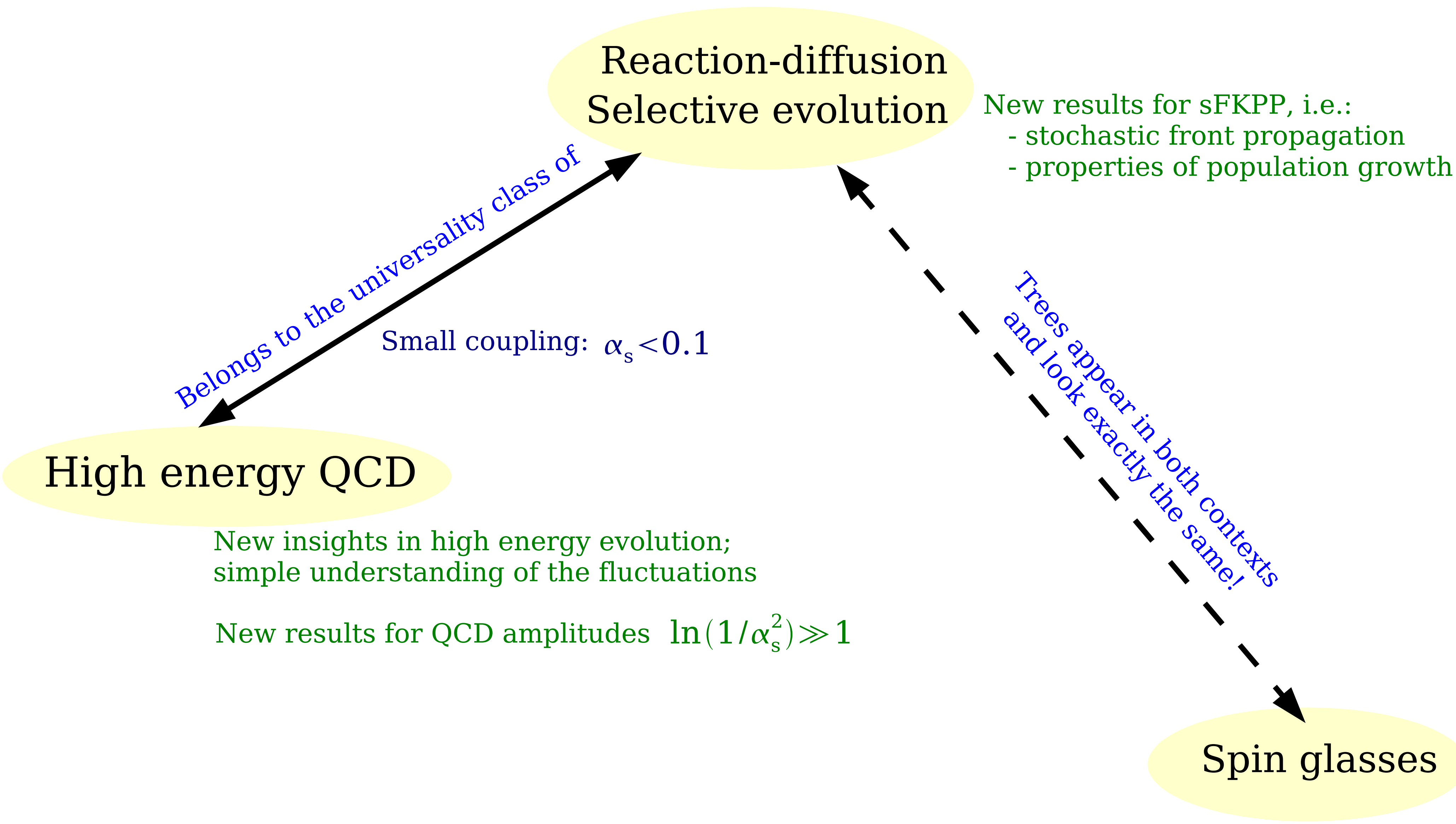
Summary



Reaction-diffusion
Selective evolution
New results for sFKPP, i.e.:
- stochastic front propagation
- properties of population growth
Belongs to the universality class of
Small coupling: $\alpha_s < 0.1$
High energy QCD
Trees appear in both contexts
and look exactly the same!
New insights in high energy evolution;
simple understanding of the fluctuations
New results for QCD amplitudes $\ln(1/\alpha_s^2) \gg 1$
Spin glasses